TOSHKENT IRRIGATSIYA VA QISHLOQ XO'JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI

FAN: **OLIY MATEMATIKA**

MAVZU

13

Chiziqli o‘zgarmas koeffisentli differensial tenglamalar sistemasi.

JUVONOV QAMARIDDIN RIZOQULOVICH

Oiy matematika kafedrasi assistenti

Chiziqli o‘zgarmas koeffisentli differensial tenglamalar sistemasi.

$y_1 = y_1(x),\ y_2 = y_2(x),\ \ldots,\ y_n = y_n(x)$ funksiyalar

$$\begin{cases} \dfrac{dy_1}{dx} = f_1(x, y_1, y_2, \ldots, y_n) \\ \dfrac{dy_2}{dx} = f_2(x, y_1, y_2, \ldots, y_n) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dfrac{dy_n}{dx} = f_n(x, y_1, y_2, \ldots, y_n) \end{cases} \qquad (1)$$

tenglamalar sistemasining yechimi bo‘lsin. Bunday differensial tenglamalar sistemasini **normal tenglamalar sistemasi** deyiladi.

Sistemaning birinchi tenglamasini x bo'yicha differensiallab

$$\frac{d^2y_1}{dx^2} = \frac{\partial f_1}{\partial x} + \frac{\partial f_1}{\partial y_1} \cdot \frac{dy_1}{dx} + \frac{\partial f_1}{\partial y_2} \cdot \frac{dy_2}{dx} + \cdots + \frac{\partial f_1}{\partial y_n} \cdot \frac{dy_n}{dx}$$

tenglikni hosil qilamiz. $\frac{dy_1}{dx}, \frac{dy_2}{dx}, \cdots, \frac{dy_n}{dx}$ larni (1) tengliklarni o'ng tomonlari bilan almashtirib,

$$\frac{d^2y_1}{dx^2} = F_2(x, y_1, y_2, \cdots, y_n)$$

tenglamani hosil qilamiz. Hosil bo'lgan tenglikni differensiallab, yuqoridagi ishni takrorlab,

$$\frac{d^3y_1}{dx^3} = F_3(x, y_1, y_2, \cdots, y_n)$$

tenglamani hosil qilamiz. Shu protsessni davom ettirib

$$\frac{d^n y_1}{dx^n} = F_n(x, y_1, y_2, \cdots, y_n)$$

tenglamani hosil qilamiz.

Shunday qilib,

$$\begin{cases} \dfrac{dy_1}{dx} = f_1(x, y_1, y_2, \cdots, y_n) \\ \dfrac{d^2 y_2}{dx^2} = F_2(x, y_1, y_2, \cdots, y_n) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \dfrac{d^n y_n}{dx^n} = F_n(x, y_1, y_2, \cdots, y_n) \end{cases} \qquad (2)$$

sistemani hosil qilamiz. Bu sistemaning birinchi $(n-1)$ tenglamasidan $y_2, y_3, \cdots, y_n$ larni x, y_1 va $\frac{dy_1}{dx}$, $\frac{d^2y_1}{dx^2}$, $\ldots$, $\frac{d^{n-1}y_1}{dx^{n-1}}$ lar orqali ifodalab

$$\begin{cases} y_2 = \varphi_2\left(x, y_1, y_1', \cdots, y_1^{(n-1)}\right) \\ y_3 = \varphi_3\left(x, y_1, y_1', \cdots, y_1^{(n-1)}\right) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ y_n = \varphi_n\left(x, y_1, y_1', \cdots, y_1^{(n-1)}\right) \end{cases} \tag{3}$$

Bu tengliklarni (2) sistemaning oxirgi tenglamasiga qo‘yib $n-$tartibli

$$\frac{d^n y_1}{dx^n} = \Phi\left(x, y_1, y_1', \cdots, y_1^{(n-1)}\right)$$

tenglamani hosil qilamiz. Bu differensial tenglamani yechib

$$y_1 = \psi_1(x, C_1, C_2, \cdots, , C_n)$$

yechimni topamiz. Bu yechimni $(n-1)$ marta differensiallab $\frac{dy_1}{dx}$, $\frac{d^2y_1}{dx^2}$, $\cdots, \frac{d^{n-1}y_1}{dx^{n-1}}$ hosilalarni topamiz. Bu hosilalarni (3) tengliklarga qo‘yib

$$\begin{cases} y_2 = \psi_2(x, C_1, C_2, \cdots, , C_n) \\ y_3 = \psi_3(x, C_1, C_2, \cdots, , C_n) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ y_n = \psi_n(x, C_1, C_2, \cdots, , C_n) \end{cases} \tag{4}$$

yechimlarni hosil qilamiz.

Misol. $\begin{cases} \dfrac{dy}{dx} = z - y \\ \dfrac{dz}{dx} = -y - 3z \end{cases}$ differensial tenglamalar sistemasini yeching.

◁ 1) Birinchi tenglamani x bo'yicha differensiallab

$$\frac{d^2y}{dx^2} = \frac{dz}{dx} - \frac{dy}{dx}$$

tenglikka ega bo‘lamiz. Bu tenglikka $\frac{dz}{dx}$, $\frac{dy}{dx}$ ifodalarni qo‘yib

$$\frac{d^2y}{dx^2} = -y - 3z - (z - y)$$

yoki

$$\frac{d^2y}{dx^2} = -4z(*)$$

tenglikka ega bo‘lamiz.

2) Berilgan sistemaning birinchi tenglamasidan $z = \frac{dy}{dx} + y(**)$ ni topib (*) tenglamaga qo‘yib,

$$y'' = -4y' - 4y$$

yoki

$$y'' + 4y' + 4y = 0$$

ikkinchi tartibli tenglamani hosil qilamiz.

Bu tenglamani yechamiz: uning xarakteristik tenglamasi

$$k^2 + 4k + 4 = 0$$

bo‘lib $k_1 = -2$, $k_2 = -2$. Shuning uchun $y'' + 4y' + 4y = 0$ tenglamaning umumiy yechimi

$$y = C_1 e^{-2x} + C_2 x e^{-2x}.$$

y dan x bo'yicha hosila olamiz:

$$\frac{dy}{dx} = -2C_1e^{-2x} + C_2e^{-2x} - 2C_2xe^{-2x} = -2C_1e^{-2x} + C_2(1-2x)e^{-2x}.$$

Buni (**) tenglikka qo'yib $z = -2C_1e^{-2x} + C_2(1-2x)e^{-2x} + C_1e^{-2x} + C_2xe^{-2x} = -C_1e^{-2x} + C_2(1-2x+x)e^{-2x} = -C_1e^{-2x} + C_2(1-x)e^{-2x}$ yechimni topamiz.▷

Endi o'zgarmas koeffitsiyentli chiziqli differensial tenglamalar sistemasi

$$\begin{cases} \frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ \frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \frac{dx_n}{dt} = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n \end{cases} \tag{5}$$

berilgan bo'lsin.

Aniqlik uchun uchta no‘malum funksiyali sistemani qaraymiz:

$$\begin{cases} \dfrac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ \dfrac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ \dfrac{dx_3}{dt} = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{cases} \tag{6}$$

Xususiy yechimni

$$x_1 = \alpha_1 e^{kt}, \;\; x_2 = \alpha_2 e^{kt}, \;\; x_3 = \alpha_3 e^{kt} \tag{7}$$

ko‘rinishda izlaymiz. (7) tengliklarni (6) sistemaga qo‘yib,

$$\begin{cases} k\alpha_1 e^{kt} = a_{11}\alpha_1 e^{kt} + a_{12}\alpha_2 e^{kt} + a_{13}\alpha_3 e^{kt} \\ k\alpha_2 e^{kt} = a_{21}\alpha_1 e^{kt} + a_{22}\alpha_2 e^{kt} + a_{23}\alpha_3 e^{kt} \\ k\alpha_3 e^{kt} = a_{31}\alpha_1 e^{kt} + a_{32}\alpha_2 e^{kt} + a_{33}\alpha_3 e^{kt} \end{cases}$$

tengliklarga ega bo‘lamiz. Har bir tenglikni e^{kt} ga bo‘lib, hamma hadlarni o‘ng tomonga o‘tkazib

$$\begin{cases} (a_{11}-k)\alpha_1 + a_{12}\alpha_2 + a_{13}\alpha_3 = 0 \\ a_{21}\alpha_1 + (a_{22}-k)\alpha_2 + a_{23}\alpha_3 = 0 \\ a_{31}\alpha_1 + a_{32}\alpha_2 + (a_{33}-k)\alpha_3 = 0 \end{cases} \tag{8}$$

bir jinsli oddiy tenglamalar sistemasini hosil qilamiz. Ma’lumki (8) sistema noldan farqli yechimlarga ega bo‘lishi uchun bu sistemaning determinanti nolga teng bo‘lishi zarur va yetarli, ya’ni

$$\begin{vmatrix} a_{11}-k & a_{12} & a_{13} \\ a_{21} & a_{22}-k & a_{23} \\ a_{31} & a_{32} & a_{33}-k \end{vmatrix} = 0 \tag{9}$$

(9) tenglama (6) sistemaning **xarakteristik tenglamasi** deyiladi.

1. Xarakteristik tenglama haqiqiy k_1, k_2, k_3 ildizlarga ega bo‘lsa, ularga (7) yechim mos kelib, uning koeffitsiyentllari α_1, α_2, α_3 (8) sistemadan aniqlanadi. Demak (6) sistemaning umumiy yechimi

$$\begin{cases} x_1 = C_1\alpha_1^{(1)}e^{k_1t} + C_2\alpha_1^{(2)}e^{k_2t} + C_3\alpha_1^{(3)}e^{k_3t} \\ x_2 = C_1\alpha_2^{(1)}e^{k_1t} + C_2\alpha_2^{(2)}e^{k_2t} + C_3\alpha_2^{(3)}e^{k_3t} \\ x_3 = C_1\alpha_3^{(1)}e^{k_1t} + C_2\alpha_3^{(2)}e^{k_2t} + C_3\alpha_3^{(3)}e^{k_3t} \end{cases}$$

ko‘rinishda bo‘ladi.

Мисол. Ушбу системанинг умумий ечимини топинг:

$$\left.\begin{aligned} \frac{dx}{dt} &= -2x - 3y \\ \frac{dy}{dt} &= -x \end{aligned}\right\} \qquad (*)$$

Ечилиши. Берилган дифференциал тенгламалар системасига мос **(9)** характеристик тенглама қуйидаги кўринишда бўлади:

$$\begin{vmatrix} -2-k & -3 \\ -1 & 0-k \end{vmatrix} = 0$$

ёки $k^2 + 2k - 3 = 0$. Унинг илдизлари: $k_1 = -3$, $k_2 = 1$

(*) системанинг хусусий ечимларини

$$x_1 = \alpha_1 e^{k_1 t}, \; y_1 = \beta_1 e^{k_1 t}; \; x_2 = \alpha_2 e^{k_2 t}, \; y_2 = \beta_2 e^{k_2 t}$$

кўринишда излаймиз.

$k_1 = -3$ да α ва β ни аниқлаш учун тенгламалар системаси қуйидагича ёзилади:

$$\left.\begin{aligned} [-2-(-3)]\,\alpha_1 - 3\beta_1 &= 0, \\ -\alpha_1 + [0-(-3)]\,\beta_1 &= 0 \end{aligned}\right\} \qquad \left.\begin{aligned} \alpha_1 - 3\beta_1 &= 0. \\ -\alpha_1 + 3\beta_1 &= 0. \end{aligned}\right\}$$

Бу система чексиз кўп ечимга эга, чунки иккинчи тенглама биринчи тенгламанинг натижасидир. Масалан, $\beta_1=1$ деб, $\alpha_1=3$ ни топамиз. Шундай қилиб, характеристик тенгламанинг $k_1=-3$ илдизига $x_1=3e^{-3t}$ ва $y_1=e^{-3t}$ хусусий ечимлар мос келади.

$k=1$ да α ва β ни аниқлаш учун тенгламалар системаси қуйидагича бўлади:

$$\left.\begin{aligned} -3\alpha_2-3\beta_2&=0, \\ -\alpha_2-\beta_2&=0 \end{aligned}\right\}$$

Бу системанинг ечимлари сифатида $\alpha_2=1$ $\beta_2=-1$ ни олиш мумкин. У ҳолда: характеристик тенгламанинг $k=1$ илдизига $x_2=e^t$ ва $y_2=-e^t$ хусусий ечимлар мос келади.

Берилган системанинг умумий ечими формулага кўра қуйидагича бўлад и

$$x(t)=3C_1e^{-3t}+C_2e^t; \; y(t)=C_1e^{-3t}-C_2e^t.$$

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E'TIBORINGIZ UCHUN RAHMAT!

JUVONOV QAMARIDDIN RIZOQULOVICH

OLIY MATEMATIKA KAFEDRASI ASSISTENTI

☎ + 998 93 588 37 89

✉ q.juvanov@tiiame.uz