TOSHKENT IRRIGATSIYA VA QISHLOQ XO'JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI

FAN: OLIY MATEMATIKA

Mavzu: **Kompleks son tushunchasi. Ular ustida chiziqli amallar. Trigonometrik ko’rinishdagi kompleks sonlar va ular ustida amallar. Kompleks sonlarning funktsiyarda qo’llanilishi.**

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Toshkent Irrigatsiya va Qishloq Xo'jaligini Mexanizatsiyalash Muhandislari Intituti

Reja:

1. Kompleks son tushunchasi
2. Trigonometrik ko'rinishdagi kompleks sonlar
3. Kompleks sonlarning funktsiyarda qo'llanilishi

Kompleks son tushunchasi

Ta'rif. x va y haqiqiy sonlar, i esa qandaydir simvol bo'lsa, $x+yi$ ifoda *kompleks son* deyiladi.

Kompleks sonni bitta harf bilan belgilaymiz, ya'ni $z=x+yi$. Kompleks sonning haqiqiy qismi x ni Re(z) bilan, mavhum qismi y ni esa Im(z) bilan belgilash qabul qilingan: $x=\mathrm{Re}(z)$, $y=\mathrm{Im}(z)$.

$i^2=-1$ tenglik bilan aniqlanadi va u *mavhum birlik* deb ataladi.

$i^2=-1,\quad i^3=-i,\quad i^4=1,\quad i^5=i$ va hokazo.

Ta'rif. Bir-biridan faqat mavhum qismlarining ishorasi bilan farq qiladigan ikki kompleks songa *o'zaro qo'shma kompleks sonlar* deyiladi.

$z=x+yi$ kompleks songa qo'shma kompleks son $\bar{z}=x-yi$ ko'rinishda yoziladi. Kompleks sonlar ustida arifmetik amallarni kiritishdan oldin quyidagi tushunchalarni kiritamiz:

a) $(x+yi)+(c+di)=(x+c)+(y+d)i$

b) $(x+yi)-(c+di)=(x-c)+(y-d)i$

v) $(x+yi)(c+di)=(xs-yd)+(xd+yc)I$

g) $\dfrac{x+yi}{c+di}=\dfrac{xc+yd}{c^2+d^2}+\dfrac{yc-xd}{c^2+d^2}i$

1-misol. $(2-i)\cdot(\frac{3}{4}+2i)$ ni hisoblang.

Yechish. $$(2-i)\cdot\left(\frac{3}{4}+2i\right)=2\cdot\frac{3}{4}+2\cdot 2i-\frac{3}{4}i-2i^2=\frac{3}{2}+4i-\frac{3}{4}i+2=\frac{7}{2}+\frac{13}{4}i$$

2-misol. $\frac{2-i}{-3+2i}$ ni hisoblang.

Yechish. $$\frac{2-i}{-3+2i}=\frac{(2-i)(-3-2i)}{(-3+2i)(-3-2i)}=\frac{-6-4i+3i-2}{9+6i-6i+4}=\frac{-8-i}{13}=-\frac{8}{13}-\frac{1}{13}i$$

z=*x*+*yi* ga qarama-qarshi son - *z*= -*x*-*yi* dir.

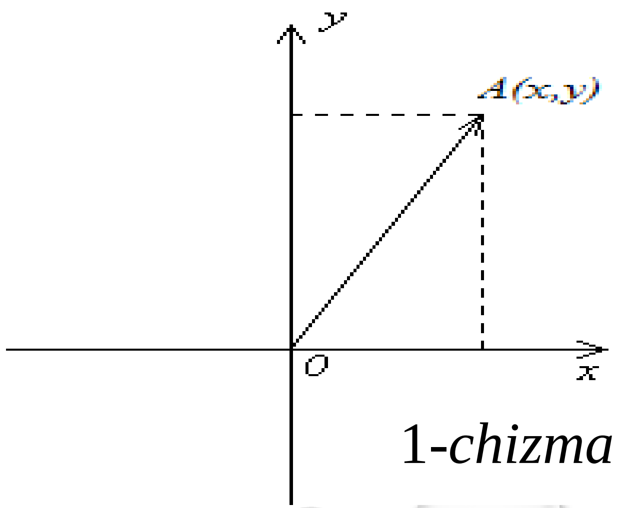
z=*x*+*yi* ga teskari bo‘lgan $\frac{1}{z}$ sonini topamiz:

$$\frac{1}{z}=\frac{1}{x+yi}=\frac{x-yi}{(x+yi)(x-yi)}=\frac{x}{x^2+y^2}-\frac{y}{x^2+y^2}i$$

Trigonometrik ko‘rinishdagi kompleks sonlar

Kompleks sonlarga oid ko‘pgina tushunchalar aniq bo‘lishi uchun kompleks sonni biror geometrik shakl sifatida qarash qulaydir.

$z=x+yi$ kompleks sonning geometrik shakli sifatida, Oxy koordinata tekisligida $A(x, y)$ nuqtani yoki boshi $O(0, 0)$ nuqtada, oxiri esa $A(x, y)$ nuqtada bo‘lgan $\vec{OA}$ vektorni qabul qilinadi. 1-chizma.



1-*chizma*

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Koordinatalar tekisligining har bir nuqtasi faqat bitta kompleks sonni tasvirlaydi va aksincha, har qanday kompleks son faqat bitta nuqta bilan tasvirlanadi. Haqiqiy sonlarga absissalar o‘qining nuqtalari, sof mavhum sonlarga esa ordinata o‘qining nuqtalari mos keladi. Shunga ko‘ra koordinatalar tekisligi kompleks tekislik, absissalar o‘qi *haqiqiy o‘q*, ordinata o‘qi esa *mavhum o‘q* deb ham ataladi. Hosil qilingan $\vec{OA}$ vektor kompleks sonning *radius vektori* deyiladi. Kompleks son radius vektorining uzunligi shu kompleks sonning *moduli* deyiladi va $|z|$ bilan yoki r bilan belgilanadi. $z=x+yi$ sonining moduli, ya’ni

$$|z| = \sqrt{x^2 + y^2}$$

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Haqiqatdan ham ikki nuqta orasidagi masofani topish formulasiga asosan,

$$|z| = |\vec{OA}| = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$$

1-misol. $z = \sqrt{3} - \sqrt{5}i$ kompleks sonning modulini toping.

Yechish. Bu yerda $x = \sqrt{3}, \quad y = -\sqrt{5}$ bo‘lgani uchun, (1) formuladan

$$|z| = \sqrt{(\sqrt{3})^2 + (-\sqrt{5})^2} = \sqrt{3+5} = \sqrt{8} = 2\sqrt{2}$$

$\vec{OA}$ vektor $z=x+yi$ kompleks sonning radius vektori bo‘lsin, markazi $O(0, 0)$ nuqtada va radiusi r bo‘lgan aylanani O nuqta atrofida $A(a, b)$ nuqta bilan ustma-ust tushadigan qilib buramiz. Bu ishni bir biridan 2π ga karrali bo‘lgan burish burchagiga farq qiladigan cheksiz ko‘p burish burchaklari yordamida amalga oshirish mumkin. Shu burilish burchaklarining har biri $z=x+yi$ kompleks sonning *argumenti* deyiladi. Kompleks sonning barcha argumentlari to‘plamini $Arg(z)$ bilan belgilanadi.

Burish burchagining sinus va kosinuslari ta’rifidan quyidagi munosabatlar o‘rinli $\cos\varphi = \dfrac{x}{|z|}, \quad \sin\varphi = \dfrac{y}{|z|}, \quad tg\varphi = \dfrac{y}{x}.$

Bundan, $x = |z|\cos\varphi, \quad y = |z|\sin\varphi, \quad \varphi = arctg\dfrac{y}{x}.$

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Kompleks sonning umumiy ko'rinishidagi *x* va *y* ning o'rniga qo'yilsa, $z = |z|(\cos\varphi + i\sin\varphi)$ ko'rinishga keladi. $|z| = r$ ekanligini hisobga olsak,

$$z = r(\cos\varphi + i\sin\varphi)$$

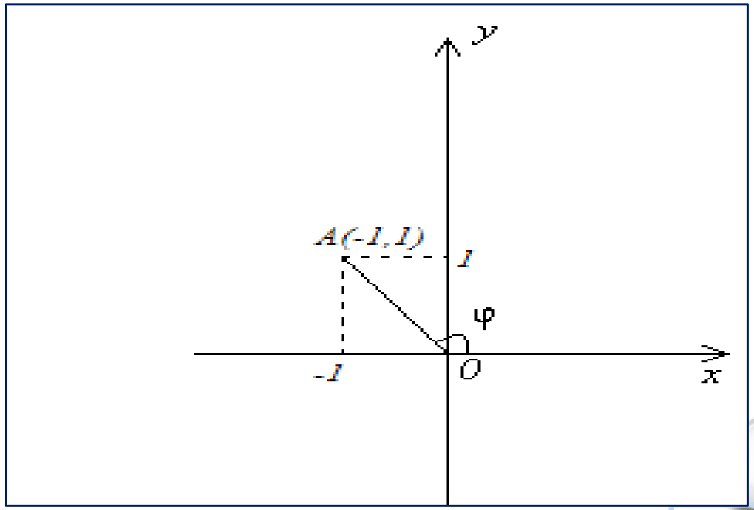
Bu ko'rinish kompleks sonning *trigonometrik shakli* deyiladi.

2-misol. $z = -1 + i$ sonini trigonometrik shaklda ifodalang.

Yechish. $|z| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$, $\varphi = arctg\left(\frac{1}{-1}\right) = \frac{3\pi}{4}$.

Bundan $z = -1 + i = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

Bu misolning geometrik shakli quyidagicha



2-rasm

Eyler formulasi

Quyidagi (2) formula *Eyler formulasi* deyiladi.

$$\cos\varphi + i\sin\varphi = e^{i\varphi} \qquad (2)$$

Eyler formulasidan foydalanib kompleks sonning yozilishining ko‘rsatkichli shakliga ega bo‘lamiz: $z = re^{i\varphi}$

$z=x+yi$ kompleks sonning $[0,\ 2\pi]$ oraliqda yotadigan argumenti shu sonning *bosh argumenti* deyiladi va *arg*(z) bilan belgilanadi.

Shunga muvofiq $z = r(\cos(\arg(z)) + i\sin(\arg(z)))$

ni kompleks sonning *bosh trigonometrik shakli* deb ataladi.

Izoh. $z=0$ sonning moduli 0 ga teng, lekin argumenti aniqlanmaydi.

Trigonometrik shaklda berilgan kompleks sonlarni ko‘paytirish

$z=r(\cos\varphi+i\sin\varphi)$ va $w=R(\cos\alpha+i\sin\alpha)$ kompleks sonlarning

trigonometrik shaklda berilgan bo‘lsa, u holda qo‘yidagi (3) formula o‘rinli.

$$zw=rR(\cos(\varphi+\alpha)+i\sin(\varphi+\alpha)) \qquad (3)$$

Misol. $z=\frac{\sqrt{2}}{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ va $w=\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}$ sonlarning ko‘paytmasini toping.

Yechish. $z=\frac{\sqrt{2}}{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ da $r=\frac{\sqrt{2}}{2}$, $\varphi=\frac{\pi}{4}$, $w=\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}$ da $R=1$, $\alpha=\frac{\pi}{8}$

(3) ga ko‘ra $zw=\frac{\sqrt{2}}{2}\cdot 1\left(\cos\left(\frac{\pi}{4}+\frac{\pi}{8}\right)+i\sin\left(\frac{\pi}{4}+\frac{\pi}{8}\right)\right)=\frac{\sqrt{2}}{2}\left(\cos\frac{3\pi}{8}+\sin\frac{3\pi}{8}\right)$

Trigonometrik shaklda berilgan kompleks sonlarni darajaga ko‘tarish

Trigonometrik shaklda berilgan kompleks sonlarni ko‘paytirish qoidasini

$z^n = z \cdot z \cdot z \cdot \ldots \cdot z$ (n ta ko‘paytuvchi) ko‘paytirish uchun ketma-ket tadbiq etib,

z^n ni hisoblash qoidasini hosil bo‘ladi. Ya’ni,

$z^n = \left(r(\cos\varphi + i\sin\varphi)\right)^n$ dan $z^n = r^n(\cos n\varphi + i\sin n\varphi)$ tenglik hosil bo‘ladi.

Bu tenglik *Muavr formulasi* deyiladi.

Misol.

$$A = \frac{\left(\sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right)^{12} \cdot \left(2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right)^{3}}{\left(2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right)^{10} \cdot \left(2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right)^{5}}$$ ni hisoblang.

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Yechish.

$$1)\left(\sqrt{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right)^{12}=2^{6}\left(\cos12\cdot\frac{\pi}{3}+i\sin12\cdot\frac{\pi}{3}\right)=2^{6}\cdot(\cos4\pi+i\sin4\pi)$$

$$2)\left(2(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})\right)^{3}=2^{3}\left(\cos3\cdot\frac{\pi}{3}+i\sin3\cdot\frac{\pi}{3}\right)=2^{3}(\cos\pi+i\sin\pi)$$

$$3)\,2^{6}(\cos4\pi+i\sin4\pi)\cdot2^{3}(\cos\pi+i\sin\pi)=2^{9}(\cos(4\pi+\pi)+i\sin(4\pi+\pi))=$$
$$=2^{9}(\cos5\pi+i\sin5\pi)$$

$$4)\left(2(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})\right)^{10}=2^{10}\left(\cos10\cdot\frac{\pi}{4}+i\sin10\cdot\frac{\pi}{4}\right)=2^{10}\left(\cos\frac{5\pi}{2}+i\sin\frac{5\pi}{2}\right)$$

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$$5)\left(2(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})\right)^5=2^5\left(\cos 5\cdot\frac{3\pi}{4}+i\sin 5\cdot\frac{3\pi}{4}\right)=2^5\left(\cos\frac{15\pi}{4}+i\sin\frac{15\pi}{4}\right)$$

$$6)\,2^{10}\left(\cos\frac{5\pi}{2}+i\sin\frac{5\pi}{2}\right)\cdot 2^5\left(\cos\frac{15\pi}{4}+i\sin\frac{15\pi}{4}\right)=2^{15}\left(\cos\left(\frac{5\pi}{2}+\frac{15\pi}{4}\right)+i\sin\left(\frac{5\pi}{2}+\frac{15\pi}{4}\right)\right)=$$

$$=2^{15}\left(\cos\frac{25\pi}{4}+i\sin\frac{25\pi}{4}\right)$$

$$7)\,\frac{2^9(\cos 5\pi+i\sin 5\pi)}{2^{15}\left(\cos\frac{25\pi}{4}+i\sin\frac{25\pi}{4}\right)}=\frac{1}{2^6}\left(\cos(5\pi-\frac{25\pi}{4})+i\sin(5\pi-\frac{25\pi}{4})\right)=$$

$$=\frac{1}{2^6}\left(\cos(-\frac{5\pi}{4})+i\sin(-\frac{5\pi}{4})\right)=\frac{1}{2^6}\left(\cos\frac{5\pi}{4}-i\sin\frac{5\pi}{4}\right)$$

Trigonometrik shaklda berilgan kompleks sonlarni bo‘lish

$z = r(\cos\varphi + i\sin\varphi)$ va $w = R(\cos\alpha + i\sin\alpha)$ kompleks sonlarning trigonometrik shakli berilgan bo‘lsa, u holda qo‘yidagi (4) formula o‘rinli.

$$\frac{z}{w} = \frac{r(\cos\varphi + i\sin\varphi)}{R(\cos\alpha + i\sin\alpha)} = \frac{r}{R}\left(\cos(\varphi - \alpha) + i\sin(\varphi - \alpha)\right) \qquad (4)$$

Misol. $z_1 = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$, $z_2 = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ kompleks sonlar berilgan bo‘lsa, $\frac{z_1}{z_2}$ ni hisoblang.

Yechish.

$$\frac{z_1}{z_2} = \frac{\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)}{2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)} = \frac{\sqrt{2}}{2}\left(\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)\right) = \frac{\sqrt{2}}{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right).$$

Trigonometrik shaklda berilgan kompleks sonlardan ildiz chiqarish

Ta'rif. z kompleks sonning n-natural darajali ildizi deb, $w^n = z$ tenglik bajariladigan har qanday w kompleks songa aytiladi.

Teorema. $z = r(\cos\varphi + i\sin\varphi) \neq 0$ kompleks sonni n ta har xil w_n kompleks ildizlarga ega va bu ildizlar quyidagi formula bilan aniqlanadi.

$$w_n = \sqrt[n]{r}\left(\cos\frac{\varphi + 2\pi k}{n} + i\sin\frac{\varphi + 2\pi k}{n}\right) \qquad k=0,1,2,\ldots,n-1 \qquad (5)$$

1-misol. $\sqrt[3]{-\sqrt{2} + i\sqrt{2}}$ ning barcha qiymatlarini toping.

Yechish. $\left|-\sqrt{2} + i\sqrt{2}\right| = \sqrt{(-\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{2+2} = 2$

$\varphi = \arg\left(-\sqrt{2} + i\sqrt{2}\right) = \frac{3\pi}{4}$ bo'lgani uchun

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$$-\sqrt{2}+i\sqrt{2}=2\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$$ ga ega bo‘lamiz.

(5) formuladan $$w_n=\sqrt[3]{2}\left(\cos\frac{\frac{3\pi}{4}+2\pi k}{3}+i\sin\frac{\frac{3\pi}{4}+2\pi k}{3}\right)$$ bo‘ladi.

k=0 da $w_0=\sqrt[3]{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$, *k*=1 da $w_1=\sqrt[3]{2}\left(\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12}\right)$

k=2 da $w_2=\sqrt[3]{2}\left(\cos\frac{19\pi}{12}+i\sin\frac{19\pi}{12}\right)$

2-misol. $z^2+4=0$ tenglamani yeching.

Yechish. $z^2+4=(z-2i)(z+2i)=0$ bundan $z_1=2i,\quad z_2=-2i$

Kompleks sonlarning va funksiyalarda qo‘llanilishi

Agar *Oxy* tekislikda chiziqlar $x=x(t)$ va $y=y(t)$ parametrik ko‘rinishda berilgan bo‘lsa $(t_1 \le t \le t_2)$ u holda

$$z = z(t) = x(t) + iy(t) \qquad (6)$$

ko‘rinishdagi tenglama *chiziqlarning kompleks shakli* deyiladi.

1-misol. Quyidagi chiziq tenglamalarining kompleks shaklini toping

a) $x = 2 + 3\cos t\ ,\ y = -1 + 3\sin t \qquad (0 \le t \le 2\pi)$

b) $2x^2 + 3y^2 = 12$

Yechish. *a*) $x = 2 + 3\cos t,\ y = -1 + 3\sin t$ bu radiusi 3 ga va markazi (2, -1) nuqtada bo‘lgan aylananing parametrik tenglamasi, bu yerdan (6) formulaga asosan quyidagini hosil qilamiz:

$$z = z(t) = 2 + 3\cos t + i(-1 + 3\sin t) = 2 - i + 3(\cos t + i\sin t) = 2 - i + 3e^{it},$$

bu yerda $\cos t + i\sin t = e^{it}$, demak izlanlayotgan tenglama $z = 2 - i + 3e^{it}$

b) $2x^2 + 3y^2 = 12$ bu tenglama, $\frac{x^2}{6} + \frac{y^2}{4} = 1$ ga teng. Bu yerda markazi

koordinatalar boshida va yarim o‘qlari $\sqrt{6}$, 2 ga teng ellips tenglamasi

ekanligi kelib chiqadi. Aslida ellipsning parametrik tenglamasi

$x = \sqrt{6}\cos t,\ y = 2\sin t$ bundan ko‘rinadiki, yuqorida berilgan parametrik ko‘rinishdagi egri chiziqning kompleks shakli quyidagicha bo‘ladi,

$$z = \sqrt{6}\cos t + i2\sin t$$

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2-misol. Kompleks shaklda berilgan chiziqning tipini aniqlang.

$a)\ z = 5e^{it} + 2e^{-it}$ $\qquad$ $b)\ z = 2t - 1 + i(1 + 2t - 4t^2)$

Yechish. *a*) Ma’lumki, $e^{\pm it} = \cos t \pm i \sin t$ bundan

$$z = 5e^{it} + 2e^{-it} = 7\cos t + i3\sin t$$

(6) formulaga asosan, $x = 7\cos t,\ y = 3\sin t$ ko‘rinishdagi parametrik tenglama hosil bo‘ladi. Bundan *t* ni topamiz va $\frac{x^2}{49} + \frac{y^2}{9} = 1$ ko‘rinishdagi ellips hosil bo‘ladi.

b) Berilgan tenglamaga asosan $x = 2t - 1,\ y = 1 + 2t - 4t^2$ ko‘rinishdagi

parametrik tenglama hosil bo‘ladi. Bu yerdan *t* ni topamiz va

$y = 1 - x - x^2$ ko‘rinishdagi uchi pastga qaragan parabola tenglamasi ekanligi kelib chiqadi.

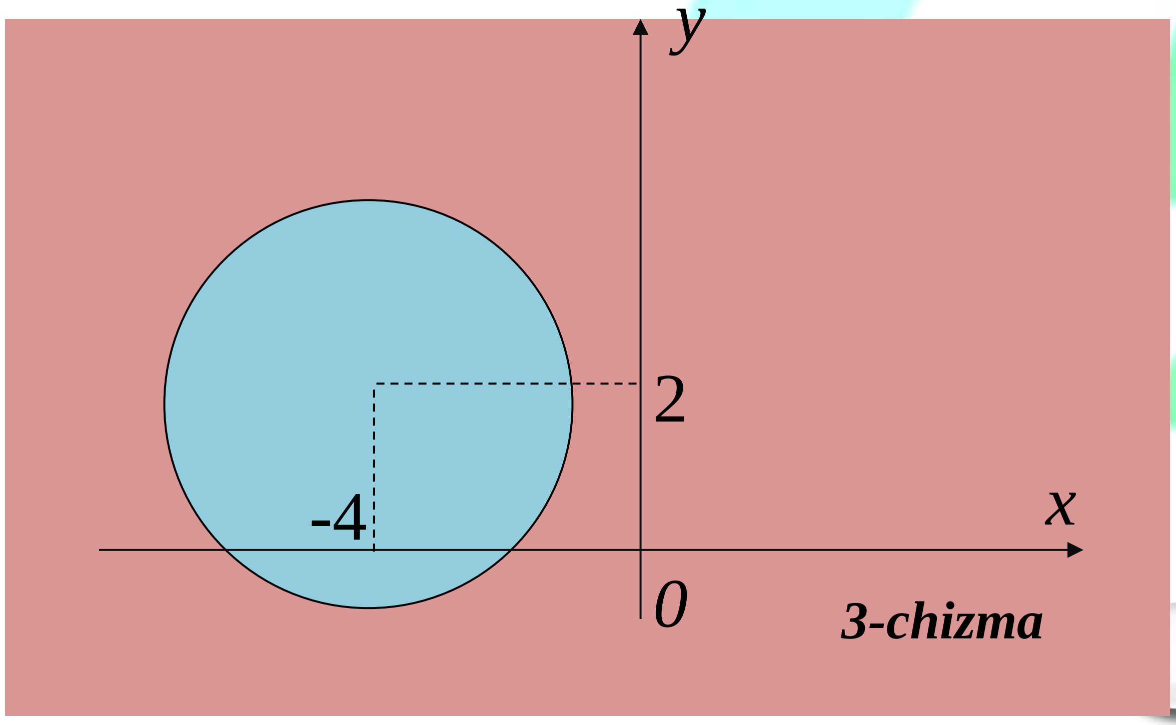
3-misol. Quyidagi tenglamalar bilan berilgan chiziqlarni *z* kompleks tekisligida yasang.

$a)\ |z + 4 - 2i| = 3 \qquad b)\ \mathrm{Re}\,\frac{z-2}{z+i} = 2$

Yechish. Modul ta’rifidan foydalanib,

$$|z+4-2i| = |x+iy+4-2i| = |(x+4)+i(y-2)| = \sqrt{(x+4)^2+(y-2)^2} = 3$$

yoki $(x+4)^2+(y-2)^2=9$, bundan radiusi 3 ga va markazi (-4, 2) nuqtada bo‘lgan aylana tenglamasi kelib chiqadi.



3-chizma

b) $z=x+iy$ dan

$$\frac{z-2}{z+i}=\frac{(x-2)+iy}{x+i(y+1)}=\frac{((x-2)+iy)(x-i(y+1))}{(x+i(y+1))(x-i(y+1))}=\frac{(x^2-2x+y^2+y+2)+i(2y-x)}{x^2+(y+1)^2}$$

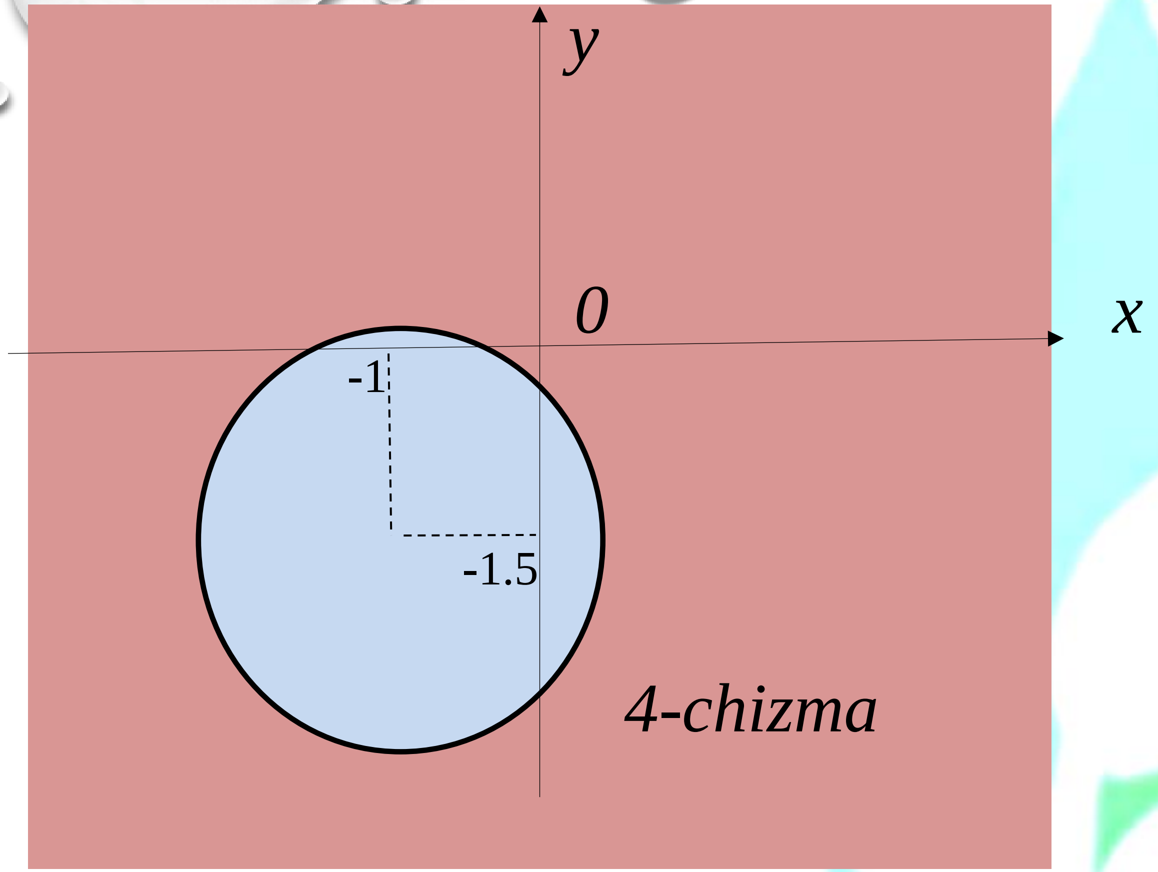
demak, $\operatorname{Re}\frac{z-2}{z+i}=\frac{x^2-2x+y^2+y+2}{x^2+(y+1)^2}=2$ bu yerdan,

$$x^2-2x+y^2+y+2=2(x^2+(y+1)^2)$$

$$x^2-2x+y^2+y+2=2x^2+2y^2+4y+2$$

$(x+1)^2+(y+1{,}5)^2=\frac{13}{4}$ ekanligi kelib chiqadi. Bu esa radiusi $\frac{\sqrt{13}}{2}$ va

markazi (-1, -1,5) nuqtada bo‘lgan aylana tenglamasidir.



4-misol. Limitlarni hisoblang

1) $\lim\limits_{n\to\infty}\left(\dfrac{2n-3}{n+1}+i\dfrac{3n^2-2n+1}{1+n^2}\right)$

2) $\lim\limits_{z\to 2-3i}\left(\dfrac{\mathrm{Re}(z^2)}{|z|^2}\right)$

Yechish.

1) $$\lim\limits_{n\to\infty}\left(\frac{2n-3}{n+1}+i\frac{3n^2-2n+1}{1+n^2}\right)=\lim\limits_{n\to\infty}\frac{2n-3}{n+1}+i\lim\limits_{n\to\infty}\frac{3n^2-2n+1}{1+n^2}=$$

$$=\lim_{n\to\infty}\frac{n(2-\frac{3}{n})}{n(1+\frac{1}{n})}+i\lim_{n\to\infty}\frac{n^2(3-\frac{2}{n}+\frac{1}{n^2})}{n^2(\frac{1}{n^2}+1)}=2+3i$$

2) $$\lim_{n\to 2-3i}\frac{\mathrm{Re}(z^2)}{|z|^2}=\left|\begin{matrix}\mathrm{Re}(z^2)=x^2-y^2, \\ |z|=\sqrt{x^2+y^2}\end{matrix}\right|=\lim_{\substack{x\to 2\\ y\to -3}}\frac{x^2-y^2}{x^2+y^2}=-\frac{5}{13}$$

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