TOSHKENT IRRIGATSIYA VA QISHLOQ XO'JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI

FAN: OLIY MATEMATIKA

MAVZU 18

Ln(1+x) funktsiyani darajali qatorga yoyish. Logarifmlarni hisoblash. Darajali qatorlarning ba’zi taqribiy hisoblashlarga tatbiqi.

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Reja:

- **Ln(1+x) funktsiyani darajali qatorga yoyish.**
- **Qatorlar yordamida funksiya qiymatlarini taqribiy hisoblash.**
- **Integralning qiymatini taqribiy hisoblash.**

Ln(1+x) funktsiyani darajali qatorga yoyish.

$\ln(1+x)$ funktsiyani Makleron qatoriga yoyish uchun cheksiz kamayuvchi

$$\frac{1}{1+x} = 1 - x + x^2 - ... + (-1)^n x^n + ... \ , \ x \in (-1,1)$$

Geometrik progressiyaning yig'indisi formulasidan foydalanamiz. Darajali qatorlarni yaqinlashish intervalida integrallash xossasidan foydalanamiz:

$$\int_0^x \frac{dx}{1+x} = \int_0^x dx - \int_0^x xdx + \int_0^x x^2 dx - ... + (-1)^n \int_0^x x^n dx + ...$$

Bundan

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - ... + (-1)^{n+1} \frac{x^n}{n+1} + ... \ ,$$

Qatorlar yordamida funksiya qiymatlarini taqribiy hisoblash.

Darajali qatorlar yordamida ko‘pgina masalalar hal etiladi. Ulardan funksiyalarning qiymatlarini topishda, aniq integrallarni taqribiy hisoblashda, differensial tenglamalarni yechishda foydalaniladi.

a) funksiya qiymatlarini taqribiy hisoblash. Ko‘p masalalarda funksiyaning biror nuqtadagi qiymatini hisoblashga to‘g‘ri keladi. Ammo funksiyaning murakkab bo‘lishi uning shu nuqtadagi qiymatini topishni juda qiyinlashtiradi. Bunday hollarda qaralayotgan funksiyani darajali qatorga yoyib, so‘ng bu yoyilmaning bir nechta hadini olib, uning yordamida funksiya qiymati taqribiy hisoblanadi.

Misollar. 1. $f(x) = \ln x$ funksiyaning $x_0 = 1.1$ nuqtadagi qiymati taqribiy hisoblansin.

$\triangleleft$ Ravshanki,

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - ... + (-1)^{n-1}\frac{x^n}{n} + ...$$

formula $x \in (-1, 1]$ da o‘rinli. Bu qatorda $x = 0.1$ deb topamiz:

$$\ln(1+0.1) = \ln 1.1 = 0.1 - \frac{0.1^2}{2} + \frac{0.1^3}{3} - \frac{0.1^4}{4} + ...$$

Oxirgi tenglikning o‘ng tomonidan to‘rtinchi hadi $\frac{0.1^4}{4}$ absolyut qiymat bo‘yicha 0.0001 dan kichik. Demak $\ln 1.1$ ni taqribiy hisoblash uchun qatorning dastlabki uchta hadini olish yetarli:

$$\ln 1.1 \approx 0.1 - \frac{0.1^2}{2} + \frac{0.1^3}{3} = 0.1 - \frac{0.01}{2} + \frac{0.001}{3} \approx 0.0953. \triangleright$$

2. Ushbu

$$f(x) = \sqrt[3]{x}$$

funksiyaning $x_0 = 130$ nuqtadagi qiymati $0,0001$ aniqlikda, taqribiy hisoblansin.

$\triangleleft$ Ravshanki,

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + ... + \frac{\alpha(\alpha-1)...(\alpha-n+1)}{n!}x^n + ...$$

Endi

$$f(130)=\sqrt[3]{130}=\sqrt[3]{125+5}=\sqrt[3]{125\left(1+\frac{1}{25}\right)}=5\left(1+\frac{1}{25}\right)^{\frac{1}{3}}$$

ni e'tiborga olib, yuqoridagi qatorga

$$x=\frac{1}{25},\quad \alpha=\frac{1}{3}$$

deb topamiz:

$$\left(1+\frac{1}{25}\right)^{\frac{1}{3}}=1+\frac{1}{3\cdot 5^2}-\frac{2}{3^2\cdot 2!\cdot 5^4}+\frac{2\cdot 5}{3^3\cdot 3!\cdot 5^6}-\dots$$

Agar bu qatorning dastlabki uchta hadini olsak, unda

$$f(130) = 5\left(1+\frac{1}{25}\right)^{\frac{1}{3}} \approx 5\left(1+\frac{1}{3\cdot 5^2}-\frac{2}{3^2\cdot 2!\cdot 5^4}\right)$$

taqribiy formula hosil bo‘ladi. Hisoblashlarni bajarib topamiz:

$$5\left(1+\frac{1}{3\cdot 5^2}-\frac{2}{3^2\cdot 2!\cdot 5^4}\right) = 5+0.06667-0.00089 = 5.06578,$$

$$\frac{2\cdot 5}{3^3\cdot 4!\cdot 5^6} = \frac{1}{81625} < 0.0001.$$

Demak,

$$f(130) = \sqrt[3]{130} \approx 5.06578. \triangleright$$

3. $\sqrt[5]{1,1}$ *ni* $0,0001$ aniqlikkacha taqribiy hisoblang.

Yechish: $\sqrt[5]{1,1}=(1+0,1)^{\frac{1}{5}}$ deb, binomial qatordan foydalansak:

$$\sqrt[5]{1,1}=(1+0,1)^{\frac{1}{5}}=1+\frac{1}{5}\cdot 0,1+\frac{{}^{1}\!/_{5}\cdot({}^{1}\!/_{5}-1)}{2!}0,01+\frac{{}^{1}\!/_{5}\cdot({}^{1}\!/_{5}-1)\cdot({}^{1}\!/_{5}-2)}{3!}0,001+$$

$$+\ldots=1+0,02-0,0008+0,000048-\ldots.$$

bo‘ladi. To‘rtinchi had $0.000048<0.0001$ bo‘lganligi uchun, hisoblashda birinchi uchta hadini olib, hisoblaymiz:

$$\sqrt[5]{1,1}\approx 1+0,02-0,0008=1,0192\,.$$

Integralning qiymatini taqribiy hisoblash.

b) integralning qiymatini taqribiy hisoblash. Fan va texnikaning turli sohalarida uchraydigan ko‘pgina masalalarni yechish ma’lum funksiyalarning integrallarini hisoblashga keltiriladi. Agar integral ostidagi funksiya murakkab bo‘lsa, tegishli aniq integralni hisoblash qiyin va uni taqribiy hisoblashga to‘g‘ri keladi. Integrallarni taqribiy hisoblash usullaridan biri integral ostida funksiyani darajali qatorga yoyishga asoslangan. Ayni paytda, shunday integrallar borki, masalan,

$$\int_a^b e^{-x^2}dx, \quad \int_a^b \sin x^2 dx, \quad \int_a^b \cos x^2 dx$$

integrallar faqat darajali qatorga yoyishi bilan hisoblanadi.

Misollar 1. Ushbu

$$\int_0^{\frac{1}{4}} e^{-x^2} dx$$

integral taqribiy hisoblansin.

◁ Ma'lumki,

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + ... + \frac{x^n}{n!} + ...$$

Shu yoyilmadan foydalanib topamiz:

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + ...$$

Bu tenglikni integrallaymiz:

$$\int_0^{\frac{1}{4}} e^{-x^2} dx = \int_0^{\frac{1}{4}} \left(1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \dots\right) dx =$$

$$= \left(x - \frac{x^3}{3} + \frac{x^5}{2! \cdot 5} - \frac{x^7}{3! \cdot 7} + \dots\right)\Bigg|_0^{\frac{1}{4}} =$$

$$= \frac{1}{4} - \frac{1}{4^3 \cdot 3} + \frac{1}{4^5 \cdot 2! \cdot 5} - \frac{1}{4^7 \cdot 3! \cdot 7} + \dots$$

Agar bu qatorning dastlabki uchta hadi olinsa, unda

$$\frac{1}{4}-\frac{1}{4^3\cdot 3}+\frac{1}{4^5\cdot 2!\cdot 5}=0.250000-0.005208+0.000098=0.244890$$

bo‘lib,

$$\int_0^{\frac{1}{4}} e^{-x^2}dx \approx 0.24489$$

bo‘ladi. Bu taqribiy tenglikning xatoligi

$$\frac{1}{4^7 3! 7}<0.0001.\triangleright$$

2. Ushbu

$$\int_0^{\frac{1}{2}} \sqrt{1+x^3}dx$$

integral 0.001 aniqlikda taqribiy hisoblansin.

◁ Avvalo integral ostidagi funksiyani darajali qatorga yoyamiz. Buning uchun

$$(1+x)^\alpha = 1+\frac{\alpha}{1!}x+\frac{\alpha(\alpha-1)}{2!}x^2+...+\frac{\alpha(\alpha-1)(\alpha-2)...(\alpha-n+1)}{n!}x^n+...$$

formulani qo‘llaymiz. U holda

$$\sqrt{1+x^3} = (1+x^3)^{\frac{1}{2}} = 1+\frac{1}{2}x^3-\frac{1}{8}x^6+\frac{1}{16}x^9-\frac{5}{128}x^{12}+...$$

bo‘ladi. Bu tenglikni $[0, \frac{1}{2}]$ oraliq bo‘yicha integrallab topamiz:

$$\int_0^{\frac{1}{2}} \sqrt{1+x^3}dx = \int_0^{\frac{1}{2}} \left(1 + \frac{1}{2}x^3 - \frac{1}{8}x^6 + \frac{1}{16}x^9 - \frac{5}{128}x^{12} + \dots\right) dx =$$

$$= \left(x + \frac{x^4}{8} - \frac{x^7}{56} + \frac{x^{10}}{160} - \frac{5x^{13}}{1664} + \dots\right)\Big|_0^{\frac{1}{2}} =$$

$$= \frac{1}{2} + \frac{1}{8} \cdot \frac{1}{2^4} - \frac{1}{56} \cdot \frac{1}{2^7} + \frac{1}{160} \cdot \frac{1}{2^{10}} - \frac{5}{1664} \cdot \frac{1}{2^{13}} + \dots$$

Modomiki,

$$\frac{1}{56 \cdot 2^7} = \frac{1}{56 \cdot 128} = \frac{1}{7168} < 0.001$$

ekan, u holda integralni ko'rsatilgan aniqlikda taqribiy hisoblash uchun oxirgi qatorning dastlabki ikkita hadini olish yetarli. Demak,

$$\int_0^{\frac{1}{2}} \sqrt{1+x^3}dx \approx \frac{1}{2} + \frac{1}{128} \approx 0.508. \triangleright$$

Adabiyotlar:

1. Soatov Yo. U. Oliy matematika 2-tom, Toshkent "O'qituvchi', 1994 y.
2. Soatov Yo. U. Oliy matematika 3-tom, Toshkent "O'qituvchi', 1996 y.
3. E.F.Fayziboyev, Z.I.Suleymenov, B.A.Xudoyorov, Oliy matematikadan misol va masalalar to'plami. Toshkent "O'qituvchi" 2005 y.
4. Farxod Rajabov, Matematika, "O'zbekiston faylasuflari milliy jamiyati" Toshkent, 2007 y.
5. V.E.Shneyder, A.I.Slutskiy, A.S.Shumov Oliy matematika qisqa kursi II-qism, Toshkent "o'qituvchi" 1987 y.
6. https://moodle.tiiame.uz/course/view.php?id=899

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