O’ZBEKISTON RESPUBLIKASI
OLIY TA’LIM, FAN VA INNOVATSIYALAR VAZIRLIGI

“TIQXMMI” MILLIY TADQIQOT UNIVERSITETI

T.G.ERGASHEV

DIFFERENSIAL TENGLAMALAR

O’quv qo’llanma

Toshkent 2023

Ushbu o'quv qo'llanma "TIQXMMI" Milliy tadqiqot universiteti Ilmiy Kengashining 2023 yil 18 martdagi majlisida ko'rib chiqilgan va chop etishga tavsiya etilgan (108 a/f – sonli buyruq).

O'quv qo'llanma 5140300 – Mexanika va matematik modellashtirish bakalavr ta'lim yo'nalishi o'quv rejasidagi "Differensial tenglamalar" fani dasturi asosida yaratilgan bolib, undan universitetlar va pedagogika institutlari hamda oliy texnika o'quv yurtlarining matematika fani chuqur o'qitiladigan bakalavr ta'lim yo'nalishlarining talabalari, o'qituvchilari va, umuman, differensial tenglamalar fanidan o'z bilimlarini yanada takomillashtirish niyatida yurganlarga mo'ljallangan.

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N.Yo'ldoshev, "TIQXMMI" Milliy tadqiqot universiteti Oliy matematika kafedrasi dotsenti, fizika-matematika fanlari nomzodi.

Ergashev T.G.

/Differensial tenglamalar/

(O'quv qo'llanma) -T:TIQXMMI MTU 2023 -384 bet.

SO’Z BOSHI

O’zbekistonda oliy ta’lim tizimini isloh qilish bilan bog’liq hujjatlarda o’qitishning sust (passiv) shakllaridan talabalar bilan faol (aktiv) ijodiy ishlashga, “yalpi” o’qitishdan ko’ra individual yondoshuvni kuchaytirishga hamda ta’lim oluvchilarning mustaqil ishlash faoliyatini kengaytirish yo’li bilan ularning ijodiy qobiliyatlarini rivojlantirishga o’tish zarurligi ta’kidlab kelinmoqda. Oliy ta’limni bu tarzda rivojlantirish va isloh qilish o’quv jarayonining yangicha uslubiy ta’minotini ko’zda tutadi: tegishli uslubiy va o’quv qo’llanmalar bilan ta’minlangan ma’ruzalar, amaliy mashg’ulotlar va laboratoriya mashg’ulotlarini o’tishda zamonaviy metodikalarni yaratish, talabalarning mustaqil ishlarini tashkil qilish va ularni nazorat qilishning yangi shakllarini va hokazolarni ishlab chiqish kerak bo’ladi.

Universitetlar va oliy texnika o’quv yurtlarining matematika fani chuqurlashtirilgan ta’lim yo’nalishlarida o’qitiladigan differentsial tenglamalar fani bo’yicha hozirgi kunda mavjud bo’lgan masalalar va mashqlar to’plamlari o’z tuzilishiga (bir xil tipdagi masalalar va mashqlarning kam sonda bo’lishiga) ko’ra o’qitishni individuallashtirish imkonini bermaydi.

Mazkur o’quv qo’llanmada talabalarning o’rganish jarayonini faollashtirish, ularda yetarli murakkablikdagi masalalarni ham yecha olish ko’nikmalarini hosil qilish uchun har bir talabaga individual uy vazifalarining berilishi, auditoriya mashg’ulotlari paytida mustaqil (nazorat) ishlarining muntazam o’tkazib borilishi va talabalar mehnatining rag’batlantirilishi (baholanishi) muhim ahamiytga ega ekanligi hisobga olingan.

Oquv qo'llanma boblarga, boblar esa mavzularga ajratilgan bo'lib, har bir mavzuning boshlanishida mavzuga doir masalalarni yechish va mashqlarni bajarishda zarur bo'ladigan nazariy ma'lumotlar (asosiy ta'riflar, tushunchalar, teoremalar va formulalar) beriladi. Ma'lumotlarning mazmuni misollar yechish yo'li bilan tushuntiriladi. Misollar yechishning boshlanishi ◄ va tugashi ► bilan belgilangan. Individual topshiriqlarga o'tishdan oldin tipik misollar yechilishlari bilan berilgan. So'ngra har biri 30 tadan misol va masalalardan iborat bo'lgan individual topshiriqlar berilgan. Individual topshiriqlar jami 60 ta bo'lib, ular 2000 ga yaqin misol va masalalarni o'z ichiga qamrab olgan. Individual topshiriqlar aksariyat qismining javoblari kitobning oxirida keltirilgan. Har bir bobning oxirida esa qiyinlik darajasi nisbatan yuqori bo'lgan amaliy xarakterdagi masalalar javoblari bilan berilgan.

Qo'lyozmani ko'rib chiqib, o'z fikr va mulohazalarini bildirgan professor A.Hasanov va dotsent N.Yo'ldoshev hamda qo'lyozmani nashrga tayyorlashda katta yordam bergan katta o'qituvchi Z.To'lakovaga muallif samimiy minnatdorchilik bildiradi.

O'quv qo'llanma to'g'risidagi kitobxonlarning tanqidiy fikr va mulohazalarini muallif mamnuniyat bilan qabul qiladi.

Muallif

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1 - BOB
BIRINCHI TARTIBLI DIFFERENSIAL TENGLAMALAR

1.1. ASOSIY TUSHUNCHALAR VA TA'RIFLAR

1.1.1. Asosiy ta`riflar. Birinchi tartibli differensial tenglama quyidagi

$$F\left(x, y, \frac{dy}{dx}\right) = 0 \qquad (1)$$

ko'rinishda bo'ladi.

Bu yerda x – erkli o'zgaruvchi (argument), $y = y(x)$ – shu x argumentning noma'lum funksiyasi, F esa x, y va $y' = \frac{dy}{dx}$ o'zgaruvchilarning berilgan funksiyasi.

(1) tenglama *hosilaga nisbatan yechilmagan differensial tenglama* deyiladi. Odatda (1) tenglamani *hosilaga nisbatan yechilgan tenglama*

$$y' = f(x, y) \qquad (2)$$

ko'rinishida yoki *differensiallar ishtirok etgan tenglama*

$$M(x, y)dx + N(x, y)dy = 0 \qquad (3)$$

ko'rinishida yozib olishga harakat qilinadi.

Biror I intervalda aniqlangan, uzluksiz differensiallanuvchi va berilgan (1) differensial tenglamani qanoatlantiradigan har qanday $y = \varphi(x)$ funksiya, ya'ni tenglamada y ni va uning hosilalarini $\varphi(x)$ va uning tegishli hosilalari bilan almashtirganda berilgan tenglamani ayniyatga aylantiradigan funksiya *differensial tenglamaning yechimi* deyiladi:

$$F\left(x, \varphi(x), \frac{d\varphi(x)}{dx}\right) \equiv 0 \quad \left(\varphi'(x) \equiv f(x, \varphi(x))\right), \ x \in I .$$

Agar (2) tenglamaning yechimi oshkormas $\Phi(x, y) = 0$ ko'rinishda topilgan bo'lsa, $\Phi(x, y) = 0$ munosabat (2) *tenglamaning integrali* deyiladi.

(2) tenglama yechimining grafigi shu *tenglamaning integral egri chizig'i,* yechim grafigining ordinatalar o'qiga proeksiyasi esa differensial *tenglamaning fazaviy egri chizig'i* (yoki *trayektoriyasi*) deyiladi.

(2) tenglamaning $\varphi(x_0)=y_0$ boshlang'ich shartni qanoatlantiruvchi $y=\varphi(x)$ yechimini topish masalasi *Koshi masalasi* deyiladi, bu yerda $x_0 \in I$.

Birinchi tartibli differensial tenglamaning *umumiy yechimi* deb ixtiyoriy C o'zgarmas miqdorga bog'liq bo'lgan hamda quyidagi shartlarni qanoatlantiruvchi $y=\varphi(x,C)$ funksiyaga aytiladi:

1) (1) yoki (2) differensial tenglamaning ixtiyoriy yechimi $y=\varphi(x,C)$ dan C o'zgarmas miqdorning aniq bir qiymatida kelib chiqadi;

2) $y(x_0)=y_0$ boshlang'ich shart berilganda $y_0=\varphi(x_0,C)$ tenglama C o'zgarmasga nisbatan yagona yechimga ega.

Umumiy yechimni oshkormas holda ifodalovchi $\Phi\left(x,y,C\right)=0$ munosabat mos differensial tenglamaning *umumiy integrali* deyiladi.

O'zgarmas C ga biror $C=C_0$ qiymat berish natijasida hosil bo'ladigan $y=\varphi(x,C_0)$ funksiya mos differensial tenglamaning *xususiy yechimi*, mos ravishda, $\Phi(x,y,C_0)=0$ munosabat tenglamaning *xususiy integrali* deyiladi.

Izoh. Differensial tenglama C o'zgarmas miqdorning hech bir qiymatida $y=\varphi(x,C)$ munosabatdan kelib chiqmaydigan $y=\varphi_k(x)$ yechim(lar)ga ham ega bolishi mumkin. Bunday holda differensial tenglamaning umumiy yechimi $y=\varphi(x,C),\ y=\varphi_k(x)$ ko'rinishda yoziladi (quyiroqda **17**, **18**, **20** va h.k. misollarga qarang).

1. $y=C\left(x^2+1\right)^{-1/2}$ funksiya $\left(x^2+1\right)y'+xy=0$ tenglamaning yechimi ekanligini ko'rsating.

◄ $y=C\left(x^2+1\right)^{-1/2}$ va $y'=-C\,x\left(x^2+1\right)^{-3/2}$ ifodalarni tenglamaga qo'yib,

$$-C\,x\left(x^2+1\right)\cdot\left(x^2+1\right)^{-\frac{3}{2}}+C\,x\left(x^2+1\right)^{-\frac{1}{2}}=0$$

ayniyatni hosil qilamiz. Demak, berilgan tenglama ko'rsatilgan tenglamaning yechimi bo'ladi.►

2. $4y\ln x + y^4 = C$ munosabat bilan aniqlanadigan $y=\varphi(x,C)$ funksiya $\left(y^3+\ln x\right)\frac{dy}{dx} = -\frac{y}{x}$ differensial tenglamaning yechimi ekanligini ko'rsating.

◀Berilgan munosabatning chap tomonini $F\left(x,y,C\right)=4y\ln x+$ $+y^4 - C$ funksiya bilan belgilab, oshkormas funksiyani differensiallash formulasi yordamida topilgan

$$\frac{dy}{dx} = -\frac{F'_x}{F'_y} = -\frac{y}{x\left(\ln x + y^3\right)}$$

ifodani tenglamaga qo'yib, ayniyatga kelamiz. Demak, berilgan oshkormas funksiya tenglamaning yechimi bo'ladi, $4y\ln x + y^4 = C$ funksiyalar esa tenglamaning umumiy integrali bo'ladi.▶

3. $x = 2t + 3t^2$, $y = t^2 + 2t^3$ parametrik ko'rinishdagi funksiya berilgan. Shu funksiya $y'^3 + xy' = 2y$ tenglamaning yechimi ekanligini ko'rsating.

◀**P**arametrik ko'rinishda berilgan $x = x(t)$, $y = y(t)$ funksiyaning hosilasi

$$y'_x = \frac{dy}{dx} = \frac{y'_t}{x'_t}$$

formula bilan hisoblanishini e'tiborga olib, $y'_x = t$ ni topamiz. Uni tenglamaga qo'ysak, ayniyat hosil bo'ladi.▶

4. Ushbu

$$y = x\cdot(C+\sin x), \quad x\neq 0 \tag{4}$$

funksiya C o'zgarmasning ixtiyoriy qiymatida

$$y = x\cdot(y' - x\cos x), \; x\neq 0 \tag{5}$$

tenglamaning yechimi ekanligi ma'lum. (5) differensial tenglama (4) yechimdan boshqa yechimga ega emasligini isbotlang.

◀Faraz qilaylik, $y=\varphi(x)$ – (5) tenglamaning (4) yechimlarining hech biri bilan ustma-ust tushmaydigan yechimi bo'lsin. U holda $\varphi(x)$ funksiya ushbu

$$\varphi(x) - x\cdot\left[\frac{d\varphi(x)}{dx} - x\cos x\right] = 0, \; x\neq 0 \tag{6}$$

ayniy tenglikni qanoatlantiradi. Endi

$$F(x)=\frac{-x\sin x+\varphi(x)}{x},\ x\neq 0$$

funksiyani qaraymiz. Bu funksiyaning hosilasi (6) tenglikka ko'ra nolga teng:

$$F'(x)=-\frac{\varphi(x)-x(\varphi'(x)-x\cos x)}{x^2}=0.$$

Bundan $F(x)=C_0$ ekanligi kelib chiqadi, bu yerda C_0- biror haqiqiy son, ya'ni

$$\frac{-x\sin x+\varphi(x)}{x}=C_0,$$

demak, $\varphi(x)=x(C_0+\sin x)$.

$y=\varphi(x)$ yechimni (4) ifodadan o'zgarmasning $C=C_0$ qiymatida olish mumkin, demak, (4) funksiya (5) differensial tenglamaning umumiy yechimi ekan.▶

1.1.2. Differensial tenglamalar tuzish. $C_i\ (i=\overline{1,n})-$biror C to'plamga tegishli bo'lgan ixtiyoriy o'zgarmas sonlar bo'lsin. Ushbu

$$\Phi(x,y,C_1,...,C_n)=0 \tag{1}$$

berilgan chiziqlar oilasi qanoatlantiradigan differensial tenglamani tuzish uchun

1) y ni x argumentning n marta uzluksiz differensiallanuvchi funksiyasi deb olib, (1) tenglikni n marta differensiallaymiz, ya'ni

$$\begin{cases}\dfrac{\partial\Phi}{\partial x}+\dfrac{\partial\Phi}{\partial y}y'=0,\\[2mm] \dfrac{\partial^2\Phi}{\partial x^2}+2\dfrac{\partial^2\Phi}{\partial x\partial y}y'+\dfrac{\partial^2\Phi}{\partial y^2}y'^2+\dfrac{\partial\Phi}{\partial y}y''=0,\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\ \dfrac{\partial^n\Phi}{\partial x^n}+...+\dfrac{\partial\Phi}{\partial y}y^{(n)}=0.\end{cases} \tag{2}$$

2) (1) va (2) munosabatlardan $C_1, C_2, ..., C_n$ o'zgarmaslarni yo'qotamiz. Natijada $x, y, y', ..., y^{(n)}$ larni bog'lovchi tenglikka ega bo'lamiz. Bu biz izlayotgan differensial tenglama bo'ladi.

Berilgan chiziqlar oilasining differensial tenglamasini tuzing (**5-9**).

5. $y^2+C\ x^2-2x=0.$

◀ $y = y(x)$ – izlanayotgan differensial tenglamaning uzluksiz differensiallanuvchi yechimi bo'lsin, C esa x ga bog'liq bo'lmagan parametr bo'lsin. U holda $x \in I \subset R$ (bu yerda I – biror to'plam)da

$$F(x, C) = y^2(x) + Cx^2 - 2x \equiv 0 \qquad (3)$$

ayniyat bajarilishi lozim. F funksiya x bo'yicha differensiallanuvchan bo'lganligi uchun

$$\frac{\partial F(x, C)}{\partial x} \equiv 2y(x)y'(x) + 2Cx - 2 \equiv 0.$$

Bundan

$$C = \frac{1 - yy'}{x} \ (x \neq 0). \qquad (4)$$

Endi (4) ni (3) ga olib borib qo'yib, $y^2 - x(1 + yy') = 0$ differensial tenglamani hosil qilamiz.▶

6. $C\,y - \cos C\,x = 0.$

◀Yuqoridagi kabi amallarni bajarib,

$$Cy'(x) + C\sin Cx \equiv 0$$

ayniyatni hosil qilamiz. $C \neq 0$ ekanligini e'tiborga olib,

$$y'^2 = \sin^2 Cx, \ C^2 y^2 = \cos^2 Cx \qquad (5)$$

tenglamalar sistemasidan topamiz:

$$C^2 = \frac{1 - y'^2}{y^2}, \ y \neq 0. \qquad (6)$$

Endi (6) ni (5) ga qo'yib,

$$1 - y'^2 = \cos^2\left(\sqrt{\frac{1 - y'^2}{y^2}}\,x\right), \text{ ya'ni } y' = -\sin\left(\frac{x\sqrt{1 - y'^2}}{|y|}\right)$$

differensial tenglamani hosil qilamiz.▶

7. $(x - C_1)^2 + C_2 y = 1.$

◀ $(x - C_1)^2 + C_2 y - 1 \equiv 0$ ayniyatni x bo'yicha ikki marta differensiallaymiz:

$$2(x - C_1) + C_2 y' \equiv 0, \quad 2 + C_2 y'' \equiv 0.$$

Uchala ayniyatlardan C_1 va C_2 o'zgarmaslarni yo'qotib, quyidagiga ega bo'lamiz:

$$y'^2 - 2yy'' = y''^2 .▶$$

8. $y = C\ln x.$

◀Tenglikni x bo’yicha differensiallaymiz: $y'(x) = C / x$, bundan $C = xy'$. Shunday qilib, berilgan chiziqlar oilasining differensial tenglamasi

$$y = xy' \ln x$$

ko’rinishda bo’ladi.▶

9. $y = C_1x^3 + C_2x^2 + C_3x.$

◀Berilgan tenglikdan x bo’yicha uch marta hosila olamiz:

$$y' = 3C_1x^2 + 2C_2x + C_3, \quad y'' = 6C_1x + 2C_2, \quad y''' = 6C_1.$$

Bu yerdan C_1, C_2 va C_3 o’zgarmaslarni topish qiyin emas:

$$C_1 = \frac{1}{6}y''', \quad C_2 = \frac{1}{2}\left(y'' - xy'''\right), \quad C_3 = y' - xy'' + \frac{1}{2}x^2y'''.$$

Topilgan C_1, C_2 va C_3 o’zgarmaslarning ifodalarini berilgan tenglikka qo’yamiz:

$$x^3y''' - 3x^2y'' + 6xy' - 6y = 0.$$ ▶

10. Ordinatalar o’qiga urinadigan barcha aylanalarning differensial tenglamasini tuzing.

◀Ma’lumki, markazi $M(C_1, C_2)$ nuqtada bo’lib, ordinatalar o’qiga urinadigan aylanalar

$$\left(x - C_1\right)^2 + \left(y - C_2\right)^2 = C_1^2 \qquad (7)$$

ko’rinishda bo’ladi. Bunday aylanalar uch marta uzluksiz differensiallanuvchi $y = y(x)$ funksiyalar bilan yoziladi, deb hisoblab, (7) dan hosil qilamiz:

$$x - C_1 + (y(x) - C_2)y'(x) \equiv 0,$$

$$1 + y'^2(x) + (y(x) - C_2)y''(x) \equiv 0.$$

Oxirgi tenglikdan $y(x) - C_2$ ifodani topib, uning yordamida $x - C_1$ ayirmani va C_1 o’zgarmasni topamiz:

$$x - C_1 = \frac{y'}{y''}\left(1 + y'^2\right), \qquad C_1 = x - \frac{1 + y'^2}{y''} \cdot y'.$$

Topilganlarni (7) tenglikka qo’yib, izlanayotgan differensial tenglamani hosil qilamiz:

$$x^2y''^2 + 2x\left(1 + y'^2\right)y'y'' = \left(1 + y'^2\right)^2.$$ ▶

11. O’qi Oy o’qqa parallel bo’lib, koordinatalar boshidan o’tadigan barcha parabolalarning differensial tenglamasini tuzing.

◀Ma'lumki, masala shartini qanoatlantiradigan parabolalar

$$y = C_1 x^2 + C_2 x \qquad (8)$$

ko'rinishda bo'ladi. Bunday parabolalar ikki marta uzluksiz differensiallanuvchi $y = y(x)$ funksiyalar bilan yoziladi deb hisoblab, (8) dan $y' = 2C_1 x + C_2$ va $y'' = 2C_1$ tengliklarni hosil qilamiz. Bu yerdan C_1 va C_2 o'zgarmaslarni topib, bir necha amallarni bajargandan so'ng ikkinchi tartibli $x^2 y'' - 2xy' + 2y = 0$ differensial tenglamaga ega bo'lamiz.▶

12. $x = C(t - \sin t),\ y = C(1 - \cos t)$ sikloidalar oilasining differensial tenglamasini tuzing.

◀ x va y dan t bo'yicha hosila olib quyidagiga ega bo'lamiz:

$$y' = \frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{\sin t}{1 - \cos t} = \frac{1}{tg\left(t/2\right)},$$

bundan $t = 2arctg\left(\frac{1}{y'}\right) + 2k\pi,\ k \in Z$ kelib chiqadi. Endi t ning qiymatini

$$(t - \sin t)y - x(1 - \cos t) = 0$$

tenglikka qo'yib, bir necha almashtirishlardan keyin

$$y' = ctg\,\frac{x + yy'}{y\left(1 + y'^2\right)}$$

differensial tenglamani hosil qilamiz.▶

13. Qutb koordinatalari sistemasida berilgan $\rho^2 + \varphi^2 = C$ chiziqlar oilasining differensial tenglamasini tuzing.

◀ Qutb va Dekart koordinatalari orasidagi

$$\rho = \sqrt{x^2 + y^2}, \qquad \varphi = arctg\,\frac{y}{x}$$

bog'lanish formulalarini e'tiborga olib, berilgan chiziqlar oilasini yozamiz:

$$x^2 + y^2 + arctg^2 z = C, \qquad (9)$$

bu yerda $y = y(x)$, shuningdek qulaylik maqsadida $y/x = z$ deb oldik.

(9) funksiyadan x bo'yicha hosila olamiz:

$$2x + 2yy' + 2arctgz \cdot \frac{xy' - y}{x^2 + y^2} = 0.$$

Bir necha almashtirishlardan keyin

$$y' = \frac{y\,arctgz - x(x^2+y^2)}{x\,arctgz + y(x^2+y^2)},\ z = \frac{y}{x}$$

differensial tenglamani hosil qilamiz.►

14. $ax + y = b,\ y^2 + z^2 = b^2$ chiziqlar oilasi qanoatlantiradigan differensial tenglamalar sistemasini tuzing, bu yerda a va b – ixtiyoriy o'zgarmas sonlar.

◄Egri chiziqning parametrik tenglamasini yozib olamiz:

$$x = x,\ y = y(x),\ z = z(x),$$

bu yerda y va z uzluksiz differensiallanuvchi funksiyalar, va ularni berilgan tenglamalarga qo'yib x ga nisbatan ayniyatlarni olamiz:

$$ax + y(x) \equiv b,\ y^2(x) + z^2(x) \equiv b^2. \qquad (10)$$

Bu tengliklarni x bo'yicha differensiallab, topamiz:

$$a + y'(x) \equiv 0,\ y(x)y'(x) + z(x)z'(x) \equiv 0,$$

bundan $a = -y'(x)$. a parametrning qiymatini (10) ayniyatlarning birinchisiga qo'ysak, $b = y - xy'$ bo'ladi. Bu munosabatni (10) dagi ikkinchi ayniyat bilan birga qarab,

$$z^2 + 2xyy' - x^2y'^2 = 0$$

tenglamaga ega bo'lamiz. Shunday qilib, izlanayotgan differensial tenglamalar sistemasi

$$yy' + zz' = 0,\ z^2 + 2xyy' - x^2y'^2 = 0$$

ko'rinishga ega ekan.►

INDIVIDUAL TOPSHIRIQLAR

M1. Quyida berilgan har bir funksiya C o'zgarmasning cheksiz ko'p qiymatida tegishli differensial tenglamaning yechimi bo'lishini tekshiring.

1. $y = Cx + \frac{C}{\sqrt{1+C^2}};\ y - xy' = \frac{y'}{\sqrt{1+y'^2}}.$
2. $y = arctg(x+y) + C;\ (x+y)^2 y' = 1.$
3. $y = x + C\sqrt{1+x^2};\ xy + 1 - \left(x^2+1\right)y' = 0.$
4. $y = Cx^m,\ x > 0;\ xy' = my.$
5. $y + \sqrt{x^2+y^2} = Cx^2;\ xy' = y + \sqrt{x^2+y^2}.$

6. $y=-2\cos^2 x+C\cos x;\ y'+y\,tgx=\sin 2x.$

7. $xy-\ln xy^3=C;\ x\left(xy-3\right)y'+xy^2-y=0.$

8. $y=\left(x+C\right)e^x;\ y'-y=e^x.$

9. $y=x\arcsin Cx;\ \ xy'-y=xtg\left(y/x\right).$

10. $y=Ce^{-x}+e^x/2;\ \ y'+y=e^x.$

11. $y=C\pm e^x;\ y'^3-y'e^{2x}=0.$

12. $y^2=2x\ln Cy;\ 2x^2y'=y^2\left(2xy'-y\right).$

13. $y^2+2x^2\ln Cy=0;\ y^2\left(y-xy'\right)=x^3y'.$

14. $4x+y-3=2tg\left(2x+C\right);\ \ y'=\left(4x+y-3\right)^2.$

15. $x=2t-\ln t,\ y=t^2-t+C;\ 2y'=x+\ln y'.$

16. $6x^3y^4+2x^3y^3+3x^2y^4=C;$

$\left(3xy+x+y\right)ydx+\left(4xy+x+2y\right)xdy=0.$

17. $x=3t^2+t^{-1},\ y=2t^3-\ln t+C;\ 3y'^3-xy'+1=0.$

18. $x=Cy^2-y^2(y+1)e^{-y};\ \left(2xe^y+y^4\right)y'=ye^y.$

19. $y=xtg\ln Cx;\ \ x^2\left(dy-dx\right)=\left(x+y\right)ydx.$

20. $e^y=x^2\ln Cx;\ xy'=x^2e^{-y}+2.$

21. $y^2=Cx^2+C^2;\ x^2\left(y-xy'\right)=yy'^2.$

22. $x=4t^3-\ln Ct,\ y=3t^4-t;\ 3y'^4=y'+y.$

23. $y\ln Cx=-x;\ \ x^2y'=y\left(x+y\right).$

24. $y=\dfrac{C}{x}-\dfrac{x}{2};\ y'+\dfrac{x+y}{x}=0.$

25. $x=t^3+t,\ 4y=3t^4+2t^2+C;\ \ x=y'^3+y'.$

26. $y=e^x\left(\ln x+C\right);\ x\left(y'-y\right)=e^x.$

27. $e^{-y}=Cx^2+x;\ x\left(e^y-y'\right)=2.$

28. $x=-y^2\ln Cx;\ 2x^2y'=y^3+xy.$

29. $y=Ce^x-x-1;\ \ y'=x+y.$

30. $y=\left(Ce^{-x/2}+x-2\right)^2;\ y'+y=x\sqrt{y}.$

M2. Quyidagi egri chiziqlar oilalarining differensial tenglamalarini tuzing.

1. $y = Cx.$ 2. $y = Cx + C^2.$ 3. $y = ax + b.$

4. $y^2 = 2Cx.$ 5. $(x - C)^2 + y^2 = 1.$ 6. $y = \sin Cx.$

7. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$ 8. $y = x\left(C + e^{x^2}\right).$ 9. $y = e^{Cx}.$

10. $y = (x - C)^3.$ 11. $y = Cx^3.$ 12. $y = \sin(x + C).$

13. $x^2 + Cy^2 = 2y.$ 14. $y^2 + Cx = x^3.$ 15. $y = C(x - C)^2.$

16. $Cy = \sin Cx.$ 17. $y = ax^2 + be^x.$ 18. $(x - a)^2 + by^2 = 1.$

19. $y = a\sin x + bx.$ 20. $x = ay^2 + by + c.$ 21. $xy\left(C - \ln^2 x\right) = 2.$

22. $y = Cx + C\ln C.$ 23. $\left(C + x^2\right)y - x = 0.$

24. $4y\cos x = 2x + \sin 2x + C.$ 25. $4y = x^4\left(C + \ln x\right)^2.$

26. $x^2 + y^2 = Cx.$ 27. $y = \sin x + C\cos x.$

28. $x + y + C(1 - xy) = 0.$

29. $x - y = Ce^{x/(y-x)}.$ 30. $6y = x^4 + Cx^{-2}.$

1.2. O'ZGARUVCHILARI AJRALADIGAN DIFFERENSIAL TENGLAMALAR

Ushbu

$$f_1(x)f_2(y)dx + g_1(x)g_2(y)dy = 0, \qquad (1)$$

tenglama *o'zgaruvchilari ajraladigan differensial tenglama* deyiladi, bu yerda f_i, g_i $(i = 1,2)$ – berilgan uzluksiz funksiyalar, $x \in (a,b)$, $y \in (c,d)$.

(1) tenglamani yechish uchun, avvalo uning ikkala tomonini $f_2(y)g_1(x) \neq 0$ ko'paytmaga bo'lib, hosil bo'lgan tenglik integrallanadi:

$$\int \frac{f_1(x)}{g_1(x)}dx + \int \frac{g_2(y)}{f_2(y)}dy = C \qquad (2)$$

Bu yerdagi integrallarni hisoblab, (1) tenglamaning umumiy integrali topiladi. Bundan tashqari, agar $g_1(x)$ va $f_2(y)$ funksiyalar, mos ravishda,

x_0 va y_0 nollarga ega bo'lsa, u holda $x = x_0$ va $y = y_0$ funksiyalar ham (1) tenglamani qanoatlantiradi. Demak, (1) tenglamaning barcha yechimlarini hosil qilish uchun $f_2(y)$ va $g_1(x)$ funksiyalarning nollarini (2) integral egri chiziqlar oilasiga qo'shib qo'yish kerak.

a, b, c – berilgan sonlar bo'lsin. Ushbu

$$y' = f(ax + by + c) \tag{3}$$

ko'rinishdagi tenglamalar $ax + by + c = z$ almashtirish yordamida o'zgaruvchilari ajraladigan tenglamaga keltiriladi.

15. Ushbu $2xdx - 3y^2dy = 0$ tenglamaning umumiy yechimini va $y(0) = 1$ shartni qanoatlantiruvchi yechimini toping.

◀Bu tenglamaning ikkala tomonini bevosita integrallab, $y = \sqrt[3]{x^2 + C}$ umumiy yechimni olamiz. Shu tenglamaning $y(0) = 1$ boshlang'ich shartni qanoatlantiruvchi yechimi $y = \sqrt[3]{x^2 + 1}$ ko'rinishda bo'ladi.▶

16. $x(y^2 + 1)dx - y(x^2 + 1)dy = 0$ tenglamani yeching.

◀Bu tenglama o'zgaruvchilari ajraladigan differensial tenglamadir. Uning ikkala tomonini $(x^2 + 1)(y^2 + 1) \neq 0$ ifodaga bo'lib, o'zgaruvchilarini ajratamiz va integrallaymiz. Natijada $y^2 + 1 = C(x^2 + 1)$ ko'rinishdagi umumiy integralni olamiz.▶

17. $xydx + (x+1)dy = 0$ tenglamani yeching.

◀Bu tenglama ham o'zgaruvchilari ajraladigan differensial tenglamadir. Uning ikkala tomonini $(x+1)y \neq 0$ ifodaga bo'lib, o'zgaruvchilarini ajratamiz va integrallaymiz. Natijada $y = C(x+1)e^{-x}$ ko'rinishdagi umumiy yechimni olamiz.

Endi $x + 1 = 0$ bo'lsin deb faraz qilamiz. $x = -1$ funksiyani berilgan tenglamaga qo'yib, uning yechim ekanligiga ishonch hosil qilamiz. $x = -1$ yechim $y = C(x+1)e^{-x}$ umumiy yechimdan C o'zgarmasning hech bir qiymatida kelib chiqmaydi. Bevosita tekshirish ko'rsatadiki, $y = 0$ funksiya ham tenglamaning yechimi bo'ladi, ammo $y = 0$ yechim umumiy yechimdan $C = 0$ da kelib chiqadi. Shunday qilib, mazkur tenglamaning barcha yechimlari $y = C(x+1)e^{-x}$, $x = -1$ ko'rinishda bo'ladi.▶

18. Ushbu $y' = 3\sqrt[3]{y^2}$ tenglamaning umumiy yechimini va $y(2) = 0$ shartni qanoatlantiruvchi yechimini toping.

◀Tenglamani $\frac{1}{3}y^{-\frac{2}{3}}dy = dx,\ y \neq 0$ ko'rinishda yozib olamiz va integrallaymiz. Natijada $y = (x - C)^3$ umumiy yechimni olamiz. $y = 0$ funksiya ham yechim bo'lishini ko'rish qiyin emas.

Endi $y(2) = 0$ shartni qanoatlantiruvchi yechimini topamiz: $y = (x-2)^3$. $y = 0$ yechim ham shu shartni qanoatlantiradi. Bundan ko'rinadiki, ikkita $y = (x-2)^3$ va $y = 0$ yechimlar masala shartini qanoatlantirar ekan.▶

19. $y' = \sqrt{4x + 2y - 1}$ tenglamani yeching.

◀Bu tenglamani $4x + 2y - 1 = z$ almashtirish yordamida o'zgaruvchilari ajraladigan tenglamaga keltirish qiyin emas: $z' = 2(\sqrt{z} + 2)$. Uning umumiy integrali $\sqrt{z} - 2\ln(\sqrt{z} + 2) = x + C$ ko'rinishda topiladi. Ammo $z = 4x + 2y - 1$ bo'lganligi uchun dastlabki tenglamaning umumiy integrali

$$\sqrt{4x + 2y - 1} - 2\ln(\sqrt{4x + 2y - 1} + 2) = x + C$$

ko'rinishda yoziladi.▶

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M3. Differensial tenglamaning umumiy yechimini toping.

1. $e^{x+3y}dy = xdx.$
2. $y'\sin 2x = 2y\ln y.$
3. $y' = (2x-1)ctg\, y.$
4. $\sec^2 x\, tg\, y\, dx + \sec^2 y\, tg\, x\, dy = 0 \quad (\sec x = 1/\cos x).$
5. $\left(1 + e^y\right)xdx - e^x dy = 0.$
6. $xe^y dx - \left(x^2 + 3\right)ydy = 0.$
7. $\sin x \cos y dx = \cos x \sin y dy.$
8. $y' = \left(2y + 1\right)tgx.$
9. $\left[\sin\left(x + y\right) + \sin\left(x - y\right)\right]dx + \dfrac{dy}{\cos y} = 0.$
10. $3e^x \sin y dx + \left(1 - e^x\right)\cos y dy = 0.$
11. $\sin x\, tg\, y\, dx - \dfrac{dy}{\sin x} = 0.$
12. $\left(1 + e^x\right)yy' = e^x.$
13. $y' = e^{2x} / \ln y.$
14. $3^{x^2+y}dy + xdx = 0.$

15. $\left[\cos\left(x-2y\right)+\cos\left(x+2y\right)\right]y'=\sec x.$

16. $y'=xe^{x^2}\left(1+y^2\right).$

17. $ctg\, x\cos^2 y dx+\sin^2 x tg\, y dy=0.$

18. $\sin x\cdot y'=y\cos x+2\cos x.$

19. $1+\left(1+y'\right)e^y=0.$

20. $y'ctgx+y=2.$

21. $\dfrac{dx}{\cos^2 y}+\dfrac{e^{-x^2}}{x}dy=0.$

22. $e^x\sin y\,dx+tg\,y\,dy=0.$

23. $\cos^3 y\cdot y'-\cos\left(2x+y\right)=\cos\left(2x-y\right).$

24. $\left[\sin\left(2x+y\right)-\sin\left(2x-y\right)\right]dx=\dfrac{dy}{\sin y}.$

25. $\cos ydx=2\sqrt{1+x^2}dy+\cos y\sqrt{1+x^2}dy.$

26. $y'\sqrt{1-x^2}-\cos^2 y=0.$

27. $e^x tg\,y\,dx=\left(1-e^x\right)\sec^2 ydy.$

28. $y'+\sin\left(x+y\right)=\sin\left(x-y\right).$

29. $yy'=x3^{y^2-x^2}.$

30. $\left(1+e^{3y}\right)xdx=e^{3y}dy.$

M4. Differensial tenglamaning umumiy yechimini toping.

1. $\left(xy+x^3y\right)y'=1+y^2.$

2. $5^{x-y}y'=3.$

3. $y-xy'=2\left(1+x^2y'\right).$

4. $y-xy'=1+x^2y'.$

5. $\left(x+4\right)dy-xydx=0.$

6. $y'+y+y^2=0.$

7. $y^2\ln xdx-\left(y-1\right)xdy=0.$

8. $\left(x+xy^2\right)dy+ydx-y^2dx=0.$

9. $y'+2y-y^2=0.$

10. $x\left(x+1\right)ydx+\left(y^2+1\right)dy=0.$

11. $x\left(y^3+1\right)dx+\left(x^2-1\right)y^2dy=0.$

12. $\left(y^2+1\right)dx-\left(x^2+1\right)ydy=0.$

13. $y'=2xy+x.$

14. $y-xy'=3\left(1+x^2y'\right).$

15. $2xyy'=1-x^2.$

16. $\left(x^2-1\right)y'-xy=0.$

17. $(x+1)y^2dy+xdx=0.$

18. $\left(1+x^3\right)y^3dx-\left(y^2-1\right)x^3dy=0.$

19. $xy'-y=y^2.$ 20. $\sqrt{y^2+1}\,dx=xydy.$

21. $y'-xy^2=2xy.$ 22. $2x^2yy'+y^2=2.$

23. $y'=\left(1+y^2\right)/\left(1+x^2\right).$ 24. $yy'\sqrt{1+y^2}=x^2.$

25. $(y+1)y'=\dfrac{y}{\sqrt{1-x^2}}+xy.$

26. $\left(1+x^2\right)y'+y\sqrt{1+x^2}=xy.$

27. $xyy'=\left(1+x^2\right)/\left(1-y^2\right).$

28. $(1-x)ydx+x(y-x)dy=0.$

29. $\left(x^2-1\right)y^2y'=y+1.$ 30. $\sqrt{1-y^2}\,dx+y\sqrt{1-x^2}\,dy=0.$

M5. Differensial tenglamaning umumiy yechimini toping.

1. $3x^2\left(y^2+1\right)dx+\left(2-x^3\right)ydy=0.$ 2. $xe^yy'=e^{2y}+1.$

3. $y\left(4+x^2\right)dy+\sqrt{1-y^2}dx=0.$ 4. $y'=x4^{x+y}.$

5. $x^2\left(1-y^2\right)dx+\left(1-x^2\right)y^2dy=0.$ 6. $\sqrt{1-x^2}dy-ydx=0.$

7. $x\left(1-y^2\right)dx+\sqrt{1-x^2}dy=0.$ 8. $\sqrt{4-x^2}yy'+\sqrt{4+y^2}=0.$

9. $\cos ye^xdx+\left(1+e^{2x}\right)\sin ydy=0.$ 10. $xy'-4=y^2.$

11. $y'-x\cos^2y\sin^2x=0.$ 12. $yy'\left(1+x^2\right)=1+y^2.$

13. $\left(1+y^4\right)dx-\sqrt{x}ydy=0.$ 14. $(x-1)yy'=1+y^2.$

15. $y(1+2x)y'=(1-2x).$ 16. $x^2yy'+3=y^2.$

17. $y\left(1+x^2\right)y'+x\left(1+y^2\right)=0.$ 18. $x^2y^2y'=y^3-1.$

19. $ydy-x\sin x\sqrt{9-y^2}dx=0.$ 20. $\left(1+x^3\right)y'=x^2(1+y).$

21. $\sqrt{1+x^2}dy=tgydx.$ 22. $x-xy^2=y'\left(4+x^2\right).$

23. $\left(1+x^2\right)dy=\sqrt{1-y^{-2}}dx.$ 24. $\left(1+x\right)yy'=e^{-y^2}.$

25. $tgxdy+\sqrt{1-y}dx=0.$ 26. $y'=2yctgx.$

27. $xx'=y\cos y^2\cdot\sqrt{1+x^2}.$ 28. $e^{1/y}y'=x^2y^2\ln x.$

29. $e^{-\sin y}dx=x\cos y\ln xdy.$ 30. $y^2dx+x\ln xdy=0.$

1.3. BIR JINSLI VA UNGA KELTIRILADIGAN DIFFERENSIAL TENGLAMALAR

Ushbu

$$y'=\varphi\left(\frac{y}{x}\right) \qquad (1)$$

ko'rinishdagi differensial tenglama *bir jinsli differensial tenglama* deyiladi, bu yerda $\varphi-$ berilgan funksiya.

(1) tenglamani yechish uchun, odatda, $y=t\,x$ almashtirish bajariladi. $y=t\,x$ va $y'=t'\,x+t$ ifodalarni (1) tenglamaga qo'yib, $t'=(\varphi(t)-t)/x$ ko'rinishdagi o'zgaruvchilari ajraladigan tenglamani hosil qilamiz. Agar biror $t=a$ sonda $\varphi(a)-a=0$ bo'lsa, u holda (1) tenglama umumiy yechimdan kelib chiqmaydigan $y=ax$ yechimga ham ega bo'ladi.

$f(x,y)$ funksiya berilgan bo'lsin. Agar shunday m haqiqiy son topilsaki, ixtiyoriy $k>0$ haqiqiy son uchun

$$f(kx,\ ky)=k^m f(x,\ y)$$

tenglik bajarilsa, u holda $f(x,y)$ funksiya m-*tartibli bir jinsli funksiya* deyiladi.

Agar $M(x,y)$ va $N(x,y)$ funksiyalar bir xil tartibli bir jinsli funksiyalar bo'lsa, u holda

$$M(x,y)dx+N(x,y)dy=0 \qquad (2)$$

tenglama bir jinsli tenglama bo'ladi. Bu tenglama ham $y=t\,x$ almashtirish yordamida o'zgaruvchilari ajraladigan tenglamaga keltirib yechiladi.

Ushbu

$$y' = f\left(\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}\right), \qquad (3)$$

bu yerda $a_i, b_i, c_i = const$, $i = 1,2$, tenglama bir jinsli tenglamaga keltiriladigan tenglamalar sirasiga kiradi. Bunda $\Delta = a_1b_2 - a_2b_1$ sonli ifodaning qiymati muhim ahamiyatga ega. Bu yerda to'rtta hol bo'lishi mumkin.

1-hol. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = k$ $(\Delta = 0)$ bo'lsin. Bunda (3) tenglama

$$y' = f\left(\frac{k(a_2x+b_2y)+c_1}{a_2x+b_2y+c_2}\right) = \varphi(a_2x+b_2y)$$

ko'rinishda yozib olinadi va $z = a_2x + b_2y$ almashtirish yordamida o'zgaruvchilari ajraladigan tenglamaga keltiriladi.

2-hol. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ $(\Delta \neq 0)$ va $c_1 = c_2 = 0$ bo'lsin. Bu holda tenglama

$$y' = f\left(\frac{a_1 + b_1\frac{y}{x}}{a_2x + b_2\frac{y}{x}}\right) = \varphi\left(\frac{y}{x}\right)$$

ko'rinishdagi bir jinsli tenglamaga keltiriladi.

3-hol. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ $(\Delta \neq 0)$ va $c_1^2 + c_2^2 \neq 0$ bo'lsin. Bu holda $\xi = x - a$, $\eta = y - b$ almashtirish bilan tenglama 2-holda o'rganilgan

$$\frac{d\eta}{d\xi} = f\left(\frac{a_1\xi + b_1\eta}{a_2\xi + b_2\eta}\right)$$

tenglamaga keltiriladi. Buning uchun a va b sifatida

$$\begin{cases} a_1\alpha + b_1\beta + c_1 = 0 \\ a_2\alpha + b_2\beta + c_2 = 0 \end{cases}$$

tenglamalar sistemasining yechimini olish yetarli.

4-hol. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = k$ bo'lsin. U holda (3) tenglama

$$y' = f(k)$$

bo'lib, uning umumiy yechimi $y = f(k)x + C$ bo'ladi.

Agar m sonini tanlash hisobiga (2) tenglamani $y = z^m$ almashtirish yordamida bir jinsli tenglamaga keltirish mumkin bo'lsa, bunday (2) tenglama *umumlashgan bir jinsli tenglama* deyiladi.

Quyidagi differensial tenglamalarni yeching (**20-26**).

20. $x^2y' = y^2 + xy$.

◄ $x = 0$ funksiya bu tenglamaning yechimi emasligini e'tiborga olib, tenglamaning ikkala tomonini $x^2 \neq 0$ ifodaga bo'lamiz va

$$y' = (y/x)^2 + y/x$$

ko'rinishdagi tenglamaga ega bo'lamiz. Bu yerda $y/x = t$ almashtirish kiritamiz va $y = tx$, $y' = t'x + t$ ifodalarni oxirgi tenglamaga qo'yamiz: $t'x = t^2$. Hosil bo'lgan o'zgaruvchilari ajraladigan tenglamani integrallab,

$$-\frac{1}{t} = \ln Cx,\ t \neq 0$$

umumiy yechimni topamiz. So'ngra x va y o'zgaruvchilarga qaytib, dastlabki tenglamaning

$$x + y\ln Cx = 0 \qquad (4)$$

ko'rinishdagi umumiy integralini topish qiyin emas. Bundan tashqari, $t = 0$, ya'ni $y = 0$ funksiya dastlabki tenglamaning yechimi bo'lib, u C o'zgarmasning hech bir qiymatida (4) umumiy yechimdan kelib chiqmaydi. Demak, berilgan tenglamaning barcha yechimlari quyidagilar: $x+y\ln Cx=0$, $y=0$.▶

21. $xy'-y=(x+y)\ln\dfrac{x+y}{x}$.

◀Tenglamaning berilishiga ko'ra, $x\neq 0$ ekanligi ravshan. Tenglamaning ikkala tomonini x ga bo'lamiz va

$$y'=\frac{y}{x}+\left(1+\frac{y}{x}\right)\ln\left(1+\frac{y}{x}\right)$$

ko'rinishdagi tenglamaga ega bo'lamiz. $y/x=t$ almashtirish kiritib, $y=tx$ va $y'=t'x+t$ ifodalarni oxirgi tenglamaga qo'yamiz. Natijada

$$t'x=(1+t)\ln(1+t)$$

ko'rinishdagi o'zgaruvchilari ajraladigan tenglama hosil bo'ladi. Uni

$$\frac{dt}{(1+t)\ \ln(1+t)}=\frac{dx}{x},\quad t>-1,\quad t\neq 0$$

ko'rinishda yozib, so'ngra integrallab, topamiz: $\ln(1+t)=Cx.$ Endi x va y o'zgaruvchilarga qaytib, dastlabki tenglamaning

$$\ln\frac{x+y}{x}=Cx \tag{5}$$

ko'rinishdagi umumiy integralini topish qiyin emas. Bundan tashqari, $t=0$, ya'ni $y=0$ funksiya dastlabki tenglamaning yechimi bo'ladi va u C o'zgarmasning $C=0$ qiymatida (5) umumiy yechimdan kelib chiqadi. Demak, berilgan tenglamaning umumiy yechimi: $\ln\dfrac{x+y}{x}=Cx$.▶

22. $(y^2-2xy)dx+x^2dy=0$

◀Bu yerda $M(x,y)=y^2-2xy$ va $N(x,y)=x^2$ funksiyalar bir jinsli bo'lib, darajalari $m=2$ ga teng. Haqiqatan ham,

$$M(kx,\ ky)=(kx)^2-2kxky=k^2(y^2-2xy)=k^2M(x,y),$$

$$N(kx,ky)=(kx)^2=k^2x^2=k^2N(x,y).$$

Endi $xt(t-1)\neq 0$ deb faraz qilamiz. $y=tx$ va $dy=tdx+xdt$ ifodalarni berilgan dastlabki (6) tenglamaga qo'yamiz va bir necha kerakli almashtirishlarni bajarganimizdan so'ng

$$\frac{dx}{x}=-\frac{dt}{t(t-1)}$$

tenglamaga ega bo'lamiz. Uni integrallab $x(t-1)=Ct$ umumiy yechimga ega bo'lamiz. So'ngra x va y o'zgaruvchilarga qaytib,

$$x(y-x)=Cy \qquad (6)$$

ko'rinishdagi umumiy yechimni topamiz. $xt(t-1)=0$ bo'lgandagi, ya'ni $xy(y-x)=0$ bo'lgan holni alohida qarab chiqamiz. Ko'rinib turibdiki, $x=0, y=0$ va $y=x$ funksiyalar berilgan tenglamaning yechimlari bo'ladi. $x=0$ va $y=x$ yechimlarni (6) umumiy yechimdan o'zgarmas-ning $C=0$ qiymatida hosil qilish mumkin, $y=0$ yechimni esa (6) umumiy yechimdan C o'zgarmasning hech bir qiymatida hosil qilib bo'lmaydi. Shu fikrlarga tayanib, berilgan tenglamaning barcha yechimlarini yozamiz: $x(y-x)=Cy$; $y=0$.►

23. $y'=\dfrac{x+y-1}{x+y+1}$.

◄Bu tenglamada $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=1$ (1-hol), shuning uchun $z=x+y$, $z'=1+y'$ desak, berilgan tenglama $z'=\dfrac{2z}{z+1}$ ko'rinishga keladi. O'zgaruvchilari ajraladigan bu tenglamaning umumiy integrali: $y-x+\ln|x+y|=C$. Bundan tashqari umumiy yechimdan kelib chiqmaydigan $y=-x$ funksiya ham yechim bo'ladi.►

24. $y'=\dfrac{x+y}{x-y}$.

◀Bu yerda $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, $c_1 = c_2 = 0$ bo'lganligi uchun (2-hol) bu tenglama bir jinslidir. $x = 0$ funksiya berilgan tenglamaning yechimi emasligini e'tiborga olib, $y/x = t$ o'rniga qo'yishni bajarsak, tenglama

$$t'x = \frac{t^2 + 1}{1 - t}$$

ko'rinishni oladi va uni o'zgaruvchilarga ajratib, so'ngra integrallaymiz:

$$\frac{1-t}{t^2+1}dt = \frac{dx}{x}, \quad arctg\, t - \ln\left|x\sqrt{1+t^2}\right| = C.$$

Dastlabki x va y o'zgaruvchilarga qaytib,

$$arctg(y/x) - \ln\sqrt{x^2 + y^2} = C$$

umumiy integralni topamiz.▶

25. $y' = \frac{2x - y + 3}{x - 2y + 3}$.

◀Bu yerda $\Delta \neq 0$ (3-hol). Avvalo

$$2\alpha - \beta + 3 = 0,\ \alpha - 2\beta + 3 = 0$$

sistemaning yechimini topamiz: $\alpha = -1$, $\beta = 1$. So'ngra bu yechim yordamida $\xi = x + 1$, $\eta = y - 1$ almashtirish bajarib,

$$\frac{d\eta}{d\xi} = \frac{2\xi - \eta}{\xi - 2\eta}$$

ko'rinishdagi bir jinsli tenglamaga ega bo'lamiz. Bu tenglamani $\eta/\xi = t$ o'rniga qo'yish yordamida o'zgaruvchilari ajraladigan tenglamaga keltirib, integrallaymiz: $\xi^2 - \xi\eta + \eta^2 = C$, $C > 0$. x va y o'zgaruvchilarga qaytib, dastlabki tenglamaning umumiy integralini yozamiz: $x^2 - xy + y^2 + 3x - 3y = C_1$, $C_1 = C - 3$.▶

26. $2x^2 y' = y^3 + xy$.

◀Berilgan tenglama bir jinsli emas. Bu tenglamaning umumlashgan bir jinsli tenglama ekanligini tekshiramiz va integrallaymiz.

$y = z^m$ deylik. U holda $y' = mz^{m-1}z'$ va tenglama ushbu ko'rinishga keladi:

$$2mx^2 z^{m-1} z' = z^{3m} + xz^m$$

Berilgan tenglama bir jinsli bo'lishi uchun $m-1+2=3m=m+1$ qo'sh tenglik yechimga ega bo'lishi kerak. $m = 1/2$ ekanligi ko'rinib turibdi. Shunday qilib, berilgan tenglama umumlashgan bir jinsli tenglama ekan. Demak, $y = \sqrt{z}$ o'rniga qo'yish berilgan tenglamani quyidagi bir jinsli tenglamaga keltiradi:

$$x^2 z' = z^2 + xz\,.$$

Bu tenglamani integrallaymiz (**20**-misolga qarang): $x + z\ln Cx = 0$, $z = 0$.Bundan oldingi o'zgaruvchilarga qaytib, umumiy integralga ega bo'lamiz: $x = -y^2 \ln Cx$, $y = 0$.▶

27. Ushbu $2y + (x^2 y + 1)\,xy' = 0$ tenglamaning $M(1,1)$ nuqtadan o'tuvchi yechimini toping.

◀Bu misolda berilgan tenglamani qanoatlantirib, shu bilan bir vaqtda berilgan M nuqtadan o'tuvchi egri chiziqni topish talab etiladi.

Avvalo berilgan tenglamaning barcha yechimlarini topamiz. Bu

tenglama bir jinsli emas. Tenglamani umumlashgan bir jinslilikka tekshiramiz.

$y = z^m$ deylik. U holda $y' = mz^{m-1}z'$ bo'lib, tenglama ushbu ko'rinishga keladi:

$$2z^m + mx^3 z^m z^{m-1} z' + mxz^{m-1} z' = 0\,.$$

Berilgan tenglama bir jinsli bo'lishi uchun $m = 2 + 2m = m$ qo'sh tenglik yechimga ega bo'lishi kerak. $m = -2$ ekanligi ko'rinib turibdi. Shunday qilib, tenglama umumlashgan bir jinsli tenglama ekan. Demak, $y = z^{-2}$ almashtirish berilgan tenglamani quyidagi bir jinsli tenglamaga keltiradi:

$$z-(x^2z^{-2}+1)xz'=0. \qquad (7)$$

Bu yerda z ni x ning funksiyasi deb emas, balki x ni z ning funksiyasi, $x=x(z)$ deb qarash qulayroq bo'ladi. $z'=1/x'$ tenglikka binoan, (7) tenglama

$$x'=\left(\frac{x}{z}\right)^3+\frac{x}{z} \qquad (8)$$

ko'rinishni oladi. (8) tenglama bir jinsli tenglama bo'lganligi uchun $x/z=t$ o'rniga qo'yish yordamida bu tenglama o'zgaruvchilarga ajraladigan $t'z+t=(t^2+1)t$ tenglamaga o'tadi. Bu tenglamani integrallab, $t^{-2}=\ln C z^{-2}$ yechimlarni hosil qilamiz. So'ngra ketma-ket amallarni bajarib eski o'zgaruvchilarga qaytamiz va

$$x^2y\ln Cy=1 \qquad (9)$$

umumiy yechimga ega bo'lamiz. Bundan tashqari, umumiy yechimdan kelib chiqmaydigan $y=0$ yechim ham bor. Demak, berilgan tenglamaning hamma yechimlari $x^2y\ln Cy=1$, $y=0$ ko'rinishda topiladi.

Endi $M(1,1)$ nuqtadan o'tuvchi egri chiziqni topamiz. Buning uchun (9) umumiy yechimda $x=1$, $y=1$ desak, $C=e$ qiymat topiladi va $x^2y(1+\ln y)=1$ xususiy integral hosil bo'ladi.►

INDIVIDUAL TOPSHIRIQLAR

M6. Differensial tenglamaning umumiy yechimini toping.

1. $y-xy'=x\sec\left(y/x\right)$.
2. $\left(y^2-3x^2\right)dy+2xydx=0$.
3. $\left(x+2y\right)dx-xdy=0$.
4. $\left(x-y\right)dx+\left(x+y\right)dy=0$.
5. $\left(y^2-2xy\right)dx+x^2dy=0$.
6. $y^2+x^2y'=xyy'$.
7. $xy'-y=xtg\left(y/x\right)$.
8. $xy'=y-xe^{y/x}$.

9. $xy' - y = (x+y)\ln\left(\dfrac{x+y}{x}\right).$ 10. $xy' = y\cos\ln\left(\dfrac{y}{x}\right).$

11. $\left(y+\sqrt{xy}\right)dx = xdy.$ 12. $xy' = \sqrt{x^2-y^2}+y.$

13. $y = x\left(y' - \sqrt[x]{e^y}\right).$ 14. $y' = \dfrac{y}{x} - 1.$

15. $xy' + x + y = 0.$ 16. $ydx + \left(2\sqrt{xy}+x\right)dy = 0.$

17. $xdy - ydx = \sqrt{x^2+y^2}\,dx.$

18. $\left(4x^2+3xy+y^2\right)dx + \left(4y^2+3xy+x^2\right)dy = 0.$

19. $(x-y)ydx - x^2dy = 0.$ 20. $xy + y^2 = \left(2x^2+xy\right)y'.$

21. $\left(x^2-2xy\right)y' = xy - y^2.$ 22. $\left(2\sqrt{xy}+y\right)dx - xdy = 0.$

23. $xy' + y\left(\ln(y/x) - 1\right) = 0.$ 24. $y' = \dfrac{x}{y} + \dfrac{y}{x}.$

25. $\left(y^2-2xy\right)dx - x^2dy = 0.$ 26. $(x+2y)dx + xdy = 0.$

27. $(4x-y)dx + (x+y)dy = 0.$ 28. $2x^3y' = y\left(2x^2-y^2\right).$

29. $x^2y' = y(x+y).$ 30. $\left(x^2+y^2\right)dx + 2xydy = 0.$

M7. Differensial tenglamaning umumiy yechimini toping.

1. $2xy' = xe^{y/x} + 2y.$ 2. $xy^2y' = x^3 + 2y^3.$

3. $(x-y)y' = 3y.$ 4. $2x^2y' = y^2 + 2xy - 4x^2.$

5. $(x+y)dx + (y-2x)dy = 0.$ 6. $\left(x^2+y^2\right)dx + x^2dy = 0.$

7. $(y-2x)dx + (y+2x)dy = 0.$ 8. $2xydy = \left(x^2-y^2\right)dx.$

9. $\left(2x^2+3y^2\right)dx = \left(y^2-x^2\right)dy.$ 10. $xy' - y = xctg(y/x).$

11. $(2y-x)dx = (y+3x)dy.$ 12. $(x-4y)y' = x + 2y.$

13. $(2x+3y)dx = (y+2x)dy.$ 14. $2xyy' = x^2 + y^2.$

15. $xy' = x\sin(y/x) + y.$ 16. $xy' = xe^{2y/x} + y.$

17. $x(x+y)y' = (4x+y)y.$ 18. $xy' = x\sec(y/x) + y.$

19. $x^2y' = y^2 - 2x^2.$

20. $xy' = y + xe^{y/x}.$

21. $(x+2y)dy + (y+2x)dx = 0.$

22. $x^2y' + y^2 = xyy'.$

23. $(x-y)y' = 2y.$

24. $2x^2 + y^2 = 2x^2y'.$

25. $(2x^2 + y^2)y' = 2xy + y^2.$

26. $(x-y)y' = x + 2y.$

27. $xy' = y\left(\ln^2(y/x) + 1\right).$

28. $xy' - y = \sqrt{y^2 + x^2}.$

29. $xy'\sin\dfrac{y}{x} + x = y\sin\dfrac{y}{x}.$

30. $xy + y^2 = y'\left(xy + 3y^2\right).$

M8. Differensial tenglamani bir jinsli tenglamaga keltirib, so'ngra umumiy yechimini (umumiy integralini) toping.

1. $y' = \dfrac{2x+3y-1}{2x+3y+1}.$

2. $y' = \dfrac{x+4y}{x-4y}.$

3. $y' = \dfrac{x-2y+9}{2x-y+9}.$

4. $y' = \dfrac{3x+4y-5}{3x+4y+5}.$

5. $y' = \dfrac{x+y}{x-2y}.$

6. $y' = \dfrac{x-5y+3}{5x-y+3}.$

7. $y' = \dfrac{2x-y-1}{2x-y+2}.$

8. $y' = \dfrac{4x+y}{x-2y}.$

9. $y' = \dfrac{7x-y+3}{x-7y+3}.$

10. $y' = \dfrac{x+3y-1}{x+3y+1}.$

11. $y' = \dfrac{x+9y}{3x-y}.$

12. $y' = \dfrac{2x-y+6}{x-2y+6}.$

13. $y' = \dfrac{8x+3y-1}{8x+3y+1}.$

14. $y' = \dfrac{4x+9y}{x-y}.$

15. $y' = \dfrac{5x-y+3}{x-5y+3}.$

16. $y' = \dfrac{-2x+3y-5}{-2x+3y+1}.$

17. $y' = \dfrac{x+4y}{2x-y}.$

18. $y' = \dfrac{4x-y+5}{x-4y+3}.$

19. $y' = \dfrac{x+7y-4}{x+7y+4}.$

20. $y' = \dfrac{4x+y}{2x-y}.$

21. $y' = \dfrac{x-3y+3}{3x-y+3}.$

22. $y' = \dfrac{3x+5y-1}{3x+5y+1}.$

23. $y' = \dfrac{x+25y}{5x-y}.$

24. $y' = \dfrac{x-2y+3}{2x-y+3}.$

25. $y' = \dfrac{6x+y-1}{6x+y+1}.$

26. $y' = \dfrac{25x+y}{x-5y}.$

27. $y' = \dfrac{4x-y+3}{x-4y+3}.$

28. $y' = \dfrac{2x+7y-7}{2x+7y+5}.$

29. $y' = \dfrac{x-y}{x+y}.$

30. $y' = \dfrac{3x-y+4}{x-3y+4}.$

1.4. CHIZIQLI TENGLAMA

Ushbu

$$y' + a(x)y = b(x) \tag{1}$$

ko'rinishdagi tenglama *birinchi tartibli chiziqli differensial tenglama* deyiladi, bu yerda $a(x)$ va $b(x)$ berilgan oraliqda aniqlangan uzluksiz funksiyalar. Chiziqli differensial tenglamalarni yechishning eng keng tarqalgan usuli *o'zgarmasni variatsiyalash usuli*dir. Shu usulni bayon qilamiz.

1.4.1. O'zgarmasni variatsiyalash usuli. Agar (1) tenglamada $b(x) = 0$ bo'lsa, u holda

$$y' + a(x)y = 0 \tag{2}$$

ko'rinishdagi tenglama hosil bo'lib, u (1) *chiziqli tenglamaga mos bir jinsli tenglama* deyiladi.

Avvalo (2) tenglamani, ya'ni (1) chiziqli tenglamaga mos bir jinsli tenglamani yechamiz. (2) tenglama o'zgaruvchilari ajraladigan tenglama bo'lib, u

$$y = C\exp\left(-\int a(x)dx\right) \tag{3}$$

ko'rinishdagi umumiy yechimga ega. (1) tenglamaning umumiy yechimini topish uchun (3) dagi C o'zgarmasni variatsiyalaymiz, ya'ni (3) formulada C o'zgarmasning o'rniga $C(x)$ funksiyani qo'yamiz:

$$y = C(x)\ \exp\left(-\int a(x)dx\right) \tag{4}$$

Endi esa (4) funksiyani va undan olingan hosilani

$$y' = C'(x)\ \exp\left(-\int a(x)dx\right) - a(x)C(x)\exp\left(-\int a(x)dx\right)$$

(1) tenglamaga qo'yamiz. Natijada oson integrallanadigan

$$\frac{dC(x)}{dx} = b(x)\ \exp\left(\int a(x)dx\right) \tag{5}$$

differensial tenglamaga kelamiz. Undan $C(x)$ funksiyani topamiz:

$$C(x) = \int \left(b(x) \exp\left(\int a(x)dx \right) \right) dx + C.$$

Topilgan $C(x)$ funksiyani (4) formulaga qo'yib, berilgan (1) chiziqli tenglamaning umumiy yechimiga ega bo'lamiz:

$$y = \exp\left(-\int a(x)dx \right) \left[\int \left(b(x) \exp\left(\int a(x)dx \right) \right) dx + C \right]. \qquad (6)$$

Birinchi tartibli (1) chiziqli tenglamaning umumiy yechimini (6) formula yordamida aniqlashda $\int a(x)dx$ va $\int \left(b(x) \exp\left(\int a(x)dx \right) \right) dx$ aniqmas integrallarning har biridagi boshlang'ich funksiyalardan bittasini olish yetarli, chunki ularga ixtiyoriy o'zgarmaslarni qo'shish faqat ixtiyoriy C o'zgarmasning qiymatini o'zgartiradi, xolos, bu esa differensial tenglamaning umumiy yechimi uchun muhim emas.

Bu usulning nomi C o'zgarmasni x argumentning funksiyasi deb qarab, uni variatsiyalaganimizdan (o'zgartirganimizdan) kelib chiqgan.

Bu yerda ko'rib chiqilgan o'zgarmasni variatsiyalash usuli bitta (1) chiziqli tenglamani integrallash masalasini o'zgaruvchilari ajraladigan ikkita (2) va (5) tenglamalarning yechimlarini izlashga olib keladi.

28. Tenglamani yeching:

$$y' + 2yctgx = 3\cos x. \qquad (7)$$

◄Avvalo berilgan chiziqli tenglamaga mos kelgan

$$y' + 2yctgx = 0 \qquad (8)$$

bir jinsli tenglamaning umumiy yechimini topamiz:

$$y = \frac{C}{\sin^2 x}. \qquad (9)$$

Berilgan (7) chiziqli tenglamani yechish uchun (9) formulada o'zgarmasni variatsiyalaymiz, ya'ni $C = C(x)$ deb olamiz, va

$$y=\frac{C(x)}{\sin^2 x} \tag{10}$$

funksiyadan (7) tenglamani qanoatlantirishni talab qilamiz:

$$\frac{C'(x)}{\sin^2 x}-\frac{2C(x)\cos x}{\sin^3 x}+2\frac{C(x)}{\sin^2 x}\cdot ctgx=3\cos x,$$

ya'ni $C'(x)=3\cos x\sin^2 x$. Bundan $C(x)=\sin^3 x+C_0$ kelib chiqadi, bu yerda C_0- yangi ixtiyoriy o'zgarmas. $C(x)$ ning topilgan ifodasini (10) formulaga qo'yib, dastlabki berilgan (7) chiziqli tenglamaning umumiy yechimini topamiz: $y=\sin x+\dfrac{C_0}{\sin^2 x}$.►

Izoh. Kelgusida yangi o'zgarmas sifatida C belgilashni olamiz. Shunday qilib, hozirgi misolda $y=\sin x+C\sin^{-2} x$ umumiy yechim, C esa o'zgarmasdir.

29. $x^2y'+xy+1=0$ tenglamaning $M(1,-1)$ nuqtadan o'tuvchi yechimini toping.

◄ $x=0$ funksiya berilgan tenglamaning yechimi emasligi aniq, shuning uchun tenglamani $y'+\dfrac{y}{x}=-\dfrac{1}{x^2}$ ko'rinishda yozib olamiz. Bu chiziqli tenglamaga mos kelgan $y'+\dfrac{y}{x}=0$ bir jinsli tenglamani yechamiz. Uning umumiy yechimi $y=C/x$ bo'ladi. Ixtiyoriy o'zgarmasni variatsiyalash usulini qo'llaymiz. Natijada $C'(x)==1/x$ tenglikka ega bo'lamiz. Bu yerdan $C(x)$ ni topamiz: $C(x)=-\ln Cx$. Shunday qilib, berilgan tenglamaning umumiy yechimi $xy=-\ln Cx$ ko'rinishda topiladi.

Endi tenglamaning $M(1,-1)$ nuqtadan o'tadigan yechimini topamiz. Buning uchun umumiy yechimning topilgan $xy=-\ln Cx$ formulasida $x=1$, $y=-1$ deb olib, $C=e$ ni topamiz. U holda $xy=-\ln|ex|$, ya'ni $xy=-1-\ln|x|$ xususiy yechim topiladi.►

30. Birinchi tartibli chiziqli differensial tenglamaning ikkita har xil y_1 va y_2 yechimlari berilgan. Shu yechimlar yordamida tenglamaning umumiy yechimini yozing.

◀Ma'lumki, $y' + a(x)y = b(x)$ chiziqli differensial tenglama

$$y = (C + \alpha(x))\,\beta(x) \tag{11}$$

umumiy yechimga ega, bu yerda

$$\alpha(x) = \int\Big(b(x)\,\exp\Big(\int a(x)dx\Big)\Big)dx = \int b(x)\,\beta^{-1}(x)dx\,,$$

$$\beta(x) = \exp\Big(-\int a(x)dx\Big).$$

Masalaning shartiga ko'ra, (11) dan

$$y_1 = (C_1 + \alpha(x))\,\beta(x)\,,\ y_2 = (C_2 + \alpha(x))\,\beta(x) \tag{12}$$

kelib chiqadi, bu yerda C_1 va C_2 – chiziqli tenglamaning y_1 va y_2 yechimlariga mos kelgan o'zgarmaslar. So'ngra, (12) tengliklardan foydalanib, α va β funksiyalarni y_1 va y_2 yechimlar orqali ifodalaymiz:

$$\alpha(x) = \frac{y_1(x) - y_2(x)}{C_1 - C_2} \quad (C_1 \neq C_2)\,,$$

$$\beta(x) = \frac{C_1 y_2(x) - C_2 y_1(x)}{y_1(x) - y_2(x)} \quad (y_1(x) \neq y_2(x)).$$

$\alpha(x)$ va $\beta(x)$ funksiyalarning ifodalarini (11) ga qo'yib, topamiz:

$$y = \frac{1}{C_1 - C_2}\big((C_1 - C)y_2(x) + (C - C_2)y_1(x)\big) =$$

$$= y_2(x) + C^*(y_2(x) - y_1(x)),$$

bu yerda $C^* = \dfrac{C_2 - C}{C_1 - C_2}$ – ixtiyoriy o'zgarmas.▶

1.4.2. O'rniga qo'yish usuli (Bernulli usuli). Birinchi tartibli chiziqli

$$y' + a(x)y = b(x) \tag{1}$$

differensial tenglamaning umumiy yechimini $y = u(x)\cdot v(x)$ ko'rinishda izlaymiz. Bu bilan y o'rniga izlanayotgan yangi o'zgaruvchi, masalan, u ni kiritgan bo'lamiz. Shu sababli ikkinchi o'zgaruvchi v ni yordamchi o'zgaruvchi deb qarab uni o'z hohishimizga ko'ra tanlashimiz mumkin.

$\frac{dy}{dx}$ hosilani hisoblab, y va $\frac{dy}{dx}$ ning u va v orqali ifodalarini (1) tenglamaga keltirib qo'yamiz.

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

bo'lganligi uchun, (1) tenglama

$$v\frac{du}{dx} + u\left(\frac{dv}{dx} + a(x)v\right) = b(x) \tag{2}$$

ko'rinishga keladi.

Yordamchi o'zgaruvchi v ni tanlash mumkinligidan foydalanib, (2) tenglikda o'rta qavs ichidagi ifoda nolga aylanadigan qilib olamiz, ya'ni

$$\frac{dv}{dx} + a(x)v = 0. \tag{3}$$

bo'lishini talab qilamiz.

(3) tenglama o'zgaruvchilari ajraladigan tenglamadir. Uning ikkala qismini v ga bo'lib va dx ga ko'paytirib,

$$\frac{dv}{v} + a(x)dx = 0$$

tenglamani hosil qilamiz. Bu tenglamani integrallab topamiz:

$$\ln v + \int a(x)dx = \ln C,$$

ya'ni

$$v = C\exp\left(-\int a(x)dx\right). \qquad (4)$$

Endi esa v ning bu ifodasini (1) chiziqli tenglamaga mos kelgan

$$y' + a(x)y = 0$$

bir jinsli tenglamaga qo'yib, u o'zgaruvchiga nisbatan o'zgaruvchilari ajraladigan tenglamani hosil qilamiz:

$$C\exp\left(-\int a(x)dx\right)\cdot\frac{du}{dx} = b(x).$$

Bu tenglamadan

$$u = \frac{1}{C}\left[\int b(x)\exp\left(\int a(x)dx\right)dx + C_1\right] \qquad (5)$$

yechimni topamiz.

(4) va (5) formulalar v va u ning x orqali ifodalarini beradi. (1) chiziqli tenglamani yechishdan maqsad y ning x ga bog'lanishini topish edi. Endi $y = uv$ bo'lganligi uchun (1) tenglamaning umumiy yechimi uzil-kesil quyidagi ko'rinishda yoziladi:

$$y = \exp\left(-\int a(x)dx\right)\left[\int\left(b(x)\,\exp\left(\int a(x)dx\right)\right)dx + C_1\right].$$

(3) tenglamani integrallashdan hosil bo'lgan ixtiyoriy C o'zgarmas u ni v ga ko'paytirish vaqtida qisqarib ketganiga e'tiborni qaratamiz. Shunday bo'lishi ham kerak edi, chunki birinchi tartibli tenglamaning umumiy yechimi faqat bitta ixtiyoriy o'zgarmasga bog'liq bo'lishi kerak. Buni avvaldan bilgan holda (4) yechimda oldindan $C = 1$ deb olish va (3) tenglamaning (4) umumiy yechimi o'rniga $v = \exp\left(-\int a(x)dx\right)$ xususiy yechimni olish mumkin, amalda (tenglamalar yechishda) shunday qilinadi.

31. $xy' + y = 2x$ tenglamani yeching.

◀Bu tenglamani o'rniga qo'yish usulida yechamiz. $y = u\cdot v$ deylik, u holda berilgan tenglama $vu' + u\left(v' + \frac{v}{x}\right) = 2$ ko'rinishga keladi.

$v' + \frac{v}{x} = 0$ tenglamaning umumiy yechimini topamiz: $v = \frac{C}{x}$.

Yuqorida aytilganidek, $v = 1/x$ xususiy yechim bilan cheklanish mumkin. v ning ifodasini almashtirilgan $vu' = 2$ tenglamaga qo'yamiz: $\frac{1}{x}u' = 2$ yoki $du = 2xdx$, bu yerdan $u = x^2 + C$. Endi umumiy yechimni yoza olamiz: $y = u \cdot v$ bo'lganligi uchun umumiy yechim $y = x + \frac{C}{x}$ ko'rinishda topiladi.►

1.4.3. Argument va funksiyaning tutgan o'rinlarini almashtirish. Berilgan tenglama $y(x)$ funksiyaga nisbatan emas, balki $x(y)$ funksiyaga nisbatan chiziqli bo'lishi ham mumkin.

32. Tenglamani yeching: $(2y\ln y + x)y' = y$.

◄Tenglama y o'zgaruvchiga nisbatan chiziqli emasligi ochiq ravshan, ammo u x o'zgaruvchiga nisbatan chiziqlidir. Shuning uchun x ni y ning funksiyasi deb hisoblaymiz.

Tenglamaning berilishiga ko'ra, $y > 0$ ekanligi tushunarli. $dy \neq 0$ deb hisoblab,

$$\frac{dx}{dy} - \frac{x}{y} = 2\ln y$$

chiziqli tenglamani hosil qilamiz. Unga mos bir jinsli

$$\frac{dx}{dy} - \frac{x}{y} = 0$$

tenglama $x = Cy$ umumiy yechimga ega. O'zgarmasni variatsiyalash usulini qo'llab, ketma-ket topamiz:

$$C'(y)y + C(y) - \frac{C(y)y}{y} = 2\ln y,$$

$$C'(y)=2\frac{\ln y}{y},\quad C(y)=\ln^2 y+C.$$

Shunday qilib, berilgan tenglamaning umumiy yechimi

$x=\left(C+\ln^2 y\right)y$ formula bilan yoziladi.►

33. Tenglamani yeching: $(1-2xy)y'=y(y-1)$.

◄Tenglama y o'zgaruvchiga nisbatan chiziqli emas, ammo u x o'zgaruvchiga nisbatan chiziqlidir. Shuning uchun x ni y ning funksiyasi deb hisoblaymiz.

$dy\neq 0$ deb hisoblab ($y=0$ va $y=1$ o'zgarmas yechimlar),

$$\frac{dx}{dy}+\frac{2x}{y-1}=\frac{1}{y(y-1)}$$

chiziqli tenglamani hosil qilamiz. Unga mos bir jinsli tenglama $x=C(y-1)^{-2}$ umumiy yechimga ega. O'zgarmasni variatsiyalash usulini qo'llab, ketma-ket topamiz:

$$\frac{C'(y)}{(y-1)^2}-2\cdot\frac{C(y)}{(y-1)^3}+\frac{2}{y-1}\frac{C(y)}{(y-1)^2}=\frac{1}{y(y-1)},$$

$$C'(y)=1-\frac{1}{y},\ C(y)=y-\ln Cy.$$

Shunday qilib, berilgan tenglamaning barcha yechimlari

$$x=\frac{y-\ln Cy}{(y-1)^2},\ \text{ya'ni}\ x(y-1)^2=y-\ln Cy,\ y=0,\ y=1$$

formulalar bilan yoziladi.►

1.4.4. Chiziqli tenglamaga keltiriladigan ba'zi tenglamalar. Quyidagi tenglamalar chiziqli tenglamalarga keltiriladi:

$$f'(y)\cdot y'+a(x)f(y)=b(x), \qquad (1)$$

$$y'+a(x)=b(x)e^{ny}. \qquad (2)$$

(1) tenglamada $f(y)=z(x)$ deb olsak, $f'(y)y'=z'(x)$ tenglik o'rinli bo'ladi. Olingan ifodalarni (1) tenglamaga qo'yib, $z'+a(x)z=b(x)$ ko'rinishdagi chiziqli tenglamani hosil qilamiz.

(2) tenglama $n=0$ da chiziqli tenglama bo'lganligi uchun $n\neq 0$ da $e^{-ny}=z(x)$ o'rniga qo'yishni bajaramiz. U holda $-ne^{-ny}y'=z'$ tenglikni e'tiborga olsak, (2) tenglama

$$-\frac{z'}{n}+a(x)z=b(x) \quad (n\neq 0)$$

ko'rinishdagi chiziqli tenglamaga o'tadi.

34. Tenglamani yeching: $x\,y'\cos y=3(x^2+\sin y)$.

◀Bu tenglama (1) ko'rinishdagi tenglamadir. Bizda $f(y)=\sin y$. Berilgan tenglamada $\sin y=z(x)$ deb olib, $\sin y=z(x)$ va $y'\cos y=z'(x)$ ifodalarni tenglamaga qo'yamiz. Natijada $z'-\frac{3z}{x}=3x$ chiziqli tenglama hosil bo'ladi. Unga mos kelgan bir jinsli tenglama $z=Cx^3$ umumiy yechimga ega. O'zgarmasni variatsiyalash usulini qo'llab topamiz: $z=Cx^3-3x^2$. Shunday qilib, berilgan tenglamaning umumiy yechimi $\sin y=Cx^3-3x^2$ ko'rinishda bo'ladi.▶

35. Tenglamani yeching: $xy'+1=e^{x-y}$.

◀Bu tenglama (2) ko'rinishdagi tenglamadir:

$$\frac{dy}{dx}+\frac{1}{x}=\frac{e^x}{x}\cdot e^{-y}.$$

Bu tenglamada $z(x)=e^y$ deb olib, ketma-ket topamiz:

$$z'=e^y y',\;\; e^{-y}z'+\frac{1}{x}=\frac{e^x}{x}\cdot e^{-y},\;\; xz'+z=e^x.$$

Hosil bo'lgan tenglama z ga nisbatan chiziqlidir. Bu tenglamani o'zgarmasni variatsiyalash yoki o'rniga qo'yish usullaridan biri yordamida yechish mumkin, albatta. Ammo bu tenglamani $(xz)' = e^x$ ko'rinishda yozib olsak, uni yechish yanada osonlashadi: $xz = e^x + C$. Endi dastlabki tenglamaning umumiy yechimini yoza olamiz: $xe^y = e^x + C$.▶

36. $y(x) = \int_0^x y(t)\cos t dt + \sin^2 x.$

◀Tenglikning ikkala tomonidan hosila olib, $y' = y\cos x + \sin 2x$ ko'rinishdagi chiziqli tenglamaga kelamiz. Bu tenglamaning umumiy yechimi $y = Ce^{\sin x} - 2(1+\sin x)$ ko'rinishda bo'ladi. Berilgan dastlabki tenglamadan kelib chiqadigan $y(0) = 0$ boshlang'ich shart bajarilishi uchun $C = 2$ ga teng bo'lishi kerak. Shunday qilib, $y = 2e^{\sin x} - 2(1+\sin x)$.▶

INDIVIDUAL TOPSHIRIQLAR

M9. Chiziqli differensial tenglamaning berilgan nuqtadan o'tuvchi xususiy yechimini (xususiy integralini) toping.

1. $\left(x^2+1\right)y' + 4xy = 3,\ y(0) = 0.$
2. $y' + ytgx = \sec x,\ y(0) = 0.$
3. $(1-x)(y'+y) = e^{-x},\ y(0) = 0.$
4. $xy' - 2y = 2x^4,\ y(1) = 0.$
5. $y' = 2x\left(x^2 + y\right),\ y(0) = 0.$
6. $y' - y = e^x,\ y(0) = 1.$
7. $xy' + y + xe^{-x^2} = 0,\ y(1) = 1/(2e).$
8. $\cos y dx = (x + 2\cos y)\sin y dy,\ y(0) = \pi/4.$
9. $x^2y' + xy + 1 = 0,\ y(1) = 0.$
10. $yx' + x = 4y^3 + 3y^2,\ y(2) = 1.$
11. $(2x+y)dy = ydx + 4\ln y dy,\ y(0) = 1.$

12. $\left(3x-y^2\right)y'=y,\ \ y(0)=1.$

13. $(1-2xy)y'=y(y-1),\ \ y(0)=1.$

14. $x(y'-y)=e^x,\ y(1)=0.$

15. $y=x(y'-x\cos x),\ \ y(\pi/2)=0.$

16. $(xy'-1)\ln x=2y,\ \ y(e)=0.$

17. $\left(2e^y-x\right)y'=1,\ y(0)=0.$

18. $xy'+(x+1)y=3x^2e^{-x},\ y(1)=0.$

19. $\left(x+y^2\right)dy=ydx,\ \ y(0)=1.$

20. $\left(\sin^2 y+xctgy\right)y'=1,\ \ y(0)=\pi/2.$

21. $(x+1)y'+y=4x^3+3x^2,\ y(0)=0.$

22. $xy'-2y+x^2=0,\ y(1)=0.$

23. $xy'+y=\sin x,\ \ y(\pi/2)=2/\pi.$

24. $\left(x^2-1\right)y'-xy=x^3-x,\ \ y\left(\sqrt{2}\right)=1.$

25. $\left(1-x^2\right)y'+xy=1,\ \ y(0)=1.$

26. $y'ctgx-y=-2\sin^2 x\cdot ctgx,\ y(0)=0.$

27. $x^2y'=2xy+3,\ y(1)=-1.$

28. $y'+2xy=xe^{-x^2},\ y(0)=0.$

29. $y'-3x^2y-x^2e^{x^3}=0,\ y(0)=0.$

30. $xy'+y=\ln x+1,\ y(1)=0.$

M10. Chiziqli differensial tenglamaning umumiy yechimini toping.

1. $\left(1-x^2\right)y'+2xy=x.$

2. $\left(x^2+4\right)y'-2xy=\left(x^2+4\right)^2.$

3. $y'-\dfrac{y}{x}=x\cos x.$

4. $\left(x^2+9\right)y'-3xy=\left(x^2+9\right)^2.$

5. $y'-2xy=\dfrac{xe^{x^2}}{x+1}.$

6. $\left(9-x^2\right)y'+2xy-x^3=0.$

7. $y'-\dfrac{y}{x-1}=(x+1)^2.$

8. $y'\sin x-y\cos x=\cos^2 x.$

9. $x^2y' + y = e^{\frac{1}{x}}x.$

10. $x^2y' - y = \frac{1}{x^2}.$

11. $x^3y' + y = e^{\frac{1}{2x^2}}\sin 2x.$

12. $y' - yctgx = \frac{\cos x}{\sin^2 x}.$

13. $y' + 2xy = x\sin xe^{-x^2}.$

14. $\left(x^3 - 1\right)y' - 3x^2y = x - 1.$

15. $xy' + 2y = x\sqrt{x}.$

16. $\left(x^4 + 1\right)y' - 4x^3y = x.$

17. $y' + \frac{y}{x} = e^{-x^2} + \frac{1}{x^3}$

18. $xy' + y = x\ln x + 2.$

19. $xy' - 2y = x^2\sqrt{x+1}.$

20. $y'\sin x - 3y\cos x = \sin 2x.$

21. $xy' + y = \sin x.$

22. $y' + 2ytgx = tgx + \sin x.$

23. $y' + ytgx = \frac{1}{\cos x}.$

24. $\left(x^2 + 4\right)y' + 2xy = 1.$

25. $y' - \frac{y}{x\ln x} = x\ln x.$

26. $x^2y' = e^{\frac{1}{x}}\sin\frac{1}{x} - y.$

27. $y' + yctgx = \sin^2 x.$

28. $y' - ytgx = \cos^{-3} x.$

29. $y' - ytgx = \cos^2 x.$

30. $2y' - 2y\sin x = \sin 2x.$

1.5. BERNULLI TENGLAMASI

Ushbu

$$\frac{dy}{dx} + a(x)y = b(x)y^n$$

ko'rinishdagi Bernulli tenglamasi deyiladi. $n = 0$ da u chiziqli tenglamaga, $n = 1$ da esa o'zgaruvchilari ajraladigan tenglamaga o'tadi. Shuning uchun kelgusida Bernulli tenglamasi bilan ishlaganda $n \neq 0$ va $n \neq 1$ deb faraz qilamiz.

Bernulli tenglamasi

$$y^{-n}y' + a(x)y^{1-n} = b(x), \quad y \neq 0$$

ko'rinishda yozib olinib, $z = y^{1-n}$ o'rniga qo'yish yordamida chiziqli tenglamaga keltiriladi:

$$\frac{1}{1-n}z' + a(x)z = b(x),\ z \neq 0.$$

Agar $n > 0$ bo'lsa, Bernulli tenglamasi $y = 0$ yechimga ham ega bo'ladi.

37. $2y' + 2y\,ctgx = 3y^{-1}\cos x.$

◀Bu Bernulli tenglamasi ($n = -1$). Tenglamaning ikkala tomonini y noma'lum funksiyaga ko'paytirib, so'ngra $y^2 = z$ deb olamiz:

$$z' + 2z\,ctgx = 3\cos x.$$

Hosil bo'lgan chiziqli tenglamani o'zgarmasni variatsiyalash usuli bilan yechamiz (**28**-misolga qarang). Bundan $z = C(x)\sin^{-2} x$ yechimni topamiz, bu yerda $C(x) = \sin^3 x + C$. Xullas, $z = \sin x + C\sin^{-2} x$.

Tenglamaning umumiy yechimi $y^2 = \sin x + C\sin^{-2} x$ ko'rinishda topiladi.▶

38. $xy' - 2x^2\sqrt{y} = 4y.$

◀Bu tenglamaning ikkala tomonini x ga bo'lamiz:

$$y' - \frac{4}{x}y = 2x\sqrt{y}.$$

Bu Bernulli tenglamasi ($n = 1/2$) bo'lib, uning ikkala tomonini $\sqrt{y}$ funksiyaga bo'lamiz va $\sqrt{y} = z$ deb olamiz: $z' - \frac{2}{x}z = x.$

Hosil bo'lgan chiziqli tenglamani o'zgarmasni variatsiyalash usuli bilan yechib, $z = x^2 \ln Cx$ umumiy yechimni topamiz.

Shunday qilib, dastlabki berilgan tenglamaning umumiy yechimi $\sqrt{y} = x^2 \ln Cx$ ko'rinishda topiladi. Bundan tashqari, tenglama $y = 0$ yechimga ham ega.

Xullas, berilgan tenglamaning barcha yechimlari $\sqrt{y} = x^2 \ln Cx$; $y = 0$ ko'rinishda yoziladi.►

39. $xy^2 y' = x^2 + y^3$.

◄Zarur almashtirishlarni bajarib, bu tenglamani

$$y' - \frac{1}{x} y = xy^{-2}$$

ko'rinishga keltiramiz. Hosil bo'lgan Bernulli tenglamasini ($n = -2$) yuqoridagi kabi usullar bilan yechish mumkin.

Ammo bu yerda tenglamani Bernulli tenglamasi ko'rinishiga keltirmasdan ham bevosita yechish mumkinligiga e'tiborni qaratamiz. Haqiqatan ham, $y^3 = z$ va $3y^2 y' = z'$ deb olsak, $xz' - 3z = 3x^2$ chiziqli tenglama hosil bo'ladi. Bu chiziqli tenglamaning umumiy yechimi $z = Cx^3 - 3x^2$ ko'rinishda topiladi.

Xullas, berilgan tenglamaning umumiy yechimi: $y^3 = Cx^3 - 3x^2$.►

40. $z' - \frac{z}{x} = \left(\frac{1}{x} - 1 \right) z^2$.

◄Bu Bernulli tenglamasini ($n = 2$) o'rniga qo'yish usulida ham yechish mumkin.

$z = 0$ funksiya bu tenglamaning yechimi ekanligi ochiq ravshan. Tenglamaning $z \neq 0$ bo'lgandagi yechimlarini topamiz. Buning uchun $z = u(x)v(x)$, $uv \neq 0$ ifodani berilgan tenglamaga qo'yamiz:

$$\left(u'(x) - \frac{1}{x} u(x) \right) v(x) + u(x)v'(x) = \left(\frac{1}{x} - 1 \right) u^2(x) v^2(x).$$

$u(x)$ sifatida $u'(x) - \frac{1}{x} u(x) = 0$ tenglamaning birorta yechimini, masalan, $u(x) = x$ funksiyani olamiz va uni

$$u(x)v'(x) = \left(\frac{1}{x} - 1 \right) u^2(x) v^2(x)$$

tenglamaga qo'yamiz:

$$xv'(x)=\left(\frac{1}{x}-1\right)x^2v^2(x).$$

Oxirgi tenglamaning ikkala tomonini $xv(x)\neq 0$ ifodaga bo'lishdan hosil bo'lgan ushbu $\frac{dv}{v^2}=(1-x)dx$ tenglamani integrallab, $v(x)=2/(x^2-2x+C)$ ko'rinishdagi yechimlarni olamiz. Endi esa $z=xv(x)$ ekanligini e'tiborga olib, natijani yozamiz: $z=\frac{2x}{x^2-2x+C}$, $z=0$.►

INDIVIDUAL TOPSHIRIQLAR

M11.Bernulli tenglamasini yeching.

1. $y'+y=x\sqrt{y}.$
2. $ydx+2xdy=2y\sqrt{x}\sec^2 ydy.$
3. $y'+2y=y^2e^x.$
4. $y'=y^4\cos x+ytgx.$
5. $xydy=\left(y^2+x\right)dx.$
6. $xy'+2y+x^5y^3e^x=0.$
7. $y'x^3\sin y=xy'-2y.$
8. $\left(2x^2y\ln y-x\right)y'=y.$
9. $2y'-\frac{x}{y}=\frac{xy}{x^2-1}.$
10. $xy'-x^2\sqrt{y}=4y.$
11. $xy^2y'=x^2+y^3.$
12. $\left(x+1\right)\left(y'+y^2\right)=-y.$
13. $y'x+y=-xy^2.$
14. $y'-xy=-y^3e^{-x^2}.$
15. $xy'-2\sqrt{x^3y}=y.$
16. $y'+xy=x^3y^3.$
17. $y'=\frac{x}{y}e^{2x}+y.$
18. $yx'+x=-yx^2.$
19. $x\left(x-1\right)y'+y^3=xy.$
20. $2x^3yy'+3x^2y^2+1=0.$
21. $\frac{dx}{x}=\left(\frac{1}{y}-2x\right)dy.$
22. $y'+x\sqrt[3]{y}=3y.$

23. $xy' + y = y^2 \ln x.$

24. $xdx = \left(\frac{x^2}{y} - y^3 \right) dy.$

25. $y' + 2xy = x^3 y^3.$

26. $y' + y = \frac{x}{y^2}.$

27. $y' - ytgx + y^2 \cos x = 0.$

28. $y' + \frac{2y}{x} = \frac{2\sqrt{y}}{\cos^2 x}.$

29. $y' - y + y^2 \cos x = 0.$

30. $y' = x\sqrt{y} + \frac{xy}{x^2 - 1}.$

M12.Bernulli tenglamasini yeching.

1. $xy' - 2y - x^2 \sqrt{y} = 0.$

2. $y' - yctgx = y^2.$

3. $y' - \frac{2xy}{x^2 + 1} + y^2 = 0.$

4. $xy' - y = x^2 y.$

5. $\left(1 + x^2\right) y' - xy = y^2.$

6. $\left(1 - x^2\right) y' - xy = x^2 y^2.$

7. $y' + \frac{y}{x-1} = xy^3.$

8. $xy' + 2y = y^2 \ln x.$

9. $xy' - y = y^4.$

10. $4xy' + 3y = -e^x x^4 y^5.$

11. $xy' - y = xy^3.$

12. $\sqrt{y}\left(3y' + 2xy\right) = x.$

13. $xy' + y = xy^2 \sin x.$

14. $\left(x^2 y - 2\right) ydx - xdy = 0.$

15. $y' + \frac{y}{x} = y^2 \ln^2 x.$

16. $y' + yctgx = y^3 \cos x.$

17. $\left(4 + x^2\right) y' - 2xy = \frac{x}{y}.$

18. $3y^2 y' - \frac{y^3}{x} = \sqrt{x} + 1.$

19. $3xy' - y = \left(x^2 + 1\right) y^{-2}.$

20. $2\left(1 + x^2\right) y' - 2xy = \frac{x}{y}.$

21. $2xy' + y = \frac{x^2}{y}.$

22. $y' - \frac{4xy}{4 + x^2} = 2\sqrt{y}\left(x + 2\right).$

23. $y' + \frac{2y}{1 + x} + 2\sqrt{y} = 0.$

24. $y' + \frac{2y}{x} = \frac{2\sqrt{y}}{\cos^2 x}.$

25. $xy' + y = y^2 \ln x.$

26. $3x^2 y' + xy + y^{-2} = 0.$

27. $xy' - 2y = 2x^3 \sqrt{y}.$

28. $2y' - ytgx + y^3 tgx = 0.$

29. $\left(1-x^2\right)y'-xy=xy^2.$ 30. $y'+\dfrac{y}{x}=y^4\left(1-x^2\right).$

1.6. RIKKATI TENGLAMASI.

Ushbu

$$y'+a(x)y+b(x)y^2=c(x) \qquad (1)$$

tenglama *Rikkati tenglamasi* deyiladi,

$$y'+a\,y^2=b\,x^{\alpha}\ \ (a, b, \alpha\ \text{–o'zgarmas sonlar}) \qquad (2)$$

tenglama esa *maxsus ko'rinishdagi Rikkati tenglamasi* deyiladi.

Rikkati tenglamalari, umuman aytganda, kvadraturalarda integrallanmaydi. Xattoki, maxsus ko'rinishdagi Rikkati tenglamasi $\alpha=\dfrac{4k}{\pm1-2k}$ (k – butun son yoki ∞) bo'lgandagina kvadraturalarga keltiriladi.

Agar $\alpha=\dfrac{4k}{1-2k}$ tenglik $k>0$ da bajarilsa, u holda (2) tenglamada $y=\dfrac{u}{x^2}+\dfrac{1}{ax}$ o'rniga qo'yishni bajarsak, (2) tenglama

$$\frac{du}{dx}+\frac{a\,u^2}{x^2}=bx^{\alpha+2}$$

ko'rinishga keladi. So'ngra $u=\dfrac{1}{v}$ deb olib,

$$\frac{dv}{dx}+bx^{\alpha+2}v^2=a\,x^{-2}$$

tenglamani olamiz. Shundan keyin $x^{\alpha+3}=z$ belgilash kiritib,

$$\frac{dv}{dz}+\frac{b}{\alpha+3}v^2=\frac{a}{\alpha+3}z^{-(\alpha+4)/(\alpha+3)}$$

tenglamaga kelamiz. Bunday almashtirishlarni o'zgaruvchilari ajraladigan tenglama hosil bo'lguncha davom ettiramiz.

Agar $\alpha=\frac{4k}{1-2k}$ tenglik $k<0$ da bajarilsa, u holda ko'rsatilgan almashtirishlarni teskari tartibda bajarish kerak.

Ushbu

$$u'=\pm u^2+w(x) \qquad (3)$$

tenglama *Rikkatining kanonik tenglamasi* deyiladi. Agar (1) tenglamada $b(x)$ funksiya ikki marta uzluksiz differensiallanuvchi bo'lsa, u holda $y=\alpha(x)z$, $z=u+\beta(x)$ almashtirishlar yordamida (1) tenglama (3) kanonik ko'rinishga keltiriladi. Ba'zan (3) yordamida (1) tenglamaning xususiy yechimini topish qiyinchilik tug'dirmaydi.

Agar $y_1(x)-(1)$ tenglamaning xususiy yechimi bo'lsa, u holda $y=y_1+z$ yoki $y=y_1+z^{-1}$ (qaysi biri qulay bo'lsa) deb olib, Rikkati tenglamasini chiziqli tenglamaga keltirish mumkin.

Xususiy yechimni tanlash usuli bilan topib, tenglamalarni yeching(**41-46**).

41. $xy'-(2x+1)y+y^2=-x^2$.

◄Xususiy yechimni $y_1(x)=ax+b$ $(a,b=const)$ ko'rinishda izlaymiz. Uni berilgan tenglamaga qo'yib, x ga nisbatan ayniyatni hosil qilamiz:

$$ax-(2x+1)(ax+b)+(ax+b)^2=-x^2,$$

bundan o'z navbatida

$$2ab-2b=0,\ a=1,\ -b+b^2=0$$

tenglamalar sistemasini olamiz. Bu yerda ikkita yechim bo'lishi mumkin: $a=b=1$ yoki $a=1,\ b=0$.

Masalan, $a=1, b=0$ bo'lsin. Shunga binoan, $y_1(x)=x$ xususiy yechimdan foydalanib, $y=x+\frac{1}{z}$ o'rniga qo'yishni bajaramiz:

$$x\left(1-\frac{z'}{z}\right)-(2x+1)\left(x+\frac{1}{z}\right)+\left(x+\frac{1}{z}\right)^2=-x^2.$$

Soddalashtirishlardan keyin $xz'+z-1=0$ ko'rinishdagi o'zgaruvchilari ajraladigan tenglama hosil bo'ladi. Bu tenglamani integrallab, topamiz: $z=1+Cx^{-1}$. Shunday qilib, berilgan tenglamaning umumiy yechimini $y=x+\frac{x}{x+C}$ ko'rinishda yoza olamiz. $y_1(x)=x$ xususiy yechimni bu umumiy yechimdan $C=\infty$ da ham hosil qilish mumkin.▶

42. $y'+\frac{2-3x}{x^2}y+\left(1-\frac{1}{x}\right)y^2=\frac{1}{x^3}-\frac{3}{x^2}.$

◀Xususiy yechimni $y_1(x)=\frac{a}{x}$ ($a=const$) ko'rinishda izlaymiz. Uni berilgan tenglamaga qo'yib,

$$a^2-4a=-3, \quad 2a-a^2=1$$

tenglamalar sistemasini hosil qilamiz. Sistemaning $a=1$ yechimi yordamida $y_1(x)=\frac{1}{x}$ xususiy yechimni yozamiz. Endi berilgan tenglamada $y=z+\frac{1}{x}$ deb olsak, u holda

$$z'-\frac{z}{x}=\left(\frac{1}{x}-1\right)z^2 \qquad (4)$$

ko'rinishdagi Bernulli tenglamasi ($n=2$) hosil bo'ladi. (4) tenglamaning umumiy yechimini (**40**-misolga qarang) bilgan holda natijani yozamiz:

$$y=\frac{2x}{x^2-2x+C}+\frac{1}{x}, \quad y_1(x)=1/x.$$▶

43. $\left(y^2-1\right)x'+x^2-2xy+1=0$.

◀Tenglamaning ikkala tomonini $y^2-1\neq 0$ ifodaga bo'lamiz:

$$x'-\frac{2y}{y^2-1}x+\frac{1}{y^2-1}x^2=-\frac{1}{y^2-1}.$$

Bu Rikkati tenglamasining xususiy yechimini $x_1(y)=ay+b$ $(a,b=$ *const*$)$ ko'rinishda topamiz: $x_1=y$. Endi $x=y+z$ deb olib,

$$-\frac{dz}{z^2}=\frac{dy}{y^2-1}$$

ko'rinishdagi o'zgaruvchilari ajraladigan tenglamani olamiz. Bu tenglamani integrallab va eski o'zgaruvchilarga qaytib, topamiz:

$$(x-y)\ln\frac{C(y-1)}{y+1}=2;\ y=x.$$ ▶

44. $y'+y^2=2x^{-4}$.

◀Bu maxsus Rikkati tenglamasi ($\alpha=-4$) bo'lganligi uchun $k=1-$ butun son bo'ladi. Shunga binoan, uni kvadraturalarga keltirish mumkin. Berilgan tenglamada $y=\frac{u}{x^2}+\frac{1}{x}$ $(a=1)$ deb olib, $u'x^2+u^2=2$ tenglamaga ega bo'lamiz. Bu tenglamani o'zgaruvchilarga ajratib integrallaymiz:

$$\frac{1}{2\sqrt{2}}\ln\left|\frac{\sqrt{2}+u}{\sqrt{2}-u}\right|+\frac{1}{x}=\ln C.$$

So'ngra eski o'zgaruvchilarga qaytamiz: $\frac{\sqrt{2}+x(xy-1)}{\sqrt{2}+x(1-xy)}e^{2\sqrt{2}/x}=C$. ▶

45. Tenglamani yeching: $y'+y^2=x^{-4/3}$.

◀Bu yerda $\alpha=-4/3$, $k=-1$. Demak, qaralayotgan tenglama yuqoridagi tenglamalarda bajarilgan almashtirishlar kabi almashtirishlarni bajarish natijasida hosil bo'lgan tenglama ekan, ya'ni tegishli almashtirishlarni teskari tartibda bajarish kerak. Shunday qilib,

$$\frac{b}{\alpha+3}=1;\ \frac{a}{\alpha+3}=1;\ \frac{\alpha+4}{\alpha+3}=\frac{4}{3}.$$

Bu tengliklardan $\alpha=0$, $a=b=3$ kelib chiqadi. Bundan ko'rinadiki,

$$y'+3y^2=3 \tag{5}$$

tenglamada

$$y=\frac{u}{x^2}+\frac{1}{3x},\ u=\frac{1}{v},\ x^3=z$$

formulalar bo'yicha almashtirishlar bajarilishi natijasida

$$\frac{dv}{dz}+v^2=z^{-4/3}$$

tenglama, ya'ni bizga berilgan dastlabki tenglama hosil bo'ladi.

(5) tenglamani yechib, topamiz:

$$\left(\frac{1+y}{1-y}\right)e^{-6x}=C,$$

ya'ni
$$\frac{v\left(3z^{2/3}+z^{1/3}\right)+3}{v\left(z^{1/3}-3z^{2/3}\right)+3}\cdot\exp\left(-6\sqrt[3]{x}\right)=C.$$

Dastlabki tenglamada y bilan funksiya, x bilan argument belgilanganligi uchun mazkur tenglamaning umumiy integrali

$$\frac{y\left(x^{1/3}+3x^{2/3}\right)+3}{y\left(x^{1/3}-3x^{2/3}\right)+3}\cdot\exp\left(-6\sqrt[3]{x}\right)=C$$

ko'rinishda topiladi.►

46. $y'+\frac{3}{x}y+xy^2=\frac{1}{x}.$

◄Tenglamani kanonik ko'rinishga keltiramiz. Avvalo $y=\alpha(x)z$ o'rniga qo'yish yordamida z^2 ning oldidagi koeffisientning 1 ga teng bo'lishiga erishamiz:

$$z'\cdot\alpha+\alpha'\cdot z+\frac{3}{x}\alpha z+x\alpha^2z^2=\frac{1}{x}, \qquad (6)$$

bundan $\alpha(x)=1/x$. Shuning uchun (6) dan $z'+\frac{2}{x}z+z^2=1$ kelib chiqadi. So'ngra $z=u+\beta(x)$ almashtirish bilan oxirgi tenglamani shunday o'zgartiramizki, natijada

$$u'+\beta'+\frac{2}{x}(u+\beta)+u^2+2u\beta+\beta^2=1$$

tenglamada izlanayotgan u funksiyaning birinchi darajasi qatnashmasin. Buning uchun $\beta(x)=-1/x$ deb olinsa, u holda Rikkatining kanonik tenglamasi hosil bo'ladi: $u'+u^2=1$. Bu tenglama $u=th(x+C)$ umumiy yechimga ega. Shunday qilib, dastlabki berilgan tenglamaning umumiy yechimi:

$$y=\frac{1}{x}\left(th(x+C)-\frac{1}{x}\right).$$►

Eslatma. $thx\equiv\frac{shx}{chx}=\frac{e^x-e^{-x}}{e^x+e^{-x}}$ – giperbolik tangens funksiya.

47. Rikkati tenglamasining umumiy yechimini uning uchta turli yechimlari yordamida yozing.

◄Agar Rikkati tenglamasining bitta $y_1(x)$ xususiy yechimi ma'lum bo'lsa, u holda uning umumiy yechimi

$$y=y_1(x)+\frac{1}{z(x)}$$

ko'rinishda bo'ladi, bu yerda $z(x)$ – mos chiziqli tenglamaning umumiy yechimi. Ma'lumki, chiziqli tenglamaning umumiy yechimi ikkita funksiya yordamida yoziladi (**30**-misolga qarang):

$$z(x) = \alpha(x) + C_1\beta(x).$$

Shuning uchun Rikkati tenglamasining umumiy yechimi

$$y = y_1(x) + \frac{1}{\alpha(x) + C_1\beta(x)} \tag{7}$$

ko'rinishda tasvirlanadi. Endi $y_2(x)$ va $y_3(x)$ – qaralayotgan Rikkati tenglamasining xususiy yechimlari bo'lsin. U holda (7) dan kelib chiqadi:

$$y_2(x) = y_1 + \frac{1}{\alpha + C_2\beta}, \quad y_3(x) = y_1 + \frac{1}{\alpha + C_3\beta}, \tag{8}$$

bu yerda $y_2(x)$ va $y_3(x)$ xususiy yechimlarga mos kelgan o'zgarmaslar mos ravishda C_2 va C_3 $(C_2 \neq C_3)$ bilan belgilangan. (8) tenglamalar sistemasini α va β ga nisbatan yechib va ularning qiymatlarini (7) ga qo'yib, topamiz:

$$y = \frac{y_2(y_3 - y_1) + C\,y_1(y_3 - y_2)}{y_3 - y_1 + C(y_3 - y_2)},$$

bu yerda $C = \dfrac{C_1 - C_2}{C_2 - C_3}$ –ixtiyoriy o'zgarmas son.►

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M13. Rikkati tenglamasining $y = y(x)$ xususiy yechimini bilgan holda uning umumiy yechimini toping.

1. $x\left(x^3 - 1\right)y' - 2xy^2 + y + x^2 = 0,\ y = x^2.$
2. $\left(x^2 + 1\right)\left(xy' - y\right) - y^2 + x^2 = 0,\ y = \pm x.$
3. $\left(x^2 - 1\right)\left(xy' - y\right) - y^2 + x^2 = 0,\ y = \pm x.$
4. $x\left(2x - 1\right)y' + y^2 - \left(4x + 1\right)y + 4x = 0,\ y = 1.$
5. $2x^2y' - 2y^2 - 3xy + 2x^2 = 0,\ 2y = 2\sqrt{x} - x.$
6. $2x^2y' - 2y^2 - xy + 2x = 0,\ y = \sqrt{x}.$

7. $x^2\left(y'+y^2\right)+4xy+2=0,\ y=-2/x.$

8. $xy'+xy^2-y-x^3=0,\ y=x.$

9. $y'+y^2-y-e^{2x}=0,\ y=e^x.$

10. $xy'+xy^2-2y-x^5=0,\ y=x^2.$

11. $y'+y^2+(xy-1)\sin x=0,\ y=1/x.$

12. $y'-y^2-xy-x+1=0,\ y=-1.$

13. $y'-y^2+y\sin x-\cos x=0,\ y=\sin x.$

14. $y'+y^2\sin x=2\sin x\cos^{-2}x,\ y=1/\cos x.$

15. $xy'+\left(y^2-x^2\right)\sin x-y=0,\ y=\pm x.$

16. $xy'+\left(y^2-x^2\right)tgx-y=0,\ y=\pm x.$

17. $y'-y^2+\left(x^2+1\right)y-2x=0,\ y=x^2+1.$

18. $xy'+\left(y^2-x^2\right)\ln x-y=0,\ y=\pm x.$

19. $\left(x^2-1\right)y'+y^2-2xy+1=0,\ y=x.$

20. $2x^2y'-2y^2-xy+8x=0,\ y=2\sqrt{x}.$

21. $2x^2y'-2y^2-3xy+18x=0,\ 2y=6\sqrt{x}-x.$

22. $xy'+\left(y^2-x^2\right)e^x-y=0,\ y=\pm x.$

23. $y'+y^2\cos x=2\cos x\sin^{-2}x,\ y=1/\sin x.$

24. $y'-y^2+y\cos x+\sin x=0,\ y=\cos x.$

25. $2xy'+2xy^2-y-2x^2=0,\ y=\sqrt{x}.$

26. $y'+y^2-3y-e^{6x}=0,\ y=e^{3x}.$

27. $(2x+1)(xy'-y)-y^2+x^2=0,\ y=\pm x.$

28. $\left(x^2-9\right)(xy'-y)-y^2+x^2=0,\ y=\pm x.$

29. $y'+y^2=2x^{-2},\ y=-1/x.$

30. $xy'-y+y^2=x^2,\ y=-x.$

M14. Birinchi tartibli differensial tenglamaning berilgan nuqtadan o’tuvchi hususiy yechimini (hususiy integralini) toping.

1. $y'\sin x=y\ln^2 y,\ \ y(\pi/2)=e.$

2. $y' + ytgx = \sec x, \; y(0) = 1.$

3. $2(y - x) = (x + 2y)y', \; y(1) = 0.$

4. $2y^2 + x^2 - x^2 y' = 0, \; y(1) = 0.$

5. $(x^2 + 4)y' = y^2 + 4, \; y(0) = \pi / 2.$

6. $(x^2 + 4)y' - 2xy = x, \quad y(0) = 1.$

7. $(xy' - y)\,arctg(y / x) = x \ln x, \; y(e) = 0.$

8. $xy' = y\left(1 - \ln^2(y / x)\right), \; y(1) = e.$

9. $2(1 + e^x)yy' = e^{x - y^2}, \; y(0) = 0.$

10. $y' + ytgx = \cos^2 x, \; y(\pi / 4) = 1 / 2.$

11. $y' - yctgx = \sin 2x \cos x, \; y(\pi / 2) = 0.$

12. $y' + ytgx = e^x \cos x, \; y(0) = 1.$

13. $y' \sin y = ctgx \cos^2 y, \; y(\pi / 2) = 0.$

14. $y + \sqrt{x^2 + y^2} - xy' = 0, \; y(1) = 1.$

15. $y' = 4 + \dfrac{y}{x} + \dfrac{y^2}{x^2}, \quad y(1) = 2.$

16. $y' \sin^2 x = y + 1, \quad y(\pi / 4) = 1.$

17. $(x^2 - y^2)dy - 2xydx = 0, \quad y(1) = 2.$

18. $y' \cos^2 x = y, \quad y(\pi / 4) = e.$

19. $2yy' = (y^2 - 1)ctgx, \quad y(\pi / 2) = 0.$

20. $tgy\,dx - x \ln x\,dy = 0, \quad y(\pi / 2) = e.$

21. $xy' - \dfrac{y}{x} = \dfrac{1}{x}, \quad y(1) = 0.$

22. $y' = \dfrac{y - x}{y + x}, \; y(1) = 0.$

23. $y' = \dfrac{y}{x} + \dfrac{x}{x^2 + 1}, \quad y(1) = 1.$

24. $y' = -\dfrac{2y^2}{x^2 + y^2}, \; y(1) = 1.$

25. $y' + 3x^2 y = 3x^5, \quad y(0) = 1.$

26. $(x + y)y' = y - 2x, \quad y(1) = 0.$

27. $xy' = y\left(1 + \ln(y / x)\right), \; y(1) = e^e.$

28. $x^2y' - y^2 = 1, \quad y(1) = 0.$

29. $(x^2+1)y' = tgy, \quad y(0) = \pi / 6.$

30. $yy' + xe^{y^2} = 0, \; y(1) = 0.$

1.7. TO’LIQ DIFFERENSIALLI TENGLAMALAR.

Quyidagi differensial tenglama berilgan bo’lsin:

$$M(x,y)dx + N(x,y)dy = 0. \tag{1}$$

Agar (1) tenglamaning chap tomoni biror $F(x,y)$ funksiyaning to’liq differensiali bo’lsa, ya’ni

$$dF(x,y) \equiv \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = M(x,y)dx + N(x,y)dy$$

munosabat o’rinli bo’lsa, (1) tenglama *to’liq differensialli tenglama* deyiladi.

(1) tenglamaning to’liq differensialli bo’lishi uchun biror sohada

$$\frac{\partial M}{\partial y} \equiv \frac{\partial N}{\partial x} \tag{2}$$

ayniyatning bajarilishi zarur va yetarli.

Agar yuqoridagi xossaga ega bo’lgan $F(x,y)$ funksiya ma’lum bo’lsa, u holda (1) tenglamaning umumiy integrali

$$F(x,y) = C, \; C = const$$

ko’rinishda yoziladi.

$F(x,y)$ funksiyani topish uchun

$$\frac{\partial F}{\partial x} = M(x,y), \quad \frac{\partial F}{\partial y} = N(x,y) \tag{3}$$

tengliklardan foydalanamiz.

Bu tengliklardan birinchisini x o'zgaruvchi bo'yicha integrallab $F(x,y)$ funksiyani ixtiyoriy differensiallanuvchi $g(y)$ funksiya aniqligida topamiz:

$$F(x,y)=\int M(x,y)dx=G(x,y)+g(y), \tag{4}$$

bu yerda $G(x,y)$ ifoda $M(x,y)$ funksiyaning x o'zgaruvchi bo'yicha boshlang'ich funksiyasi.

Endi (4) tenglikni y bo'yicha differensiallab, (3) tengliklardan ikkinchisini e'tiborga olsak, $g(y)$ funksiyani aniqlash uchun ushbu tenglamani hosil qilamiz:

$$\frac{\partial G(x,y)}{\partial y}+g'(y)=N(x,y).$$

Bu yerdan $g(y)$ funksiyani topib, uni (4) munosabatga qo'yamiz va $F(x,y)$ funksiyani topamiz.

Yuqorida aytilganlarni quyidagi formulalar ko'rinishida ifodalash ham mumkin:

$$F(x,y)=\int_{x_0}^{x} M(t,y)dt+\int_{y_0}^{y} N(x_0,t)dt \tag{5}$$

yoki

$$F(x,y)=\int_{x_0}^{x} M(t,y_0)dt+\int_{y_0}^{y} N(x,t)dt\,.$$

48. Tenglamani yeching: $x^2y^3dx+\left(x^3y^2-y\right)dy=0.$

◀Bu yerda

$$M(x,y)=x^2y^3 \text{ va } N(x,y)=x^3y^2-y;$$

$$\frac{\partial M}{\partial y}=3x^2y^2 \text{ va } \frac{\partial N}{\partial x}=3x^2y^2.$$

Shunday qilib, $\frac{\partial M}{\partial y} \equiv \frac{\partial N}{\partial x}$, ya'ni berilgan tenglama to'liq differensialli bo'lib, uning chap tomoni haqiqatan ham biror $F(x, y)$ funksiyaning to'liq differensiali bo'lar ekan.

Izlanayotgan $F(x, y)$ funksiyani topish uchun ushbu

$$\frac{\partial F}{\partial x} = x^2 y^3, \quad \frac{\partial F}{\partial y} = x^3 y^2 - y$$

tenglamalardan birinchisini x bo'yicha integrallaymiz

$$F(x,\, y) = \int \frac{\partial F}{\partial x} dx = \int x^2 y^3 dx = \frac{1}{3} x^3 y^3 + g(y).$$

Topilgan $F(x, y)$ funksiyani tenglamalardan ikkinchisiga qo'yamiz:

$$\frac{\partial F}{\partial y} = x^3 y^2 + g'(y) = x^3 y^2 - y.$$

Bu tenglikdan $g(y)$ funksiyani topish qiyin emas: $g(y) = -\frac{1}{2} y^2$. Shunga binoan, $F(x,\, y) \equiv \frac{1}{3} x^3 y^3 - \frac{1}{2} y^2$ bo'ladi. Berilgan tenglamaning umumiy integrali $2x^3 y^3 - 3y^2 = C$ ko'rinishda yoziladi.▶

49. Tenglamani yeching: $\left(2 - 9xy^2\right) x dx + \left(4y^2 - 6x^3\right) y dy = 0.$

◀Tekshirish mumkinki, $\frac{\partial}{\partial y}\left(2x - 9x^2 y^2\right) = \frac{\partial}{\partial x}\left(4y^3 - 6x^3 y\right) = -18x^2 y.$

Shuning uchun qaralayotgan tenglamaning chap tomoni haqiqatan ham qandaydir $F(x, y)$ funksiyaning to'liq differensiali bo'lar ekan. (5) formuladan foydalanamiz:

$$F(x, y) = \int_{x_0}^{x} \left(2t - 9t^2 y^2\right) dt + \int_{y_0}^{y} \left(4t^3 - 6x_0^3\, t\right) dt,$$

bu yerda x_0 va y_0 – ixtiyoriy o'zgarmaslar. Integrallab topamiz:

$$F(x,y)=x^2-3x^3y^2+y^4-x_0^2+3x_0^3y_0^2-y_0^4.$$

Endi esa x_0 va y_0 ixtiyoriy o’zgarmaslardan iborat ifodani $C=x_0^2-3x_0^3y_0^2+y_0^4$ bilan belgilaymiz. Natijada $x^2-3x^3y^2+y^4=C$ umumiy integralni olamiz.►

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M15. To’la differensialli tenglamalarni yeching:

1. $\frac{x^2-3y^2}{x^4}dx+\frac{2y}{x^3}dy=0.$
2. $e^{y/x}\left(1-\frac{y}{x}\right)dx+\left(1+e^{y/x}\right)dy=0.$
3. $x\left(2x^2+y^2\right)dx+y\left(x^2+2y^2\right)dy=0.$
4. $\left(4x^3+6xy^2\right)dx+\left(3y^2+6x^2y\right)dy=0.$
5. $\left(\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{x}-\frac{y}{x^2}\right)dx+\left(\frac{y}{\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{1}{y}\right)dy=0.$
6. $\left(y^3\sec^2x+4x^3+\frac{3x^2}{y^2}\right)dx+\left(3y^2tgx-\frac{2x^3}{y^3}\right)dy=0.$
7. $\left(\frac{y}{x^2}+\frac{1}{y}\right)dx-\left(2y+\frac{1}{x}+\frac{x}{y^2}\right)dy=0.$
8. $\left(x-\frac{\sin^2y}{x^2}\right)dx+\left(\frac{\sin 2y}{x}+y\right)dy=0.$
9. $\left(2x-y+3x^2\right)dx-\left(x+2y-3y^2\right)dy=0.$
10. $\frac{1}{y}dx-\frac{x}{y^2}dy=0.$
11. $\frac{ydx-xdy}{x^2+y^2}=0.$
12. $(2x-y-1)dx+(2y-x+1)dy=0.$

13. $\left(x-\frac{y}{x^2+y^2}\right)dx+\left(y+\frac{x}{x^2+y^2}\right)dy=0.$

14. $\frac{xdx}{\sqrt{y^2-x^2}}-\left(\frac{y}{\sqrt{y^2-x^2}}-1\right)dy=0.$

15. $\left(\frac{1}{2}\sqrt{\frac{y}{x}}+y^2\sin xy^2\right)dx+\left(4+\frac{1}{2}\sqrt{\frac{x}{y}}+2xy\sin xy^2\right)dy=0.$

16. $\left(3-y\cdot 3^{xy}\ln 3\right)dx-x\cdot 3^{xy}\ln 3dy=0.$

17. $\left(7x^6y^3+\frac{1}{x-y}\right)dx+\left(3x^7y^2-\frac{1}{x-y}\right)dy=0.$

18. $\left(\frac{1}{y^2}+y\cos xy\right)dx+\left(\frac{2x}{y^3}+x\cos xy\right)dy=0.$

19. $\frac{ydx}{\sqrt{1-x^2y^2}}+\left(\frac{x}{\sqrt{1-x^2y^2}}-2y\right)dy=0.$

20. $\left(4x^3y^5-3x^2\right)dx+\left(5x^4y^4+28y^6\right)dy=0.$

21. $\left(2xe^{x^2+y^2}-3\right)dx+\left(2ye^{x^2+y^2}+2\right)dy=0.$

22. $\left(2x-3x^2\sin 3y\right)dx-\left(7+3x^3\cos 3y\right)dy=0.$

23. $\left(\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{y}\right)dx+\left(\frac{y}{\sqrt{x^2+y^2}}-\frac{x}{y^2}\right)dy=0.$

24. $\left(y^3+3x^2y\right)dx+\left(x^3+3xy^2\right)dy=0.$

25. $x\left(x^2+y^2-a^2\right)dx+y\left(x^2+y^2+a^2\right)dy=0.$

26. $\left(y\cos x-\cos y+\frac{1}{x}\right)dx+\left(\sin x+x\sin y+\frac{1}{y}\right)dy=0.$

27. $\left(\frac{y}{\cos^2 xy}-\sin x\right)dx+\frac{x+\sin y\cos^2 xy}{\cos^2 xy}dx=0.$

28. $(1-\cos xy)ydx+\left(x+3y^2-x\cos xy\right)dy=0.$

29. $\left(16x+\frac{y}{x^2}e^{y/x}\right)dx+\left(12y^3-\frac{1}{x}e^{y/x}\right)dy=0.$

30. $\frac{e^x}{1+y^2}dx+\frac{2y\left(1-e^x\right)}{\left(1+y^2\right)^2}dy=0.$

M16. To'la differensialli tenglamalarni yeching:

1. $\frac{1}{x}dy-\frac{y}{x^2}dx=0.$

2. $\frac{xdy-ydx}{x^2+y^2}=0.$

3. $\left(2x-y+1\right)dx+\left(2y-x-1\right)dy=0.$

4. $xdx+ydy+\frac{ydx-xdy}{x^2+y^2}=0.$

5. $\left(\frac{x}{\sqrt{x^2-y^2}}-1\right)dx-\frac{ydy}{\sqrt{x^2-y^2}}=0.$

6. $\frac{2x\left(1-e^y\right)}{\left(1+x^2\right)^2}dx+\frac{e^y}{1+x^2}dy=0.$

7. $\frac{2x}{y^3}dx+\frac{y^2-3x^2}{y^4}dy=0.$

8. $\left(e^{\frac{x}{y}}-1\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0.$

9. $x\left(2x^2+y^2\right)+y\left(x^2+2y^2\right)y'=0.$

10. $\left(3x^2+6xy^2\right)dx+\left(6x^2y+4y^3\right)dy=0.$

11. $\left(\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{1}{y}\right)dx+\left(\frac{y}{\sqrt{x^2+y^2}}+\frac{1}{y}-\frac{x}{y^2}\right)dy=0.$

12. $\left(3x^2tgy-\frac{2y^3}{x^3}\right)dx+\left(x^3\sec^2 y+4y^3+\frac{3y^2}{x^2}\right)dy=0.$

13. $\left(\frac{x^2+y^2}{x^2+y}+2x\right)dx=\frac{x^2+y^2}{xy^2}dy.$

14. $\left(\frac{\sin 2x}{y}+x\right)dx+\left(y-\frac{\sin^2 x}{y^2}\right)dy=0.$

15. $\left(3x^2-2x-y\right)dx+\left(2y-x+3y^2\right)dy=0.$

16. $\frac{xdx+ydy}{\sqrt{x^2+y^2}}+\left(\frac{y}{\sqrt{x^2+y^2}}+\frac{1}{y}-\frac{1}{y^2}\right)dy=0.$

17. $\left(3x^2y+y^3\right)dx+\left(x^3+3xy^2\right)dy=0.$

18. $y\left(x^2+y^2+a^2\right)dy+x\left(x^2+y^2-a^2\right)dx=0.$

19. $\left(\sin y+y\sin x+\frac{1}{x}\right)dx+\left(x\cos y-\cos x+\frac{1}{y}\right)dy=0.$

20. $\frac{y+\sin x\cos^2 xy}{\cos^2 xy}dx+\left(\frac{x}{\cos^2 xy}-\sin y\right)dy=0.$

21. $\left(3x^2+y-y\cos xy\right)dx+\left(x-x\cos xy\right)dy=0.$

22. $\left(12x^3-\frac{1}{y}e^{x/y}\right)dx+\left(16y+\frac{x}{y^2}e^{x/y}\right)dy=0.$

23. $\left(\frac{y}{2\sqrt{xy}}+2xy\sin x^2y+4\right)dx+\left(\frac{x}{2\sqrt{xy}}+x^2\sin x^2y\right)dy=0.$

24. $y\cdot 3^{xy}\ln 3dx+\left(x\cdot 3^{xy}\ln 3-3\right)dy=0.$

25. $\left(\frac{1}{x-y}+3x^2y^7\right)dx+\left(7x^3y^6-\frac{1}{x-y}\right)dy=0.$

26. $\left(\frac{2y}{x^3}+y\cos xy\right)dx+\left(x\cos xy-\frac{1}{x^2}\right)dy=0.$

27. $\left(\frac{y}{\sqrt{1-x^2y^2}}-2x\right)dx+\frac{xdy}{\sqrt{1-x^2y^2}}=0.$

28. $\left(5x^4y^4+28x^6\right)dx+\left(4x^5y^3-3y^2\right)dy=0.$

29. $\left(2xe^{x^2+y^2}+2\right)dx+\left(2ye^{x^2+y^2}-3\right)dy=0.$

30. $\left(3y^3\cos 3x+7\right)dx+\left(3y^2\sin 3x-2y\right)dy=0.$

M17. To'la differensialli tenglamalarni yeching:

1. $\left(4x^3y^3+3x^2y^2+2xy\right)dx+\left(3x^4y^2+2x^3y+x^2\right)dy=0.$

2. $\left(4x^3y^2+3x^2y+2x\right)dx+\left(2x^4y+x^3+2y\right)dy=0.$

3. $\left(\ln x+2xy^2\right)dx+\left(2x^2y+\ln y\right)dy=0.$

4. $\left(\cos x\sin y+xe^x\right)dx+\left(\sin x\cos y+ye^y\right)dy=0.$

5. $(\ln x+y)dx+(\ln y+x)dy=0.$

6. $(arctgx+\ln y)dx+\left(\frac{y}{1+y^2}+\frac{x}{y}\right)dy=0.$

7. $\left(2x\sin y+3x^2\right)dx+\left(x^2\cos y+\frac{1}{y}\right)dy=0.$

8. $\left(3x^2e^y+\frac{x}{\sqrt{1+x^2}}\right)dx+\left(x^3e^y+y^3\right)dy=0.$

9. $\left(y+\sin x\cos^2 x\right)dx+\left(x+\sin^3 y\cos y\right)dy=0$

10. $\left(2x\cos\left(x^2+y^2\right)+x^2\right)dx+\left(2y\cos\left(x^2+y^2\right)+y\right)dy=0.$

11. $\left[x(y+2)e^{xy}+2x\right]dx+\left[2x+x^2e^{xy}\right]dy=0.$

12. $(y\cos x+\cos y)dx+(\sin x-x\sin y+2y)dy=0.$

13. $\left(2x\sin y+4x^3\right)dx+\left(x^2\cos y-\sin y\right)dy=0.$

14. $\left(y-x\sqrt{1+x^2}\right)dx+\left(y\sqrt{y^2-1}+x\right)dy=0.$

15. $\left(2xe^{x^2+y}+\cos x\right)dx+\left(e^{x^2+y}-\sin y\right)dy=0.$

16. $\left(\frac{2x-1}{1+x}+2xy\right)dx+\left(\frac{\cos\sqrt{y}}{\sqrt{y}}+x^2\right)dy=0.$

17. $\frac{\cos(\ln x)+\ln y}{x}dx+\frac{\ln x-\sin(\ln y)}{y}dy=0.$

18. $\left(e^{x-y}+y^2+3x^2\right)dx+\left(2xy-e^{x-y}\right)dy=0.$

19. $\left(2xe^{x^2+y^2}+3x^2\right)dx+\left(2ye^{x^2+y^2}-3y^2\right)dy=0.$

20. $\left(4x^3y^2+2xy^3\right)dx+\left(2x^4y+3x^2y^2+4y^3\right)dy=0.$

21. $\left(3x^2y+y^2+2x\right)dx+\left(x^3+2xy\right)dy=0.$

22. $(\sin x+y)dx+\left(y\cos y^2+x\right)dy=0.$

23. $\left(\ln x+e^{x+y}\right)dx+\left(e^x+e^y\right)e^y dy=0.$

24. $\left(2xe^y+y^3e^x+2\right)dx+\left(x^2e^y+3y^2e^x\right)dy=0.$

25. $\left(x^{-1}+2xy^2\right)dx+\left(y^{-1}+2x^2y\right)dy=0.$

26. $\left(4x^3\sin y+2x\cos y\right)dx+\left(x^4\cos y-x^2\sin y\right)dy=0.$

27. $\left(y^2+3x^2y^4+2x\right)dx+\left(2xy+4x^3y^3-3y^2\right)dy=0.$

28. $\left(\frac{y}{x}+\ln y+2x\right)dx+\left(\ln x+\frac{x}{y}+2y\right)dy=0.$

29. $\left(\frac{x^3}{\sqrt{1-x^4}}+y\right)dx+\left(x-\frac{y^2}{\sqrt{1-y^3}}\right)dy=0.$

30. $\left(\sin^2 x+2xy^2\right)dx+\left(2x^2y-\cos^2 y\right)dy=0.$

1.8. INTEGRALLOVCHI KO’PAYTUVCHI

Agar

$$M(x,y)dx+N(x,y)dy=0 \qquad (1)$$

tenglama uchun

$$\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x} \qquad (2)$$

munosabat o’rinli bo’lsa, u holda (1) tenglama to’liq differensialli bo’lmaydi.

Agar shunday $m(x,y)\neq 0$ funksiya topilsaki, $m(x,y)$ funksiyani (1) tenglamaning ikkala tomoniga ko’paytirish natijasida hosil bo’lgan tenglama to'liq differensialli bo'lsa, bunday $m(x,y)$ funksiya (1) tenglamaning *integrallovchi ko'paytuvchisi* deyiladi.

M va N funksiyalar ikkalasi bir vaqtda nolga teng bo'lmagan funksiyalar, ya'ni $M^2+N^2\neq 0$ bo'lsin. Agar M va N - uzluksiz funksiyalar bo'lib, uzluksiz xususiy hosilalarga ega bo'lsa, u holda integrallovchi ko'paytuvchi mavjud bo'ladi (yetarli shart).

Agar $m_0(x,y)$ funksiya (1) tenglamaning integrallovchi ko'paytuvchisi bo'lib, $u_0(x,y)$ esa (1) tenglamaning shu integrallovchi ko'paytuvchiga mos integrali, ya'ni

$$m_0(Mdx+Ndy)=du_0$$

tenglik o'rinli bo'lsa, u holda ixtiyoriy φ differensiallanuvchi funksiya uchun $m=m_0(x,y)\varphi(u_0)$ funksiya ham berilgan tenglamaning integral-lovchi ko'paytuvchisi bo'ladi.

Integrallovchi ko'paytuvchining bu xossasi ko'p hollarda berilgan *tenglamani ikki qismga ajratish usuli* bilan integrallovchi ko'paytuvchini topish imkonini beradi.

Usulning mohiyatini bayon qilamiz. Ushbu ikkita $M_1\,dx+N_1\,dy=0$ va $M_2\,dx+N_2\,dy=0$ tenglamalarning umumiy integrallari va integrallovchi ko'paytuvchilari, mos ravishda, $u_1(x,y)=C_1$, $m_1(x,y)$ va $u_2(x,y)=C_2$, $m_2(x,y)$ ko'rinishda bo'lsin. U holda, yuqoridagi xossaga ko'ra, $m_1^*=m_1\cdot\varphi_1(u_1)$ va $m_2^*=m_2\cdot\varphi_2(u_2)$ funksiyalar, mos ravishda, birinchi va ikkinchi tenglamalarning integrallovchi ko'paytuvchilari bo'ladi. Endi φ_1 va φ_2 funksiyalarni shunday tanlaymizki, $m_1\cdot\varphi_1(u_1)=m_2\cdot\varphi_2(u_2)$ munosabat o'rinli bo'lsin. U holda $m=m_1\cdot\varphi_1(u_1)=m_2\cdot\varphi_2(u_2)$ funksiya

$$(M_1+M_2)dx+(N_1+N_2)dy=0$$

tenglama uchun integrallovchi ko'paytuvchi bo'ladi. Amalda φ_1 va φ_2 funksiyalarni 1 ga teng qilib olish mumkin.

Integrallovchi ko’paytuvchining ta’rifidan ko’rinadiki, agar $m(x,y)$ funksiya (1) tenglamaning integrallovchi ko’paytuvchisi bo’lsa, u holda

$$m(x,y)M(x,y)dx + m(x,y)N(x,y)dy = 0$$

tenglama to’liq differensialli bo’ladi va (2) shartga ko’ra

$$\frac{\partial}{\partial y}\big(m(x,y)M(x,y)\big) \equiv \frac{\partial}{\partial x}\big(m(x,y)N(x,y)\big).$$

munosabat o’rinli bo’ladi. Shunday qilib, integrallovchi ko’paytuvchini

$$m(x,y)\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = N(x,y)\frac{\partial m}{\partial x} - M(x,y)\frac{\partial m}{\partial y} \tag{3}$$

ko’rinishdagi birinchi tartibli xususiy hosilali differensial tenglamaning yechimi sifatida aniqlash mumkin.

Agar $m = m(\omega)$ ekanligi ma’lum bo’lsa (bu yerda $\omega = \omega(x,y)$ - ma’lum differensiallanuvchi funksiya), u holda m integrallovchi ko’paytuvchi ushbu differensial tenglamani qanoatlantiradi:

$$\left(N\frac{\partial \omega}{\partial x} - M\frac{\partial \omega}{\partial y}\right)\frac{dm}{d\omega} = \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)m. \tag{4}$$

Biz birinchi tartibli xususiy hosilali differensial tenglamalar bilan tanish emasmiz. Shuning uchun integrallovchi ko’paytuvchini (3) yoki (4) differensial tenglamalar yordamida topishning ba’zi xususiy hollariga to’xtalamiz. Bunday hollarda $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$ ifodaning ko’rinishi muhimdir. Kelgusida yozuvda qulay bo’lishi uchun

$$k(x,y) = \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$$

belgilash kiritamiz. Shubhasiz, $k(x,y) \neq 0$ va $k(x,y) \neq const$ bo’lishi kerak.

1-hol. Agar $\frac{k(x,y)}{N(x,y)}$ ifoda o’zgarmas son yoki faqat x o’zgaruvchiga bog'liq funksiya bo'lsa, integrallovchi ko'paytuvchi $m(x,y)=m(x)$ ko'rinishda, ya'ni faqat x o'zgaruvchiga bog'liq funksiya bo'ladi va u ushbu

$$\frac{1}{m(x)}\frac{dm(x)}{dx}=\frac{k(x,y)}{N(x,y)}$$

tenglamadan

$$m(x)=\exp\left(\int\psi(x)dx\right),\ \ \psi(x)=\frac{k(x,y)}{N(x,y)}$$

formula bo'yicha topiladi.

2-hol. Agar $\frac{k(x,y)}{M(x,y)}$ ifoda o'zgarmas son yoki faqat y o'zgaruvchiga bog'liq funksiya bo'lsa, u holda integrallovchi ko'paytuvchi $m(x,y)=m(y)$ ko'rinishda, ya'ni faqat y o'zgaruvchiga bog'liq funksiya bo'ladi va u ushbu

$$\frac{1}{m(y)}\frac{dm(y)}{dy}=-\frac{k(x,y)}{M(x,y)}$$

tenglamadan

$$m(y)=\exp\left(\int\varphi(y)dy\right),\ \varphi(y)=-\frac{k(x,y)}{M(x,y)}$$

ko'rinishda topiladi.

3-hol. Agar $\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=N\psi_1(x)-M\psi_2(y)$ tenglikni qanoatlantiruvchi qandaydir $\varphi_1(x)$ va $\varphi_2(y)$ funksiyalar topilsa, u holda integrallovchi ko'paytuvchi $m(x,y)=m_1(x)\cdot m_2(y)$ ko'paytma ko'rinishida bo'ladi. $m_1(x)$ va $m_2(y)$ funksiyalar, mos ravishda,

$$m_1(x)=\exp\left(\int\psi_1(x)dx\right)\ \text{ va }\ m_2(y)=\exp\left(\int\psi_2(y)dy\right)$$

formulalar yordamida topiladi.

4-hol. Agar $M(x,y)$ va $N(x,y)$ funksiyalar bir xil tartibli bir jinsli funksiyalar bo'lsa (bir jinsli funksiyalar haqida **1.3-**bandga qarang), u holda integrallovchi ko'paytuvchi

$$m(x,y)=\frac{1}{x\cdot M(x,y)+y\cdot N(x,y)}$$

ko'rinishda topiladi.

Integrallovchi ko'paytuvchi $m=m(x)$ yoki $m=m(y)$ ko'rinishda ekanligini bilgan holda quyidagi tenglamalarni yeching (**50-51**).

50. $xdx=(xdy+ydx)\sqrt{1+x^2}$

◀Berilgan tenglamani quyidagicha yozib olamiz:

$$\left(y\sqrt{1+x^2}-x\right)dx+x\sqrt{1+x^2}dy=0. \qquad (5)$$

Bu holda

$$M(x,y)=y\sqrt{1+x^2}-x \text{ va } N(x,y)=x\sqrt{1+x^2};$$

$$\frac{\partial M}{\partial y}=\sqrt{1+x^2} \text{ va } \frac{\partial N}{\partial x}=\sqrt{1+x^2}+\frac{x^2}{\sqrt{1+x^2}}.$$

Ko'rinib turibdiki, $\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$, ya'ni berilgan tenglama to'liq differensialli emas. $m(x,y)$ integrallovchi ko'paytuvchini topish maqsadida, avvalo, quyidagi ifodalarni qaraymiz:

$$k(x,y)\equiv\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=-\frac{x^2}{\sqrt{1+x^2}}; \quad \frac{k(x,y)}{N}=-\frac{x}{1+x^2}.$$

Shunga binoan, integrallovchi ko'paytuvchi faqat x o'zgaruvchiga bog'liq funksiya bo'ladi (1-hol):

$$\frac{dm(x)}{m(x)}=-\frac{x}{1+x^2}dm$$

Bu tenglamani integrallab, $m(x)=\frac{C}{\sqrt{1+x^2}}$ funksiyani topamiz. Bizni birorta integrallovchi ko'paytuvchi qiziqtirayotganligi uchun $C=1$ deb olish bilan cheklanamiz:

$$m(x)=\frac{1}{\sqrt{1+x^2}} \qquad (6)$$

Endi (5) tenglamaning ikkala tomoniga (6) integrallovchi ko'paytuvchini ko'paytiramiz. Natijada

$$\left(y-\frac{x}{\sqrt{1+x^2}}\right)dx+xdy=0 \qquad (7)$$

to'liq differensialli tenglama hosil bo'ladi. Haqiqatan ham, agar

$$M^*(x,y)=y-\frac{x}{\sqrt{1+x^2}}\ ,\ \ N^*(x,y)=x$$

deb olsak, u holda $\frac{\partial M^*}{\partial y}=\frac{\partial N^*}{\partial x}=1$ bo'ladi. Demak, (7) tenglamaning chap tomoni biror $F(x,y)$ funksiyaning to'liq differensiali bo'lar ekan. Bu $F(x,y)$ funksiyani topish qiyin emas: $F(x,y)=xy-\sqrt{1+x^2}$. Shunday qilib, berilgan tenglamaning umumiy integrali $xy-\sqrt{1+x^2}=C$ ko'rinishda bo'ladi.►

51. $x^2y(x'y+x)=1.$

◄Berilgan tenglamani quyidagicha yozib olamiz:

$$x^2y^2dx+(x^3y-1)dy=0. \qquad (8)$$

Bu holda

$$M(x,y)=x^2y^2 \text{ va } N(x,y)=x^3y-1;\quad \frac{\partial M}{\partial y}=2x^2y \text{ va } \frac{\partial N}{\partial x}=3x^2y.$$

Berilgan tenglamaning to'liq differensialli emasligi ravshan. Ushbu

$$\frac{k(x,y)}{N} = -\frac{x^2 y}{x^3 y - 1} \text{ va } \frac{k(x,y)}{-M} = \frac{1}{y}$$

tengliklardan ko'rinadiki, integrallovchi ko'paytuvchi faqat y o'zgaruvchiga bog'liq funksiya ekan (2-hol):

$$\frac{1}{m(y)} \frac{dm(y)}{dy} = \frac{1}{y}.$$

Bu tenglamani integrallash natijasida topilgan $m(y) = y$ integrallovchi ko'paytuvchini (8) tenglamaning ikkala tomoniga ko'paytirib,

$$x^2 y^3 dx + \left(x^3 y^2 - y\right) dy = 0$$

ko'rinishdagi to'liq differensialli tenglamani hosil qilamiz. Bu **48-** misolda o'rganilgan tenglamadir. Shuning uchun uning umumiy integrali

$$2x^3 y^3 - 3y^2 = C, \ y \neq 0$$

ko'rinishda bo'ladi. Bu misolda $y = 0$ funksiya berilgan $x^2 y(x'y + x) = 1$ tenglamaning yechimi emasligiga e'tibor berish kerak.▶

52. Integrallovchi ko'paytuvchi $m = m(x+y)$ yoki $m = m(x-y)$ ko'rinishda ekanligini bilgan holda $\left(y - \frac{a\,y}{x} + x\right) dx + a\,dy = 0$ tenglamani integrallovchi ko'paytuvchi usulida yeching.

◀Integrallovchi ko'paytuvchini $m = m(x+y)$ ko'rinishda izlaymiz. U holda (4) formula yordamida topamiz ($\omega = x + y$):

$$\left(a - y + \frac{a\,y}{x} - x\right) \frac{dm}{d\omega} = \left(1 - \frac{a}{x}\right) m,$$

ya'ni

$$\left[(a-x)x - y(x-a)\right] m' = (x-a)\,m;$$

$$(x+y)m' + m = 0;$$

$$\omega m' + m = 0.$$

Shunday qilib, integrallovchi ko'paytuvchi $m=\frac{1}{\omega}$, ya'ni $m=\frac{1}{x+y}$ ga teng. Zarur amallarni bajargandan so'ng berilgan tenglama

$$\left(1-\frac{a\,y}{x(x+y)}\right)dx+\frac{a}{x+y}dy=0$$

ko'rinishdagi to'liq differensialli tenglamaga o'tadi. Bu tenglamadan umumiy integralni topamiz: $e^x\left|1+\frac{y}{x}\right|^\alpha=C.$ ▶

Integrallovchi ko'paytuvchini topib, tenglamalarni yeching (**53-55**).

53. $\frac{dx}{x}+\left(x-\frac{1}{y}\right)dy=0$.

◀Bu tenglamada $M(x,y)=\frac{1}{x}$, $N(x,y)=x-\frac{1}{y}$. Integrallovchi ko'paytuvchi $m=m(x)$ yoki $m=m(y)$ ko'rinishda emasligiga bevosita tekshirish yordamida ishonch hosil qilish mumkin. Shuning uchun uni $m(x,y)=m_1(x)\cdot m_2(y)$ ko'rinishda izlab ko'ramiz (3-hol). Buning uchun berilgan tenglamadan hosil bo'ladigan $\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=-1$ ifoda qandaydir $\psi_1(x)$ va $\psi_2(y)$ funksiyalarda $N(x,y)\psi_1(x)-M(x,y)\psi_2(y)=-1$ tenglikni qanoatlantirishi lozim. Haqiqatan ham, $\psi_1(x)=-\frac{1}{x}$ va $\psi_2(y)=\frac{1}{y}$ funksiyalar $\left(x-\frac{1}{y}\right)\psi_1(x)-\frac{1}{x}\psi_2(y)=-1$ tenglikni qanoatlantirishini ko'rish qiyin emas. Shunday qilib, integrallovchi ko'paytuvchi $m(x,y)=m_1(x)\cdot m_2(y)$ ko'rinishda bo'ladi, bu yerdagi $m_1(x)$ va $m_2(y)$ funksiyalar, mos ravishda,

$$m_1(x)=\exp\left(\int\psi_1(x)dx\right) \text{ va } m_2(y)=\exp\left(\int\psi_2(y)dy\right)$$

formulalar yordamida topiladi: $m_1(x) = 1/x$, $m_2(y) = y$. Demak, $m(x,y) = y/x$.

Bu integrallovchi ko'paytuvchi yordamida berilgan tenglama

$$\frac{ydx}{x^2} + \left(y - \frac{1}{x}\right)dy = 0$$

ko'rinishdagi to'liq differensialli tenglamaga o'tadi. Uning umumiy integralini topish qiyin emas: $x(y^2 - C) = 2y$. ▶

54. $y^3dx - x(x^2 + y^2)dy = 0.$

◀Ko'rinib turibdiki, $M(x,y) = -y^3$ va $N(x,y) = x^3 + xy^2$ funksiyalar bir xil tartibli bir jinsli funksiyalardir. Bu holda integrallovchi ko'paytuvchi $m(x,y) = \left[x \cdot M(x,y) + y \cdot N(x,y)\right]^{-1}$ formula yordamida topiladi: $m(x,y) = x^{-3}y^{-1}$, $x\,y \neq 0$. Mazkur integrallovchi ko'paytuvchi tenglamaning ikkala tomoniga ko'paytirilsa,

$$\frac{y^2}{x^3}dx - \frac{x^2 + y^2}{x^2 y}dy = 0$$

ko'rinishdagi to'liq differensialli tenglama hosil bo'ladi va uning umumiy integralini

$$F(x,y) = \int_{x_0}^{x} \frac{y^2}{t^3}dt - \int_{y_0}^{y} \frac{x_0^2 + t^2}{t\,x_0^2}dt$$

formula yordamida topamiz: $\left(y/x\right)^2 + 2\ln|y| = C$, bu yerda $C = C(x_0,\, y_0)$ – ixtiyoriy o'zgarmas.

$x = 0$ va $y = 0$ funksiyalar berilgan tenglamaning yechimi ekanligini e'tiborga olib, oxirgi natijani yozamiz: $\left(y/x\right)^2 + 2\ln|y| = C$, $x = 0$, $y = 0$.▶

55. $y(x + y^2)dx + x^2(y - 1)dy = 0$.

◀Berilgan tenglamaning to'liq differensialli emasligi ravshan, shuning uchun uni quyidagicha yozib olamiz:

$$y^3dx + x^2ydy + xydx - x^2dy = 0$$

So'ngra ushbu

$$y^3dx + x^2ydy = 0,\ xydx - x^2dy = 0 \qquad (9)$$

tenglamalarni qaraymiz. Bu tenglamalarning integrallovchi ko'paytuvchi-lari va umumiy integrallarini topish uncha qiyin bo'lmaganligi uchun, ularni topishni o'quvchiga havola qilib, natijani yozamiz:

$$m_1(x,y) = \frac{1}{x^2y^3},\quad \frac{xy}{x+y} = C_1;\quad m_2(x,y) = \frac{1}{x^2y},\quad \frac{y}{x} = C_2$$

ko'rinishda bo'ladi. U holda (9) tenglamalarning barcha integrallovchi ko'paytuvchilari, mos ravishda,

$$m_1^*(x,y) = \frac{1}{x^2y^3}\varphi_1\left(\frac{xy}{x+y}\right) \quad \text{va} \quad m_2^*(x,y) = \frac{1}{x^2y}\varphi_2\left(\frac{y}{x}\right)$$

formulalar bilan ifodalanadi, bu yerda φ_1 va φ_2 - ixtiyoriy funksiyalar.

φ_1 va φ_2 funksiyalarning ixtiyoriyligidan foydalanib, ularni shunday tanlaymizki, natijada

$$\frac{1}{x^2y^3}\varphi_1\left(\frac{xy}{x+y}\right) = \frac{1}{x^2y}\varphi_2\left(\frac{y}{x}\right)$$

tenglik bajarilsin. Bundan

$$\varphi_2\left(\frac{y}{x}\right) = \frac{1}{y^2}\varphi_1\left(\frac{xy}{x+y}\right).$$

Aytaylik, $\varphi_1(z) = z^2$ bo'lsin. U holda

$$\frac{1}{y^2}\varphi_1\left(\frac{xy}{x+y}\right) = \frac{1}{y^2}\cdot\left(\frac{xy}{x+y}\right)^2 = \frac{x^2}{(x+y)^2} = \frac{1}{(1+y/x)^2}.$$

Demak, $\varphi_2(t)=(1+t)^{-2}$. Olingan ma'lumotlar asosida dastlabki tenglamaning integrallovchi ko'paytuvchisini yozamiz:

$$m(x,y)=\frac{1}{x^2y}\cdot\frac{1}{(1+y/x)^2}=\frac{1}{y(x+y)^2}.$$

Mana shu integrallovchi ko'paytuvchini dastlabki berilgan tenglamaning ikkala tomoniga ko'paytirib, to'liq differensialli tenglamaga ega bo'lamiz:

$$\frac{x+y^2}{(x+y)^2}dx+\frac{x^2(y-1)}{y(x+y)^2}dy=0.$$

Bu tenglamani integrallash uchun **1.7**-banddagi (5) formuladan foydalanamiz:

$$F(x,y)=\int_{x_0}^{x}\frac{t+y^2}{(t+y)^2}dt+\int_{y_0}^{y}\frac{x_0^2(t-1)}{t(x_0+t)^2}dt \qquad (10)$$

Ma'lumki, bu formuladagi x_0 va y_0- ixtiyoriy sonlardir. Shuning uchun, agar, masalan, $x_0=0$ deb olsak, u holda (10) formuladagi ikkinchi qo'shiluvchi nolga aylanib, $F(x,y)$ ni topish ancha osonlashadi:

$$F(x,y)=\int_{0}^{x}\frac{t+y^2}{(t+y)^2}dt=\ln\left|\frac{x+y}{y}\right|+\frac{x(y-1)}{x+y}.$$

Shunday qilib, dastlabki berilgan tenglamaning umumiy integrali

$$\ln\left|\frac{x+y}{y}\right|+\frac{x(y-1)}{x+y}=C \qquad (11)$$

ko'rinishda bo'ladi. Bundan tashqari, tenglama $x=0$ va $y=0$ trivial yechimlarga ham ega. $x=0$yechim (11) umumiy integralda bor ($C=0$).

Endi oxirgi natijani yozamiz: $\ln\left|\frac{x+y}{y}\right|+\frac{x(y-1)}{x+y}=C$, $y=0$.▶

To'liq differensialli bo'lmagan tenglamalarni yeching (**56-58**).

56. $(x^2+3\ln y)y\,dx = x\,dy.$

◀Bu yerda $\ln y = u$ almashtirish bajarib, $(x^2+3u)\,dx - x\,du = 0$ tenglamani hosil qilamiz va uning integrallovchi ko'paytuvchisini $m=m(x)$ ko'rinishda izlab, $m(x)=x^{-4}$ integrallovchi ko'paytuvchini topamiz. Shundan keyin hosil bo'ladigan

$$\left(\frac{1}{x^2}+\frac{3u}{x^4}\right)dx - \frac{du}{x^3} = 0$$

to'liq differensialli tenglamaning umumiy integrali $x^2+\ln y = Cx^3$ ko'rinishda topiladi. Bundan tashqari, berilgan tenglama $x=0$ trivial yechimga ham ega.▶

57. $y^2dx+(e^x-y)dy=0.$

◀Ketma-ket $e^x=u$ va $u=zy$ almashtirishlar natijasida

$$\frac{y}{u}du+\left(\frac{u}{y}-1\right)dy=0;\;\; ydz+z^2dy=0$$

tenglamalarni hosil qilamiz. Oxirgi tenglamani integrallab va avvalgi o'zgaruvchilarga qaytib, $\ln|y|-ye^{-x}=C$ umumiy integralni olamiz.

Bundan tashqari, tenglama $y=0$ trivial yechimga ham ega.▶

58. $(x^2+1)(2xdx+\cos ydy)=2x\sin ydx.$

◀ $x^2+1=u$ va $\sin y = v$ deb olib, tenglamani

$$(u-v)du+udv=0$$

ko'rinishga keltiramiz va uni $u\neq 0$ bo'lganda

$$\frac{du}{u}+\frac{udv-vdu}{u^2}=0,\text{ ya'ni } d\left(\ln u+\frac{v}{u}\right)=0$$

ko'rinishda yozib olamiz. Bundan $v+u\ln u=Cu$ kelib chiqadi. Xullas,

$(x^2+1)\ln(x^2+1)+\sin y = C(x^2+1).$▶

$M(x,y)dx + N(x,y)dy = 0$ tenglama uchun

$m = m(x,y)$ integrallovchi ko'paytuvchini topish

JAD VALI

№	M va N funksiyalarga shartlar	Integrallovchi ko'paytuvchi
1	$M = y\varphi(xy), \quad N = x\psi(xy)$	$m = 1/\left(xM - yN\right),$
2	$M_x = N_y, \quad M_y = -N_x$	$m = 1/\left(M^2 + N^2\right)$
3	$\dfrac{M_y - N_x}{N} = \varphi(x)$	$m = \exp\left[\int \varphi(x)dx\right]$
4	$\dfrac{M_y - N_x}{M} = \varphi(y)$	$m = \exp\left[-\int \varphi(y)dy\right]$
5	$\dfrac{M_y - N_x}{N - M} = \varphi(x+y)$	$m = \exp\left[\int \varphi(z)dz\right],$ $z = x + y$
6	$\dfrac{M_y - N_x}{yN - xM} = \varphi(xy)$	$m = \exp\left[\int \varphi(z)dz\right], z = xy$
7	$\dfrac{x^2(M_y - N_x)}{yN + xM} = \varphi\left(\dfrac{y}{x}\right)$	$m = \exp\left[-\int \varphi(z)dz\right], \ z = \dfrac{y}{x}$
8	$\dfrac{M_y - N_x}{xN - yM} = \varphi\left(x^2 + y^2\right)$	$m = \exp\left[\dfrac{1}{2}\int \varphi(z)dz\right],$ $z = x^2 + y^2$
9	$M_y - N_x = \varphi(x)N - \psi(y)M$	$m = \exp\left[\int \varphi(x)dx + \int \psi(y)dy\right]$
10	$\dfrac{M_y - N_x}{N\omega_x - M\omega_y} = \varphi(\omega)$	$m = \exp\left[\int \varphi(\omega)d\omega\right]$

Izoh. Jadvalda $\varphi(x)$, $\psi(x)$ - ixtiyoriy funksiyalar, $\omega = \omega(x,y)$ esa ikki o'zgaruvchili ixtiyoriy funksiya.

INDIVIDUAL TOPSHIRIQLAR

M18. Integrallovchi ko'paytuvchini topib, differensial tenglamani yeching:

1. $2xydx+\left(y^2-3x^2\right)dy=0.$

2. $\left(1+e^{x/y}\right)ydx+e^{x/y}\left(y-x\right)dy=0.$

3. $\left(2x^2+y^2\right)dx+\frac{y}{x}\left(x^2+2y^2\right)dy=0.$

4. $\left(3x+6y^2\right)dx+\left(6xy+\frac{4y^3}{x}\right)dy=0.$

5. $\left(\frac{xy^2}{\sqrt{x^2+y^2}}+\frac{y^2}{x}+y\right)dx+\left(\frac{y^3}{\sqrt{x^2+y^2}}+y-x\right)dy=0.$

6. $\left(3x^5tgy-2y^3\right)dx+\left(x^6\sec^2 y+4x^3y^3+3xy^2\right)dy=0.$

7. $\left(2x^3+\frac{x^2}{y}+y\right)dx-\left(\frac{x^3}{y^2}+x\right)dy=0.$

8. $\left(y\sin 2x+xy^2\right)dx+\left(y^3-\sin^2 x\right)dy=0.$

9. $\left(3x-2-\frac{y}{x}\right)dx+\left(2\frac{y}{x}+3\frac{y^2}{x}-1\right)dy=0.$

10. $\left(\frac{x^2}{\sqrt{x^2+y^2}}-\frac{y}{x}\right)dx+\left(\frac{xy}{\sqrt{x^2+y^2}}+1\right)dy=0.$

11. $\left(3x^2+y^2\right)dx+\left(\frac{x^3}{y}+3xy\right)dy=0.$

12. $\left(x^2+y^2+a^2\right)dy+\frac{x}{y}\left(x^2+y^2-a^2\right)dx=0.$

13. $\left(x\sin y+xy\sin x+1\right)dx+\left(x^2\cos y-x\cos x+\frac{x}{y}\right)dy=0.$

14. $\frac{y+\sin x\cos^2 xy}{x\cos^2 xy}dy+\left(\frac{1}{\cos^2 xy}-\frac{\sin y}{x}\right)dy=0.$

15. $\left(3x-\frac{y}{x}\cos xy+\frac{y}{x}\right)dx+\left(1-\cos xy\right)dy=0.$

16. $\left(12x^3y^2-ye^{x/y}\right)dx+\left(16y^3+xe^{x/y}\right)dy=0.$

17. $\left(\frac{4}{x}+\frac{y}{2x\sqrt{xy}}+2y\sin x^2y\right)dx+\left(\frac{1}{2\sqrt{xy}}+x\sin x^2y\right)dy=0.$

18. $3^{xy}\ln 3dx-\left(\frac{3}{y}-\frac{x}{y}\cdot 3^{xy}\ln 3\right)dy=0.$

19. $\left(\frac{1}{(x-y)y^6}+3x^2y\right)dx+\left(7x^3-\frac{1}{(x-y)y^6}\right)dy=0.$

20. $\left(2y+x^3y\cos xy\right)dx+\left(x+x^4\cos xy\right)dy=0.$

21. $\left(\frac{y}{x\sqrt{1-x^2y^2}}-2\right)dx+\frac{dy}{\sqrt{1-x^2y^2}}=0.$

22. $\left(5x^4y+\frac{28x^6}{y^3}\right)dx+\left(4x^5-\frac{3}{y}\right)dy=0.$

23. $\left(2xe^{y^2}+2e^{-x^2}\right)dx+\left(2ye^{y^2}-3e^{-x^2}\right)dy=0.$

24. $\left(\frac{7}{y}+3y^2\cos 3x\right)dx-\left(2-3y\sin 3x\right)dy=0.$

25. $\left(x^2y\cos xy+y\right)dx+\left(x^3\cos xy-x\right)dy=0.$

26. $\frac{dx}{x^2+y^2}+\frac{ydy}{x\left(x^2+y^2\right)}=0.$

27. $\left(2-\frac{y}{x}+\frac{1}{x}\right)dx+\left(2\frac{y}{x}-\frac{1}{x}-1\right)dy=0.$

28. $\left(x^2+\frac{xy}{x^2+y^2}\right)dx+\left(xy-\frac{x^2}{x^2+y^2}\right)dy=0.$

29. $\left(\frac{1}{\sqrt{x^2-y^2}}-\frac{1}{x}\right)dx-\frac{ydy}{x\sqrt{x^2-y^2}}=0.$

30. $2x\left(1-e^{y}\right)dx+\left(1+x^{2}\right)e^{y}dy=0.$

M19. Integrallovchi ko'paytuvchini topib, differensial tenglamani yeching:

1. $\left(\frac{2x-1}{1+x}+2xy\right)\sqrt{y}dx+\left(\cos\sqrt{y}+x^{2}\sqrt{y}\right)dy=0.$

2. $\left(\cos\left(\ln x\right)+\ln y\right)dx+\frac{x\ln x-x\sin\left(\ln y\right)}{y}dy=0.$

3. $\left(e^{x}+y^{2}e^{y}+3x^{2}e^{y}\right)dx+\left(2xye^{y}-e^{x}\right)dy=0.$

4. $\left(2xe^{x^{2}}+3x^{2}e^{-y^{2}}\right)dx+\left(2ye^{x^{2}}-3y^{2}e^{-y^{2}}\right)dy=0.$

5. $\left(4x^{2}y^{2}+2y^{3}\right)dx+\left(2x^{3}y+3xy^{2}+\frac{4y^{3}}{x}\right)dy=0.$

6. $\left(3x^{2}+y+\frac{2x}{y}\right)dx+\left(\frac{x^{3}}{y}+2x\right)dy=0.$

7. $\left(\frac{\sin x}{y}+1\right)dx+\left(\cos y^{2}+\frac{x}{y}\right)dy=0.$

8. $\left(e^{-y}\ln x+e^{x}\right)dx+\left(e^{x}+e^{y}\right)dy=0.$

9. $\left(2xe^{y-x}+y^{3}+2e^{-x}\right)dx+\left(x^{2}e^{y-x}+3y^{2}\right)dy=0.$

10. $\left(1+2x^{2}y^{2}\right)dx+\left(xy^{-1}+2x^{3}y\right)dy=0.$

11. $\left(4x^{3}tgy+2x\right)dx+\left(x^{4}-x^{2}tgy\right)dy=0.$

12. $\left(y+3x^{2}y^{3}+\frac{2x}{y}\right)dx+\left(2x+4x^{3}y^{2}-3y\right)dy=0.$

13. $\left(y+x\ln y+2x^{2}\right)dx+\left(x\ln x+\frac{x^{2}}{y}+2xy\right)dy=0.$

14. $\left(\frac{x^{2}}{\sqrt{1-x^{4}}}+\frac{y}{x}\right)dx+\left(1-\frac{y^{2}}{x\sqrt{1-y^{3}}}\right)dy=0.$

15. $\left(\frac{\sin^2 x}{y}+2xy\right)dx+\left(2x^2-\frac{\cos^2 y}{y}\right)dy=0.$

16. $\left(4x^2y^3+3xy^2+2y\right)dx+\left(3x^3y^2+2x^2y+x\right)dy=0.$

17. $\left(4x^3y+3x^2+\frac{2x}{y}\right)dx+\left(2x^4+\frac{x^3}{y}+2\right)dy=0.$

18. $\left(\frac{\ln x}{y}+2xy\right)dx+\left(2x^2+\frac{\ln y}{y}\right)dy=0$

19. $\left(e^{-x}\cos x\sin y+x\right)dx+\left(e^{-x}\sin x\cos y+ye^{y-x}\right)dy=0.$

20. $\left(x\ln x+xy\right)dx+\left(x\ln y+x^2\right)dy=0.$

21. $\left(arctgx+\ln y\right)ydx+\left(\frac{y^2}{1+y^2}+x\right)dy=0.$

22. $\left(2x\sin y+3x^2\right)ydx+\left(x^2y\cos y+1\right)dy=0.$

23. $\left(3x^2e^y\sqrt{1+x^2}+x\right)dx+\left(x^3e^y+y^3\right)\sqrt{1+x^2}dy=0.$

24. $\left(\frac{2y}{\cos x}+\sin 2x\right)dx+\frac{2x+\sin 2y\cos y}{\cos x}dy=0.$

25. $\left(2\cos\left(x^2+y^2\right)+x\right)dx+\frac{2y\cos\left(x^2+y^2\right)+y}{x}dy=0.$

26. $\left[(y+2)e^{xy}+2\right]dx+\left[2+xe^{xy}\right]dy=0.$

27. $\left(y\sin 2x+2\sin x\cos y\right)dx+\sin x\left(2\sin x-2x\sin y+4y\right)dy=0.$

28. $\left(2\sin y+4x^2\right)dx+\left(x\cos y-\frac{\sin y}{x}\right)dy=0.$

29. $\left(1-\frac{x}{y}\sqrt{1+x^2}\right)dx+\left(\sqrt{y^2-1}+\frac{x}{y}\right)dy=0.$

30. $\left(2xe^{x^2}+e^{-y}\cos x\right)dx+\left(e^{x^2}-e^{-y}\sin y\right)dy=0.$

1.9. HOSILAGA NISBATAN YECHILMAGAN TENGLAMALAR. MAXSUS YECHIMLAR

Hosilaga nisbatan yechilmagan $F(x,y,y')=0$ tenglama quyidagi usullar bilan yechilishi mumkin.

1.9.1. y' hosilaga nisbatan yechib olish usuli. Bunda $F(x,y,y')=0$ tenglamadan y' ni x va y orqali ifodalab olinadi. Natijada $y'=f(x,y)$ ko'rinishdagi bitta yoki bir necha tenglamalar hosil bo'ladi. Bu tenglamalarning har birini yechib, berilgan tenglama yechimlariga ega bo'lamiz.

Tenglamalarni yeching (**59-60**).

59. $y'^2-(y+e^x)y'+5ye^x-2e^{2x}-2y^2=0.$

◀Berilgan tenglamani y' hosilaga nisbatan kvadrat tenglama sifatida yechib, ikkita birinchi tartibli differensial tenglamalarni hosil qilamiz:

$$y'=2y-e^x,\;\; y'=-y+2e^x.$$

Ikkala tenglamalar ham chiziqli tenglamalar bo'lib, ularning umumiy yechimlarini topish qiyin emas:

$$y=e^x+C_1e^{2x},\; y=e^x+C_2e^{-x},$$

bundan

$$e^{-2x}\left(y-e^x\right)=C_1,\;\; e^x\left(y-e^x\right)=C_2$$

umumiy integrallarni topamiz. Endi dastlabki tenglamaning umumiy integralini yozamiz: $\left(e^{-2x}\left(y-e^x\right)-C_1\right)\left(e^x\left(y-e^x\right)-C_2\right)=0.$▶

60. $y'^3-xy'^2-\left(2x^2-xy+y^2\right)y'-2xy(x-y)=0.$

◀Bu tenglamani y' ga nisbatan uchinchi darajali algebraik tenglama sifatida qaraymiz. Bunday tenglamalar nazariyasiga ko'ra, $-2xy(x-y)$ ozod hadning bo'luvchilari berilgan tenglamaning ildizlari bo'lishi mumkin. Shunga asoslangan holda $y'=2x$ ifoda berilgan tenglamaning yechimi ekanligiga bevosita tekshirish bilan ishonch hosil qilamiz. Berilgan tenglamaning qolgan yechimlarini $y'^2+xy+(x-y)\,y=0$ tenglamadan topamiz: $y'=-y$, $y'=y-x$. Hosil bo'lgan uchala differensial tenglamalarning umumiy yechimlari mos ravishda $y=x^2+C_1$, $y=C_2e^{-x}$, $y=C_3e^x+x+1$ ko'rinishlarda topiladi. Shunday qilib, dastlabki berilgan tenglamaning umumiy integrali

$$\left(y-x^2-C_1\right)\left(ye^x-C_2\right)\left(e^{-x}(y-x-1)-C_3\right)=0$$

ko'rinishda yoziladi.►

1.9.2. Parametr kiritish usuli. Maxsus yechimlar. $F(x,y,y')=0$ tenglama har doim y' hosilaga nisbatan yechilavermaydi va y' hosilaga nisbatan yechilgan $y'=f(x,y)$ ko'rinishdagi tenglama(lar) ham ko'p hollarda oson integrallanmaydi. Bunday hollarda hosilaga nisbatan yechilmagan tenglama, odatda, *parametr kiritish usuli* bilan yechiladi. Shu usulni bayon qilamiz.

Hosilaga nisbatan yechilmagan $F(x,y,y')=0$ tenglamani y ga nisbatan yechish, ya'ni $y=f(x,y')$ ko'rinishda yozish mumkin bo'lsin. U holda

$$p=\frac{dy}{dx}=y' \qquad (1)$$

parametr kiritib,

$$y=f(x,p) \qquad (2)$$

ifodani hosil qilamiz. (2) tenglikning ikkala tomonidan to'la differensial olamiz:

$$dy=\frac{\partial f(x,y)}{\partial x}dx+\frac{\partial f(x,p)}{\partial p}dp.$$

So'ngra, (1) ga ko'ra, $dy=p\,dx$ ekanligini e'tiborga olib,

$$pdx = \frac{\partial f(x,p)}{\partial x}dx + \frac{\partial f(x,p)}{\partial p}dp,$$

ya'ni

$$M(x,p)dx + N(x,p)dp = 0$$

ko'rinishdagi differensiallarda yozilgan tenglamani hosil qilamiz, bu yerda $M(x,p) = p - \frac{\partial f(x,p)}{\partial x}$, $N(x,p) = -\frac{\partial f(x,p)}{\partial p}$ – ma'lum funksiyalar.

Agar bu tenglamaning yechimi $x = \varphi(p,C)$ ko'rinishda topilsa, u holda (2) tenglikdan foydalanib, dastlabki tenglamaning yechimi parametrik ko'rinishda yoziladi: $x = \varphi(p,C)$, $y = f\left(\varphi(p,C), p\right)$.

$x = f(y,y')$ ko'rinishdagi tenglama ham xuddi shu kabi yechiladi.

Endi $F(x,y,y') = 0$ tenglama uchun $y(x_0) = y_0$ boshlang'ich shartli Koshi masalasini o'rganamiz. Agar Koshi masalasi yagona yechimga ega bo'lsa, ya'ni (x_0, y_0) nuqta orqali $F(x,y,y') = 0$ tenglamaning faqat bitta yechimi o'tsa, bunday nuqta *oddiy nuqta* deyiladi. Oddiy nuqtaga mos kelgan yechim *oddiy yechim*, integral egri chiziq esa *oddiy integral egri chiziq* deyiladi.

Agar biror (x_0, y_0) nuqtada Koshi masalasi uchun yechimning yagonaligi o'rinli bo'lmasa, ya'ni shu nuqtadan bir xil urinmali bittadan ortiq integral egri chiziqlar o'tsa, bunday nuqta $F(x,y,y') = 0$ tenglamaning *maxsus nuqta*si deyiladi. Maxsus nuqtalarning majmuasi *maxsus to'plam*, maxsus yechimga mos kelgan integral egri chiziq esa *maxsus integral egri chiziq* deyiladi.

Agar $F(x,y,y')$ funksiya x bo'yicha uzluksiz hamda y va y' bo'yicha uzluksiz differensiallanuvchi bo'lsa, u holda

$$F(x,y,y') = 0 \qquad (3)$$

tenglamaning $y = y(x)$ maxsus yechimi (agar maxsus yechim bor bo'lsa) ushbu

$$F(x,\ y,\ y')=0,\ \frac{\partial F(x,\ y,\ y')}{\partial y'}=0 \qquad (4)$$

tenglamalar sistemasini qanoatlantiradi. Shunga ko'ra, (3) tenglamaning maxsus yechimlarini topish uchun (4) tenglamalar sistemasidan y' hosilani yo'qotib, biror $\psi(x,y)=0$ tenglamani hosil qilish kerak. $\psi(x,y)=0$ tenglama *diskriminant egri chiziq tenglamasi* deyiladi. Diskriminant egri chiziqlar (1) tenglamaning yechimi bo'lishi ham, qisman yechimi bo'lishi ham yoki umuman yechimi bo'lmasligi ham mumkin. Shuning uchun diskriminant egri chiziqning har bir tarmog'i (agar u bir necha tarmoq(bo'lak)lardan iborat bo'lsa) berilgan $F(x,y,y')=0$ tenglamaning yechimi bo'lishi yoki bo'lmasligini, agar yechim bo'lsa, uning maxsus yechim bo'lishi yoki bo'lmasligini, ya'ni uning har bir nuqtasiga boshqa yechimlar urinishi yoki urinmasligini alohida tekshirib ko'rish kerak.

Aytaylik, $y=y_1(x,C)$ – (3) tenglamaning umumiy yechimi, $y=y_2(x)$ esa diskriminant egri chiziqlarning (3) tenglamaning yechimi bo'ladigan tarmog'i bo'lsin, ya'ni $y=y_2(x)$ funksiya (4) sistemani qanoatlantirsin. Agar $y=y_2(x)$ funksiya (3) tenglamaning maxsus yechimi bo'lsa, u holda bu funksiya quyidagi sistemani tenglamaning aniqlanish sohasiga tegishli bo'lgan ixtiyoriy x_0 nuqtada qanoatlantiradi:

$$y_1(x_0,C)=y_2(x_0),\ \frac{\partial y_1(x_0,C)}{\partial x}=\frac{dy_2(x_0)}{dx}. \qquad (5)$$

Ushbu $\Phi(x,y,C)=0$ bir parametrli silliq chiziqlar oilasi berilgan bo'lsin, bu yerda C - har xil qiymatlarni qabul qiladigan parametr. Agar biror ℓ chiziq o'zining har bir nuqtasida shu oila chiziqlaridan birortasi bilan umumiy urinmaga ega bo'lsa, u holda ℓ chiziq $\Phi(x,y,C)=0$ *chiziqlar oilasining o'ramasi* deyiladi.

Maxsus yechimlarni (3) tenglamaning $\Phi(x,y,C)=0$ integral egri chiziqlari oilasining o'ramasi yordamida ham topish mumkin. O'rama diskriminant egri chiziq tarkibiga kiradi. Diskriminant egri chiziq esa

$$\Phi(x,y,C)=0,\ \frac{\partial\Phi(x,y,C)}{\partial C}=0 \qquad (6)$$

tenglamalar sistemasidan aniqlanadi.

$y=\varphi(x)$ chiziq diskriminant egri chiziqning biror tarmog'i bo'lsin. Agar $y=\varphi(x)$ chiziqda Φ_x va Φ_y xususiy hosilalar moduli bo'yicha chegaralangan hamda $\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial y}\right)^2\neq 0$ bo'lsa, u holda diskriminant egri chiziqning $y=\varphi(x)$ tarmog'i o'rama bo'ladi.

Tenglamalarning barcha yechimlarini toping; maxsus yechimlarni (agar ular bor bo'lsa) ajrating (**61-65**).

61. $(y'+1)^3=27(x+y)^2$.

◀Bu tenglamada $x+y=u(x)$ deb olib va hosilaga nisbatan yechib, $u'=3u^{2/3}$ tenglamani olamiz, bundan $u=(x+C)^3$, yoki $y=(x+C)^3-x$ umumiy yechim kelib chiqadi. So'ngra ushbu

$$\begin{cases}\Phi(x,\ y,\ C)\equiv x+y-(x+C)^3=0,\\ \dfrac{\partial\Phi(x,\ y,\ C)}{\partial C}\equiv -3(x+C)^2=0\end{cases}$$

tenglamalar sistemasida C ni yo'qotib, $y=-x$ to'g'ri chiziqni, ya'ni

integral egri chiziqlar oilasining diskriminant chizig'ini topamiz. Birinchidan, $y=-x$ to'g'ri chiziq $y=(x+C)^3-x$ integral egri chiziqlar oilasiga tegishli emas, ya'ni C o'zgarmasning hech bir qiymatida bu oiladan $y=-x$ to'g'ri chiziqni hosil qilib bo'lmaydi. Ikkinchidan, $y=-x$ diskriminant chiziqda

$$\left(\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial y}\right)^2\right)_{y=-x}=\left.\left(\left(1-3(x+C)^2\right)^2+1^2\right)\right|_{y=-x}$$

$$=\left.\left(\left(1-3(x+C)^2\right)^2+1^2\right)\right|_{x+C=0}=2\neq 0$$

munosabat o’rinli. Shunday qilib, $y=-x$ to’g’ri chiziq $y=(x+C)^3-x$ oilaning o’ramasi, demak, $y=-x$ to’g’ri chiziq maxsus yechim bo’ladi.►

62. $y'^3+y^2=y\,y'(y'+1).$

◄Tenglamani hosilaga nisbatan yechib, uchta differensial tenglamalarni olamiz: $y'=y,\;\; y'=\pm\sqrt{y}$. Ularni integrallab, berilgan tenglamaning $y=C_1e^x$ va $4y=(x+C_2)^2$ umumiy yechimlarini topamiz.

Diskriminant chiziqni topish uchun $\Phi_1(x,y,C_1)=y-C_1e^x$ yechimlar oilasiga mos kelgan sistemani tuzamiz:

$$\begin{cases}\Phi_1(x,\;y,\;C_1)\equiv y-C_1e^x=0,\\ \dfrac{\partial\Phi_1(x,\;y,\;C_1)}{\partial C_1}\equiv -e^x\neq 0.\end{cases}$$

Ko’rinib turibdiki, bu tenglamalar sistemasi yechimga ega emas. Shuning uchun bu holda diskriminant chiziq mavjud emas.

Endi $\Phi_2(x,y,C_2)=4y-(x+C_2)^2$ oilaga mos sistemani tuzamiz:

$$\begin{cases}\Phi_2(x,\;y,\;C_2)\equiv 4y-(x+C_2)^2=0,\\ \dfrac{\partial\Phi_2(x,\;y,\;C_2)}{\partial C_2}\equiv -2(x+C_2)=0.\end{cases}$$

Bu sistemani yechib, $y=0$ ko’rinishdagi diskriminant chiziqni topamiz. Bundan oldingi misoldagi kabi, birinchidan, $y=0$ to’g’ri chiziq $\Phi_2(x,y,C_2)=4y-(x+C_2)^2$ oilaga tegishli emas, ikkinchidan, $y=0$ chiziqda

$$\left.\left[\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial y}\right)^2\right]\right|_{y=0}=\left.\left[4(x+C_2)^2+16\right]\right|_{y=0}\neq 0\,.$$

Shunday qilib, $y=0$ to’g’ri chiziq $4y=(x+C_2)^2$ oilaning o’ramasi, shunga binoan, $y=0$ to’g’ri chiziq maxsus yechim bo’ladi.►

63. $y'^2 + xy = y^2 + xy'$.

◀Berilgan tenglamani quyidagicha yozib olamiz:

$$(y' - \frac{1}{2}x)^2 - (y - \frac{1}{2}x)^2 = 0, \text{ ya'ni } (y' - y)(y' + y - x) = 0.$$

Tenglamani hosilaga nisbatan yechib, ikkita differensial tenglamalarni olamiz: $y' = y$, $y' = x - y$. Ularni integrallab, berilgan tenglamaning $y = C_1 e^x$ va $y = C_2 e^{-x} + x - 1$ umumiy yechimlarini topamiz.

Yuqoridagi (**62**-misolga qarang) mulohazalarni takrorlab, berilgan tenglama maxsus yechimga ega emasligini ko'rish qiyin emas.

Ko'p hollarda tenglamani maxsus yechimga tekshirish (4) va (5) sistemalar yordamida bevosita (o'ramani topmasdan) amalga oshiriladi. Hozir biz berilgan tenglamani shu usul yordamida o'rganib, tenglama haqida olingan tasdiqning to'g'riligiga yana bir bor ishonch hosil qilamiz.

Berilgan tenglamani maxsus yechimga tekshirish uchun

$$\begin{cases} F(x,\ y,\ y') \equiv y'^2 - xy' + xy - y^2 = 0, \\ \dfrac{\partial F(x, y,\ y')}{\partial y'} \equiv y' - \dfrac{1}{2}x = 0 \end{cases}$$

sistemani tuzamiz. Bu sistemani yechib, $\psi(x, y) \equiv y - 0{,}5x = 0$ diskriminant chiziq (bu yerda to'g'ri chiziq)ni olamiz. Bevosita tekshirish ko'rsatadiki, $y = \frac{1}{2}x$ funksiya berilgan tenglamani qanoatlantirmaydi. O'z-o'zidan ravshanki, bu funksiya maxsus yechim bo'lmaydi.

Shunday qilib, berilgan tenglama umumiy integralining ko'rinishi $(ye^{-x} - C_1)\left((y - x + 1)e^x - C_2\right) = 0$ va maxsus yechimlar yo'q, deb xulosa qilamiz.▶

64. $y(xy' - y)^2 = y - 2xy'$.

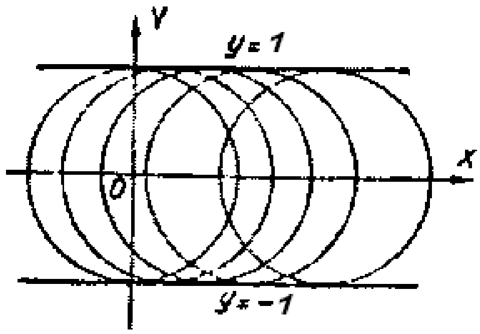
◄ Bu tenglamani $x^2yy'^2 - 2x(y^2-1)y' + y(y^2-1) = 0$ ko'rinishda yozib, uni y' hosilaga nisbatan kvadrat tenglama sifatida yechib, bir necha amallarni bajargandan so'ng quyidagi tenglamaga ega bo'lamiz:

$$\frac{ydy}{\sqrt{1-y^2}\left(\sqrt{1-y^2}\pm 1\right)} = -\frac{dx}{x}\ , \quad y \neq \pm 1,\ y \neq 0$$

Bu tenglamani integrallab, $(Cx+1)^2 + y^2 = 1$ umumiy integralni olamiz. Bundan tashqari, $y = \pm 1$ funksiyalar ham berilgan tenglamaning yechimlaridir. Shuni ta'kidlash kerakki, berilgan tenglamaning umumiy integrali

$$\frac{\left(x+C_1\right)^2}{C_1^{\ 2}} + y^2 = 1,\ C_1 = \frac{1}{C}$$

ko'rinishdagi ellipslar oilasidir (1-rasm).



1-rasm.

Endi maxsus yechimga tekshiramiz. Diskriminant chiziqni topish maqsadida quyidagi sistemani tuzamiz:

$$\begin{cases} F(x,\ y,\ y') \equiv y(xy'-y)^2 - y + 2xy' = 0, \\ F_{y'}(x, y,\ y') \equiv 2xy(xy'-y) + 2x = 0. \end{cases}$$

Bu sistemaning tenglamalaridan y' hosilani yo'qotib, $y = \pm 1$

diskriminant chiziqlarni topamiz. $(Cx+1)^2 + y^2 = 1$ umumiy integralning $x = 0,\ \ y = 0$ nuqtasi ham bu sistemani qanoatlantiradi, lekin bu nuqta diskriminant chiziqlar tarkibiga kirmaydi. Diskriminant chiziqning $y = 1$ tarmog'i berilgan tenglamaning yechimi bo'lishini bevosita tekshirish tasdiqlaydi. Endi $y = 1$ to'g'ri chiziqning har bir nuqtasidan berilgan tenglamaning boshqa yechimlari o'tishini ko'rsatamiz. Buning uchun $y_1 = \sqrt{1-(Cx+1)^2}$ va $y_2 = 1$ deb olib, (5) sistemani tuzamiz:

$$\sqrt{1-\left(Cx_0+1\right)^2} = 1, \qquad \frac{-C\left(Cx_0+1\right)^2}{\sqrt{1-\left(Cx_0+1\right)^2}} = 0\,.$$

Bu sistemani $Cx_0+1=0$ munosabat bilan bog'langan hamma (x_0,C) juftliklar qanoatlantiradi. Bu esa $y=1$ to'g'ri chiziqning ixtiyoriy (x_0,y_0) nuqtasiga $\frac{(x+C_1)^2}{C_1^2}+y^2=1$ ellipslardan bittasi, aniqrog'i $C_1=-x$ bo'lgandagisi urinib o'tishini bildiradi. Xuddi shunday mulohazalarni $y_1=-\sqrt{1-(Cx+1)^2}$ va $y_2=-1$ haqida ham ayta olamiz. Shunday qilib, $y=\pm1$ to'g'ri chiziqlar maxsus yechimlardir.

Xullas, berilgan tenglamaning barcha yechimlari $(Cx+1)^2+y^2=1,$ $y=\pm1$ ko'rinishda bo'lib, bulardan $y=\pm1$ − to'g'ri chiziqlar maxsus yechimlar bo'ladi.►

65. $yy'^2+y'(x-y)-x=0.$

◄Tenglamani $y\neq0$ bo'lganda y' hosilaga nisbatan yechib, $y'=1$ va $y'=-x/y$ tenglamalarga ega bo'lamiz. Bu tenglamalarning o'ng tomonlari va ularning y bo'yicha xususiy hosilalari faqat $y=0$ da uzilishga ega bo'lib, $y=0$ funksiya berilgan tenglamaning yechimi emas. Demak, tenglamaning maxsus yechimlari yo'q.►

Tenglamalarni parametr kiritish usuli bilan yeching (**66-67**).

66. $y=y'^2+2y'^3.$

◄ $y'=p$, ya'ni $p=\frac{dy}{dx}$ parametr kiritib, so'ngra hosil bo'lgan $y=p^2+2p^3$ ifodadan to'la differensial olamiz:

$$dy=2p(1+3p)dp. \quad (7)$$

Endi $dy=p\,dx$ ekanligini e'tiborga olsak, (7) tenglama

$$(dx-2(1+3p)dp)\,p=0$$

ko'rinishni oladi. Bundan $x=2p+3p^2+C$ va $p=0$ yechimlarni olamiz. $p=0$ tenglamaning yechimlari $y=C_1$ to'g'ri chiziqlardan iborat, ammo $y=C_1$ funksiyalar berilgan tenglamani $C_1=0$ bo'lgandagina qanoatlantirishini ko'rish qiyin emas. Shunday qilib, berilgan tenglamaning barcha yechimlari

$$x = 2p + 3p^2 + C,\ y = p^2 + 2p^3;\ y = 0$$

ko'rinishda yoziladi.►

67. $x = y'^3 + y'.$

◄ $y' = p$, ya'ni $p = \frac{dy}{dx}$ parametr kiritib, so'ngra hosil bo'lgan $x = p^3 + p$ ifodadan to'la differensial olamiz:

$$dx = (3p^2 + 1)dp. \tag{8}$$

Endi $dx = \frac{1}{p}dy$ ekanligini e'tiborga olsak, (8) tenglama

$$\frac{1}{p}dy - (3p^2 + 1)dp = 0$$

ko'rinishni oladi. O'zgaruvchilari ajraladigan bu tenglamaning umumiy yechimi $4y = 3p^4 + 2p^2 + C$ ko'rinishda topiladi.

Shunday qilib, berilgan tenglamaning umumiy yechimi parametrik ko'rinishda yoziladi: $x = p^3 + p$, $4y = 3p^4 + 2p^2 + C$.►

68. Ushbu

$$y'^2 = |y|^m,\ \ m = const \tag{9}$$

tenglamani yeching. m parametrning qanday qiymatlarida tenglama maxsus yechimga ega bo'ladi?

◄Aniqlik uchun, $y \ge 0$ bo'lsin. Bu holda (9) tenglama

$$y'^2 = y^m \tag{10}$$

ko'rinishda yoziladi.

(10) tenglamani yechish m parametrning qiymatlariga bog'liqliligi tushunarli. Haqiqatan ham, masalan, $y = 0$ funksiya $m > 0$ bo'lganda (10)

tenglamaning yechimi, $m \le 0$ bo'lganda esa yechimi emas. (10) tenglamaning qolgan yechimlarini topish uchun uni quyidagicha yozib olamiz:

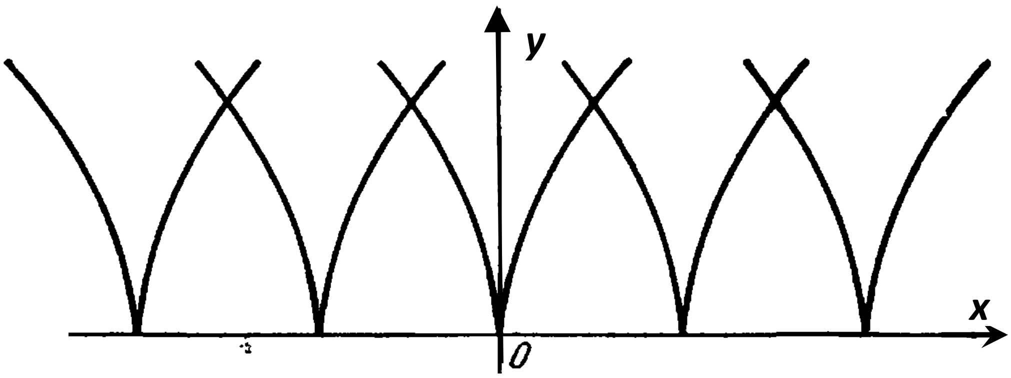
$$(y' - y^{m/2})(y' + y^{m/2}) = 0.$$

Bu yerda bir necha hol bo'lishi mumkin.

1-hol. $m \le 0$ bo'lsin. Bu holda, yuqorida aytilganidek, $y = 0$ funksiya (10) tenglamaning yechimi emas. $y' \pm y^{m/2} = 0$ tenglamalarning umumiy integrallari

$$x \pm \frac{2}{2-m} y^{(2-m)/2} = C \qquad (11)$$

ko'rinishda yoziladi (2-rasm).



2-rasm.

Maxsus yechimlarni topish uchun $F(x, y, y') = y'^2 - y^m$ belgilash kiritib,

$$F(x,\ y,\ y') \equiv y'^2 - y^m = 0,\ \frac{\partial F(x, y,\ y')}{\partial y'} \equiv 2y' = 0 \qquad (12)$$

sistemani tuzamiz.

$m \le 0$ bo'lganligi uchun bu sistema yechimga ega emas. Shunday qilib, $m \le 0$ bo'lganda (10) tenglamaning maxsus yechimi yo'q.

2-hol. $0 < m < 2$ bo'lsin. Bu holda (10) tenglamaning umumiy integrali (11) ko'rinishda bo'ladi. $m > 0,\ m \ne 2$ bo'lganda, $m \le 0$ holdagidan farqli ravishda, (12) sistemani yechib topiladigan $y = 0$

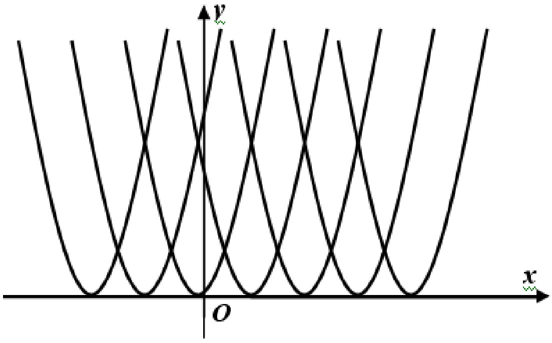
diskriminant chiziq (10) tenglamani qanoatlantiradi. Demak, $y=0$ to'g'ri chiziq maxsus yechim bo'lib qolishi mumkin. Buni aniqlash maqsadida

$$y_1(x)=\left(\frac{2-m}{2}\right)^{2/(2-m)}\left(C\pm x\right)^{2/(2-m)},\ y_2(x)=0$$

funksiyalar uchun (5) shartlarning bajarilishini tekshiramiz. Haqiqatan ham,

$$\begin{cases}\left(\frac{2-m}{2}\right)^{2/(2-m)}\left(C\pm x_0\right)^{2/(2-m)}=0,\\ \pm\left(\frac{2-m}{2}\right)^{m/\ (2-m)}\left(C\pm x_0\right)^{m/\ (2-m)}=0.\end{cases}$$

Bu sistemani $0<m<2$ bo'lganda $C\pm x_0=0$ munosabat bilan bog'langan hamma (x_0, C) juftliklar qanoatlantiradi. Bu esa o'z navbatida $y=0$ to'g'ri chiziqning ixtiyoriy nuqtasiga (11) chiziqlardan $0<m<2$ va $C=\pm x_0$ bo'lgandagisi urinib o'tishini bildiradi (3-rasm). Shunday qilib, $0<m<2$ bo'lganda $y=0$ to'g'ri chiziq (11) chiziqlar oilasining o'ramasi, ya'ni maxsus yechim bo'ladi.

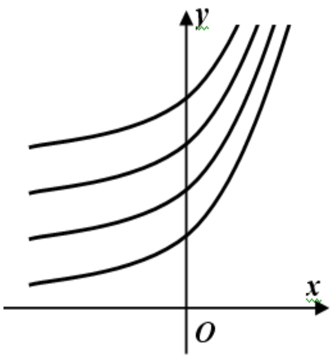


3-rasm.

3-hol. $m=2$ bo'lsin. Bu holda $y'^2=y^2$ tenglamaning umumiy yechimi $y=Ce^{\pm x}$ ko'rinishda bo'ladi (4-rasm). Ushbu

$$y'^2-y^2=0,\ 2y'=0$$

sistemadan y' ni yo'qotib, diskriminant chiziqni topamiz: $y=0$. Shu $y=0$ to'g'ri chiziq maxsus yechim bo'lib qolishi mumkin. Buni aniqlash



4-rasm.

maqsadida $y_1(x)=Ce^{\pm x}$ $(C\neq 0)$ va $y_2(x)=0$ deb olib, (5) sistemani tuzamiz:

$$Ce^{\pm x_0}=0,\ \ \pm Ce^{\pm x_0}=0.$$

Bu sistema $C\neq 0$ da yechimga ega emas. Demak, $m=2$ bo'lganda $y'^2=y^2$ tenglamaning maxsus yechimi yo'q.

4-hol. Endi $m>2$ bo'lsin. Bu holda (10) tenglama

$$y=\left(\frac{2}{m-2}\right)^{2/(m-2)}\cdot\frac{1}{\left(x\pm C\right)^{2/(m-2)}}$$

ko'rinishdagi umumiy yechimga ega bo'ladi. Yuqoridagi mulohazalarni takrorlab, (10) tenglama $m>2$ bo'lganda maxsus yechimga ega emasligini aniqlash qiyinchilik tug'dirmaydi.

(9) tenglamani $y\leq 0$ bo'lganda tadqiq qilish yuqoridagi kabi amalga oshiriladi.

Shunday qilib, (9) tenglama $m\neq 2$ da $x\pm\dfrac{2}{2-m}|y|^{(2-m)/2}=C,\ \ y=0$; $m=2$ da $y=Ce^{\pm x}$ yechimlarga ega; $0<m<2$ da $y=0-$ maxsus yechim; boshqa hollarda maxsus yechim yo'q.►

69. Agar biror differensial tenglama yechimlarining $xy=Cy-C^2$ oilasi berilgan bo'lsa, shu tenglamaning maxsus yechimini toping.

◄ $\Phi(x,y,C)=xy-Cy+C^2$ funksiya uzluksiz differensiallanuvchi bo'lganligi uchun $xy-Cy+C^2=0$ integral egri chiziqlar oilasining diskriminant chizig'i

$$\Phi(x,\ y,\ C)\equiv xy-Cy+C^2=0,$$

$$\frac{\partial\Phi(x,\ y,\ C)}{\partial C}\equiv -y+2C=0$$

tenglamalar sistemasini qanoatlantiradi. Ikkinchi tenglamadan $C=y/2$ ni topib va uni birinchisiga qo'yib, $y^2-4xy=0$ diskriminant chiziqni olamiz. Bu diskriminant chiziq ikkiga ajraydi: $y=4x$ va $y=0$. Birinchi chiziqda

$$\left.\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial y}\right)^2\right|_{y=4x}=\left.\left(y^2+(x-C)^2\right)\right|_{y=4x}=$$

$$=\left.\left(y^2+\left(x-\frac{y}{2}\right)^2\right)\right|_{y=4x}=17x^2\neq 0,$$

ikkinchi chiziqda esa

$$\left.\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial y}\right)^2\right|_{y=4x}=\left.\left(y^2+(x-C)^2\right)\right|_{y=0}=$$

$$=\left.\left(y^2+\left(x-\frac{y}{2}\right)^2\right)\right|_{y=0}=x^2\neq 0$$

$y=4x$ to'g'ri chiziq C o'zgarmasning hech bir qiymatida $\Phi(x,y,C)\equiv xy-Cy+C^2=0$ chiziqlar oilasiga tegishli bo'lmaganligi uchun bu to'g'ri chiziq tegishli differensial tenglamaning maxsus yechimi bo'ladi. Ikkinchi $y=0$ $(x\neq 0)$ to'g'ri chiziq esa bu yechimlar oilasiga tegishli, shuning uchun uning maxsus yechim bo'lishi yoki bo'lmasligi masalasi hozircha hal emas. Bu savolga javob berish uchun oilaning egri chiziqlarini

$$y=\frac{C^2}{C-x},\ x\neq C \tag{13}$$

ko'rinishda tasvirlaymiz. $C\neq 0$ da $y'(x_0)\neq 0$ bo'lganligi uchun, (13) oilaning chiziqlari $y=0$ $(x\neq 0)$ to'g'ri chiziqqa urinmaydi. Demak,

$y=0\,(x\neq 0)$ to'g'ri chiziq, ta'rifga ko'ra, maxsus yechim emas.

Demak, yechimlarining oilasi $xy=Cy-C^2$ ko'rinishda bo'lgan differensial tenglamaning maxsus yechimi $y=4x$ to'g'ri chiziqdan iborat ekan.►

INDIVIDUAL TOPSHIRIQLAR

M20. Quyidagi tenglamalarni avval y' hosilaga nisbatan yechib olib, so'ngra hosil bo'lgan tenglama(lar)ning umumiy yechimini ma'lum usullar yordamida toping; maxsus yechimlarni (agar ular bor bo'lsa) toping.

1. $y(y')^2 + 2xy' = y.$
2. $(y')^2 + xy' - x^2 = 0.$
3. $x^3 + (y')^2 = x^2.$
4. $(y')^2 - y^2 = 0.$
5. $8(y')^3 = 27y.$
6. $y^2(y'^2 + 1) = 1.$
7. $(y')^2 - 4y^3 = 0.$
8. $(y')^2 = 4y^3(1-y).$
9. $x(y')^2 = y.$
10. $y(y')^3 + x = 1.$
11. $4(1-y) = (3y-2)^2(y')^2.$
12. $xy'(xy' + y) = 2y^2.$
13. $x(y')^2 - 2yy' + x = 0.$
14. $x(y')^2 = y(2y' - 1).$
15. $(y')^2 + x = 2y.$
16. $(y')^3 + (x+2)e^y = 0.$
17. $(y')^2 - 2xy' = 8x^2.$
18. $(xy' + 3y)^2 = 7x.$
19. $(y')^2 - 2yy' = y^2(e^x - 1).$
20. $y'(2y - y') = y^2 \sin^2 x.$
21. $(y')^4 + y^2 = y^4.$
22. $x(y - xy')^2 = x(y')^2 - 2yy'.$
23. $yy'(yy' - 2x) = x^2 - 2y^2.$
24. $(y')^2 + 4xy' - y^2 - 2x^2y = x^4 - 4x^2.$
25. $y(y - 2xy')^2 = 2y'.$
26. $yy'^2 - 2xy' + y = 0.$
27. $y'^2 - (y - x)yy' - xy^3 = 0.$
28. $x^4y'^2 - xy' - y = 0.$
29. $y'^2 + (\sin x - 2xy)y' - 2xy\sin x = 0.$
30. $y'^2 - 2xy'^2 + y' = 2x.$

M21. Quyidagi tenglamalarni $y' = p$ paramatr usulida yeching; maxsus yechimlarni (agar ular bor bo'lsa) toping.

1. $y\sqrt{1 + (y')^2} = y'.$
2. $x(1 + y'^2) = 1.$

3. $y = y'^2 + 2y'^3.$

4. $x = 6y' + 4\sqrt{1+y'^2}.$

5. $x = 2y' + 3y'^2.$

6. $y = y' + \sqrt{1+y'^2}.$

7. $x = y'\sin y'.$

8. $y = y'^2 e^{y'}.$

9. $x = y'\sqrt{1+y'^2}.$

10. $x\left(y'^2 - 1\right) = 2y'.$

11. $y'\left(x - \ln y'\right) = 1.$

12. $y = \ln\left(1+y'^2\right).$

13. $\left(y'+1\right)^3 = \left(y'-y\right)^2.$

14. $y = \left(y'-1\right)e^{y'}.$

15. $y'^4 - y'^2 = y^2.$

16. $y'^2 - y'^3 = y^2.$

17. $y'^4 = 2yy' + y^2.$

18. $y'^2 - 2xy' = x^2 - 4y.$

19. $5y + y'^2 = x\left(x + y'\right).$

20. $x^2 y'^2 = xyy' + 1.$

21. $y'^3 + y^2 = xyy'.$

22. $2xy' - y = y'\ln yy'.$

23. $y' = e^{xy'/y}.$

24. $y = xy' - x^2 y'^3.$

25. $y\left(y - 2xy'\right)^3 = y'^2.$

26. $y = 2xy' + y^2 y'^3.$

27. $x = 8y' + 10\sqrt{1+y'^2}.$

28. $x = 4y' + 9y'^2.$

29. $x = 2y' + 14\sqrt{1+y'^2}.$

30. $x = 8y' + 9y'^2.$

1.10. LAGRANJ VA KLERO TENGLAMALARI.

Hosilaga nisbatan yechilmagan tenglamalarga Lagranj va Klero tenglamalari misol bo'la oladi.

Ushbu

$$y = x\varphi(y') + \psi(y') \qquad (1)$$

ko'rinishdagi differensial tenglama *Lagranj tenglamasi* deyiladi, bu yerda $\varphi(y')$ va $\psi(y')$ – biror intervalda differensiallanuvchi funksiyalar.

Agar (1) Lagranj tenglamasida $\varphi(y') \equiv y'$ bo'lsa, *Klero tenglamasi* deb ataluvchi

$$y = xy' + \psi(y') \qquad (2)$$

tenglama hosil bo’ladi.

Klero tenglamasini qaraymiz. $y' = p$ deb olsak, (2) tenglama

$$y = xp + \psi(p) \tag{3}$$

ko’rinishga keladi.

(3) tenglamaning ikkala tomonini x bo’yicha differensiallab, so’ngra $dy = pdx$ ekanligini e’tiborga olsak,

$$p = x\frac{dp}{dx} + p + \psi'(p)\frac{dp}{dx}, \text{ ya'ni } \left[x + \psi'(p)\right]\frac{dp}{dx} = 0$$

tenglamaga ega bo’lamiz. Oxirgi tenglamani yechish ushbu

$$\frac{dp}{dx} = 0, \; x + \psi'(p) = 0 \tag{4}$$

sodda tenglamalarni yechishga teng kuchli.

Bu (4) tenglamalardan birinchisini integrallashdan topilgan $p = C$ $(C = const)$ yechimni (3) ifodaga qo’yib,

$$y = Cx + \psi(C) \tag{5}$$

*Klero tenglamasining umumiy yechimi*ni topamiz. (2) Klero tenglamasining (5) ko’rinishdagi yechimlari geometrik nuqtai nazardan to’g’ri chiziqlar oilasini tasvirlaydi.

Agar (4) tenglamalarning ikkinchisidan p parametrni x o’zgaruvchining funksiyasi sifatida topib va $p = p(x)$ funksiyani (3) ifodaga qo’ysak, u holda

$$y = xp(x) + \psi\left(p(x)\right) \tag{6}$$

funksiya hosil bo’ladi. (6) ifodadagi $p(x)$ funksiya $x + \psi'(p) = 0$ shartni qanoatlantiridigan biror funksiya ekanligini bilgan holda (6) funksiya Klero tenglamasining yechimi bo’lishini ko’rsatish qiyin emas.

(6) yechim (5) umumiy yechimdan C o’zgarmasning hech bir qiymatida hosil bo’lmaydi. Yuqorida bayon qilingan maxsus yechimlar

nazariyasini qo'llab, (6) ko'rinishdagi $y = xp(x) + \psi[p(x)]$ yechim (2) Klero tenglamasining maxsus yechimi bo'lishini ko'rsatish qiyin emas. Bu maxsus yechim $y = xp + \psi(p)$ va $x + \psi'(p) = 0$ tenglamalardan p parametrni yoki $y = Cx + \psi(C)$ va $x + \psi'(C) = 0$ tenglamalardan C parametrni yo'qotish natijasida hosil bo'ladi. Shunday qilib, Klero tenglamasining maxsus yechimi (5) formula bilan berilgan integral to'g'ri chiziqlar oilasining o'ramasini aniqlaydi (5-rasm).

Lagranj tenglamasi ham $y' = p$ parametr kiritish yo'li bilan yechiladi. Buning natijasida (1) Lagranj tenglamasi

$$y = x\varphi(p) + \psi(p) \tag{7}$$

ko'rinishni oladi. (7) ifodaning ikkala tomonini x bo'yicha differensial-lab,

$$p - \varphi(p) = \left[x\varphi'(p) + \psi'(p)\right]\frac{dp}{dx} \tag{8}$$

tenglamani hosil qilamiz. Bu tenglamadan ba'zi yechimlarni birdaniga topish mumkin: bu tenglama p parametrning $p_i - \varphi(p_i) = 0$ shartni qanoatlantiradigan har qanday o'zgarmas $p = p_i$ qiymatida ayniyatga aylanadi. Aytaylik, $p = p_i$ son $p - \varphi(p) = 0$ funksional tenglamaning yechimi bo'lsin. U holda

$$y = x\varphi(p_i) + \psi(p_i) \tag{9}$$

funksiya (1) Lagranj tenglamasining yechimi bo'lishini bevosita tekshirib ko'rish mumkin.

Endi umumiy yechimni topamiz. Buning uchun (8) tenglamani

$$\frac{dx}{dp} - \frac{\varphi'(p)}{p - \varphi(p)}x = \frac{\varphi'(p)}{p - \varphi(p)}$$

ko'rinishda yozib olamiz va x ni p ning funksiyasi deb qaraymiz. Hosil bo'lgan tenglama p o'zgaruvchining x funksiyasiga nisbatan chiziqli differensial tenglama bo'ladi. Bu tenglamani yechib, $x = \Phi(p, C)$ umumiy yechimni topamiz. $x = \Phi(p, C)$ va $y = x\varphi(p) + \psi(p)$ ifodalardan p

parametrni yo’qotib, (1) Lagranj tenglamasining umumiy integralini $\Psi(x,y,C)=0$ ko’rinishda hosil qilamiz. Agar p parametrni yo’qotib bo’lmasa, u holda (1) tenglamaning umumiy yechimi parametr ko’rinishida yoziladi: $x=\Phi(p,C)$, $y=x\varphi(p)+\psi(p)$. Lagranj tenglamasining yechimlari qatoriga (9) ko’rinishdagi $y=x\varphi(p_i)+\psi(p_i)$ to’g’ri chiziqlarni ham (agar shunday yechimlar bor bo’lsa) qo’shib qo’yish kerak.

Klero va Lagranj tenglamalarini yeching (**70-73**).

70. $y'^3=3(xy'-y)$.

◀ Bu Klero tenglamasi: $y=xy'-\frac{1}{3}y'^3$; $\psi(y')\equiv-\frac{1}{3}y'^3$.

Tenglamani parametr kiritish usuli bilan yechamiz:

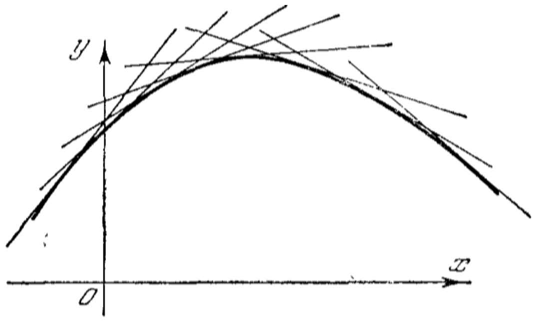
$$y'=p; y=px-\frac{1}{3}p^3;\ \ dy=p\,dx;\ (x-p^2)\frac{dp}{dx}=0.$$

Agar $\frac{dp}{dx}=0$ bo’lsa, $p=C$ bo’ladi. Uni $y=px-\left(p^3/3\right)$ ifodaga qo’ysak, u holda $y=Cx-\left(C^3/3\right)$, ya’ni $C^3=3(Cx-y)$ ko’rinishdagi bir parametrli integral to’g’ri chiziqlar oilasiga ega bo’lamiz. Bu berilgan tenglamaning yechimi bo’ladi.

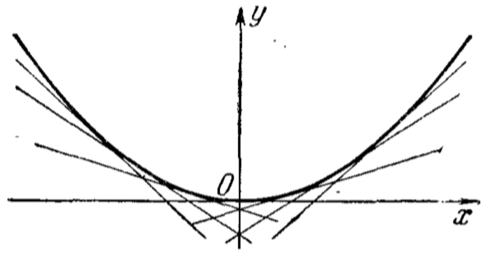
Endi $x-p^2=0$ va $p=C$ tenglamalardan $x-C^2=0$ munosabatni olamiz. So’ngra $3y=3Cx-C^3$ bir parametrli integral to’g’ri chiziqlar oilasining o’ramasini topish uchun $3y=3Cx-C^3$ va $x-C^2=0$ tenglamalardan C ni yo’qotamiz: $9y^2=4x^3$ – o’rama tenglamasi. Shunday qilib, berilgan tenglamaning hamma yechimlari

$$C^3=3(Cx-y);\ \ 9y^2=4x^3$$

ko’rinishda bo’ladi.▶



5-rasm.



6-rasm.

71. $y = xy' - y'^2$.

◀Bu ham Klero tenglamasi. Integral to'g'ri chiziqlarning oilasi $y = Cx - C^2$ ko'rinishda bo'ladi. Bundan tashqari, bu oilaning $y = Cx - C^2$ va $x - 2C = 0$ tenglamalardan topiladigan o'ramasi ham integral egri chiziq bo'ladi. C ni yo'qotib, $4y = x^2$ o'ramani topamiz (6-rasm).▶

72. $y = 2xy' - y'^3$.

◀Bu Lagranj tenglamasi: $\varphi(y') = 2y'$; $\psi(y') = y'^3$. $y' = p$ parametr kiritib,

$$y = 2px - p^3 \tag{10}$$

ifodani olamiz. Uni differensiallab, topamiz:

$$p = 2p + 2x\frac{dp}{dx} - 3p^2\frac{dp}{dx}. \tag{11}$$

Bu tenglikni $\frac{dp}{dx}$ ga bo'lgandan keyin hosil bo'lgan

$$p\frac{dx}{dp} = -2x + 3p^2 \tag{12}$$

chiziqli tenglamani integrallab, $x = \frac{C}{p^2} + \frac{3}{4}p^2$ umumiy yechimni olamiz. Shunday qilib, berilgan tenglamaning integral egri chiziqlari

$$y = 2px - p^3,\ x = \frac{C}{p^2} + \frac{3}{4}p^2$$

tenglamalar bilan aniqlanadi.

(12) chiziqli tenglamani hosil qilishda (11) tenglamaning ikkala tomoni $p - \varphi(p) \neq 0$ ifodaga bo'linganligi uchun, $p - \varphi(p) = 0$ bo'lganda tenglamaning yechimlari yo'qolishi mumkin. Shuning uchun, $p - \varphi(p) = 0$ tenglamaning $p = p_i$ ildizlarini topish kerak. Biz qarayotgan tenglamada $\varphi(p) = 2p$, shunga binoan, $p - \varphi(p) \equiv p - 2p = 0$, bundan $p = 0$ ni topib, (10) ga qo'yamiz: $y = 0$ - yechim. Shunday qilib, berilgan tenglamaning hamma yechimlari

$$x = \frac{C}{p^2} + \frac{3}{4}p^2,\ y = 2px - p^3;\ \ y = 0. \blacktriangleright$$

73. $y = 2xy' - \ln y'$.

◄Bu Lagranj tenglamasi: $\varphi(y') = 2y';\ \psi(y') = -\ln y'$. Endi $y' = p$ parametr kiritib, $y = 2px - \ln p$ ifodaga ega bo'lamiz. Aniqlanishiga ko'ra, $p > 0$. Hosil bo'lgan

$$\frac{dx}{dp} + \frac{2}{p}x = p^{-2}$$

chiziqli differensial tenglamaning umumiy yechimi $x = (p + C)p^{-2}$ ko'rinishda bo'ladi. Shunday qilib, berilgan tenglamaning integral egri chiziqlari

$$y = 2px - \ln p,\ x = (p + C)p^{-2}$$

tenglamalar bilan aniqlanadi. Buni unga teng kuchli bo'lgan

$$xp^2 = p + C,\ \ y = 2 + 2Cp^{-1} - \ln p$$

ko'rinishda ham yozsa bo'ladi.

72-misoldagi kabi, $p - \varphi(p) = 0$ tenglamani o'rganamiz. Bu yerda ham $p - \varphi(p) \equiv p - 2p = 0$ bo'lib, $p = 0$ bo'lishi kerak, ammo berilgan tenglama aniqlanishiga ko'ra $p>0$ bo'lganligi uchun, $p-\varphi(p)=0$ tenglama yechimga ega emas.►

74. Quyidagi xossaga ega bo'lgan chiziqni toping: Bu chiziqqa o'tkazilgan har bir urinma koordinatalar o'qlarida shunday kesmalarni ajratadiki, bu kesmalarning uzunliklariga teskari miqdorlar kvadratlarining yig'indisi 1 ga teng.

◄$M(x,y)$ nuqtaga o'tkazilgan urinma tenglamasiga ko'ra, urinmaning Ox o'q bilan kesishish nuqtasining abssissasi $x_0=x-(y/y')$ va Oy o'q bilan kesishish nuqtasining ordinatasi $y_0=y-xy'$ ga teng. Masala shartiga ko'ra,

$$\frac{1}{x_0^2}+\frac{1}{y_0^2}=1, \text{ ya'ni } \frac{y'^2}{(xy'-y)^2}+\frac{1}{(y-xy')^2}=1.$$

Oxirgi tenglamani y ga nisbatan yechib va $y'=p$ deb olib,

$$y=xp\pm\sqrt{1+p^2} \tag{13}$$

ifodaga ega bo'lamiz. (13) tenglamaning ikkala tomonini differensiallaymiz. Bunda $dy=p\,dx$ tenglikni e'tiborga olsak,

$$\left(x\pm p/\sqrt{1+p^2}\right)dp=0$$

tenglama hosil bo'ladi, bundan $p=C$ va $x=\mp p/\sqrt{1+p^2}$ yechimlarni olamiz. p ning qiymatini (13) ga qo'ysak, $y=Cx\pm\sqrt{1+C^2}$ ko'rinishidagi to'g'ri chiziqlarni hosil qilamiz. To'g'ri chiziqlar trivial yechim bo'lganligi uchun, ularni qaramaymiz. Endi x ning qiymatini (13) ga qo'yamiz va sodda almashtirishlardan keyin $y=\pm1/\sqrt{1+p^2}$ ifodani olamiz. Xullas, qo'yilgan masalaning yechimi $x=\mp p/\sqrt{1+p^2}$, $y=\pm1/\sqrt{1+p^2}$ parametrik ko'rinishda topildi. Bu tenglamalardan p ni yo'qotib $x^2+y^2=1$ aylana tenglamasini hosil qilamiz. Demak, masalada aytilgan xossaga ega bo'lgan chiziq birlik aylana ekan.►

INDIVIDUAL TOPSHIRIQLAR

M22. Lagranj va Klero tenglamalarini yeching.

1. $y = xy'^2 + 2y'$.
2. $y = 2xy' + y'^2$.
3. $y = xy'^2 + y'^3$.
4. $y = xy' - y'^2$.
5. $y = xy' + \arcsin y'$.
6. $y = xy' + y' - y'^2$.
7. $y = xy' + \dfrac{2}{y'}$.
8. $y = xy' + \sqrt{1 + y'^2}$.
9. $x = \dfrac{y}{y'} + \dfrac{1}{y'^2}$.
10. $y = 2y'^2 + (x-1)y'$.
11. $y' = 3xy' - 7y'^3$.
12. $y + xy' = 4\sqrt{y'}$.
13. $y = xy' - (y' + 2)$.
14. $x(y'^2 + 1) = 2yy'$.
15. $y = xy'^2 - 2y'^3$.
16. $xy' - y = \ln y'$.
17. $xy'(y' + 2) = y$.
18. $2y'^2(y - xy') = 1$.
19. $y = xy' + \sin y'$.
20. $\ln y' + 2(xy' - y) = 0$.
21. $y = 2xy' - 4y'^2$.
22. $y = xy'^2 + y'^2$.
23. $y = x(1 + y') + y'^2$.
24. $y = 2xy' + y'^2$.
25. $y = xy' - \dfrac{1}{y'^2}$.
26. $y = xy' + \dfrac{1}{y'}$.
27. $xy' + y' = y$.
28. $xy' + \sqrt{1 - y'^2} = y$.
29. $y = xy' + \dfrac{3}{y'^2}$.
30. $y = \dfrac{3}{2}xy' + e^{y'}$.

1.11.AMALIY MASALALAR

Geometrik masalalarni yechishda chizmalardan hamda hosila va integralning geometrik talqinidan foydalanish maqsadga muvofiqdir.

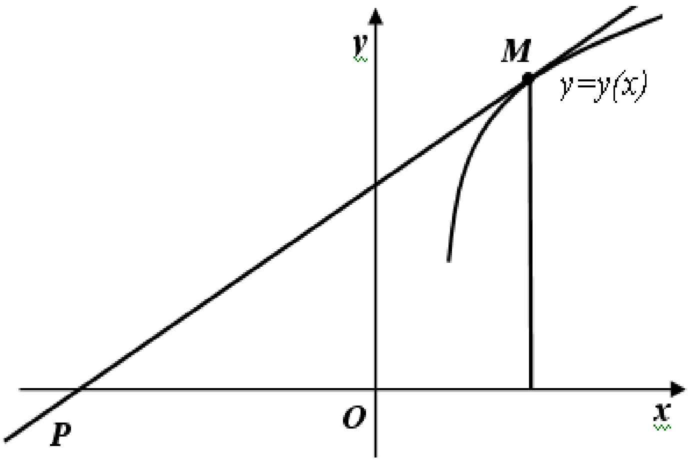
Fizik jarayonlarni tavsiflovchi differensial tenglamalarni tuzishda fizik qonunlar bilan bir qatorda hosilaning, qandaydir miqdorning o’zgarish tezligi sifatidagi fizik ma’nosidan foydalanish zarur.

75. Ixtiyoriy nuqtasining urinma osti shu nuqta abssissasining ikkilanganiga teng bo'lgan egri chiziqlarni toping.

◀ $M(x,\ y)$ - izlanayotgan egri chiziqning ixtiyoriy nuqtasi bo'lsin (7-rasm). Ko'rinib turibdiki, ΔMNP - to'g'ri burchakli uchburchak va, demak, $MN = PN \cdot tg\alpha$. Masala shartiga ko'ra, $MN = y,\ PN = 2x$ va $tg\alpha = y'$ ekanligini e'tiborga olib,

$$y = 2xy' \qquad (1)$$

ko'rinishdagi differensial tenglamani hosil qilamiz. Masalaning mohiyatiga ko'ra, izlanayotgan egri chiziq $y(x) \neq 0$ bo'lishi kerak.



7-rasm.

O'zgaruvchilari ajraladigan (1) tenglamaning umumiy yechimi $y^2 = Cx$, $C \neq 0$ ko'rinishda bo'ladi. Shunday qilib, qo'yilgan masalaning yechimi o'qi Ox bo'lgan va koordinatalar boshidan o'tuvchi parabolalar oilasi ekan.▶

76. Shunday egri chiziqlarni topingki, shu chiziqlarga o'tkazilgan urinma, urinish nuqtasining ordinatasi va abssissalar o'qidan tuzilgan uchburchakning yuzi o'zgarmas a^2 ga teng bo'lsin.

◀Izlanayotgan $y = f(x)$ chiziqning ixtiyoriy $M(x, y)$ nuqtasida chiziqqa urinma o'tkazamiz (8-rasm). Qaralayotgan uchburchakning yuzi $S = \frac{1}{2}|NK|y$ ga teng. Hosilaning geometrik ma'nosiga ko'ra, $tg\alpha = y'$ bo'lganligi uchun $S = \frac{y^2}{2y'}$, $y' > 0$. Shunday qilib,

$$\frac{y^2}{2} = a^2 y'$$

tenglamani hosil qilamiz va uning umumiy yechimini

$$y = -\frac{2a^2}{Ca^2 + x}$$

ko’rinishda topamiz.

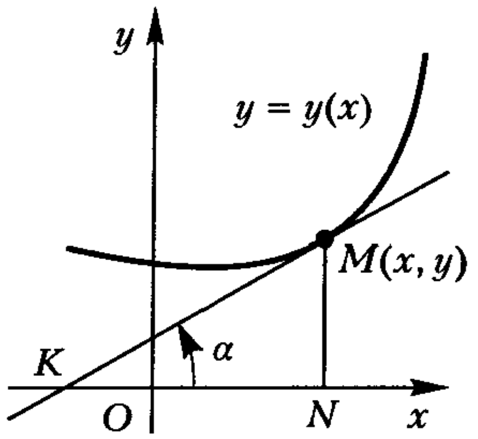
Agar $y' < 0$ bo’lsa (9-rasm), u holda $S = -\frac{y^2}{2y'} = a^2$. Bu tenglamani integrallab,

$$y = \frac{2a^2}{x - Ca^2}$$

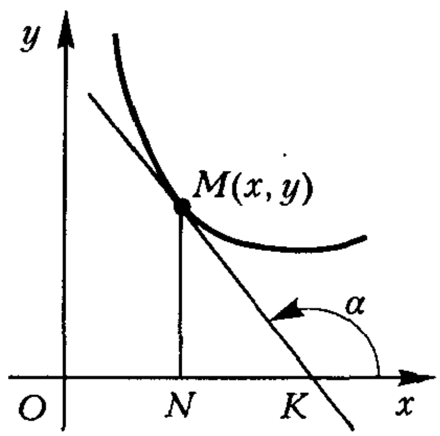
yechimni olamiz.

So’ngra $Ca^2 = -C_1$ belgilash kiritib, ikkala javobni birlashtiramiz:

$y = \frac{2a^2}{C_1 \pm x}$, ya’ni $(C_1 \pm x)y = 2a^2$.►



8-rasm.



9-rasm.

77. Quyidagi xossalarga ega bo’lgan egri chiziqlarni toping: agar chiziqlarning ixtiyoriy nuqtasidan koordinata o’qlariga parallel to’g’ri chiziqlar o’tkazilib, ular koordinata o’qlari bilan kesishguncha davom ettirilsa, u holda hosil bo’lgan to’g’ri to’rtburchakni mazkur chiziq 1:2 nisbatda bo’ladi.

◄Integralning geometrik talqiniga ko’ra (10-rasm),

$$S_2 = \int_0^x y(t)dt. \qquad (2)$$

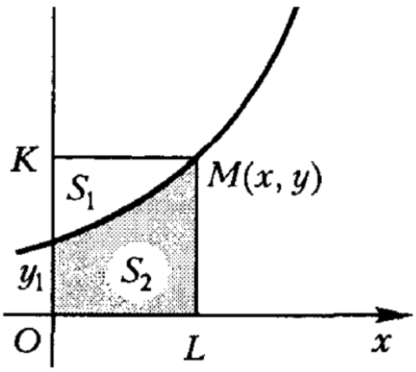
So'ngra, $S_1 + S_2 = xy$ va $S_2 = 2S_1$ bo'lganligi uchun, (2) ni e'tiborga olib,

$$S_2 = \frac{2}{3}xy = \int_0^x y(t)dt$$

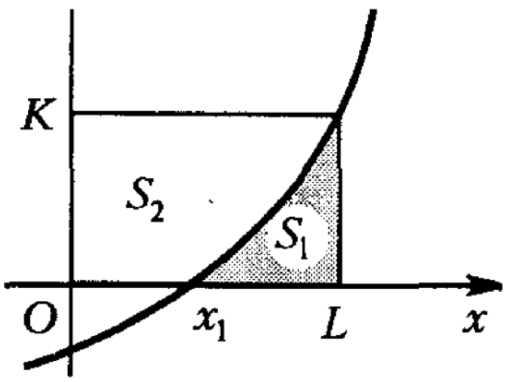
tenglikka ega bo'lamiz. Bu tenglikni x bo'yicha differensiallab, topamiz:

$$\frac{2}{3}(xy' + y) = y, \text{ ya'ni } \frac{dy}{y} = \frac{dx}{2x}.$$

Bundan $y = C\sqrt{x},$ ya'ni $x = Cy^2$ ekanligi kelib chiqadi.



10-rasm.



11-rasm.

Agar x va y o'zgaruvchilarning o'rinlarini almashtirsak masala $y = Cx^2$ yechimga ham ega bo'ladi.►

1-izoh. Yechimni topish vaqtida x va y o'zgaruvchilarni musbat deb faraz qildik. Ammo, ko'rinib turibdiki, ular manfiy bo'lishi ham mumkin, ya'ni ikkala yechimlarda C o'zgarmas ixtiyoriy haqiqiy qiymat qabul qiladi, deb hisoblash mumkin.

2-izoh. Agar 10-rasmning o'rniga 11-rasmdan foydalansak, yana shu natijani olamiz, ya'ni hamma hollarda ham uchlari koordinatalar boshida bo'lgan parabolalar oilasi hosil bo'ladi.

78. Shunday chiziqlarni topingki, bu chiziqlarning ixtiyoriy nuqtasiga o'tkazilgan urinmalar qutb radiusi va qutb o'qi bilan bilan bir xil burchak tashkil etsin.

◄ α va φ burchaklar orasidagi munosabatlardan (12-rasm) va hosilaning geometrik talqinidan

$$\frac{dy}{dx}=\frac{y'(\varphi)}{x'(\varphi)}=\frac{\rho'\sin\varphi+\rho\cos\varphi}{\rho'\cos\varphi-\rho\sin\varphi}=-tg\alpha,\quad \alpha=\frac{1}{2}(\pi-\varphi),$$

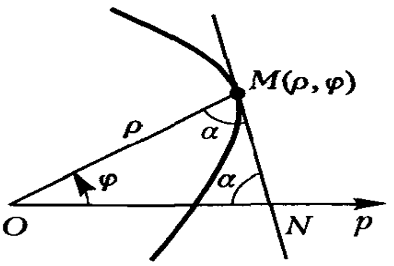
foydalanib, bir necha sodda almashtirishlardan so'ng

$$\frac{\rho'}{\rho}=tg\frac{\varphi}{2}$$

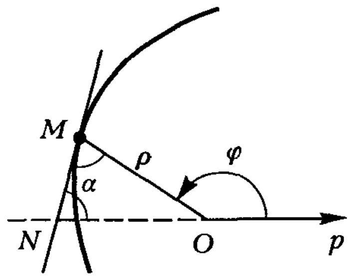
differensial tenglamani olamiz. Bu tenglamaning umumiy yechimi

$$\ln\rho=-2\ln\left|\cos\frac{\varphi}{2}\right|+\ln 2C, \tag{3}$$

ya'ni $\rho=\dfrac{C}{1+\cos\varphi}$ ko'rinishda topiladi.►



12-rasm.



13-rasm.

Izoh. Agar 12-rasm o'rniga 13-rasmni olsak, u holda $\dfrac{\rho'}{\rho}=-ctg\dfrac{\varphi}{2}$ differensial tenglamani olamiz. Uning umumiy yechimi $\rho=\dfrac{C}{1-\cos\varphi}$ ko'rinishga ega bo'ladi.

Bu yechimlar oilasini (3) dan φ ning o'rniga $\varphi+\pi$ qo'yish yo'li bilan hosil qilish mumkin edi. Shunday qilib, qo'yilgan masalaning yechimi $\rho=\dfrac{C}{1\pm\cos\varphi}$ ko'rinishda yoziladi.

79. 20 litr sig'imli idishda havo (80 % azot va 20 % kislorod) bor. Idishga har sekundda 0,1 litr azot qo'shilmoqda (quyilmoqda) va uzluksiz aralashmoqda hamda shuncha miqdordagi aralashma chiqib ketmoqda. Qancha vaqtdan keyin idishda 99 foiz azot bo'ladi?

◀Azotning havo bilan aralashishi boshlangandan keyingi t vaqt momentida idishdagi azotning litrlardagi miqdori $Q(t)$ bo'lsin. U holda $0{,}1dt$ litrli aralashmada $\frac{0{,}1Q\,dt}{20}$ litr azot bor. Masalaning shartiga ko'ra, dt vaqt mobaynida idishga $0{,}1dt$ litr azot qo'shiladi, $\frac{0{,}1Q\,dt}{20}$ litr azot chiqib ketadi. Demak, dt vaqt mobaynida idishga dQ miqdorda azot qo'shiladi va idishda $0{,}1\left(1-\frac{Q}{20}\right)dt$ litr azot qoladi. Shunday qilib,

$$dQ = 0{,}1\left(1-\frac{Q}{20}\right)dt,$$

ya'ni

$$\frac{dQ}{20-Q} = \frac{dt}{200}$$

tenglama hosil bo'ladi, uni integrallab topamiz: $Q(t) = 20 - Ce^{-0{,}005t}$.

Endi C o'zgarmasni aniqlash uchun $Q(t)\big|_{t=0} = 16$ litr shartdan foydalanamiz. $C = 4$ hosil bo'ladi, natijada

$$Q(t) = 20 - 4e^{-0{,}005t} \tag{4}$$

funksiya qo'yilgan masalaning yechimi bo'ladi. (4) ifodada $t = T$ va

$Q = 19{,}8$ litr (20 litrning 99 foizi 19,8 litrga teng) deb olsak,

$$T = 200\ln 20\,c = 599{,}2\,c \approx 10 \text{ minut},$$

ya'ni shuncha vaqtdan keyin idishda azot 99 foizni tashkil etadi.▶

80. Nyuton qonuniga asosan, jismning havoda sovish tezligi jism va havo haroratlari ayirmasiga proporsional ($k=-\frac{1}{9}\ln 2$). Agar havo harorati 20° va jism harorati dastlab 100° bo'lsa, qancha vaqtdan keyin u 30° gacha soviydi?

◀Jism haroratini T va vaqtni t bilan belgilaymiz. Masala shartiga ko'ra,

$$\frac{dT}{dt}=k(t-20),\ T(0)=100.$$

Bu Koshi masalasining yechimini $T(t)=20+80\cdot 2^{-t/9}$ ko'rinishda topamiz.

Endi jism qancha vaqtdan keyin 30° gacha sovishini aniqlaymiz:

$$30=20+80\cdot 2^{-t/9}.$$

Bundan $t=27$ minut ekanligi kelib chiqadi. Demak, jism 27 minutda 30° gacha soviydi.▶

81. Harorati 20° bo'lgan 1 kg suvi bor idishga 0,5 kg massali alyuminiy predmet tushirildi. Bu alyuminiyning solishtirma issiqlik sig'imi 0,2 va harorati 75°. Bir minutdan so'ng suv 2° ga isidi. Qachon suv va predmet haroratlarining farqi 1° bo'ladi? Idishni qizdirishga sarflanadigan issiqlik sarfi va boshqalarni hisobga olmang.

◀Bundan oldingi masaladagi kabi,

$$\frac{dT_n}{dt}=k_n(T_n-T_c),\ \frac{dT_c}{dt}=k_c(T_c-T_n),$$

bu yerda T_n va T_c –predmet va suvning haroratlari, k_n va k_c – o'zgarmas koeffitsientlar. Birinchi munosabatdan ikkinchisini hadma-had ayirib va $R=T_n-T_c$ belgilash kiritib, yozamiz:

$$\frac{dR}{dt}=kR,\ k=k_n+k_c.$$

Bundan $R = Ce^{kt}$ umumiy yechimni topamiz. Masala shartiga ko'ra, vaqtning boshlang'ich $t = 0$ momentida $R = 55^\circ$ bo'lganligi uchun $C = 55$ bo'ladi. Shuning uchun $R = 55e^{kt}$. k koeffitsientni topish uchun issiqlik balansi tenglamasidan foydalanamiz:

$$Q = cm(T_к - T_n),$$

bu yerda c –jismning solishtirma issiqlik sig'imi, m – uning massasi. Bu formulada $Q_1 = 2C_{H_2O}$ va $Q_2 = 0,2 \cdot 0,5(75 - T)C_{H_2O}$ haroratlarning farqi $T_к - T_n$ bilan belgilangan. Bu yerda Q_1 –suvdagi issiqlik miqdori, Q_2 – predmet T temperaturagacha soviganda ajralib chiqqan issiqlik miqdori. Shartga ko'ra, $Q_1 = Q_2$ bo'lganligi uchun $T = 55^\circ$. Demak, bir minutdan keyin $R = 55^\circ - 22^\circ = 33^\circ$ bo'ladi. U holda $33 = 55e^k$, bundan $k = \ln 0,6$. Shuning uchun suv va jism temperaturalarining bir-biriga yaqin kelish qonuni $R = 55 \cdot (0,6)^t$ ko'rinishda bo'ladi. $1 = 55 \cdot (0,6)^t$ tenglikdan

$$t = \frac{\ln 55}{\ln 5 - \ln 3} \approx 8 \text{ minut},$$

ya'ni 8 minut o'tgandan keyin jismning harorati suvning haroratidan 1° ga yuqori bo'ladi.►

82. Jismning harakat tezligi u bosib o'tgan yo'lga proporsional (proporsionallik koeffitsienti $k = 2$). Agar jism 10 sekundda 100 metr yo'l bosib o'tsa, uning yo'l formulasini toping.

◄Jism tezligini $v(t)$, jism bosib o'tgan yo'lni esa $s(t)$ bilan belgilaymiz. Masala shartiga ko'ra, $v(t) = 2s(t)$. $v(t) = \frac{ds}{dt}$ ekanligini va masala shartini e'tiborga olib,

$$s'(t) = 2s(t),\ s(10) = 100$$

ko'rinishdagi Koshi masalasiga ega bo'lamiz. Bu masalani yechib, jismning yo'l formulasini topamiz: $s(t) = 100e^{2t-20}$.►

83. Qayiq suvning qarshiligi ta’sirida o’z harakatini sekinlashtiradi. Suvning qarshiligi qayiqning tezligiga proporsional. Qayiqning boshlang’ich tezligi 1,5 m/sek va 4 sekunddan keyin 1 m/sek bo’ldi. Qachon tezlik 1 sm/sek bo’ladi? Qayiq to’xtab qolguncha qancha yo’l bosib o’tadi?

◀Qayiqning harakat boshlangandan keyingi t vaqt momentidagi tezligi $v(t)$ bo’lsin. U holda $\frac{dv}{dt}$ esa uning tezlanishi bo’ladi. Nyutonning ikkinchi qonuniga ko’ra

$$m\frac{dv}{dt}=F, \qquad (5)$$

bu yerda F – suvning qarshilik kuchi. Masala shartiga ko’ra, $F=kv$, shuning uchun (5) tenglik

$$\frac{dv}{dt}=\frac{k}{m}v=bv \quad (b=const)$$

ko’rinishni oladi. Bu tenglamani integrallab, topamiz:

$$v(t)=Ce^{bt} \qquad (6)$$

$v(0)=1{,}5$ shartdan foydalanib, $C=1{,}5$ ni olamiz. U holda (6) formula

$$v(t)=1{,}5e^{bt}$$

ko’rinishni oladi, bu yerda t sekundlarda hisoblanadigan miqdor. $v(4)=1$ m/sek bo’lganligi uchun $1=1{,}5e^{4b}$ tenglikdan $b=0{,}25\ln(2/3)$ kelib chiqadi. Shuning uchun qayiqning harakat tezligi

$$v(t)=\left(\frac{2}{3}\right)^{\frac{t}{4}-1} \text{ m/sek} \qquad (7)$$

formula bilan ifodalanadi. Bu yerga $v=1$ sm/sek$=0{,}01$ m/sek qo’yib, tegishli vaqt momentini topamiz:

$$t_1 = 4\left(1 + \frac{\ln 0,01}{\ln(2/3)}\right) \approx 50 \text{ sekund.}$$

So'ngra, $v(t) = \dfrac{ds(t)}{dt}$ bo'lganligi uchun, $s(t)-$ bosib o'tilgan yo'l, (7) dan

$$s(t) = \frac{4}{\ln(2/3)}\left(\frac{2}{3}\right)^{\frac{t}{4}-1} + s_0$$

tenglik kelib chiqadi, bu yerda s_0 – integrallash o'zgarmasi.

Endi $s(0) = 0$ bo'lsin. U holda

$$s_0 = -\frac{4}{\ln(2/3)}\left(\frac{2}{3}\right)^{-1}$$

bo'lib, qayiqning harakat qonuni

$$s(t) = \frac{6}{\ln(2/3)}\left(\left(\frac{2}{3}\right)^{\frac{t}{4}} - 1\right)$$

ko'rinishda chiqadi.

Endi s_1 ni, ya'ni qayiq to'xtab qolguncha bosib o'tadigan yo'lni aniqlaymiz. (7) dan $\lim\limits_{t\to+\infty} v(t) = 0$ ekanligi ko'rinib turibdi. Shuning uchun qayiqning harakat qonunidan

$$s_1 = \lim_{t\to+\infty} s(t) = \frac{6}{\ln(3/2)} \approx 15 \text{ metr}$$

kelib bo'ladi.▶

84. Tajribadan ma'lumki, bir yil davomida radiyning har bir grammidan 0,44 mg yemiriladi. Necha yildan keyin radiyning yarmi yemiriladi?

◀Radiaktiv yemirilish qonunidan foydalanamiz: radiaktiv moddaning birlik vaqt mobaynida yemiriladigan miqdori shu moddaning qaralayotgan vaqt momentidagi miqdoriga proporsionaldir.

Radiaktiv moddaning yemirilish boshlangandan o'tgan vaqtning t momentidagi miqdori $Q(t)$ bo'lsin. U holda, radiaktiv yemirilish qonuniga ko'ra, $q(t) = k\,Q(t)$, bu yerda k – proporsionallik koeffisienti, $q(t)$ – moddaning birlik vaqt mobaynida yemiriladigan miqdori. Demak, t dan $t + \Delta t$ gacha vaqt mobaynida $k\,Q(t_1)\,\Delta t$ modda yemiriladi, bu yerda $t_1 \in (t,\, t + \Delta t)$, $Q(t_1)$ esa moddaning $Q(t)$ va $Q(t + \Delta t)$ miqdorlari orasidagi biror qiymatidir. Ikkinchi tomondan, $Q(t + \Delta t) - Q(t) = k\,Q(t_1)\,\Delta t$, ya'ni

$$\frac{Q(t + \Delta t) - Q(t)}{\Delta t} = k\,Q(t).$$

$Q(t)$ funksiyani uzluksiz differensiallanuvchi deb hisoblab va oxirgi tenglikda $\Delta t \to 0$ limitga o'tamiz, natijada

$$\frac{dQ(t)}{dt} = k\,Q(t)$$

differensial tenglamani hosil qilamiz. Bu tenglamaning umumiy yechimi $Q(t) = C\,e^{kt}$ ko'rinishda bo'lishi tushunarli. Demak, $Q(t) = Q(0)\,e^{kt}$ funksiya yemirilish qonunini ifodalaydi, bu yerda t – yillar bilan o'lchanadigan vaqt. Endi k koeffitsientni aniqlash uchun masalaning shartidan foydalanamiz: Agar $Q(0) = 1$gramm bo'lsa, u holda 1 yildan keyin $Q = 999{,}56$ milligramm qoladi. Bundan $e^k = 0{,}99956$. Shunday qilib, $Q(t) = Q(0)\,(0{,}99956)^t$. Bu yerda $Q(t_1) = 0{,}5\,Q(0)$ deb olib, t_1 vaqtni topamiz:

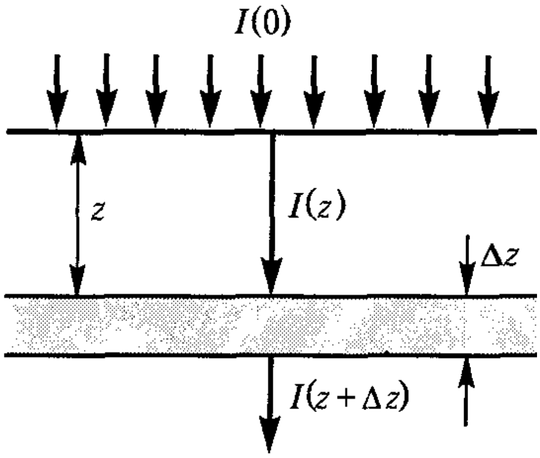
$$t_1 = -\frac{\ln 2}{\ln 0{,}99956} \approx 1600 \;\; \text{yil.}$$ ▶

85. Kichik qalinlikdagi suv qatlami yutayotgan yorug'lik miqdori shu qatlamga tushayotgan yorug'lik miqdori va qatlam qalinligiga proporsional. 35 sm qalinlikdagi suv qatlami unga tushayotgan yorug'lik miqdorining yarmini yutadi. 2 m qalinlikdagi qatlam qancha yorug'likning necha foizini yutadi?

◄ z qalinlikdagi suv qatlamidan o'tayotgan yorug'lik miqdori $I(z)$ bo'lsin (14-rasm). U holda, shartga ko'ra, $I(z+\Delta z)-I(z)$ yutilgan yorug'likning miqdori $k\,I(z)\Delta z\;(k=const)$ ga teng. Shunday qilib, yutilgan yorug'lik miqdori $I(z)$ quyidagi differensial tenglamani qanoatlantiradi:

$$\frac{dI}{dz}=k\,I\,.$$

Tenglamaning yechimi $I(z)=I(0)e^{kz}$ ko'rinishda bo'lib, $I(35)=0,5I(0)$ shartdan $k=-\frac{1}{35}\ln 2$



14-rasm.

kelib chiqadi. Shuning uchun $I(z)=I(0)0,5^{\frac{z}{35}}$, bu yerda z santimetrlarda o'lchanadi. Endi $I(z)$ yechimda $z=2\text{m}=200\,\text{sm}$ deb olsak, $\frac{I(200)}{I(0)}=\left(\frac{1}{2}\right)^{\frac{40}{7}}$ tenglikka ega bo'lamiz.

Bundan

$$\frac{I(0)-I(200)}{I(0)}=1-\left(\frac{1}{2}\right)^{\frac{40}{7}}\approx 0,98.$$

Shunday qilib, 2 m qalinlikdagi qatlamning yuzasiga tushayotgan yorug'likning 98 foizi yutiladi.►

86. Silindrik idishning diametri $2R=1{,}8$ m va balandligi $H=2{,}45$ m. Idishdagi hamma suv idish tubidagi $2r=6\,\text{sm}$ diametrli teshikdan qancha vaqtda oqib chiqadi? Silindrning o'qi vertikal joylashgan.

◄Idishdagi suyuqlikning $t>0$ vaqt momentidagi sath balandligi $h(t)$ bo'lsin. Δt vaqt oralig'ida suyuqlik sathi $h(t+\Delta t)$ qiymatgacha pasayadi. Demak, idishdan $(h(t)-h(t+\Delta t))\pi R^2$ ga teng suyuqlik oqib chiqib ketadi. Boshqa tomondan, idishdagi teshikdan $\pi r^2 v(t_1)\Delta t$ miqdordagi suyuqlik oqib chiqadi, bu yerda $t_1 \in (t, t+\Delta t)$, $v(t_1)$ esa suyuqlikning $(t, t+\Delta t)$ intervalda oqish tezligining biror oraliq qiymati. Massaning saqlanish qonuniga asosan,

$$h(t+\Delta t)-h(t)=-\left(\frac{r}{R}\right)^2 v(t_1)\Delta t$$

bo'ladi. Bu tenglikning ikkala tomonini Δt ga bo'lib va $h(t)$ funksiyani differensiallanuvchi, $v(t)$ funksiyani esa uzluksiz deb faraz qilib, Δt ni nolga intiltiramiz. U holda

$$\frac{dh}{dt}=-k^2 v(t),\ k=\frac{r}{R},\ v=0,6\sqrt{2gh}$$

differensial tenglamani olamiz. Bu tenglamaning yechimi

$$h(t)=\left(C-0,3\sqrt{2g}\,k^2 t\right)^2,\ C=const$$

ko'rinishda topiladi.

Masala shartiga ko'ra, $h(0)=H$ bo'lganligi uchun $C=\sqrt{H}$ bo'ladi.

Bundan $t=\dfrac{10\sqrt{H}}{3\sqrt{2g}}\dfrac{R^2}{r^2}\approx 1050$ sekund =17,5 minutda $h(t)=0$ bo'lishi kelib chiqadi.►

87. Agar Yer sathida atmosfera bosimi 1kG/sm^2 va havoning zichligi 0,0012 g/sm^3 bo'lsa, h balandlikdagi atmosfera bosimini toping. Boyl-Mariotta qonuniga ko'ra, zichlik bosimga proporsional (ya'ni balandlikka bog'liq tarzda havo haroratining o'zgarishini hisobga olinmasin).

◄Yerning sirtidan z balandlikda havoning bosimi $P(z)$ bo'lsin. U holda $P(z)-P(z+\Delta z)$ bosimlar ayirmasi asosining yuzasi 1 sm^2 va balandligi Δz bo'lgan havoli ustunchaning og'irligiga, ya'ni

$\rho(z+\theta\cdot\Delta z)\,g\,\Delta z$ miqdorga teng, bu yerda ρ —havoning o'rtacha zichligi, $0<\theta<1$. Shuning uchun

$$P(z)-P(z+\Delta z)=\rho(z+\theta\cdot\Delta z)\,g\cdot\Delta z.$$

Bu yerda $\Delta z\to 0$ da limitga o'tib, differensial tenglamani hosil qilamiz:

$$\frac{dP}{dz}=-g\,\rho(z). \qquad (8)$$

Boyl-Mariotta qonuniga ko'ra, o'zgarmas temperaturada havoning zichligi bosimga proporsional, ya'ni $\rho(z)=kP(z)$. Shu tenglikni e'tiborga olib, (8) dan differensial tenglama va uning yechimini topamiz:

$$\frac{dP}{P}=-k\,g\,,\ P=P_0\,e^{-k\,g\,z}\ \text{kG/sm}^2.$$

$z=0$ da $P=1$ kG/sm^2 bo'lganligi uchun $P=e^{-k\,g\,z}$ bo'ladi. Shuningdek, erkin tushish tezlanish $g=9{,}8$ m/sek^2 va

$$0{,}0012\ \text{g/sm}^3=1000\ \text{G/sm}^2\cdot k=1000\ \text{g /sm}^2\,g\,k$$

bo'lganligi uchun $k\,g=0{,}12\cdot10^{-5}\,\text{sm}^{-1}$=0,12 (km)$^{-1}$ bo'ladi. Shunday qilib, h km balandlikda havoning bosimi $P=e^{-0{,}12h}$ kG/sm^2 ga teng bo'ladi.►

88. Shunday egri chiziqni topingki, bu chiziqqa o'tkazilgan istalgan urinmaning abssissa o'qi bilan kesishish nuqtasi urinish nuqtasi va koordinatalar boshidan bir xil uzoqlashgan bo'lsin.

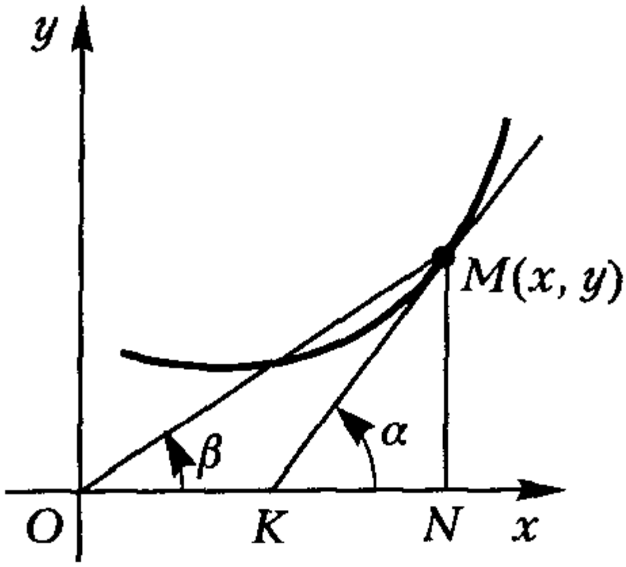
◄Masala shartiga ko'ra, $|OK|= \;=|KM|$ (15-rasm). Shuning uchun $\alpha=2\beta$ va $tg\alpha=\dfrac{2tg\beta}{1-tg^2\beta}$. Lekin $tg\beta=\dfrac{y}{x}$ bo'lganligi uchun $tg\alpha=\dfrac{2xy}{x^2-y^2}$ tenglik o'rinli.

Hosilaning geometrik ma’nosidan foydalanib,

$$y' = \frac{2xy}{x^2 - y^2}$$

tenglamani hosil qilamiz. Bu tenglama bir jinsli tenglama bo’lganligi uchun $y = xt(x)$ deb olamiz va

$$\frac{dx}{x} = \frac{1-t^2}{t+t^3}dt$$



15-rasm.

tenglikni integrallab, topamiz:
$x(t^2+1) = 2Ct$, ya’ni $x^2 + y^2 = 2Cy$.

Xullas, markazi $M(0,C)$ nuqtada va radiusi $R = |C|$ bo’lgan barcha aylanalar qo’yilgan masala shartini qanoatlantiradi.►

INDIVIDUAL TOPSHIRIQLAR

M23. Quyidagi masalalarni differensial tenglamalar yordamida yeching.

Agar egri chiziqning ixtiyoriy nuqtasiga o’tkazilgan urinmaning burchak koeffisienti shu nuqtaning orditasidan k marta katta bo’lsa, $A(x_0, y_0)$ nuqta orqali o’tuvchi egri chiziq tenglamasini tuzing:

1. $A(0,2)$, $k = 3$.
2. $A(0,5)$, $k = 7$.
3. $A(-1,3)$, $k = 2$.
4. $A(-2,4)$, $k = 6$.
5. $A(-2,1)$, $k = 5$.
6. $A(3,-2)$, $k = 4$.

Agar egri chiziqning ixtiyoriy nuqtasiga o’tkazilgan urinmaning burchak koeffisienti shu nuqta koordinatalar boshi bilan tutashtiruvchi to’g’ri chiziqning burchak koeffisientidan n marta katta bo’lsa, $A(x_0, y_0)$ nuqta orqali o’tuvchi egri chiziq tenglamasini tuzing.

7. $A(2,5)$, $n = 8$.
8. $A(3,-1)$, $n = 3/2$.

9. $A(-6,4),\ n=9.$ 10. $A(-8,-2),\ n=3.$

Agar egri chiziqning ixtiyoriy nuqtasidan o'tkazilgan normalning ordinatalar o'qidan ajratgan kesmasining uzunligi shu nuqtadan koordinatalar boshigacha bo'lgan masofaga teng bo'lsa, $A(x_0, y_0)$ nuqta orqali o'tuvchi egri chiziq tenglamasini tuzing.

11. $A(0,4)$. 12. $A(0,-8)$. 13. $A(0,1)$. 14. $A(0,-3)$.

Quyidagi xossaga ega bo'lgan va $A(x_0, y_0)$ nuqtadan o'tuvchi egri chiziq tenglamasini tuzing: egri chiziqqa o'tkazilgan urinmaga koordinatalar boshidan tushirilgan perpendikulyarning uzunligi urinish nuqtasining absissasiga teng.

15. $A(2,3)$. 16. $A(-4,1)$. 17. $A(1,-2)$.

18. $A(-2,-2)$. 19. $A(4,-3)$. 20. $A(5,0)$.

Quyidagi xossaga ega bo'lgan va $A(x_0, y_0)$ nuqtadan o'tuvchi egri chiziq tenglamasini tuzing: egri chiziqqa ixtiyoriy nuqtada o'tkazilgan urinma Oy o'qda ajratgan kesmaning uzunligi urinish nuqtasi abssissasining kvadratiga teng.

21. $A(4,1)$. 22. $A(-2,5)$. 23. $A(3,-2)$.

24. $A(-2,-4)$. 25. $A(3,0)$. 26. $A(2,8)$.

Agar egri chiziqning ixtiyoriy nuqtasidan o'tkazilgan urinmaning ordinatalar o'qidan ajratgan kesmasining uzunligi urinish nuqtasi koordinatalarining yarim yig'indisiga teng bo'lsa, $A(x_0, y_0)$ nuqta orqali o'tuvchi egri chiziq tenglamasini tuzing.

27. $A(9,-4)$. 28. $A(4,10)$.

29. $A(18,-2)$. 30. $A(1,-7)$.

M24. Quyidagi masalalarni differensial tenglamalar yordamida yeching.

1. Egri chiziqlar orasidan urinma ostisining uzunligi o‘zgarmas a bo‘lganini toping.

2. Egri chiziqlar ichidan normal ostisining uzunligi o‘zgarmas p bo‘lganini toping.

3. $(2,0)$ nuqta orqali o‘tuvchi shunday egri chiziqni topingki, unga o‘tkazilgan urinmaning urinish nuqtasi va Oy o‘q orasida joylashgan kesmasining uzunligi o‘zgarmas 2 ga teng bo‘lsin.

4. Shunday egri chiziqni topingki, bu egri chiziqning har bir nuqtasiga o’tkazilgan normal ostining uzunligi shu nuqta koordinatalari kvadratlarining o’rta arifmetigiga teng bo’lsin.

5. Ixtiyoriy egri chiziqlarga urinmalar o‘tkazilgan. Shu egri chiziqlar ichidan normal ostisining uzunligi urinish nuqtasining abssissa va ordinatasi yig‘indisiga teng bo‘lganini toping.

6. Barcha urinmalari koordinatalar boshidan o‘tadigan egri chiziqlarni toping.

7. Ixtiyoriy egri chiziqlarga urinmalar o‘tkazilgan. Shu egri chiziqlar ichidan urinma ostisining uzunligi urinish nuqtasining abssissa va ordinatasi yig‘indisiga teng bo‘lganini toping.

8. Shunday egri chiziqni topingki, uning normal ostisi urinish nuqtasining radius-vektori va absissasining ayirmasiga teng bo‘lsin.

9. Shunday egri chiziqni topingki, unga o‘tkazilgan normal urinish nuqtasining radius-vektori bilan ustma-ust tushsin.

10. Egri chiziqqa o’tkazilgan normal va koordinata o’qlaridan tashkil topgan uchburchak Ox o’q hamda shu egri chiziqqa o’tkazilgan urinma va normaldan tuzilgan uchburchakka tengdosh ekanligi ma’lum. Shunday xossaga ega bo’lgan egri chiziqni toping.

11. Barcha urinmalari berilgan (a,b) nuqta orqali o‘tadigan egri chiziqni toping.

12. Shunday egri chiziqni topingki, bu chiziqqa o‘tkazilgan har bir urinma $y=1$ to‘g‘ri chiziqni absissasi urinish nuqtasining absissasidan ikki marta katta bo‘lgan nuqtada kesib o‘tsin.

13. Shunday egri chiziqni topingki, bu chiziqqa (x,y) nuqtada o‘tkazilgan urinma $\left(x^2,y^2\right)$ nuqta orqali o‘tsin.

14. Quyidagi xossalarga ega bo‘lgan egri chiziqlarni toping: biror boshlang‘ich nuqtadan chiqadigan egri chiziqning uzunligi shu egri chiziq chegaralaydigan egri chiziqli trapetsiyaning yuziga teng.

15. $(1,2)$ nuqta orqali o‘tuvchi shunday egri chiziqni topingki, egri chiziqdagi ixtiyoriy nuqtaning radius-vektori, shu nuqtaga o’tkazilgan

urinma va absissalar o’qi tashkil etgan uchburchakning uzi 2 ga teng bo’lsin.

16. Shunday egri chiziqni topingki, bu chiziqqa o‘tkazilgan urinmaning Oy o‘qda ajratgan kesmasi xuddi shu urinma Ox o‘qda ajratgan kesmaning teskari qiymatiga teng bo‘lsin.

17. Shunday egri chiziqlarni topingki, bu chiziqlar normalining koordinata o‘qlari orasidagi kesmasining uzunligi o‘zgarmas bo‘lsin.

18. Shunday egri chiziqlarni topingki, bu chiziqlarga o‘tkazilgan urinmaning koordinata o‘qlari orasidagi kesmasining uzunligi o‘zgarmas a bo‘lsin.

19. $(2,3)$ nuqta orqali o‘tuvchi shunday egri chiziqni topingki, unga o‘tkazilgan ixtiyoriy urinmaning koordinata o‘qlari orasida joylashgan kesmasi urinish nuqtasida teng ikkiga bo‘linsin.

20. $(1,1)$ nuqta orqali o‘tuvchi shunday egri chiziqni topingki, unga o‘tkazilgan ixtiyoriy urinma og‘ish burchagining tangensi urinish nuqtasi ordinatasining kvadratiga proportsional bo‘lsin.

21. Nuqta orqali o‘tuvchi shunday egri chiziqni topingki, chiziqning ixtiyoriy nuqtasida o‘tkazilgan urinma og‘ish burchagining tangensi shu nuqta va koordinatalar boshi orqali o‘tuvchi to‘g‘ri chiziqning og‘ish burchagi tangensidan n marta katta bo‘lsin.

22. Qutb koordinatalarida shunday egri chiziqni topingki, chiziq ixtiyoriy nuqtasi radius vektorining urinma bilan tashkil etgan burchagining tangensi radius vektorning kvadratiga teng bo‘lsin.

23. Qutb koordinatalarida shunday egri chiziqni topingki, urinish nuqtasiga o’tkazilgan radius-vektor va shu nuqtada o’tkazilgan urinma orasidagi burchakning tangensi qutb burchakka teng bo’lsin.

24. Shunday egri chiziqni topingki, bu chiziqqa o‘tkazilgan urinma, urinish nuqtasining ordinatasi va abssissa o‘qidan tuzilgan to‘g‘ri burchakli uchburchak katetlarining yig‘indisi o‘zgarmas b songa teng bo‘lsin.

25. Quyidagi xossalarga ega bo‘lgan egri chiziqlarni toping: Egri chiziqning ixtiyoriy nuqtasidan o‘tkazilgan urinma va normalning abssissa o‘qida hosil qiladigan kesmasi $2a$ ga teng.

26. Shunday egri chiziqni topingki, bu chiziqqa o‘tkazilgan ixtiyoriy urinmaning absissa o‘qi bilan kesishish nuqtasi urinish nuqtasi absissasidan ikki marta kichik absissaga ega bo‘lsin.

27. Shunday egri chiziqlarni topingki, bu chiziqlarning ixtiyoriy nuqtasiga o‘tkazilgan urinmalar qutb radiusi va qutb o‘qi bilan bir xil burchak tashkil etsin.

28. Shunday egri chiziqni topingki, bu chiziqqa o‘tkazilgan ixtiyoriy urinmadan koordinatalar boshigacha bo‘lgan masofa urinish nuqtasining abssissasiga teng bo‘lsin.

29. Koordinatalar boshidan o‘tuvchi shunday egri chiziqni topingki, uning ixtiyoriy nuqtasiga o‘tkazilgan normalning koordinata o‘qlari orasidagi kesmasining uzunligi 2 ga teng bo‘lsin.

30. Shunday egri chiziqni topingki, bu chiziqning ixtiyoriy nuqtasiga o‘tkazilgan urinma va koordinata o‘qlari tashkil etgan uchburchakning yuzi $2a^2$ ga teng bo‘lsin.

M25. Amaliy masalalarni differensial tenglamalar yordamida yeching.

1. Idishda tarkibida 10 kg tuzi bo‘lgan 100 litr aralashma bor. Idishga minutiga 5 litr suv uzluksiz quyilmoqda va mavjud aralashma bilan aralashmoqda hamda xuddi shu tezlikda aralashma oqib chiqib ketmoqda. Bir soatdan so‘ng idishda qancha tuz bo‘ladi.
2. Bitta nuqtadan chiqadigan barcha nurlarni berilgan yo’nalishga parallel akslantiruvchi ko’zguning shaklini toping.
3. Quiydagi xossaga ega bo’lgan egri chiziqni toping: koordinatalar boshidan egri chiziqning istalgan nuqtasigacha bo’lgan masofaning kvadrati bilan bu nuqtadan o’tgan normalning absissalar o’qidan ajratgan kesmaning ko’paytmasi shu nuqta absissasining kubiga teng.
4. a gradus haroratli metal parchasi pechkaga joylashtirildi. Pechkaning harorati bir soatda a gradusdan b gradusgacha tekis ko‘tariladi. Pechka va metal haroratlarining ayirmasi T gradus bo‘lganda metall minutiga kT gradus tezlik bilan isiydi. Metalning harorati bir soatdan so‘ng qancha bo‘ladi?
5. Massasi m bo‘lgan moddiy nuqtani ikkita markazning har biri ham masofaga proporsional kuch bilan o’ziga tortadi. Proporsionallik koeffisiyenti k ga teng. Markazlar orasidagi masofa $2c$.. Dastlabki paytda nuqta markazlarni tutashtiruvchi chiziqda, bu chiziqning o’rtasidan a masofada turadi. Boshlang’ich tezlik nolga teng. Nuqta harakatining tenglamasini tuzing.
6. Tog‘ rudasining parchasida 100 mg uran va 14 mg uran qo‘rg‘oshini bor. Ma'lumki, uran $4,5\cdot 10^9$ yilda yarim yemiriladi va 238 g uran to‘la

yemirilganda 206 g uran qo‘rg‘oshini hosil bo‘ladi. Tog‘ rudasining yoshini aniqlang. Hosil bo‘lgan paytda tog‘ rudasida qo‘rg‘oshin yo‘q edi, deb hisoblang hamda uran va qo‘rg‘oshin orasidagi radiaktiv moddalar yo‘q edi deb hisoblang (chunki ular urandan tezroq yemiriladi).

7. 20 litr suvi bor idishga har minutda tarkibida 0,2 kg tuz bo’lgan 5 litr aralashma (nomakob) uzluksiz aralashmoqda. Nomakob idishda aralashib, huddi shu tezlikda idishdan chiqib ketmoqda. 4 minutdan keyin idishda qancha tuz bo’ladi?

8. Vazni 0,4 kG bo‘lgan to‘p 20 m/sek tezlik bilan yuqoriga tepildi. Havoning qarshiligi tezlikning kvadratiga proporsional bo‘lib, u (havoning qarshiligi) 1 m/sek tezlikda 0,48 G ga teng. To‘pning ko‘tarilishiga ketgan vaqtni va eng yuqori balandlikni toping. Agar havoning qarshiligi e'tiborga olinmasa, bu natijalar qanday o‘zgaradi?

9. Boshlang‘ich tezlik nolga teng bo‘lganda havoning qarshiligi hisobga olgan holda 16,3 m balandlikdan to‘pning tushishiga ketgan vaqtni hisoblang. Tushishning oxiridagi tezlikni toping.

10. Silindrik idishning diametri $2R = 1{,}8$ m va balandligi $H = 2{,}45$ m. Agar silindrning o‘qi gorizontal joylashgan bo‘lsa, u holda idishdagi hamma suv idish tubidagi $2r = 6$ sm diametrli teshikdan qancha vaqtda oqib chiqadi?

11. A va B moddalar o’rtasidagi kimyoviy reaksiya natijasida C modda hosil bo’ladi. Agar reaksiya boshlanishi oldidan A va B moddalarning miqdorlari mos ravishda a va b teng ekanligi ma’lum bo’lsa, C modda massasining vaqtga bog’liq formulasini toping. Reaksiya tezligi reaksiyaga kirishuvchi moddalar massalarining ko’paytmasiga proporsional ekanligi ma’lum.

12. Idish konus shaklida bo‘lib, radiusi $R = 6$ sm va balandligi $H = 10$ sm hamda konusning uchi pastda joylashgan. Suv bilan to‘ldirilgan idishdagi suv idishning uchidagi 0,5 smli teshikdan qancha vaqtda oqib tushadi?

13. O‘lchamlari 60 sm x75 sm va balandligi 80 sm bo‘lgan idishga sekundiga l,8 litr suv quyilmoqda. Idishning tubida yuzasi 2,5 sm^2 bo‘lgan teshik bor. Idish qancha vaqtda to‘ladi? Idish teshiksiz bo‘lganda qancha vaqtda to‘lgan bo‘lar edi?

14. Uzunligi 1 m bo‘lgan rezina ip (arqon) f kG kuch ta'sirida kf metrga cho‘ziladi. Uzunligi l va vazni P bo‘lgan xuddi shunday ip

(arqon) bir uchidan osib qo‘yilgan bo‘lsa, o‘z vazni ta'sirida qanchaga cho‘ziladi?

15.Daryo kemalarini pristanda to‘xtatib olish uchun kemadan turib pristandagi ustunga arqon tashlaydilar. Arqonning ustunga tortishish koeffisienti $1/3$ ga teng va pristanda turgan ishchi arqonning bo‘sh uchini 10 kG kuch bilan tortadi. Agar arqon ustunga uch marta o‘ralgan bo‘lsa, kemani qanaqa kuch to‘xtatib qoladi?

16.Hajmi v m^3 bo‘lgan usti yopiq binoda suv quyilgan usti ochiq idish bor. Suvning bug‘lanish tezligi $q_1 - q$ ayirmaga proporsional, bu yerda q_1 - berilgan haroratda 1 m^3 havodagi suv bug‘ining miqdori, q - qaralayotgan momentdagi 1 m^3 havodagi suv bug‘ining miqdori (havo va suvning harorati hamda bug‘lanish yuz berayotgan yuzaning miqdorini o‘zgarmas deb hisoblaymiz). Dastlab idishda m_0 gramm suv, 1 m^3 havoda esa q_0 gramm bug‘ bor edi. t vaqtdan so‘ng idishda qancha suv qoladi?

17.Raketaning to‘ldirilgan yonilg‘i bilan massasi M, yonilg‘isiz esa m, yonish mahsulotlarining sarflanish tezligi c, raketaning boshlang‘ich tezligi nolga teng. Raketaning yonilg‘i yonib bo‘lganidan keyingi massasini toping. Og‘irlik kuchi va havoning qarshiligini hisobga olmang (Siolkovskiy formulasi).

18.Aylanma jism ko‘rinishidagi idish berilgan. Idishning tubidagi r radiusli teshikdan suyuqlik bir tekisda oqib chiqishi uchun idishining shakli qanaqa bo‘lishi kerak.

19.$v_0 = 400$ m/sek tezlik bilan harakatlanayotgan o’q $h = 0,02$ m qalinlikdagi devorni teshib o‘tib, undan 100 m/sek tezlik bilan chiqib ketadi. Devorning qarshilik kuchi o‘qning harakat tezligi kvadratiga proporsional ekanligini bilgan holda o‘qning devor ichidagi harakat vaqti T ni toping.

20.Raketa boshlang‘ich tezlik bilan yuqoriga vertikal uchirildi. Havoning qarshiligi natijasida uning harakati sekinlashdi va manfiy tezlanish oldi. Bu tezlanish raketa tezligining kvadratiga $\left(-kv^2\right)$ proporsional ekanligini bilgan holda raketa yuqori balandlikka erishish uchun sarflagan vaqtini toping.

21.m massali M moddiy nuqta vertikal o‘q atrofida o‘zgarmas burchak tezlik aylanayotgan $y = y(x)$ egri chiziq ustida joylashgan. Agar moddiy nuqta har qanday holda ham egri chiziq ustida muvozanatda turgan bo‘lsa, egri chiziqning tenglamasini toping.

22.Motorli qayiq ko‘lda 20 km/soat tezlik bilan harakatlanmoqda. Agar dvigatel' o‘chirib qo‘yilsa, uning tezligi 40 sekunddan keyin 8 km/soatga tushadi. Suvning qarshiligi qayiqning harakat tezligiga proporsional. Dvigatel o‘chirib qo‘yilganidan 2 minutdan keyin qayiqning tezligi qanday bo‘ladi?

23.Esayotgan shamol daraxtlarning qarshiligi tufayli o‘rmon ichida o‘z tezligini kamaytiradi. Cheksiz kichik masofada bu yo‘qotish shamolning yo‘l boshidagi tezligiga va yo‘lning uzunligiga proporsional. Shamolning boshlang‘ich tezligi 12 m/sek. Agar o‘rmon ichida $s=1$ m masofani bosib o‘tganda shamolning tezligi bo‘lib qolganligi ma'lum bo‘lsa, o‘rmon ichida 150 m yo‘l bosib o‘tgan shamolning tezligini toping.

24.Idishda 100 l hajmida 10 foizli namakob bor. Izdishga har minutda 30 litr suv quyilmoqda va 20 litr aralashma chiqib ketmoqda. 10 minutdan idishda qancha miqdorda tuz qoladi?

25.m massali moddiy nuqtaga o‘zgarmas kuch ta'sir qilib, unga a tezlanish beradi. Atrof muhitning harakatlanayotgan nuqtaga qarshiligi nuqtaning tezligiga proporsional bo‘lib, proporsionallik koeffisienti γ ga teng. Agar boshlang‘ich vaqtda nuqta muvozanatda turgan bo‘lsa, vaqt o‘tishi bilan uning tezligi qanday o‘zgaradi?

26.Moddiy nuqtaning to‘g‘ri chiziq bo‘ylab harakat tezligi bosib o‘tilgan yo‘lga teskari proporsional. Harakatning boshlang‘ich vaqtida nuqta yo‘l boshidan 5 m masofada joylashgan bo‘lib, $v_0=20$ m/sek tezlikka ega bo‘lgan bo‘lsa, harakat boshlanganidan 10 sekunddan keyin nuqtaning bosib o‘tgan yo‘lini va tezligini toping.

27.Suvning qarshiligi qayiqning tezligiga proporsional bo‘lib, shu qarshilik tufayli qayiqning harakati sekinlashadi. Qayiqning boshlang‘ich tezligi 2 m/sek, uning 4 sekunddan keyingi tezligi esa 1 m/sek. Qancha vaqtdan keyin qayiqning tezligi 0,25 m/sek bo‘ladi? Qayiq to‘xtab qolguncha qancha yo‘l bosib o‘tishi kerak?

28.Lokomotivning boshlang‘ich tezligi v_0 ga teng, tezlanishi esa F tortish kuchiga proporsional va poezdning m massasiga teskari proporsional. Lokomotivning tortish kuchi $F(t)=b-kv(t),$ bu yerda $v(t)$

- lokomotivning t vaqt momentidagi tezligi, b va k esa o‘zgarmas kattaliklar. Lokomotiv tortish kuchining t vaqtga bog‘lanishini toping.

29. m massali moddiy nuqta koordinata to‘g‘ri chizig‘i Ox bo‘ylab harakatlanmoqda. Nuqtaga ta'sir qilayotgan kuch bajargan ish harakat boshlangan t vaqtga proporsional (proporsionallik koeffisienti k ga teng). Agar boshlang‘ich vaqtda ($t=0$ da) nuqta sanoq boshidan s_0 masofada muvozanatda turgan bo‘lsa, nuqtaning harakat qonunini toping.

30.Parashyutchi $R=4$m radiusli yarim sfera ko‘rinishidagi parashyutda pastga tushmoqda. Uning parashyut massasi bilan birgalikdagi massasi 82 kg. Tushish boshlanganidan 2 sekund o‘tganidan keyin parashyutchining tezligini toping. Havoning qarshilik kuchini $F_1 = 0,00081 s v^2$ deb oling, bu yerda s - harakat yo‘nalishiga perpendikulyar bo‘lgan eng katta kesimning yuzasi; v harakat tezligi.

I BOBNI TAKRORLASHGA DOIR ARALASH MASALALAR

Tenglamalarning yechimlarini izoklinalar yordamida (taxminan) tasvirlang:

1. $y' = y - x^2$. **2.** $2(y+y') = x+3$. **3.** $x^2 + y^2 y' = 1$. **4.** $(x^2+y^2)y' = 4x$.

Berilgan chiziqlar oilalarining differensial tenglamalarini tuzing:

5. $y = e^{Cx}$. **6.** $x^2 + Cy^2 = 2y$. **7.** $y = ax^2 + be^x$. **8.** $x = ay^2 + by + c$.

9. Markazlari $y = 2x$ to`g’ri chiziqda joylashgan barcha birlik aylanalarning differensial tenglamasini tuzing.

10. $x^2 + z^2 = y^2 - 2by,\ z = ax + b$ chiziqlar oilasini qanoatlantiradigan differensial tenglamalar sistemasini tuzing.

Berilgan chiziqlar oilasini berilgan φ burchak ostida kesib o`tadigan trayektoriyalarning differensial tenglamalarini tuzing (**11-16**):

11. $x^2 + y^2 = 2ax, \varphi = 90^\circ$. **12.** $y = kx, \varphi = 60^\circ$.

13. $3x^2 + y^2 = C, \varphi = 30^\circ$. **14.** $\rho = a\cos^2\theta,\ \varphi = 90^\circ$.

15. $\rho = a\sin\theta,\ \ \varphi = 45^{\circ}.$ **16.** $y = x\ln x + Cx,\ \ \varphi = arctg\,2.$

Tenglamalarni yeching. Berilgan boshlang'ich shartlarni qanoatlantiruvchi yechimlarni toping (**17-22**):

17. $(x^2-1)y' + 2xy^2 = 0;\ y(0) = 1.$ **18.** $2x^2yy' + y^2 = 2.$

19. $y' - xy^2 = 2xy.$ **20.** $x\dfrac{dx}{dt} + t = 1.$

21. $y' = \cos(y-x)$**.** **22.** $(x+2y)y' = 1;\ \ y(0) = -1.$

23. $y' = \sqrt{\ln(1+y)/\sin x}$ tenglamaning integral egri chiziqlarini koordinatalar boshining atrofida o'rganing. Birinchi koordinata burchagi chegaralarining har bir nuqtasidan shu burchakning ichidan o`tuvchi bitta integral egri chiziq o`tishini ko`rsating.

24. Quyidagi xossalarga ega bo`lgan chiziqlarni toping: Chiziqning ixtiyoriy nuqtasidan o`tkazilgan urinma va normalning abssissa o`qida hosil qiladigan kesmasi $2a$ ga teng.

25. Hajmi 200 m^3 bo`lgan xonaning havosida $0{,}15\%$ karbonat angidrid $(CO_2)$ gazi bor. Ventilyator har minutda tarkibida $0{,}04\%$ $CO_2$ bo`lgan 20 m^3 havoni tozalayapti. Qancha vaqtdan keyin xonadagi karbonat angidrid gazi 3 marta kamayadi?

26. Jism 10 minut ichida 100° dan 60° gacha sovidi. Tashqi muhit havosining harorati 20°. Jism qancha vaqtdan keyin 25° gacha soviydi?

27. 30 kun davomida radioaktiv modda dastlabki miqdorining 50 %i emirildi. Qancha vaqtdan so`ng dastlabki miqdorning 1%i qoladi?

28. Parashyutchi 1,5 km balandlikdan sakradi va parashyutni 0,5 km balandlikda ochdi. Parashyutchi parashyutni ochgunga qadar qancha vaqt sarfladi? Ma`lumki, normal zichlikdagi havoda odamning eng tez tushish tezligi 50 m/sekundga teng. Zichlik o`zgarmaydi deb hisoblanadi. Havoning qarshiligi tezlikning kvadratiga proporsional.

29. Silindrik idish vertikal o`rnatilgan va tubida teshik bor. Suvning yarmi to`la idishdan 5 minutda oqib chiqadi. Qancha vaqtdan keyin hamma suv oqib chiqib ketadi?

Tenglamalarni yeching (**30-35**):

30. $(x+2y)dx - xdy = 0.$ **31.** $y^2 + x^2 y' = xyy'.$

32. $\left(y+\sqrt{xy}\right)dx = xdy.$ **33.** $x - y - 1 + (y - x + 2)y' = 0.$

34. $(y'+1)\ln\frac{y+x}{x+3} = \frac{y+x}{x+3}.$ **35.** $2y' + x = 4\sqrt{y}.$

36. Shunday chiziqni topingki, bu chiziqqa o`tkazilgan istalgan urinmadan koordinatalar boshigacha bo`lgan masofa urinish nuqtasining absissasiga teng bo'lsin.

Tenglamalarni yeching (**37-42**):

37. $(2x+1)y' = 4x + 2y$**.** **38.** $y = x(y' - x\cos x).$

39. $(x + y^2)dy = ydx$**.** **40.** $y' = y/(3x - y^2).$

41. $y' = y^4 \cos x + y\,tgx.$ **42.** $xy' + 2y + x^5 y^3 e^x = 0.$

Rikkati tenglamasining xususiy yechimini tanlash yo`li bilan topib va Bernulli tenglamasiga keltirib, ularni yeching (**43-45**):

43. $x^2 y' + xy + x^2 y^2 = 4.$ **44.** $y' + y^2 = 2x^{-2}.$

45. $y' + 2ye^x - y^2 = e^{2x} + e^x$**.**

Tenglamalar to`liq differensialli ekanligini tekshiring va ularni yeching (**46-49**):

46. $e^{-y}dx - (2y + xe^{-y})dy = 0.$ **47.** $\frac{y}{x}dx + (y^3 + \ln x)dy = 0.$

48. $2x\left(1+\sqrt{x^2 - y}\right)dx - \sqrt{x^2 - y}\,dy = 0.$

49. $(1 + y^2 \sin 2x)dx - 2y\cos^2 x dy = 0.$

Biror usul bilan integrallovchi ko`paytuvchini topib yoki o`zgaruvchilarni almashtirib tenglamalarni yeching (**50-55**):

50. $(x^2+y^2+x)dx+y\,dy=0.$ **51.** $y^2dx-(xy+x^3)dy=0.$

52. $y(x+y)dx+(xy+1)dy=0.$ **53.** $x^2y(ydx+xdy)=2ydx+xdy.$

54. $x(\ln y+2\ln x-1)dy=2ydx.$ **55.** $(x^2-y)dx+x(y+1)dy=0$**.**

56. Berilgan tenglamaning berilgan boshlang'ich shartni qanoatlantiradigan yechimiga y_0, y_1, y_2 ketma-ket yaqinlashishlarni tuzing:

a) $y'=y^2+3x^2-1,\ y(1)=1.$ b) $y'=y+e^{y-1},\ y(0)=1.$

c) $y'=1+x\sin y,\ y(\pi)=2\pi.$

57. Boshlang'ich shartli masalaning yechimi mavjud bo`ladigan biron-bir kesmani ko`rsating:

a) $y'=2y^2-x,\ y(1)=1.$ b) $\dfrac{dx}{dt}=y^2,\ \dfrac{dy}{dt}=x^2;\ x(0)=1, y(0)=2.$

58. a ning qanday nomanfiy qiymatlarida $y'=|y|^a$ tenglama yechimlarining yagonaligi buziladi va qaysi nuqtalarda buziladi?

59. Yagonalikning zarur va yetarli sharti yordamida $y'=f(y)$ ko`rinishidagi tenglamalarni tadqiq qiling:

a) $y'=\sqrt[3]{y^2}.$ b) $y'=y\sqrt[3]{y+1}.$

c) $y'=y\ln y.$ d) $y'=y\ln^2 y.$

60. Boshlang'ich shartlar qanday bo`lganda quyidagi tenglamalar va sistemalar yagona yechimga ega bo`ladi:

a) $y''=tg\,y+\sqrt[3]{x}.$ b) $(x+1)y''=y+\sqrt{y}.$

c) $\dfrac{dx}{dt}=y^2+\sqrt[3]{t},\ \dfrac{dy}{dt}=\sqrt[3]{x}.$

61. $y^{(n)}=x+y^2$ tenglamaning bir vaqtda ikkita $y(0)=1, y'(0)=2$ shartlarni qanoatlantiradigan nechta yechimlari mavjud? $n=1, 2, 3$ hollarni alohida-alohida o'rganing.

62. Quyidagi tenglama va sistemalarning yechimlari koordinatalar boshining atrofida nechanchi tartibli hosilalarga ega?

a) $y' = x + y^{7/3}$. b) $y'' = \left|x^3\right| + y^{5/3}$. c) $\dfrac{dx}{dt} = t + y,\ \dfrac{dy}{dt} = x + t^2\left|t\right|$.

63. a ning qanday qiymatlarida har bir yechim $-\infty < x < +\infty$ intervalga davom ettiriladi:

a) $y' = \left|y\right|^a$. b) $y' = \left|y\right|^{a-1} + \left(x\sqrt[3]{y}\right)^{2a}$.

64. Quyidagi tenglamalar uchun ixtiyoriy $y(x_0) = y_0$ boshlang'ich shartni qanoatlantiradigan yechim $x_0 \le x < +\infty$ da mavjud bo'lishini isbotlang:

a) $y' = x^3 - y^3$. b) $y' = xy + e^{-y}$.

Tenglamalarning hamma yechimlarini toping; maxsus yechimlarni (agar ular bor bo'lsa) ajrating (**65-68**):

65. $y'^2 - y^2 = 0$. **66.** $y'^2 - 4y^3 = 0$. **67.** $xy'^2 = y$. **68.** $yy'^3 + x = 1$.

Tenglamalarni y' ga nisbatan yechib olib, so'ngra odatdagi usullar bilan umumiy yechimni toping. Maxsus yechimlarni (agar ular bor bo'lsa) ajrating (**69-74**):

69. $y'^2 + xy' - x^2 = 0$. **70.** $xy'^2 - 2yy' + x = 0$.

71. $y'^2 - 2xy' = 8x^2$. **72.** $y'^2 - 2yy' = y^2(e^x - 1)$.

73. $y'(2y - y') = y^2 \sin^2 x$. **74.** $yy'(yy' - 2x) = x^2 - 2y^2$.

Tenglamalarni parametr kiritish usuli bilan yeching (**75-84**):

75. $x = y'\sqrt{y'^2 + 1}$. **76.** $y'(x - \ln y') = 1$.

77. $y = (y' - 1)e^{y'}$. **78.** $y'^4 - y'^2 = y^2$.

79. $y'^2 - 2xy' = x^2 - 4y$. **80.** $5y + y'^2 = x(x + y')$.

81. $y'^3 + y^2 = xyy'$. **82.** $y' = e^{xy'/y}$.

83. $y = xy' - x^2 y'^3$. **84.** $y(y - 2xy')^3 = y'^2$.

Klero va Lagranj tenglamalarini yeching (**85-90**):

85. $y + xy' = 4\sqrt{y'}$. **86.** $y = 2xy' - 4y'^3$. **87.** $y = xy' - (2 + y')$.

88. $y = xy'^2 - 2y'^3$. **89.** $xy' - y = \ln y'$. **90.** $xy'(y' + 2) = y$.

91. Differensial tenglama yechimlarining oilasidan foydalanib, shu tenglamaning maxsus yechimini toping:

a) $y = Cx^2 - C^2$. b) $Cy = (x - C)^2$. c) $y = C(x - C)^2$.

92. Shunday chiziqni topingki, bu chiziqning istalgan nuqtasiga o`tkazilgan urinma va koordinata o`qlari tashkil etgan uchburchakning yuzi $2a^2$ ga teng bo`lsin.

93. Koordinatalar boshidan o`tuvchi shunday chiziqni topingki, uning ixtiyoriy nuqtasiga o`tkazilgan normalning birinchi chorak ichidagi kesmasi uzunligi 2 ga teng bo`lsin.

2-BOB
YUQORI TARTIBLI DIFFERENSIAL TENGLAMALAR

2.1. TARTIBI PASAYADIGAN TENGLAMALAR

2.1.1. $F\left(x, y^{(n)}\right)=0$ **ko'rinishdagi tenglamalar.** Agar $F\left(x,u\right)=0$ ko`rinishdagi tenglamani $u=\varphi(x)$ yoki $x=\psi(u)$ ga nisbatan yechish mumkin bo'lsa, $F\left(x, y^{(n)}\right)=0$ ko`rinishdagi differensial tenglamani ham integrallash mumkin.

Haqiqatan ham, berilgan tenglamani $u=\varphi(x)$ ga nisbatan yechish mumkin bo`lgan holda

$$y^{(n)}=\varphi(x),$$

$$y=\frac{1}{(n-1)!}\int_{x_0}^{x}(x-t)^{n-1}\varphi(t)dt+C_1x^{n-1}+C_2x^{n-2}+...+C_{n-1}x+C_n \qquad (1)$$

bo`ladi, bu yerda $C_j\ (j=\overline{1,n})$ – ixtiyoriy o'zgarmaslar.

Berilgan tenglama $x=\psi(u)$ ga nisbatan yechilgan holda $y^{(n)}=t$ deb olamiz. U holda $x=\psi(t)$ va $d(y^{n-1})=t\,dx=t\psi'(t)dt$ bo`lib, bundan

$$y^{(n-1)}=\int t\psi'(t)dt+C_1$$

kelib chiqadi. Xuddi shunga o'xshash, $y^{(n-1)}, y^{(n-2)}, ..., y=g(t)+$ $+\omega(t,C_1,C_2,...,C_n)$ ifodalarni topamiz, bu yerda g va ω – ma'lum funksiyalar. Shunday qilib, tenglamaning umumiy yechimi

$$x=\psi(t),\ y=g(t)+\omega(t,C_1,C_2,...,C_n) \qquad (2)$$

parametrik ko`rinishda topiladi.

Ba`zan $F\left(x, y^{(n)}\right)=0$ tenglamani $x=\alpha(t),\ y^{(n)}=\beta(t)$ parametrik tenglamalar ham qanoatlantiradi, ya'ni $t\in(t_0,t_1)$ da $F(\alpha(t),\beta(t))\equiv 0$

bo'ladi. Bu holda yuqoridagi kabi amallarni bajarib umumiy yechimning (2) ko'rinishdagi parametrik tenglamasini hosil qilamiz.

Tenglamalarni yeching va boshlang'ich shartlar berilgan hollarda xususiy yechimlarni toping (**89-90**).

89. $y^{IV}=x^2-\sin x.$

◀Tenglamaning ikkala tomonini ketma-ket to`rt marta integrallaymiz:

$$y'''=\int(x^2-\sin x)dx+C_1=\frac{1}{3}x^3+\cos x+C_1,$$

$$y''=\int\left(\frac{1}{3}x^3+\cos x+C_1\right)dx+C_2=\frac{1}{12}x^4+\sin x+C_1x+C_2,$$

$$y'=\int\left(\frac{1}{12}x^4+\sin x+C_1x+C_2\right)dx+C_3=$$

$$=\frac{1}{60}x^5-\cos x+C_1\frac{x^2}{2}+C_2x+C_3,$$

$$y=\frac{1}{360}x^6-\sin x+C_1\frac{x^3}{6}+C_2\frac{x^2}{2}+C_3x+C_4.$$▶

90. $y'''=24x+8e^{2x}$, $x_0=0$ da $y_0=1,\ y_0'=2$ va $y_0''=2$ bo`lsin.

◀Avvalo tenglamaning umumiy yechimini topamiz:

$$y''=12x^2+4e^{2x}+C_1,\quad y'=4x^3+2e^{2x}+C_1x+C_2,$$

$$y=x^4+e^{2x}+C_1\frac{x^2}{2}+C_2x+C_3.$$

So'ngra o'zgarmaslarning qiymatlarini shunday tanlaymizki, berilgan boshlang'ich shartlar bajarilsin:

$$2=4+C_1,\ 2=2+C_2,\ 1=1+C_3,$$

bulardan $C_1=-2,\ C_2=C_3=0$ qiymatlarni topamiz. Demak, berilgan boshlang’ich shartlarni qanoatlantiradigan xususiy yechim $y = x^4 - x^2 + e^{2x}$ ko’rinishda bo`ladi.▶

2.1.2. $F\left(y^{(n-1)}, y^{(n)}\right) = 0$ **ko’rinishdagi tenglamalar.** Agar parametrik ko’rinishda berilgan $u = \alpha(t), \quad v = \beta(t)$ tenglamalar $F(u,v) = 0$ tenglamani qanoatlantirsa, u holda $F\left(y^{(n-1)}, y^{(n)}\right) = 0$ ko’rinishdagi differensial tenglamani integrallash mumkin. Haqiqatan ham, $y^{(n-1)} = \alpha(t), \;\; y^{(n)} = \beta(t)$ va $d\left(y^{(n-1)}\right) = \beta(t)dx,$ ya’ni $\alpha'(t)dt = \beta(t)dx$ deb olib, x ni aniqlaymiz:

$$x = \int \frac{\alpha'(t)}{\beta(t)} dt + C_1 \;.$$

y funksiyani $y^{(n-1)} = \alpha(t)$ tenglamadan **2.1.1-**bandda bayon qilingan usulda topamiz.

$F\left(y^{(n-1)}, y^{(n)}\right) = 0$ tenglamani yechishning yana boshqa usuli $y^{(n-1)} = z(x)$ belgilash kiritishdir. Bunda masala $F(z, z') = 0$ ko`rinishdagi birinchi tartibli differensial tenglamani yechishga keltiriladi.

Tenglamalarni yeching (**91-92**).

91. $y''' - e^{-y''} = 0.$

◀Bu tenglamada $n = 3$ va $F(u,v) \equiv u - e^{-v} = 0.$ $y'' = t, \; y''' = e^{-t}$ deb olamiz. U holda $d\left(y''\right) = e^{-t}dx,$ yoki $dt = e^{-t}dx.$ Oxirgi tenglamani integrallab, $x = e^t + C_1$ ni topamiz. y funksiyani topish uchun $y'' = t$ tenglamadan foydalanamiz. Buning uchun $d(y') = t\,dx = t\,e^t dt$ deb yozib, topamiz:

$$y' = \int t\,e^t dt + C_2 = e^t(t-1) + C_2.$$

Yana bir marta integrallaymiz: $y = \dfrac{e^{2t}}{2}\left(t - \dfrac{3}{2}\right) + C_2 e^t + C_3.$

Oxirgi natijani yozamiz:

$$x = e^t + C_1, \quad y = \frac{e^{2t}}{2}\left(t - \frac{3}{2}\right) + C_2 e^t + C_3. \blacktriangleright$$

92. $y''' = 3\sqrt[3]{y''^{\,2}}$.

◀Bu tenglamada $y'' = z(x)$ deb olsak, $z' = 3\sqrt[3]{z^2}$ ko'rinishdagi birinchi tartibli tenglamaga ega bo`lamiz, uning yechimlarini topish qiyin emas (**18**-misolga qarang): $z = (x - C_1)^3$; $z = 0$. Demak, $y'' = (x - C_1)^3$, $y'' = 0$. Bu tenglamalarni integrallab, natijani yozamiz:

$$y = \frac{1}{20}(x - C_1)^5 + C_2 x + C_3, \quad y = C_1 x + C_2. \blacktriangleright$$

2.1.3. $F\left(x, y^{(k)}, y^{(k+1)}, ..., y^{(n)}\right) = 0$ **ko'rinishdagi tenglamalar.** Agar tenglama $F\left(x, y^{(k)}, y^{(k+1)}, ..., y^{(n)}\right) = 0$ $(k = 1, 2, ...)$ ko'rinishda bo'lsa, u holda $y^{(k)} = z(x)$ o`rniga qo'yish yordamida $F\left(x, z, z', ..., z^{(n-k)}\right) = 0$ ko'rinishdagi ($n - k$)-tartibli tenglama hosil qilinadi.

Tenglamalarni yeching (**93-94**).

93. $x^2 y'' = y'^{\,2}$.

◀Bu tenglamada $k = 1$, $n = 2$, shuning uchun $y' = z(x)$ deb olamiz. U holda $n - k = 1$-tartibli $x^2 z' = z^2$ differensial tenglama hosil bo'ladi. O'zgaruvchilari ajraladigan bu tenglamani integrallab, topamiz:

$$z = \frac{x}{1 - C_1 x} = y'; \quad z = y'.$$

Oxirgi tenglamani integrallash natijasi unda qatnashayotgan C_1 ixtiyoriy o'zgarmasning qabul qiladigan qiymatlariga bog'liq: agar $C_1 \neq 0$ bo'lsa, u holda $y \equiv \int \frac{x\,dx}{1 - C_1 x} + C_2 = -\frac{1}{C_1}x - \frac{1}{C_1^2}\ln\left|C_1 x - 1\right| + C_2$, aks holda, ya'ni agar $C_1 = 0$ bo`lsa, u holda $2y = x^2 + C$ bo`ladi.▶

94. $x y''' = y'' - x y''$.

◀Bu tenglamada $k=2, n=3$, shuning uchun $y''=z(x)$ deb olamiz. U holda $n-k=1$-tartibli $xz'=(1-x)z$ differensial tenglama hosil bo'ladi. O'zgaruvchilari ajraladigan bu tenglamani integrallab, topamiz:

$$z=C_1xe^{-x};\ y''=C_1xe^{-x}.$$

Hosil bo'lgan oxirgi tenglamani ikki marta integrallab, natijani olamiz: $y=C_1e^{-x}(x+2)+C_2x+C_3$.▶

2.1.4. $F\left(y,y',...,y^{(n)}\right)=0$ **ko`rinishdagi tenglamalar.** Agar tenglamada $x$ erkli o`zgaruvchi oshkor qatnashmasa, ya'ni tenglama $F\left(y,y',...,y^{(n)}\right)=0$ ko`rinishda bo'lsa, u holda y ni yangi erkli o'zgaruvchi, $y'=p(y)$ ni esa noma'lum funksiya sifatida olish kerak.

Shunga binoan,

$$y''=\frac{d}{dx}(y')=\frac{dy'}{dy}\frac{dy}{dx}=p\frac{dp}{dy}=p\,p',$$

$$y'''=\frac{d}{dx}\left(p\frac{dp}{dy}\right)=\frac{d}{dy}\left(p\frac{dp}{dy}\right)\cdot\frac{dy}{dx}=p\frac{d}{dy}\left(p\frac{dp}{dy}\right)$$

$$=p\left(\left(\frac{dp}{dy}\right)^2+p\frac{d^2p}{dy^2}\right)=p\left(p'^2+p\,p''\right)$$

va shu kabi tengliklar o`rinli bo`ladi. Ularni berilgan tenglamaga qo'ysak, $(n-1)-$tartibli differensial tenglama hosil bo`ladi.

Tenglamalarni yeching (**95-96**).

95. $yy''=y'^2-y'^3$.

◀ $y'=p(y)$ deb olamiz. U holda $y''=p\dfrac{dp}{dy}$ va $yp\dfrac{dp}{dy}=p^2-p^3$. Hosil bo'lgan tenglama ikkita tenglamaga ajraydi: $p=0$ va $y\dfrac{dp}{dy}=p-p^2$. Endi $p=y'$ ekanligini e'tiborga olsak, birinchi tenglamadan $y=C$ tenglik, ikkinchi tenglamadan esa $y'=y/(C_1+y)$ tenglama kelib chiqadi. Oxirgi tenglamani integrallab, $x=C_1\ln|y|+y+C_2$ yechimga ega bo'lamiz. Demak, tenglamaning umumiy yechimi: $x=C_1\ln|y|+y+C_2$, $y=C$.►

96. $y''^2-2y'y'''+1=0.$

◄ $y'=p(y)$ deb olamiz. U holda $y''=p\,p'$, $y'''=p\left(p'^2+p\,p''\right)$. Bularni berilgan tenglamaga qo'ysak, $p^2p'^2-2p^2\left(p'^2+pp''\right)+1=0$ kelib chiqadi. Bundan quyidagi ikkinchi tartibli tenglama hosil bo`ladi: $p'^2+2pp''-p^{-2}=0$. Bu tenglamada $y$ argument oshkor qatnashmayapti, shuning uchun $2+3m+m-2=2=4m$ deb olamiz. U holda $p''=u\dfrac{du}{dp}$ bo'lib, oxirgi tenglama $u^2+2puu'-p^{-2}=0$, ya'ni $w+pw'=p^{-2}$ ko`rinishga keladi, bu yerda $w=u^2$. Bu tenglamaning umumiy yechimi $w=\dfrac{C_1}{p}-\dfrac{1}{p^2}$, ya`ni $p'^2=\dfrac{C_1}{p}-\dfrac{1}{p^2}$ bo'lib, bundan p' ni topamiz:

$$p'=\pm\sqrt{\frac{C_1}{p}-\frac{1}{p^2}}. \qquad (1)$$

(1) tenglamani integrallab, quyidagiga ega bo`lamiz:

$$\pm\int\frac{pdp}{\sqrt{C_1p-1}}=y+C_2, \text{ ya`ni } \quad \pm\frac{2}{3C_1^2}\sqrt{C_1p-1}(C_1p+2)=y+C_2.$$

Endi $dx=\dfrac{dy}{p}$ ekanligidan va (1) tenglamadan foydalansak,

$$dx=\pm\frac{dp}{\sqrt{C_1p-1}}, \quad x=\pm\frac{2}{C_1}\sqrt{C_1p-1}+C_3$$

bo'ladi. So'ngra x va y larning ifodalaridan p parametrni yo'qotib, natijani yozamiz:

$$12(C_1y-x)=C_1^2(x-C_3)^3-12(C_3+C_1C_2),\ \text{ya`ni}$$
$$12(C_1y-x)=C_1^2(x-C_3)^3+C_2^*,$$

bu yerda $C_2^*=-12(C_3+C_1C_2)$.▶

Izoh. Bu tenglamada y o'zgaruvchi oshkor qatnashmayotganligi uchun uni boshqa usul bilan ham yechish mumkin. Haqiqatan ham, $y'=z(x)$ deb olsak, berilgan $y''^2-2y'y'''+1=0$ tenglama $z'^2-2zz''+1=0$ ko`rinishni oladi. Bu tenglamada $x$ erkli o`zgaruvchi oshkor qatnashmayotganligi uchun yuqoridagi kabi $z'=p(z)$ deb olamiz. Buning natijasida hosil bo'lgan $p^2-2zpp'+1=0$ tenglamaning umumiy yechimini $p=\pm\sqrt{C_1z-1}$, ya`ni $z'=\pm\sqrt{C_1z-1}$ ko'rinishda topamiz. Bu tenglamani integrallaymiz:

$$\pm2\sqrt{C_1z-1}=C_1(x+C_2),\ \text{ya`ni}\ 4(C_1y'-1)=C_1^2(x+C_2)^2.$$

Oxirgi tenglamani yana bir marta integrallab,

$$12(C_1y-x)=C_1^2(x-C_2)^3+C_3$$

natijani olamiz.

2.1.5. Noma'lum funksiya va uning hosilalariga nisbatan bir jinsli differensial tenglamalar. Agar ixtiyoriy k haqiqiy sonda

$$F\left(x,ky,ky',...,ky^{(n)}\right)=k^{\alpha}F\left(x,y,y',...,y^{(n)}\right)$$

ayniyat o`rinli bo`lsa, u holda $F\left(x,y,y',...,y^{(n)}\right)=0$ tenglama *funksiya va uning hosilalariga nisbatan bir jinsli tenglama* deyiladi. Bunday tenglamalarni yechish uchun $y'=y\,z(x)$ deb olish kerak, bu yerda $z=z(x)$ – yangi noma`lum funksiya.

Shunday qilib,

$$y'=y\,z(x),\quad y''=(y\,z(x))'=y'z+y\,z'=y(z^2+z'),$$

$$y'''=\left(y(z^2+z')\right)'=y'(z^2+z')+y(2zz'+z'')=y(z^3+3zz'+z'')$$

va h.k. hosilalarni funksiya va uning hosilalariga nisbatan bir jinsli tenglamaga qo'ysak, tenglamaning tartibi bittaga pasayadi.

Tenglamalarning bir jinsliligidan foydalanib tartibini pasaytiring va ularni yeching (**97-98**).

97. $xyy'' - xy'^2 = yy'.$

◀Bu tenglamada $F(x,y,y',y'') \equiv xyy'' - xy'^2 - yy'$ bo'lib,

$$x \cdot ky \cdot ky'' - x \cdot (ky')^2 - ky \cdot ky' = k^2 \left(xyy'' - xy'^2 - yy' \right)$$

ayniyat o`rinli bo'lganligi uchun bu tenglama funksiya va uning hosilalariga nisbatan bir jinsli tenglamadir. $y' = y\,z(x)$, $y \neq 0$ deb olamiz. U holda $xy \cdot y(z^2 + z') - x(y\,z)^2 = y \cdot yz$, ya`ni $xz' = z$ tenglamaga ega bo'lamiz. Oxirgi tenglamani integrallab, $z = C_1 x$, ya`ni $y' = C_1 xy$ ko`rinishdagi o'zgaruvchilari ajraladigan tenglamaga ega bo'lamiz. Bu tenglama $y = C_2 \exp(C_1^* x^2)$ umumiy yechimga ega, bu yerda $C_1^* = C_1 / 2.$

Yuqorida $y \neq 0$ deb olingan edi. $y = 0$ bo'lgan holni alohida ko'rib chiqamiz. $y = 0$ bo'lganda $y' = y\,z(x)$ tenglikka ko'ra $y' = 0$ bo'ladi. Bevosita tekshirish ko'rsatadiki, $y' = 0,$ ya`ni $y = C$ funksiyalar tenglamani qanoatlantiradi va bu yechimlar umumiy yechimda bor. Demak, berilgan tenglamaning umumiy yechimi $y = C_2 \exp(C_1 x^2)$ ko'rinishda bo'ladi.▶

98. $x^2(y'^2 - 2yy'') = y^2.$

◀Bu tenglamada $F(x,y,y',y'') \equiv x^2(y'^2 - 2yy'') - y^2$ bo'lib,

$$x^2 \left((ky')^2 - 2ky \cdot ky'' \right) - (ky)^2 = k^2 \left(x^2 (y'^2 - 2yy'') - y^2 \right)$$

ayniyat o'rinli bo'lganligi uchun bu tenglama funksiya va uning hosilalariga nisbatan bir jinsli tenglamadir. $y' = y\,z(x)$, $y \neq 0$ deb olamiz. U holda $2z' + z^2 = -x^{-2}$ Rikkati tenglamasiga (**1.6-**bandga qarang) ega bo'lamiz. Rikkati tenglamasining $z_1(x) = 1/x$ xususiy yechimini tanlash yo'li bilan topib, $z = u + x^{-1}$ almashtirish yordamida uni Bernulli tenglamasiga keltiramiz: $u' + \frac{u}{x} = -\frac{1}{2}u^2$. Bernulli tenglamasining yechimini $u = \frac{2}{x\ln C_1 x}$, $u = 0$, ya`ni $z = \frac{2}{x\ln C_1 x} + \frac{1}{x}$, $z = \frac{1}{x}$ ko'rinishda topamiz.

Endi $y' = y\,z(x)$ ekanligini e`tiborga olib, so'ngra

$$\frac{dy}{y} = \left(\frac{2}{x\ln C_1 x} + \frac{1}{x}\right)dx,\ \ \frac{dy}{y} = \frac{1}{x}dx$$

tenglamalarni integrallab, natijani yozamiz:

$$y = C_2 x\left(\ln C_1 x\right)^2;\ y = Cx. \blacktriangleright$$

2.1.6. Umumlashgan bir jinsli differensial tenglamalar. Agar ixtiyoriy k va biror α haqiqiy sonda

$$F\left(kx, k^{\alpha}y, k^{\alpha-1}y', k^{\alpha-2}y'', ..., k^{\alpha-n}y^{(n)}\right) = k^{\beta}F\left(x, y, y', ..., y^{(n)}\right)$$

ayniyat o'rinli bo'lsa, u holda $F\left(x, y, y', ..., y^{(n)}\right) = 0$ tenglama *umumlashgan bir jinsli tenglama* deyiladi.

Agar $F\left(x, y, y', ..., y^{(n)}\right) = 0$ tenglama umumlashgan bir jinsli tenglama bo'lsa, u holda $x = e^t$, $y = e^{\alpha t}z(t)$ o'rniga qo'yish yordamida hosil bo'lgan yangi tenglamada t o'zgaruvchi oshkor qatnashmaydi. Tenglamaning tartibi shu yo'l bilan pasaytiriladi.

Shunday qilib,

$$y' = \frac{d(e^{\alpha t}z)}{dx} = e^{-t}\frac{d}{dt}(e^{\alpha t}z) = e^{-t+\alpha t}(\alpha z + z'),$$

$$y'' = \frac{dy'}{dx} = e^{-t}\frac{dy'}{dt} = e^{-t}\frac{d}{dt}\left(e^{(\alpha-1)t}(\alpha z + z')\right) =$$

$$= e^{(\alpha-2)t}\left((\alpha-1)\alpha z + (2\alpha-1)z' + z''\right)$$

va h.k. hosilalarni umumlashgan bir jinsli tenglamaga qo'ysak, tenglamaning tartibi pasayadi.

Tenglamalarni yeching (**99-100**).

99. $4x^2y^3y''=x^2-y^4$.

◀Tenglamani umumlashgan bir jinslilikka tekshiramiz. Shu niyatda $F(x,y,y',y'')=4x^2y^3y''-x^2+y^4$ funksiyaning ifodasida x,y,y',y'' o'zgaruvchilar o'rniga mos ravishda $kx,\ k^my,\ k^{m-1}y',\ k^{m-2}y''$ qo'yamiz va m ning qiymatini shunday tanlaymizki (agar tanlash imkoni bo'lsa), natijada

$$4(kx)^2(k^my)^3\cdot k^{m-2}y''-(kx)^2+(k^my)^4=k^{\alpha}(4x^2y^3y''-x^2+y^4)$$

ayniyat o'rinli bo'lsin. $2+3m+m-2=2=4m$ qo'sh tenglik $m=1/2$ da yechimga ega. Shunday qilib, berilgan tenglama umumlashgan bir jinsli va uni integrallash uchun $x=e^t,\ y=e^{t/2}u(t)$ o'rniga qo'yishdan foydalanamiz. Bunda

$$y'=\frac{dy}{dx}=\frac{e^{t/2}\left(\frac{u(t)}{2}+u'(t)\right)}{e^t}=e^{-t/2}\left(\frac{u}{2}+u'\right),$$

$$y''=\frac{dy'}{dx}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=e^{-t}\frac{d}{dt}\left(e^{-t/2}\left(\frac{u}{2}+u'\right)\right)=e^{-3t/2}\left(u''-\frac{u}{4}\right).$$

x va y dan olingan hosilalarni dastlabki tenglamaga qo'yib, bir necha amallarni bajargandan so'ng $4u^3u''=1,\ u\neq 0$ tenglamaga ega bo'lamiz. Bu tenglamada t o'zgaruvchi oshkor qatnashmaydi, shuning uchun $u'=p(u)$ o'rniga qo'yish yordamida tenglamaning tartibini pasaytiramiz: $4pu^3\frac{dp}{du}=1$. Bu tenglamani integrallab, bir necha amallarni bajargandan so'ng

$$u'=\pm\sqrt{C_1-\frac{1}{4u^2}} \qquad (1)$$

tenglamaga ega bo'lamiz va uni $4C_1u^2-1\neq 0$ bo'lgan holda integrallaymiz:

$$\pm 2\int\frac{udu}{\sqrt{4C_1u^2-1}}=t+C_2,\ \text{ya`ni}\ \pm\frac{1}{2C_1}\sqrt{4C_1u^2-1}=t+C_2,$$

bundan o'z navbatida kelib chiqadi: $u^2=C_1(t+C_2)^2+\dfrac{1}{4C_1}.$

Shunday qilib, berilgan tenglamaning yechimi

$$x=e^t,\ y^2=e^t\left(C_1(t+C_2)^2+\frac{1}{4C_1}\right)$$

parametrik ko'rinishda topiladi. Yechimni x va y o'zgaruvchilar yordamida yozishimiz ham mumkin. Buning uchun $e^t=x$, ya`ni $t=\ln x$ ekanligini e`tiborga olish kerak. Yechimni

$$4C_1^*y^2=4x+x\left(C_1^*\ln C_2^*x\right)^2$$

ko'rinishda olamiz, bu yerda $C_1^*=4C_1,\ C_2^*=e^{C_2}$ - yangi o'zgarmaslar.

Endi (1) tenglamani $4C_1u^2-1=0$ holda alohida qarab chiqamiz. Bunda $u'=0,$ ya`ni $u=C$ bo'ladi. $x=e^t,\ y=e^{t/2}u(t)$ munosabatlardan $y^2=xu^2$ tenglamaga ega bo'lamiz, ammo bevosita tekshirish ko'rsatadiki, $y^2=C^2x$ funksiya dastlabki tenglamaning yechimi emas.►

100. $x^2(2yy''-y'^2)=1-2xyy'.$

◄Tenglamani umumlashgan bir jinslilikka tekshiramiz. Shu maqsadda $F\left(x,y,y',y''\right)=x^2(2yy''-y'^2)-1+2xyy'$ funksiyaning ifodasida x,y,y',y'' o'zgaruvchilar o'rniga mos ravishda $kx,k^my,k^{m-1}y',$ $k^{m-2}y''$ qo'yamiz va $m=0$ qiymatni olamiz. Shunday qilib, berilgan tenglama umumlashgan bir jinsli va uni integrallash uchun $x=e^t,\ y=u(t)$ o'rniga qo'yishdan foydalanamiz. Bir necha amallarni bajargandan so'ng $2uu''-u'^2=1,\quad u\neq 0$ tenglamaga ega bo'lamiz. Bu tenglamada t

o'zgaruvchi oshkor qatnashmaydi, shuning uchun $u' = p(u)$ o'rniga qo'yish yordamida

$$p' - \frac{1}{2u} p = \frac{1}{2u} p^{-1}$$

ko'rinishdagi Bernulli tenglamasini olamiz. Uni integrallash natijasida $p = \pm\sqrt{C_1 z - 1}$, ya`ni $z' = \pm\sqrt{C_1 z - 1}$ birinchi tartibli tenglama hosil bo'ladi. Bu tenglamani integrallab va zarur amallarni bajarib, dastlabki tenglamaning yechimini topamiz: $4(C_1 y - 1) = C_1^2 \ln^2 C_2 x$.►

2.1.7. To'la hosila ko'rinishiga keltiriladigan tenglamalar. Agar algebraik almashtirishlar yordamida

$$F\left(x, y, y', ..., y^{(n)}\right) = 0$$

tenglamani

$$\left(\varphi(x, y, y', ..., y^{(n-1)})\right)' = 0$$

ko'rinishga keltirish mumkin bo'lsa, u holda integrallash yordamida tenglamaning tartibini bittaga pasaytirish mumkin:

$$\varphi\left(x, y, y', ..., y^{(n-1)}\right) = C_1,$$

bu yerda φ – ma'lum funksiya.

Tenglamalarni to'la hosila ko'rinishiga keltiring va yeching (**101-102**).

101. $y''' + y' = 0.$

◄Tenglamaning ikkala tomoniga $2y''$ ni ko'paytirib, $2y''y''' + 2y''y' = 0$ $(y'' \neq 0)$ tenglamani olamiz. Bu yerdan $\left(y''^2\right)' + \left(y'^2\right)' = 0$, so'ngra $y''^2 + y'^2 = C_1^2$ tenglamaga ega bo'lamiz. Endi $y' = C_1 \sin t,\ y'' = C_1 \cos t$ deb olamiz. $d(y') = y''dx$ bo'lganligi uchun $d(\sin t)=\cos t dt$, yoki $dx=dt$, bundan $x=t+C_2$ kelib chiqadi. $y'=C_1\sin t$ tenglamadan topamiz: $y=-C_1\cos t+C_3$.

Endi oxirgi natijani yozamiz:

$x=t+C_2$, $y=-C_1\cos t+C_3$, ya`ni $y=C_1\cos(x+C_2)+C_3$.▶

102. $y'y'''=2y''^2$.

◀Tenglamani $y'y'''-y''^2=y''^2$ ko'rinishda yozib olib, ikkala tomonini $y''^2\neq 0$ ifodaga bo'lib, $\left(y'/y''\right)'=-1$ tenglamaga ega bo'lamiz. Bu tenglama $\dfrac{y'}{y''}=-x+C$, ya`ni $\dfrac{y''}{y'}=\dfrac{1}{C-x}$ umumiy integralga ega. Endi oxirgi tenglamadan $y'=\dfrac{1}{C_1x+C_2}$ $\left(C_1/C_2=-C\right)$ ko'rinishdagi birinchi tartibli tenglamani olish qiyin emas.

Bu tenglamani integrallaymiz: $C_1y=\ln\left|C_1x+C_2\right|+C_3$. Bundan tashqari, $y''=0$, ya`ni $y=C_1x+C_2$ to'g'ri chiziqlar ham tenglamaning yechimi bo'ladi.▶

103. $2y'''-3y'^2=0$ tenglamaning $y(0)=-3,\ y'(0)=1,\ y''(0)=-1$ boshlang'ich shartlarni qanoatlantiradigan yechimini toping.

◀Tenglamaning ikkala tomoniga $y''\neq 0$ ifodani ko'paytirib, uni $\left(y''^2-y'^3\right)'=0$ ko'rinishda yozib olamiz. Bundan $y''^2-y'^3=C_1$ tenglamaga ega bo'lamiz. So'ngra $y'(0)=1,\ \ y''(0)=-1$ shartlardan foydalansak, $C_1=0$ natijani olamiz. Endi $y''^2-y'^3=0$ tenglamani integrallash uchun bu tenglamada $y'=z(x)$ deb olamiz. Yangi $z'^2-z^3=0$ tenglama $z=\dfrac{4}{(x+C_2)^2}$, ya`ni $y'=\dfrac{4}{(x+C_2)^2}$ umumiy yechimga ega. Oxirgi tenglamani integrallashdan oldin $y'(0)=1$ boshlang'ich shartni e`tiborga olsak, $C_2=\pm 2$ qiymatni olamiz. Shundan so'ng $y'=4(x\pm 2)^{-2}$ tenglamaning $y(0)=-3$ boshlang'ich shartni qanoatlantiradigan ikkita

$y=-\dfrac{4}{x-2}-5$ va $y=-\dfrac{4}{x+2}-1$ yechimlarini topamiz. Ammo bulardan $y=-\dfrac{4}{x-2}-5$ funksiya masalaning $y''(0)=-1$ boshlang'ich shartini qanoatlantirmasligiga bevosita tekshirish bilan ishonch hosil qilish mumkin. Shunday qilib, qo'yilgan boshlang'ich shartli masalaning yechimi $(x+2)y=-x-6$ ko'rinishda topiladi.►

104. $y''+2yy'^{2}=\left(2x+\dfrac{1}{x}\right)y'$ tenglamaning tartibini pasaytirib, birinchi tartibli tenglamaga keltiring.

◄Tenglamaning ikkala tomonini $y'\neq 0$ ifodaga bo'lamiz:

$$\frac{y''}{y'}+2yy'=2x+\frac{1}{x}.$$

Hosil bo'lgan tenglamani $\left(\ln|y'|+y^2-x^2+\ln|x|\right)'=0$ ko'rinishda yozib olish mumkin. Bundan $\ln|y'|+y^2-x^2+\ln|x|=C_1$ umumiy integralni olamiz. Bir necha sodda almashtirishlardan keyin bu birinchi tartibli tenglama $xy'=C\exp(x^2-y^2)$ ko'rinishni oladi, bu yerda C-ixtiyoriy o'zgarmas.►

INDIVIDUAL TOPSHIRIQLAR

M26.Differensial tenglamaning berilgan shartlarni qanoatlantiruvchi yechimini toping va topilgan yechimning $x=x_0$ nuqtadagi qiymatini verguldan keyingi ikki xonali son aniqligida toping.

1. $y'''=\sin x,\ x_0=\pi/2, y(0)=1, y'(0)=0, y''(0)=0.$
2. $y'''=1/x,\ x_0=2,\ y(1)=1/4,\ y'(1)=y''(1)=0.$
3. $y''=1/\cos^2 x,\ x_0=\pi/3,\ y(0)=1,\ y'(0)=3/5.$
4. $y'''=6/x^3,\ x_0=2,\ y(1)=0,\ y'(1)=5,\ y''(1)=1.$
5. $y''=4\cos 2x,\ x_0=\pi/4,\ y(0)=1,\ y'(0)=3.$

6. $y'' = 1/\left(1+x^2\right),\ x_0 = 1,\quad y(0) = 0,\quad y'(0) = 0.$

7. $xy''' = 2,\ x_0 = 2,\ y(1) = 1/2,\quad y'(1) = y''(1) = 0.$

8. $y''' = e^{2x},\ x_0 = \dfrac{1}{2},\ y(0) = \dfrac{9}{8},\ y'(0) = \dfrac{1}{4},\ y''(0) = -\dfrac{1}{2}.$

9. $y''' = \cos^2 x,\ x_0 = \pi,\ y(0) = 1,\ y'(0) = -1/8,\ y''(0) = 0.$

10. $y'' = 1/\sqrt{1-x^2},\ x_0 = 1,\ y(0) = 2,\ y'(0) = 3.$

11. $y'' = \dfrac{1}{\sin^2 x},\ x_0 = \dfrac{5\pi}{4},\ y\left(\dfrac{\pi}{4}\right) = \dfrac{\pi}{4},\quad y'\left(\dfrac{\pi}{4}\right) = 1.$

12. $y'' = x + \sin x,\ x_0 = 5,\ y(0) = -3,\quad y'(0) = 0.$

13. $y'' = arctgx,\ x_0 = 1,\ y(0) = y'(0) = 0.$

14. $y'' = tgx \cdot \dfrac{1}{\cos^2 x},\ x_0 = \dfrac{\pi}{4},\ y(0) = \dfrac{1}{2},\ y'(0) = 0.$

15. $y''' = e^{x/2} + 1,\ x_0 = 2,\ y(0) = 8,\ y'(0) = 5,\ y''(0) = 2.$

16. $y'' = xe^{-2x},\ x_0 = -\dfrac{1}{2},\ y(0) = \dfrac{1}{4},\quad y'(0) = -\dfrac{1}{4}.$

17. $y'' = \sin^2 3x,\ x_0 = \dfrac{\pi}{12},\ y(0) = -\dfrac{\pi^2}{16},\quad y'(0) = 0.$

18. $y''' = x\sin x,\ x_0 = \pi/2,\ y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$

19. $y''' = \dfrac{\sin 2x}{\sin^4 x},\ x_0 = \dfrac{5\pi}{2},\ y\left(\dfrac{\pi}{2}\right) = \dfrac{\pi}{2},\ y'\left(\dfrac{\pi}{2}\right) = 1,\ y''\left(\dfrac{\pi}{2}\right) = -1.$

20. $y'' = e^{-x} + \cos x,\ x_0 = \pi,\quad y(0) = -e^{-\pi},\quad y'(0) = 1.$

21. $y'' = \sin^3 x,\ x_0 = \dfrac{5\pi}{2},\quad y\left(\dfrac{\pi}{2}\right) = -\dfrac{7}{9},\quad y'\left(\dfrac{\pi}{2}\right) = 0.$

22. $y''' = \sqrt{x} - \sin 2x,\ x_0 = 1,\ y(0) = -\dfrac{1}{8},\ y'(0) = \dfrac{\cos 2}{8},\ y''(0) = \dfrac{1}{2}.$

23. $y'' = \cos^{-2}(x/2),\ x_0 = 4\pi,\quad y(0) = 0,\quad y'(0) = 1.$

24. $y'' = 2\sin x\cos^2 x,\ x_0 = \dfrac{\pi}{2},\ y(0) = -\dfrac{5}{9},\quad y'(0) = -\dfrac{2}{3}.$

25. $y'' = 2\sin^2 x\cos x,\ x = \pi,\quad y(0) = 1/9,\quad y'(0) = 1.$

26. $y'' = 2\sin x\cos^2 x - \sin^3 x,\ x_0 = \pi/2,\ y(0) = 0,\ y'(0) = 1.$

27. $y'' = 2\cos x\sin^2 x - \cos^3 x,\ x_0 = \dfrac{\pi}{2},\ y(0) = \dfrac{2}{3},\ y'(0) = 2.$

28. $y'' = x - \ln x,\ x_0 = 2,\ y(1) = -\dfrac{5}{12},\ y'(1) = \dfrac{3}{2}.$

29. $y'' = 1/x^2,\ x_0 = 2,\ y(0) = 3,\ y'(1) = 1.$

30. $y''' = \cos 4x,\ x_0 = \pi,\ y(0) = 2,\ y'(0) = \dfrac{15}{16},\ y''(0) = 0.$

M27. Tenglamaning umumiy yechimini toping:

1. $y'' = \sin x / \cos^2 x.$
2. $y'' = 4\cos^2 5x.$
3. $y''\sqrt{1-x^2} + x = 0.$
4. $y'' = xe^x.$
5. $y''\left(x^2+4\right) = 2.$
6. $y'' = 2x\, arctgx.$
7. $y'' = arctg 2x.$
8. $y''\sqrt{9-x^2} = 1.$
9. $y'' = x\ln x.$
10. $y'' = x\sin^2 x.$
11. $y''\left(1-x^2\right) = x^3.$
12. $y'' = 3x^2 + \ln x.$
13. $y''\sqrt{x^2-4} = x.$
14. $y'' = x\sin 2x.$
15. $y'' = 6x\, arctgx.$
16. $y'' = \sin^3 x.$
17. $y'' = \cos^3 x.$
18. $y'' = xe^{2x}.$
19. $y'' = 4\sin^2 x.$
20. $xy'' = 1 + x^2.$
21. $y''\sqrt{1+x} = x.$
22. $y'' = x\sqrt{x-1}.$
23. $y''\cos^3 x = \sin x.$
24. $y'' = e^x(x+1).$
25. $y'' = \sqrt{x} + \ln x.$
26. $y'' = \cos^4 x.$
27. $x^2 y'' = \ln x.$
28. $y'' = (x+3)e^x.$
29. $(x-1)^2 y'' = x^2 - 2x.$
30. $2\sqrt{x}\left(\sqrt{x}+1\right)^2 y'' = 1.$

M28. $y' = z(x)$ almashtirishdan foydalanib, tenglamaning umumiy yechimini toping:

1. $\left(x^2+9\right)y'' + 2xy' = 0.$
2. $xy'' = y' + x.$

3. $y'' - 2y'ctgx = \cos x.$ 4. $x(x+1)y'' = 2(2x+1)y'.$

5. $y'' - 2y'ctgx = \sin^3 x.$ 6. $(x^2+2)y'' + 2xy' = x^2.$

7. $xy'' + x(y')^2 - 2y' = 0.$ 8. $y''\sin x - y'\cos x = \sin x.$

9. $xy'' - y' = x^2e^x.$ 10. $y'' + 4y'tgx = \cos^2 x.$

11. $xy'' + y' = (y')^2.$ 12. $xy'' = y' + x^3.$

13. $xy''\ln x = y'.$ 14. $y'' + 2y'tgx = \cos^3 x.$

15. $y'' - 2xy' = 4x.$ 16. $xy'' - y' = x^2\cos x.$

17. $y'' - 2y'ctgx = 0.$ 18. $xy'' = y' + xe^x.$

19. $(x^2+1)y'' = 2x(y'+1).$ 20. $xy'' = y' + x^2\sin x.$

21. $(x^2+1)y'' = 4x(y'-1).$ 22. $x(2+\ln x)y'' = y'.$

23. $xy'' = y'\ln(y'/x).$ 24. $(1-x^2)y'' = 2xy'.$

25. $xy'' = y' + x^2.$ 26. $(x^2+1)y'' = 2xy'.$

27. $xy'' + y' = x.$ 28. $y'' - 2y'tgx = \cos x.$

29. $x^2y'' + xy' = 1.$ 30. $y'' - y'/(x-1) = x(x-1).$

M29. $y' = p(y)$ almashtirishdan foydalanib, tenglamaning umumiy yechimini toping:

1. $yy'' = (y')^2.$ 2. $2yy'' + (y')^2 = 0.$

3. $yy'' = 2y' + (y')^2.$ 4. $yy'' + 2(y')^2 = 0.$

5. $yy'' - 2(y')^2 = 2y^3y'.$ 6. $y^3y'' + 2y' = 0.$

7. $yy'' = 3(y')^2.$ 8. $y'' = (y')^2 + 2y'.$

9. $y''y^3 = 2y'.$ 10. $3\cdot\sqrt[3]{y^2}\,y'' = y'.$

11. $y'' = 2yy'.$ 12. $2y^2y'' = y'.$

13. $y^3y'' = 6.$ 14. $y^2y'' = y'.$

15. $y'' - 2yy' = 0.$ 16. $y'' - y' = (y')^3.$

17. $4\sqrt{y}\,y'' = 1.$ 18. $2yy'' - (y')^2 = y'.$

19. $yy''+(y')^2+1=0.$

20. $2yy''+(y')^2+y'=0.$

21. $yy''=2(y')^2-(y')^3.$

22. $y''=2y'(y+1).$

23. $yy''-(y')^2=y^2y'.$

24. $yy''-2yy'\ln y=(y')^2.$

25. $3y''=y^{-5/3}.$

26. $yy''=2(y')^2+y'.$

27. $y''-(y')^2=y'.$

28. $y^3y''+9=0.$

29. $y''+(y')^2=2e^{-y}.$

30. $(y+1)y''-(y')^2=y'.$

M30. Tenglamaning tartibini pasaytirib, umumiy yechimini toping:

1. $(1-x^2)y''-xy'=2.$

2. $2xy'y''=y'^2-1.$

3. $x^3y''+x^2y'=1.$

4. $y''+y'tgx=\sin 2x.$

5. $xy''\ln x=y'.$

6. $xy''-y'=x^2e^x.$

7. $y''x\ln x=2y'.$

8. $x^2y''+xy'=1.$

9. $y''=-x/y'.$

10. $xy''=y'.$

11. $y''=y'+x.$

12. $xy''=y'+x^2.$

13. $xy''=y'\ln(y'/x).$

14. $xy''+y'=\ln x.$

15. $y''tgx=y'+1.$

16. $y''+2xy'^2=0.$

17. $2xy'y''=y'^2+1.$

18. $y''-y'/(x-1)=x(x-1).$

19. $y''-2y'ctgx=\sin^3 x.$

20. $y''+4y'=2x^2.$

21. $xy''-y'=2x^2e^x.$

22. $x(y''+1)+y'=0.$

23. $y''+4y'=\cos 2x.$

24. $y''+y'=\sin x.$

25. $x^2y''=y'^2.$

26. $2xy'y''=y'^2-4.$

27. $xy'''\ln x=y''.$

28. $y''ctgx+y'=2.$

29. $(1+x^2)y''=2xy'.$

30. $y'''+y''tgx=\sec x.$

M31.Differensial tenglamaning berilgan shartlarni qanoatlantiruvchi yechimini toping:

1. $y''=y'e^y,\qquad y(0)=0,\quad y'(0)=1.$

2. $y'^2+2yy''=0,\qquad y(0)=1,\quad y'(0)=1.$

3. $yy''+y'^2=0,\qquad y(0)=1,\quad y'(0)=1.$

4. $y'' + 2yy'^3 = 0, \quad y(0) = 2, \quad y'(0) = \frac{1}{3}.$

5. $y'' tgy = 2y'^2, \quad y(1) = \frac{\pi}{2}, \quad y'(1) = 2.$

6. $2yy'' = y'^2, \quad y(0) = 1, \quad y'(0) = 1.$

7. $yy'' - y'^2 = y^4, \quad y(0) = 1, \quad y'(0) = 1.$

8. $y'' = -\frac{1}{2y^3}, \quad y(0) = \frac{1}{2}, \quad y'(0) = \sqrt{2}.$

9. $y'' = 1 - y'^2, \quad y(0) = 0, \quad y'(0) = 0.$

10. $y''^2 = y', \quad y(0) = \frac{2}{3}, \quad y'(0) = 1.$

11. $2yy'' - y'^2 = 1, \quad y(0) = 2, \quad y'(0) = 1.$

12. $y'' = 2 - y, \quad y(0) = 2, \quad y'(0) = 2.$

13. $y'' = y^{-3}, \quad y(0) = 1, \quad y'(0) = 0.$

14. $yy'' - 2y'^2 = 0, \quad y(0) = 1, \quad y'(0) = 2.$

15. $y'' = y' + y'^2, \quad y(0) = 0, \quad y'(0) = 1.$

16. $y'' + \frac{2}{1-y} y'^2 = 0, \quad y(0) = 0, \quad y'(0) = 1.$

17. $y''(1+y) = 5y'^2, \quad y(0) = 0, \quad y'(0) = 1.$

18. $y''(2y+3) - 2y'^2 = 0, \quad y(0) = 0, \quad y'(0) = 3.$

19. $4y''^2 = 1 + y'^2, \quad y(0) = 1, \quad y'(0) = 0.$

20. $2y'^2 = (y-1)y'', \quad y(0) = 2, \quad y'(0) = 2.$

21. $1 + y'^2 = yy'', \quad y(0) = 1, \quad y'(0) = 0.$

22. $y'' + yy'^3 = 0, \quad y(0) = 1, \quad y'(0) = 2.$

23. $yy'' - y'^2 = 0, \quad y(0) = 1, \quad y'(0) = 2.$

24. $yy'' - y'^2 = y^2 \ln y, \quad y(0) = 1, \quad y'(0) = 1.$

25. $y(1 - \ln y) y'' + (1 + \ln y) y'^2 = 0, \quad y(0) = y'(0) = 1.$

26. $y''(1+y) = y'^2 + y', \quad y(0) = y'(0) = 2.$

27. $\sqrt{y}y'' = y'$, $y(0)=1$, $y'(0)=2$.

28. $1/\left(1+y'^2\right) = y''$, $y(0)=0$, $y'(0)=0$.

29. $yy'' - 2yy'\ln y = y'^2$, $y(0)=y'(0)=1$.

30. $\sqrt{y}y'' = 1$, $y(0)=0$, $y'(0)=0$.

2.2. O'ZGARMAS KOEFFITSIENTLI CHIZIQLI BIR JINSLI DIFFERENSIAL TENGLAMALAR

O'zgarmas koeffitsientli chiziqli bir jinsli

$$y^{(n)} + a_1 y^{(n-1)} + \ldots + a_{n-1}y' + a_n y = 0,\ a_j = const\ (j=1,\ldots,n), \tag{1}$$

tenglamani yechish uchun

$$\lambda^n + a_1\lambda^{n-1} + \ldots + a_{n-1}\lambda + a_n = 0 \tag{2}$$

xarakteristik tenglamani tuzish va uning $\lambda_1,\ \lambda_2,\ldots,\lambda_n$ ildizlarini topish kerak.

Agar λ_j ildiz (2) xarakteristik tenglamaning oddiy ildizi bo'lsa, u holda (1) tenglamaning bu ildizga mos kelgan umumiy yechimi $C_j e^{\lambda_j x}$ ko'rinishda bo'ladi. Agar λ ildiz (2) tenglamaning k karrali ildizi bo'lsa, u holda (1) tenglamaning bu ildizga mos kelgan umumiy yechimi

$$\left(C_{m+1} + C_{m+2}x + C_{m+3}x^2 + \ldots + C_{m+k}x^{k-1}\right)e^{\lambda x} \tag{3}$$

ko'rinishda bo'ladi. Bu yerda barcha C_j – ixtiyoriy o'zgarmaslar. (1) tenglamaning koeffitsientlari va λ ildizlar haqiqiy yoki kompleks sonlar bo'lishi mumkin.

Agar (1) tenglamaning hamma koeffitsientlari haqiqiy bo'lsa, u holda λ ildizlar kompleks bo'lgan holda ham yechimni haqiqiy ko'rinishda yozish mumkin. Haqiqatan ham, agar bu kompleks ildiz (2) tenglamaning oddiy ildizi bo'lsa, u holda kompleks qo'shma ildizlarning har bir $\lambda = \alpha \pm \beta i$ juftiga

$$D_1 e^{\alpha x}\cos\beta x + D_2 e^{\alpha x}\sin\beta x$$

umumiy yechim mos keladi, bu yerda D_1, D_2 – ixtiyoriy haqiqiy sonlar.

Agar bu kompleks ildiz (2) tenglamaning k karrali ildizi bo'lsa, u holda kompleks qo'shma ildizlarning har bir $\lambda = \alpha \pm \beta i$ juftiga

$$P_{k-1}(x)e^{\alpha x}\cos\beta x + Q_{k-1}(x)e^{\alpha x}\sin\beta x$$

umumiy yechim mos keladi. Bu yerda P_{k-1} va Q_{k-1} – $(k-1)$-darajali ko'phadlar bo'lib, ularning koeffitsientlari ixtiyoriy haqiqiy sonlardir.

Tenglamalarni yeching va boshlang'ich shartlar berilgan hollarda xususiy yechimlarni toping (**105-110**).

105. $y'' - y' - 2y = 0.$

◀Xarakteristik tenglamani tuzamiz: $\lambda^2 - \lambda - 2 = 0.$ Uning ildizlari $\lambda_1 = -1$ va $\lambda_2 = 2$. λ_1 ildizga $y_1 = e^{-x}$, λ_2 ildizga esa $y_2 = e^{2x}$ xususiy yechimlar mos keladi. Bu yechimlarning ixtiyoriy chiziqli kombinatsiyasi berilgan tenglamaning umumiy yechimi bo'ladi: $y = C_1e^{-x} + C_2e^{2x}$.▶

106. $y'' - 4y' = 0,\ y(0) = 0, y'(0) = 4.$

◀Berilgan differensial tenglamaga mos kelgan $\lambda^2 - 4\lambda = 0$ xarakteristik tenglama $\lambda_1 = 0$ va $\lambda_2 = 4$ ildizlarga ega, shuning uchun dastlabki tenglamaning umumiy yechimi $y = C_1 + C_2e^{4x}$ ko'rinishda bo'ladi.

Berilgan boshlang'ich shartlarni qanoatlantiruvchi xususiy yechimni topish uchun umumiy yechimdan hosila olamiz: $y' = 4C_2e^{4x}$ bo'ladi. So'ngra umumiy yechimning ifodasida x, y, y' o'rniga mos ravishda ularning $0, 0, 4$ qiymatlarini qo'yamiz: $C_1 + C_2 = 0, 4C_2 = 4,$ bundan $C_1 = -1, C_2 = 1$ qiymatlarni olamiz. Izlanayotgan xususiy yechim $y = e^{4x} - 1$ ko'rinishda topiladi.▶

107. $y''' - 64y = 0.$

◀Berilgan differensial tenglamaga mos kelgan $\lambda^3 - 64 = 0$

xarakteristik tenglama $\lambda_1 = 4$ va $\lambda_{2,3} = -2 \pm 2\sqrt{3}\, i$ ildizlarga ega. $\lambda_1 = 4$ haqiqiy ildizga mos kelgan umumiy yechim $C_1 e^{4x}$, $\lambda_{2,3} = -2 \pm 2\sqrt{3}\, i$ kompleks qo'shma ildizlar juftiga mos keladigan umumiy yechim esa $C_2 e^{-2x} \cos 2\sqrt{3}\, x + C_3 e^{-2x} \sin 2\sqrt{3}\, x$ ko'rinishda bo'ladi.

Shunday qilib, dastlabki tenglamaning umumiy yechimini yoza olamiz:

$$y = C_1 e^{4x} + e^{-2x}\left(C_2 \cos 2\sqrt{3}\, x + C_3 \sin 2\sqrt{3}\, x\right).$$ ►

108. $y'' - 6y' + 9y = 0.$

◄Berilgan differensial tenglamaga mos $\lambda^2 - 6\lambda + 9 = 0$ xarakteristik tenglamani $(\lambda - 3)^2 = 0$ shaklda yozish mumkin. Bundan xarakteristik tenglama 2 karrali $\lambda = 3$ ildizga ega ekanligi ma`lum bo'ladi. Berilgan tenglamaning umumiy yechimi $y = \left(C_1 + C_2 x\right) e^{3x}$ ko'rinishda bo'ladi.►

109. $y^{IV} + 8y'' + 16y = 0.$

◄Berilgan differensial tenglamaga mos $\lambda^4 + 8\lambda^2 + 16 = 0$ xarakteristik tenglamani yechib, $\lambda_1 = \lambda_2 = 2i$, $\lambda_3 = \lambda_4 = -2i$ ildizlarni olamiz. Xarakteristik tenglama 2 karrali $\pm 2i$ kompleks ildizlarga $(\alpha = 0,$ $\beta = 2)$ ega bo'lganligi uchun dastlabki tenglamaning umumiy yechimi

$$y = (C_1 + C_2 x)\cos 2x + (C_3 + C_4 x)\sin 2x$$

ko'rinishda bo'ladi.►

110. $y^{VI} - y^{V} + 9y^{IV} - 9y''' = 0.$

◄Berilgan tenglamaga mos kelgan $\lambda^6 - \lambda^5 + 9\lambda^4 - 9\lambda^3 \equiv \lambda^3(\lambda - 1)(\lambda^2 + 9) = 0$ xarakteristik tenglamani yechib, $\lambda_1 = \lambda_2 = \lambda_3 = 0$, $\lambda_4 = 1$, $\lambda_{5,6} = \pm 3i$ ildizlarni olamiz. 3 karrali $\lambda_{1,2,3} = 0$ ildizga mos kelgan umumiy yechim $C_1 + C_2 x + C_3 x^2$, $\lambda_4 = 1$ ildizga mos kelgani $C_4 e^x$ va kompleks $\lambda_{5,6} = \pm 3i$ ildizlarga mos kelgani esa

$C_5 \cos 3x + C_6 \sin 3x$ ekanligini bilgan holda dastlabki tenglamaning umumiy yechimini yozamiz:

$$y = C_1 + C_2 x + C_3 x^2 + C_4 e^x + C_5 \cos 3x + C_6 \sin 3x . ▶$$

INDIVIDUAL TOPSHIRIQLAR

M32. Differensial tenglamaning umumiy yechimini (umumiy integralini) toping.

1. $a)\ y'' + 4y = 0,\ b)\ y'' - 10y' + 25y = 0.$
2. $a)\ y'' - y' - 2y = 0,\ b)\ y'' + 9y = 0.$
3. $a)\ y'' - 4y' = 0,\ b)\ y'' - 4y' + 13y = 0.$
4. $a)\ y'' - 5y' + 6y = 0,\ b)\ y'' + 3y' = 0.$
5. $a)\ y'' - 2y' + 10y = 0,\ b)\ y'' + y' - 2y = 0.$
6. $a)\ y'' - 4y = 0,\ b)\ y'' + 2y' + 17y = 0.$
7. $a)\ y'' + y' - 6y = 0,\ b)\ y'' + 9y' = 0.$
8. $a)\ y'' - 49y = 0,\ b)\ y'' - 4y' + 5y = 0.$
9. $a)\ y'' + 7y' = 0,\ b)\ y'' - 5y' + 4y = 0.$
10. $a) y'' - 6y' + 8y = 0, \quad b) y'' + 4y' + 5y = 0.$
11. $a) 4y'' - 8y' + 3y = 0, \quad b) y'' - 3y' = 0.$
12. $a) y'' + 4y' + 20y = 0,\ b) y'' - 3y' - 10y = 0.$
13. $a) 9y'' + 6y' + y = 0, \quad b) y'' - 4y' - 21y = 0.$
14. $a) 2y'' + 3y' + y = 0,\ b) y'' + 4y' + 8y = 0.$
15. $a) y'' - 10y' + 21y = 0,\ b) y'' - 2y' + 2y = 0.$
16. $a) y'' + 6y' = 0,\ b) y'' + 10y' + 29y = 0.$
17. $a) y'' + 25y = 0,\ b) y'' + 6y' + 9y = 0.$
18. $a) y'' - 3y' = 0,\ b) y'' - 7y' - 8y = 0.$
19. $a) y'' - 3y' - 4y = 0,\ b) y'' + 6y' + 13y = 0.$
20. $a) y'' + 25y' = 0,\ b) y'' - 10y' + 16y = 0.$
21. $a) y'' - 3y' - 18y = 0, \quad b) y'' - 6y' = 0.$
22. $a) y'' - 6y' + 13y = 0,\ b) y'' - 2y' - 15y = 0.$
23. $a) y'' + 2y' + y = 0,\ b) y'' + 6y' + 25y = 0.$
24. $a) y'' + 10y' = 0,\ b) y'' - 6y' + 8y = 0.$

25. $a)\ y'' + 5y = 0, \quad b)\ 9y'' - 6y' + y = 0.$

26. $a)\ y'' + 6y' + 10y = 0, \quad b)\ y'' - 4y' + 4y = 0.$

27. $a)\ y'' - y = 0, \quad b)\ 4y'' + 8y' - 5y = 0.$

28. $a)\ y'' + 8y' + 25y = 0, \quad b)\ y'' + 9y' = 0.$

29. $a)\ 6y'' + 7y' - 3y = 0, \quad b)\ y'' + 16y = 0.$

30. $a)\ 9y'' - 6y' + y = 0, \quad b)\ y'' + 12y' + 37y = 0.$

M33. Differensial tenglamaning umumiy yechimini (umumiy integralini) toping.

1. $a)\ y'' + 3y' + 2y = 0, \quad b)\ 2y'' - y' - y = 0.$
2. $a)\ y'' + 4y' + 4y = 0, \quad b)\ 2y'' - 3y' - 5y = 0.$
3. $a)\ y'' - 3y' + 2y = 0, \quad b)\ 4y'' + 4y' + 5y = 0.$
4. $a)\ y'' + 2y' + 5y = 0, \quad b)\ 4y'' - 8y' + 5y = 0.$
5. $a)\ y'' - 2y' = 0, \quad b)\ 4y'' - 12y' + 9y = 0.$
6. $a)\ y'' - y' - 12y = 0, \quad b)\ 3y'' - 4y' + 4y = 0.$
7. $a)\ y'' - 4y' + 20y = 0, \quad b)\ 9y'' - 12y' + 4y = 0.$
8. $a)\ y'' + 2y' - 3y = 0, \quad b)\ 5y'' - 6y' + 5y = 0.$
9. $a)\ y'' + 16y = 0, \quad b)\ 5y'' + 2y' - 7y = 0.$
10. $a)\ y'' + 5y' = 0, \quad b)\ 3y'' - 5y' - 8y = 0.$
11. $a)\ y'' + 2y' + 10y = 0, \quad b)\ y'' - 9y = 0.$
12. $a)\ y'' - 16y = 0, \quad b)\ 9y'' + 12y' + 4y = 0.$
13. $a)\ y'' + 144y = 0, \quad b)\ y'' - 2y' - 3y = 0.$
14. $a)\ y'' - 6y' + 9y = 0, \quad b)\ y'' - 4y' + 29y = 0.$
15. $a)\ y'' + 4y' = 0, \quad b)\ 25y'' - 10y' + y = 0.$
16. $a)\ y'' - 8y' + 7y = 0, \quad b)\ 4y'' - 12y' + 9y = 0.$
17. $a)\ y'' + 2y' + 2y = 0, \quad b)\ 36y'' - 12y' + y = 0.$
18. $a)\ y'' + 4y' + 13y = 0, \quad b)\ 16y'' - 8y' + y = 0.$
19. $a)\ y'' + 2y' = 0, \quad b)\ 8y'' - 4y' + y = 0.$
20. $a)\ y'' - 8y' + 16y = 0, \quad b)\ 2y'' + y' - y = 0.$
21. $a)\ y'' + 2y' + 5y = 0, \quad b)\ 2y'' + 3y' - 5y = 0.$
22. $a)\ y'' - 8y' = 0, \quad b)\ 4y'' - 8y' + 5y = 0.$
23. $a)\ y'' - 4y = 0, \quad b)\ 5y'' + 6y' + 5y = 0.$

24. $a)\ 4y''+4y'+y=0,\ \ b)\ \ 3y''+5y'-8y=0.$
25. $a)\ y''+6y'+8y=0,\ \ b)\ \ 5y''-2y'-7y=0.$
26. $a)\ y''-5y'=0,\ \ b)\ \ 25y''+10y'+y=0.$
27. $a)\ y''-6y'+10y=0,\ \ b)\ \ y''-12y'+11y=0.$
28. $a)\ 9y''+3y'-2y=0,\ \ b)\ \ y''-6y'+5y=0.$
29. $a)\ 4y''-4y'+y=0,\ \ b)\ \ y''+7y'-8y=0.$
30. $a)\ y''+12y'=0,\ \ b)\ \ y''-10y'+9y=0.$

M34. Differensial tenglamaning boshlamg'ich shartlarni qanoatlantiruvchi yechimini toping.

1. $y'''-7y''+6y'=0,\ y(0)=y'(0)=0, y''(0)=30.$
2. $y^{V}-9y'''=0,\ y(0)=1, y'(0)=-1, y''(0)=0.$
3. $y'''-y''=0, y(0)=y'(0)=0, y''(0)=-1.$
4. $y'''-4y'=0, y(0)=0,\ y'(0)=2, y''(0)=4.$
5. $y'''+y'=0, y(0)=0,\ y'(0)=-1, y''(0)=1.$
6. $y'''-y'=0, y(0)=0,\ y'(0)=2, y''(0)=4.$
7. $y'''-y'=0, y(0)=3,\ y'(0)=-1, y''(0)=1.\ \ y=2+e^{-x}$
8. $y'''+y''-5y'+3y=0,\ \ y(0)=0,\ y'(0)=1,\ y''(0)=-14.$
9. $y'''+y''=0,\ \ y(0)=0,\ \ y'(0)=1,\ y''(0)=-1.$
10. $y'''-5y''+8y'-4y=0,\ y(0)=1,\ y'(0)=-1,\ y''(0)=0.$
11. $y'''+3y''+2y'=0,\ y(0)=0,\ y'(0)=0,\ \ y''(0)=2.$
12. $y'''+3y''+3y'+y=0,\ y(0)=-1,\ y'(0)=0,\ y''(0)=1.$
13. $y'''-2y''+9y'-18y=0,\ y(0)=2,\ y'(0)=5,\ y''(0)=-5.$
14. $y'''+9y'=0,\ y(0)=0,\ y'(0)=9,\ y''(0)=-18.$
15. $y'''-13y''+12y'=0,\ y(0)=0,\ y'(0)=1,\ \ y''(0)=133.$
16. $y^{IV}-5y''+4y=0,\ y(0)=0,\ y'(0)=3,\ y''(0)=0,\ y'''(0)=9.$
17. $y^{IV}-10y''+9y=0,\ y(0)=y'(0)=0,\ \ y''(0)=8,\ y'''(0)=24.$
18. $y'''-y''+y'-y=0,\ y(0)=0,\ y'(0)=1,\ \ y''(0)=0.$
19. $y'''-3y''+3y'-y=0,\ y(0)=0,\ y'(0)=0,\ y''(0)=4.$

20. $y''' - y'' + 4y' - 4y = 0,\ y(0) = -1,\ y'(0) = 0,\ y''(0) = -6.$

21. $y^{IV} - 2y''' + y'' = 0,\ y(0) = y'(0) = 0,\ y''(0) = 1,\ y'''(0) = 2.$

22. $y^{IV} - y = 0,\ y(0) = 0,\ y'(0) = 3,\ y''(0) = 0,\ y'''(0) = -1.$

23. $y^{IV} - 16y = 0,\ y(0) = 0,\ y'(0) = 4,\ y''(0) = 0,\ y'''(0) = 0.$

24. $y''' + y'' - 4y' - 4y = 0,\ y(0) = 0,\ y'(0) = 0,\ y''(0) = 12.$

25. $y''' + 2y'' + 9y' + 18y = 0,\ y(0) = 1,\ y'(0) = -3,\ y''(0) = -9.$

26. $y^{V} - 6y^{IV} + 9y''' = 0, y(0) = y'(0) = y'''(0) = 0, y''(0) = -1, y^{IV}(0) = 27.$

27. $y''' + 2y'' + y' = 0,\ y(0) = 0,\ y'(0) = 2,\ y''(0) = -3.$

28. $y''' - y'' - y' + y = 0,\ y(0) = -1,\ y'(0) = 0,\ y''(0) = 13.$

29. $y^{IV} + 5y'' + 4y = 0, y(0) = 1, y'(0) = 4, y''(0) = -1, y'''(0) = -16.$

30. $y^{IV} + 10y'' + 9y = 0, y(0) = 1, y'(0) = 3, y''(0) = -9, y'''(0) = -27.$

M35. Differensial tenglamaning umumiy yechimini (umumiy integralini) toping.

1. $y''' + 2y'' + 4y' + 8y = 0.$
2. $y^{(IV)} - 2y'' - 3y = 0.$
3. $y''' + y'' + 4y = 0.$
4. $y''' - y'' - 8y' + 12y = 0.$
5. $y''' + y'' + y' - 3y = 0.$
6. $2y''' + 3y'' + 18y' - 10y = 0.$
7. $y^{(IV)} - 5y''' + 20y' - 16y = 0.$
8. $y^{(IV)} + 7y''' + 11y'' + 7y' + 10y = 0.$
9. $y^{(IV)} + 16y = 0.$
10. $y^{(IV)} + y''' - 8y' - 8y = 0.$
11. $y^{(IV)} + 2y''' + y'' - 2y' - 2y = 0.$
12. $y^{(IV)} - 7y''' + 5y'' + 4y' + 12y = 0.$
13. $y^{(VI)} + 64y = 0.$
14. $y^{(8)} - y = 0.$
15. $y^{(VI)} + 64y = 0.$
16. $y^{(VI)} - 7y''' - 8y = 0.$
17. $y''' - 3y'' + 4y' - 2y = 0.$
18. $y''' + y' - 10y = 0.$
19. $y''' - 2y'' - 3y' + 10y = 0.$
20. $y^{(IV)} + 18y'' + 81y = 0.$
21. $y^{(IV)} - 2y''' + 5y'' - 8y' + 4y = 0.$
22. $y^{(IV)} + 3y''' - 8y' + 24y = 0.$
23. $y^{(IV)} - 2y''' - 8y'' + 19y' - 6y = 0.$

24. $y^{(8)} - 15y^{(IV)} - 16y = 0.$

25. $y^{(V)} - y''' + 4y'' - 4y' = 0.$ 26. $y^{(VI)} + 8y^{(IV)} + 16y'' = 0.$

27. $y^{(V)} - 10y''' + 9y' = 0.$ 28. $y^{(V)} - 4y^{(IV)} + 4y''' = 0.$

29. $y^{(IV)} - 8y' = 0.$ 30. $y^{(IV)} - y = 0.$

2.3. O'ZGARMAS KOEFFITSIENTLI CHIZIQLI BIR JINSLI BO'LMAGAN DIFFERENSIAL TENGLAMALAR

Aniqmas koeffitsientlar usuli. O'zgarmas koeffitsientli chiziqli bir jinsli bo'lmagan

$$y^{(n)} + a_1 y^{(n-1)} + ... + a_{n-1} y' + a_n y = f(x), \tag{1}$$

tenglama berilgan bo'lsin, bu yerda $a_j = const\ (j = 1,...,n)$ va $f(x)$ – ma`lum funksiya. Bir jinsli bo'lmagan tenglamaning umumiy yechimini aniqlash uchun avvalo (1) tenglamaga mos bir jinsli

$$y^{(n)} + a_1 y^{(n-1)} + ... + a_{n-1} y' + a_n y = 0 \tag{2}$$

tenglamaning y_0 umumiy yechimini topib, so'ngra bir jinsli bo'lmagan (1) tenglamaning bitta y_1 xususiy yechimini topish kerak. y_0 umumiy va y_1 xususiy yechimlarning $y_0 + y_1$ yig'indisi aynan bir jinsli bo'lmagan (1) tenglamaning umumiy yechimini ifodalaydi. (1) tenglamaning xususiy yechimini topish bilan shug'ullanamiz.

Agar o'zgarmas koeffitsientli chiziqli bir jinsli bo'lmagan tenglamaning o'ng tomoni $b_0 + b_1 x + b_2 x^2 + ... + b_m x^m$, $e^{\alpha x}$, $\cos\beta x$, $\sin\beta x$ funksiyalarning ko'paytmalari va yig'indilaridan iborat bo'lsa, u holda bir jinsli bo'lmagan tenglamaning xususiy yechimini aniqmas koeffitsientlar usuli yordamida topish mumkin.

Agar tenglamaning o'ng tomoni $(b_0 + b_1 x + b_2 x^2 + ... + b_m x^m)e^{\gamma x}$ ko'rinishda bo'lsa, u holda xususiy yechim

$$y_1 = x^s Q_m(x) e^{\gamma x} \tag{3}$$

ko'rinishda bo'ladi, bu yerda $Q_m(x)$ – $m-$darajali va koeffitsientlari hozircha ixtiyoriy bo'lgan ko'phad. Agar γ soni

$$\lambda^n + a_1\lambda^{n-1} + ... + a_{n-1}\lambda + a_n = 0 \qquad (4)$$

xarakteristik tenglamaning ildizi bo'lmasa, ya`ni u bironta ham λ_i ga teng bo'lmasa, u holda (3) formulada $s=0$ deb olinadi. Agar γ soni (2) xarakteristik tenglamaning k karrali ildizi bo'lsa, u holda (3) formulada $s=k$ deb olinadi. $Q_m(x)$ ko'phadning koeffitsientlarini aniqlash uchun (3) yechimni (1) tenglamaga qo'yib, chap va o'ng tomonlardagi o'xshash hadlar oldidagi koeffitsientlarni bir-biriga tenglashtirish kerak.

Agar tenglamaning o'ng tomonida sinus va kosinus qatnashsa, u holda ularni Eyler formulalari

$$\cos\beta x = \frac{e^{i\beta x} + e^{-i\beta x}}{2}, \quad \sin\beta x = \frac{e^{i\beta x} - e^{-i\beta x}}{2} \qquad (5)$$

yordamida ko'rsatkichli funksiya orqali ifodalab olib, masalani ko'rilgan holga keltirish mumkin.

Agar tenglama chap tomonining koeffitsientlari haqiqiy sonlar bo'lsa, u holda (5) formulalardan foydalanmasa ham bo'ladi. O'ng tomoni

$$e^{\alpha x}(P_j(x)\cos\beta x + Q_l(x)\sin\beta x) \qquad (6)$$

bo'lgan tenglamaning xususiy yechimini

$$y_1 = x^s e^{\alpha x}(R_m(x)\cos\beta x + T_m(x)\sin\beta x) \qquad (7)$$

ko'rinishda izlaymiz. Agar $\alpha+\beta i$ soni (2) xarakteristik tenglamaning ildizi bo'lmasa, u holda (7) formulada $s=0$ deb olinadi. Agar $\alpha+\beta i$ soni (2) xarakteristik tenglamaning k karrali ildizi bo'lsa, u holda (7) formulada $s=k$ deb olinadi. m esa j va l sonlaridan kattasiga teng, ya`ni $m=\max\{j,l\}$.

Agar tenglamaning o'ng tomoni $P_j(x)e^{\gamma x}$ va (6) ko'rinishdagi bir necha funksiyalarning yig'indisiga teng bo'lsa, xususiy yechim quyidagi qoida bo'yicha izlanadi.

O'ng tomoni $f_1 + \ldots + f_p$ bo'lgan tenglamaning xususiy yechimi o'ng tomonlari $f_1, \ldots, f_p$ bo'lgan tenglamalar xususiy yechimlarining yig'indisiga teng.

Tenglamalarni yeching (**111-115**).

111. $2y'' + y' - y = 2e^x.$ (8)

◀Avvalo berilgan bir jinsli bo'lmagan differensial tenglamaga mos

$$2y'' + y' - y = 0 \quad (9)$$

bir jinsli tenglamaning umumiy yechimini topamiz. Bu tenglamaga mos

$$2\lambda^2 + \lambda - 1 = 0 \quad (10)$$

xarakteristik tenglamaning ildizlari $\lambda_1 = -1$ va $\lambda_2 = 1/2$ bo'lganligi uchun bir jinsli (9) tenglamaning umumiy yechimi

$$y_0 = C_1 e^{-x} + C_2 e^{x/2} \quad (11)$$

ko'rinishda bo'ladi.

(8) tenglamaning o'ng tomoni $f(x) = 2e^x$ bo'lib, $\gamma = 1$ soni (10) xarakteristik tenglamaning ildizlaridan birontasiga ham teng emas va $P_0(x) = 2$ - nolinchi darajali ko'phaddir. Shuning uchun $y_1 = x^s Q_m(x) e^{\gamma x}$ xususiy yechim formulasida $s = 0$ va $Q_0(x) = a$ deb olamiz. Shunday qilib, bir jinsli bo'lmagan tenglamaning xususiy yechimini

$$y_1 = a e^x \quad (12)$$

ko'rinishda izlaymiz. a ni aniqlash uchun (12) ni (8) tenglamaga qo'yamiz va $a = 1$ ni topamiz. Demak, $y_1 = e^x$. Berilgan bir jinsli bo'lmagan tenglamaning umumiy yechimini yozamiz:

$$y = C_1 e^{-x} + C_2 e^{x/2} + e^x . ▶$$

112. $y'' - 7y' + 6y = 74 \sin x.$ (13)

◀Berilgan tenglamaga mos kelgan bir jinsli tenglamaning umumiy yechimini topish qiyin emas:

$$y_0 = C_1e^x + C_2e^{6x}, \quad (\lambda_1 = 1, \ \lambda_2 = 6).$$

(13) tenglamaning o'ng tomoni $f(x) = \sin x$ bo'lib, $\alpha = 0, \beta = 1$, $P_j(x) = 0$, $Q_l(x) = 74$, $j = 0, l = 0$. Demak, $m \equiv \max\{j, l\} = 0$ va $\gamma \equiv \alpha + \beta i = i$ soni xarakteristik tenglamaning ildizlaridan birontasiga ham teng bo'lmaganligi uchun, (7) formulada $s = 0$, $R_0(x) = a$ va $T_0(x) = b$ deb olamiz. Shunday qilib, bir jinsli bo'lmagan tenglamaning hususiy yechimini $y_1 = a\cos x + b\sin x$ ko'rinishda izlaymiz. a va b koeffitsientlarni aniqlash uchun y_1 ni (13) tenglamaga qo'yamiz. $\cos x$ va $\sin x$ funksiyalar oldidagi koeffitsientlarni taqqoslab, $a = 7$, $b = 5$ qiymatlarni topamiz. Demak, $y_1 = 7\cos x + 5\sin x$.

Nihoyat, berilgan (13) tenglamaning umumiy yechimini yozamiz:

$$y = C_1e^x + C_2e^{6x} + 7\cos x + 5\sin x. \blacktriangleright$$

113. $y''' - 3y'' + 3y' - y = 6e^x.$ (14)

◀Berilgan tenglamaga mos kelgan bir jinsli tenglamaning umumiy yechimini topish qiyin emas ($\lambda_1 = 1$ - xarakteristik tenglamaning uch karrali ildizi): $y_0 = (C_1 + C_2x + C_3x^2)e^x$.

(14) tenglamaning o'ng tomoni $f(x) = 2e^x$ bo'lib, $\gamma = 1$ soni xarakteristik tenglamaning uch karrali ildiziga teng va $P_0(x) = 2$ - nolinchi darajali ko'phaddir. Shuning uchun xususiy yechimning (3) formulasida $s = 3$ va $Q_0(x) = a$ deb olamiz. Shunday qilib, bir jinsli bo'lmagan tenglamaning hususiy yechimini $y_1 = ax^3e^x$ ko'rinishda izlaymiz. a ni aniqlash uchun y_1 ni (14) tenglamaga qo'yamiz va $a = 1$ ni topamiz. Demak, $y_1 = x^3e^x$.

Berilgan tenglamaning umumiy yechimini yozamiz:

$$y = (C_1 + C_2x + C_3x^2 + x^3)e^x. \blacktriangleright$$

114. $y'' + y = 4x\cos x$

◀$\lambda^2+1=0$ xarakteristik tenglama $\lambda_{1,2}=\pm i$ kompleks ildizlarga ega. Shuning uchun bir jinsli tenglamaning umumiy yechimi:

$$y_0=C_1\cos x+C_2\sin x.$$

Berilgan tenglamaning o'ng tomoni $f(x)=4x\cos x$ bo'lib,

$\alpha=0,\ \beta=1,\ P_j(x)=4x,\ Q_l(x)=0,\ j=1, l=0.$

Shunga binoan, $m\equiv\max\{j,l\}=1$ va $\gamma\equiv\alpha+\beta i=i$ soni xarakteristik tenglamaning bir karrali $\lambda_1=i$ ildiziga teng, (7) formulada $s=1$, $R_1(x)=ax+b$ va $T_1(x)=cx+d$ deb olamiz. Shunday qilib, bir jinsli bo'lmagan tenglamaning xususiy yechimini

$$y_1=x[(ax+b)\cos x+(cx+d)\sin x]$$

ko'rinishda izlaymiz. a, b, c, d koeffitsientlarni aniqlash uchun y_1 ni dastlabki tenglamaga qo'yamiz va $a=0$, $b=1$, $c=1$, $d=0$ qiymatlarni topamiz. Demak, $y_1=x\cos x+x^2\sin x$ – tenglamaning xususiy yechimi.

Berilgan tenglamaning umumiy yechimini yozamiz:

$$y\equiv y_0+y_1=(C_1+x)\cos x+(C_2+x^2)\sin x.$$ ▶

115. $y''+2y'+y=e^{-x}\cos x+12xe^{-x}.$

◀Xarakteristik tenglamani yechamiz: $\lambda_1=\lambda_2=-1$ - ikki karrali ildiz. Bir jinsli tenglamaning umumiy yechimi $y_0=(C_1+C_2x)e^{-x}$ bo'ladi.

Qaralayotgan tenglamaning o'ng tomoni $f(x)=e^{-x}\cos x+xe^{-x}$ ikkita funksiyalarning yig'indisidan iborat bo'lganligi uchun xususiy yechimni

$$y''+2y'+y=e^{-x}\cos x, \qquad (15)$$

$$y''+2y'+y=12xe^{-x} \qquad (16)$$

tenglamalarning mos y_1 va y_2 xususiy yechimlarining yig'indisi ko'rinishida izlaymiz.

(15) tenglamaning o'ng tomoni $f(x)=e^{-x}\cos x$ bo'lib, $\alpha=-1, \beta=1$, $P_j(x)=1$, $Q_l(x)=0$, $j=0$, $l=0$. Demak, $m\equiv\max\{j,l\}=0$ va $\gamma\equiv\alpha+\beta i=-1+i$ soni xarakteristik tenglamaning bironta ham ildiziga teng emas, (7) formulada $s=0$, $R_0(x)=a$ va $T_0(x)=b$ deb olamiz. Shunday qilib, (15) tenglamaning xususiy yechimini $y_1=-e^{-x}\cos x$ ko'rinishda topamiz ($a=-1, b=0$).

(16) tenglamaning o'ng tomoni $f(x)=12xe^{-x}$ bo'lib, $\gamma=1$ soni xarakteristik tenglamaning ikki karrali ildiziga teng va $P_1(x)=12x$- birinchi darajali ko'phaddir. Shuning uchun xususiy yechimning $y_2=x^s Q_m(x)e^{\gamma x}$ formulasida $s=2$ va $Q_1(x)=ax+b$ deb olamiz. Shunday qilib, (16) tenglamaning xususiy yechimi $y_2=2x^3e^{-x}$ ko'rinishda topiladi ($a=2$, $b=0$).

Izlanayotgan umumiy yechim $y=y_0+y_1+y_2$ ko'rinishida bo'ladi:

$$y=\left(C_1+C_2x+2x^3-\cos x\right)e^{-x}.\blacktriangleright$$

116. $y^{IV}-4y'''+5y''=x^2\cos 2x+xe^x\sin 2x+e^{2x}\sin x$ tenglamaning xususiy yechimi ko'rinishini aniqmas koeffitsientlar bilan yozing (koeffitsientlarning sonli qiymatlarini topish shart emas).

◀Xarakteristik tenglamani tuzib, ildizlarini topamiz:

$$\lambda^4-4\lambda^3+5\lambda^2=0,\ \lambda_1=\lambda_2=0,\ \lambda_{3,4}=2\pm i.$$

Xususiy yechimni quyidagi

$$y^{IV}-4y'''+5y''=x^2\cos 2x,\quad y^{IV}-4y'''+5y''=xe^x\sin 2x,$$

$$y^{IV}-4y'''+5y''=e^{2x}\sin x$$

uchta tenglamalar y_1, y_2 va y_3 xususiy yechimlarining yig'indisi ko'rinishida izlaymiz.

Birinchi tenglamaning o'ng tomoni $f_1(x)=x^2\cos 2x$ bo'lib, $\alpha=0$, $\beta=2$, $P_j(x)=x^2$, $Q_l(x)=0$, $j=2$, $l=0$. Demak, $m\equiv\max\{j,l\}=2$ va $\gamma\equiv\alpha+\beta i=2i$ soni xarakteristik tenglamaning bironta ham ildiziga teng emas ($s=0$). Shunga asoslanib, birinchi tenglama xususiy yechimining ko'rinishini yozamiz:

$$y_1=(a_0x^2+a_1x+a_2)\cos 2x+(b_0x^2+b_1x+b_2)\sin 2x.$$

Ikkinchi tenglamaning o'ng tomoni $f_2(x)=xe^x\sin 2x$ bo'lib, $\alpha=1$, $\beta=2$, $P_j(x)=0$, $Q_l(x)=x$, $j=0$, $l=1$. Demak, $m\equiv\max\{j,l\}=1$ va $\gamma\equiv\alpha+\beta i=1+2i$ soni xarakteristik tenglamaning bironta ham ildiziga teng emas ($s=0$). Shunday qilib, ikkinchi xususiy yechim $y_2=[(c_0x+c_1)\cos 2x+(d_0x+d_1)\sin 2x]e^x$ ko'rinishda izlanadi.

Uchinchi tenglamaning o'ng tomoni $f_3(x)=e^{2x}\sin x$ bo'lib, $\alpha=2$, $\beta=1$, $P_j(x)=0$, $Q_l(x)=1$, $j=0$, $l=0$. Demak, $m\equiv\max\{j,l\}=0$ va $\gamma\equiv\alpha+\beta i=2+i$ soni xarakteristik tenglamaning bir karrali $\lambda_3=2+i$ ildiziga teng ($s=1$). Shunga ko'ra, uchinchi tenglamaning xususiy yechimi $y_3=x(p\cos x+q\sin x)e^{2x}$ ko'rinishda izlanadi.

Xullas, berilgan tenglamaning xususiy yechimi $y=y_1+y_2+y_3$ ko'rinishda bo'ladi.►

INDIVIDUAL TOPSHIRIQLAR

M36. Differensial tenglamaning umumiy yechimini (umumiy integralini) toping.

1. $y''+y'=2x-1.$
2. $y''-2y'+5y=20e^{-x}\cos 2x.$
3. $y''-2y'-8y=12\sin 2x-36\cos 2x.$
4. $y''-12y'+36y=14e^{6x}.$
5. $y''-3y'+2y=(34-12x)e^{-x}.$
6. $y''-6y'+10y=51e^{-x}.$
7. $y''+y=2\cos x-(4x+4)\sin x.$
8. $y''+6y'+10y=74e^{3x}.$
9. $y''-3y'+2y=3\cos x+19\sin x.$

10. $y'' + 6y' + 9y = 8(6x+1)e^{x}$.

11. $y'' + 5y' = 72e^{2x}$.

12. $y'' - 5y' - 6y = 3\cos x + 19\sin x$.

13. $y'' - 8y' + 12y = 36x^4 - 96x^3 + 24x^2 + 16x - 2$.

14. $y'' + 8y' + 25y = 90e^{5x}$.

15. $y'' - 9y' + 20y = 126e^{-2x}$.

16. $y'' + 36y = 36 + 66x - 36x^3$.

17. $y'' + y = -4\cos x - 2\sin x$.

18. $y'' + 2y' - 24y = 6\cos 3x - 33\sin 3x$.

19. $y'' + 6y' + 13y = -75\sin 2x$.

20. $y'' + 5y' = 39\cos 3x - 105\sin 3x$.

21. $y'' - 4y' + 29y = 104\sin 5x$.

22. $y'' - 4y' + 5y = (24\sin x + 8\cos x)e^{-2x}$.

23. $y'' + 16y = 8\cos 4x$.

24. $y'' + 9y = 9x^4 - 12x^2 - 27$.

25. $y'' - 12y' + 40y = 4e^{6x}$.

26. $y'' + 4y' = (24\cos 2x + 2\sin 2x)e^{x}$.

27. $y'' + 2y' + y = 6e^{-x}$.

28. $y'' + 2y' + 37y = 37x^2 - 33x + 74$.

29. $6y'' - y' - y = 3e^{2x}$.

30. $2y'' + 7y' + 3y = 222\sin 3x$.

M37. Differensial tenglamaning umumiy yechimini (umumiy integralini) toping.

1. $y'' - 8y' + 17y = 10e^{2x}$.

2. $y'' + y' - 6y = (6x+1)e^{3x}$.

3. $y'' - 7y' + 12y = 3e^{4x}$.

4. $y'' - 2y' = 6 + 12x - 24x^2$.

5. $y'' - 6y' + 34y = 18\cos 5x + 60\sin 5x$.

6. $y'' - 2y' = (4x+4)e^{2x}$.

7. $y'' + 2y' + y = 4x^3 + 24x^2 + 22x - 4$.

8. $y'' - 4y' = 8 - 16x$.

9. $y'' - 2y' + y = 4e^x.$

10. $y'' - 8y' + 20y = 16(\sin 2x - \cos 2x).$

11. $y'' - 6y' + 13y = 156e^{-3x} \sin 2x.$

12. $y'' + 2y' - 3y = (12x^2 + 6x - 4)e^x.$

13. $y'' + 4y' + 4y = 6e^{-2x}.$ 14. $y'' + 3y' = 10 - 6x.$

15. $y'' + 10y' + 25y = 40 + 52x - 240x^2 - 200x^3.$

16. $y'' + 4y' + 20y = 4\cos 4x + 52\sin 4x.$

17. $y'' + 4y' + 5y = 5x^2 - 32x + 5.$

18. $y'' + 2y' + y = (12x - 10)e^{-x}.$

19. $y'' - 4y = (-24x - 10)e^{2x}.$

20. $y'' + 6y' + 9y = 72e^{3x}.$

21. $y'' + 16y = 80e^{2x}.$ 22. $y'' + 4y' = 15e^x.$

23. $y'' + y' - 2y = 9\cos x - 7\sin x.$

24. $y'' + 2y' + y = (18x + 8)e^{-x}.$

25. $y'' - 14y' + 49y = 98\sin 7x.$ 26. $y'' + 9y = 18e^{3x}.$

27. $4y'' - 4y' + y = -25\cos x.$

28. $3y'' - 5y' - 2y = 6\cos 2x + 38\sin 2x.$

29. $y'' + 4y' + 29y = 26e^{-x}.$

30. $4y'' + 3y' - y = 11\cos x - 7\sin x.$

M38. Differensial tenglamaning boshlamg'ich shartlarini qanoatlantiruvchi yechimini toping.

1. $y'' - 2y' + y = 12\cos 2x - 9\sin 2x,$ $y(0) = -2,$ $y'(0) = 0.$

2. $y'' - 6y' + 9y = 9x^2 - 39x + 65,$ $y(0) = -1,$ $y'(0) = 1.$

3. $y'' + 2y' + 2y = 2x^2 + 8x + 6,$ $y(0) = 1,$ $y'(0) = 4.$

4. $y'' - 6y' + 25y = 9\sin 4x - 24\cos 4x,$ $y(0) = 2,$ $y'(0) = -2.$

5. $y'' - 14y' + 53y = 53x^3 - 42x^2 + 59x - 14,$ $y(0) = 0,$ $y'(0) = 7.$

6. $y'' + 16y = (\cos 4x - 8\sin 4x)e^x,$ $y(0) = 0,$ $y'(0) = 5.$

7. $y'' - 4y' + 20y = 16xe^{2x},$ $y(0) = 1,$ $y'(0) = 7.$

8. $y'' - 12y' + 36y = 32\cos 2x + 24\sin 2x,$ $y(0) = 2,$ $y'(0) = 4.$

9. $y'' + y = x^3 - 4x^2 + 7x - 10, \quad y(0) = 2, \quad y'(0) = 3.$

10. $y'' - y = (14 - 16x)e^{-x}, \quad y(0) = 0, \quad y'(0) = -1.$

11. $y'' + 8y' + 16y = 16x^2 - 16x + 66, \; y(0) = 3, \quad y'(0) = 0.$

12. $y'' + 10y' + 34y = -9e^{-5x}, \; y(0) = 0, \quad y'(0) = 6.$

13. $y'' - 6y' + 25y = 12(1 - 3x)\cos 3x + (32x - 12)\sin 3x.$
$y(0) = 4, \quad y'(0) = 0.$

14. $y'' + 25y = (\cos 5x - 10\sin 5x)e^{x}, \; y(0) = 3, \quad y'(0) = -4.$

15. $y'' + 2y' + 5y = -8e^{-x}\sin 2x, \; y(0) = 2, \; y'(0) = 6.$

16. $y'' - 10y' + 25y = 2e^{5x}, \; y(0) = 3, \quad y'(0) = 13.$

17. $y'' + y' - 12y = (16x + 26)e^{4x}, \; y(0) = 3, \; y'(0) = 5.$

18. $y'' - 2y' + 5y = 5x^2 + 6x - 12, \; y(0) = 0, \quad y'(0) = 2.$

19. $y'' + 8y' + 16y = 16x^3 + 24x^2 - 10x + 8, \quad y(0) = 1, \; y'(0) = 3.$

20. $y'' - 2y' + 37y = 36e^{x}\cos 6x, \; y(0) = 0, \; y'(0) = 6.$

21. $y'' - 8y' = 16 + 48x^2 - 128x^3, \; y(0) = -1, \; y'(0) = 14.$

22. $y'' + 12y' + 36y = 72x^3 - 28, \quad y(0) = 1, \quad y'(0) = 0.$

23. $y'' + 3y' = (40x + 58)e^{2x}, \; y(0) = 0, \quad y'(0) = -2.$

24. $y'' - 9y' + 18y = 26\cos x - 8\sin x, \; y(0) = 0, \; y'(0) = 2.$

25. $y'' + 8y' = 18x + 60x^2 - 32x^3, \; y(0) = 5, \quad y'(0) = -16.$

26. $y'' - 3y' + 2y = -7\cos x - \sin x, \; y(0) = 2, \; y'(0) = 7.$

27. $y'' + 2y' = 6x^2 + 2x - 2, \; y(0) = 2, \quad y'(0) = 2.$

28. $y'' + 16y = 32e^{4x}, \; y(0) = 2, \quad y'(0) = 0.$

29. $y'' + 5y' + 6y = 52\sin 2x, \; y(0) = -2, \quad y'(0) = -5.$

30. $y'' - 4y = 8e^{2x}, \quad y(0) = 1, \quad y'(0) = -8.$

M39. Differensial tenglamaning umumiy yechimini (umumiy integralini) toping.

1. $y'' - 2y' + y = xe^{x}.$
2. $y'' + y = x\sin x.$
3. $y'' - 6y' + 9y = e^{x}\sin x.$
4. $y'' + 2y' + 5y = e^{x}\sin 2x.$
5. $y'' - 6y' + 9y = 2x^2 - 2x + 3.$
6. $y'' - 2y' + 2y = e^{x}\cos x.$
7. $y'' + 4y' + 5y = x^2 + 3.$
8. $y'' - 3y' + 2y = 2xe^{x}.$

9. $y'' + 2y' = 2 + x - x^2.$

10. $4y'' - 16y' + 15y = e^{1,5x}.$

11. $y'' + 4y' + 4y = e^{-2x} + e^x.$

12. $y'' - y' = 2x^2.$

13. $y'' + y' = e^{-x} + x + 1.$

14. $y'' + 2y' + 10y = xe^{-x}.$

15. $y'' + y' - 2y = e^x \cos x.$

16. $y'' + 2y' + y = e^{-x} + \sin x.$

17. $y'' + 4y = \cos 2x.$

18. $y'' + 9y = xe^{3x}.$

19. $y'' - 2y' + 2y = e^x \sin 2x.$

20. $y'' + 9y = e^{3x}.$

21. $y'' - 2y' + y = e^x.$

22. $y'' - 3y' = 1 - 2x - x^2.$

23. $y'' + 4y' - 5y = xe^x.$

24. $y'' - 4y' + 4y = e^{2x}.$

25. $y'' + 2y' + 2y = (x+1)e^x.$

26. $y'' + 2y' + 5y = \cos x.$

27. $y'' + 4y' + 3y = e^{-x} + x^2.$

28. $y'' + 4y' + 8y = \cos 2x.$

29. $y'' - 3y' + 2y = e^{2x}.$

30. $y'' - 8y' = x + \sin 4x.$

M40. Ikkinchi tartibli differensial tenglama uchun Koshi masalasining yechimini toping.

1. $y'' + 4y' + 8y = \sin 4x, \quad y(0) = 0, \quad y'(0) = 1.$
2. $y'' - 3y' + 2y = e^x, \quad y(0) = 2, \quad y'(0) = 1.$
3. $y'' + y' + y = \cos 2x, \quad y(0) = -1, \quad y'(0) = 3.$
4. $y'' - 4y' + 3y = x^2 - 3x, \quad y(0) = 2, \quad y'(0) = 4.$
5. $y'' + 2y' + 5y = e^{2x}, \quad y(0) = 0, \quad y'(0) = 0.$
6. $y'' - 10y' + 9y = xe^x, \quad y(0) = 1, \quad y'(0) = 0.$
7. $y'' - 4y' + 4y = e^{2x}, \quad y(0) = 1, \quad y'(0) = 1.$
8. $y'' - 2y' + 2y = 3x - 2, \quad y(0) = -2, \quad y'(0) = 2.$
9. $y'' + 5y' - 6y = e^{4x}, \quad y(0) = 3, \quad y'(0) = 2.$
10. $y'' + 2y' + 10y = x^2 - 4, \quad y(0) = 0, \quad y'(0) = 4.$
11. $y'' - 4y' = x^2 - 5x + 2, \quad y(0) = 0, \quad y'(0) = -1.$
12. $y'' - 2y' + y = e^x, \quad y(0) = 3, \quad y'(0) = 5.$
13. $y'' + 9y = \sin 3x, \quad y(0) = 2, \quad y'(0) = -1.$
14. $y'' + 2y' + 2y = e^x \sin 3x, \quad y(0) = 1, \quad y'(0) = 3.$
15. $y'' + 2y' + y = e^{-x}, \quad y(0) = 4, \quad y'(0) = 0.$
16. $y'' + y' - 2y = e^{2x} \sin x, \quad y(0) = -5, \quad y'(0) = 1.$

17. $y'' + 6y' + 9y = e^{-3x}$, $y(0) = -3$, $y'(0) = 2$.

18. $y'' + 4y' + 5y = e^{2x}$, $y(0) = 2$, $y'(0) = 6$.

19. $4y'' - 16y' + 15y = x^2 - 1$, $y(0) = 3$, $y'(0) = -1$.

20. $4y'' + 4y' + 5y = xe^x$, $y(0) = 4$, $y'(0) = -1$.

21. $y'' - 3y' + 2y = e^{2x}$, $y(0) = 1$, $y'(0) = 0$.

22. $y'' + 4y' + 5y = x^2 + 2x$, $y(0) = 1$, $y'(0) = 4$.

23. $y'' - 2y' + 2y = e^x \cos x$, $y(0) = 2$, $y'(0) = -5$.

24. $y'' - 6y' + 9y = 2x^2 + 5$, $y(0) = 0$, $y'(0) = 3$.

25. $2y'' - y' - y = x + e^x$, $y(0) = 0$, $y'(0) = 0$.

26. $y'' - 2y' + 10y = \cos x$, $y(0) = -1$, $y'(0) = -3$.

27. $4y'' - 8y' + 5y = xe^x$, $y(0) = 2$, $y'(0) = -4$.

28. $3y'' - 12y' + 4y = e^x \sin 2x$, $y(0) = 1$, $y'(0) = 5$.

29. $y'' - 4y' = 2x^2 + 3x - 1$, $y(0) = 6$, $y'(0) = -2$.

30. $y'' - 8y' + 16y = e^{4x}$, $y(0) = 3$, $y'(0) = 8$.

M41. Differensial tenglamaning umumiy yechimini toping.

1. $2y'' - 7y' + 3y = f(x)$ $a)$ $f(x) = (2x+1)e^{3x}$, $b)$ $f(x) = \cos 3x$.

2. $3y'' - 7y' + 2y = f(x)$

$a)$ $f(x) = 3xe^{2x}$, $b)$ $f(x) = \sin 2x - 3\cos 2x$.

3. $2y'' + y' - y = f(x)$ $a)$ $f(x) = (x^2 - 5)e^{-x}$, $b)$ $f(x) = x \sin x$.

4. $2y'' - 9y' + 4y = f(x)$ $a)$ $f(x) = -2e^{4x}$, $b)$ $f(x) = e^x \cos 4x$.

5. $y'' + 49y = f(x)$ $a)$ $f(x) = x^3 + 4x$, $b)$ $f(x) = 3\sin 7x$.

6. $3y'' + 10y' + 3y = f(x)$

$a)$ $f(x) = e^{-3x}$, $b)$ $f(x) = 2\cos 3x - \sin 3x$.

7. $y'' - 3y' + 2y = f(x)$ $a)$ $f(x) = x + 2e^x$, $b)$ $f(x) = 3\cos 4x$.

8. $y'' - 4y' + 4y = f(x)$ $a)$ $f(x) = \sin 2x + 2e^x$, $b)$ $f(x) = x^2 - 4$.

9. $y'' - 2y' + 2y = f(x)$ $a)$ $f(x) = e^x \cos x$, $b)$ $f(x) = 7x + 2$.

10. $y'' - 3y' = f(x)$ $a)$ $f(x) = 2x^2 - 5x$, $b)$ $f(x) = e^{-x} \sin 2x$.

11. $y'' + 3y' - 4y = f(x)$ $a)$ $f(x) = 3xe^{-4x}$, $b)$ $f(x) = x \sin x$.

12. $y'' + 36y = f(x)$ $a)$ $f(x) = 4xe^{-x}$, $b)$ $f(x) = 2\sin 6x$.

13. $y''-6y'+9y=f(x)$ $\quad a)$ $f(x)=(x-2)e^{3x}$, $\quad b)$ $f(x)=4\cos x$.

14. $4y''-5y'+y=f(x)$ $\quad a)$ $f(x)=2(2x+1)e^{x}$, $b)$ $f(x)=e^{x}\sin 3x$.

15. $4y''+7y'-2y=f(x)$ $\quad a)$ $f(x)=3e^{-2x}$, $\quad b)$ $f(x)=(x-1)\cos 2x$.

16. $y''-y'-6y=f(x)$ $\quad a)$ $f(x)=2xe^{3x}$, $\quad b)$ $f(x)=9\cos x-\sin x$.

17. $y''-16y=f(x)$ $\quad a)$ $f(x)=-3e^{4x}$, $\quad b)$ $f(x)=\cos x-4\sin x$.

18. $y''-4y'=f(x)$ $\quad a)$ $f(x)=(x-2)e^{4x}$, $\quad b)$ $f(x)=3\cos 4x$.

19. $y''-2y'+2y=f(x)$ $\quad a)$ $f(x)=(2x-3)e^{4x}$, $\quad b)$ $f(x)=e^{x}\sin x$.

20. $5y''-6y'+y=f(x)$ $\quad a)$ $f(x)=x^{2}e^{x}$, $\quad b)$ $f(x)=\cos x-\sin x$.

21. $5y''+9y'-2y=f(x)$

$a)$ $f(x)=x^{3}-2x$, $\quad b)$ $f(x)=2\sin 2x-3\cos 2x$.

22. $y''-2y'-15y=f(x)$ $\quad a)$ $f(x)=4xe^{3x}$, $\quad b)$ $f(x)=x\sin 5x$.

23. $y''-3y'=f(x)$ $\quad a)$ $f(x)=2x^{3}-4x$, $\quad b)$ $f(x)=2e^{3x}\cos x$.

24. $y''-7y'+12y=f(x)$ $\quad a)$ $f(x)=xe^{3x}+2e^{x}$, $\quad b)$ $f(x)=3x\sin 2x$.

25. $y''+9y'=f(x)$ $\quad a)$ $f(x)=x^{2}+4x-3$, $\quad b)$ $f(x)=xe^{2x}\sin x$.

26. $y''-4y'+5y=f(x)$

$a)$ $f(x)=-2xe^{x}$, $\quad b)$ $f(x)=x\cos 2x-\sin 2x$.

27. $y''+3y'+2y=f(x)$

$a)$ $f(x)=(3x-7)e^{-x}$, $b)$ $f(x)=\cos x-3\sin x$.

28. $y''-8y'+16y=f(x)$

$a)$ $f(x)=2xe^{4x}$, $\quad b)$ $f(x)=\cos 4x+2\sin 4x$.

29. $y''+y'-2y=f(x)$ $\quad a)$ $f(x)=(2x-1)e^{x}$, $\quad b)$ $f(x)=3x\cos 2x$.

30. $y''+3y'-4y=f(x)$ $\quad a)$ $f(x)=6xe^{-x}$, $\quad b)$ $f(x)=x^{2}\sin 2x$.

M42. Differensial tenglamaning umumiy yechimini (umumiy integralini) toping.

1. $y'''-y''+9y'-9y=4e^{x}$. $\qquad$ 2. $y^{(4)}+16y=3\sin 2x$.

3. $y'''-2y''+y'-2y=9\sin x$. $\qquad$ 4. $y^{(4)}-5y''+4y=xe^{2x}$.

5. $y'''+2y''+4y'+8y=12e^{-2x}$. $\qquad$ 6. $y^{(4)}+8y''-9y=4\sin x+\cos x$.

7. $y'''-4y''+5y'-2y=xe^{x}$. $\qquad$ 8. $y^{IV}+4y''+4y=12\sin 2x$.

9. $y'''-3y''-y'+3y=xe^{3x}$. $\qquad$ 10. $y'''+5y''+7y'+3y=16(x+1)e^{-x}$.

11. $y''' - y'' - 4y' + 4y = 15e^x$.   12. $y^{(4)} - 13y'' + 36y = \sin 2x$.

13. $y''' - 4y'' + y' + 6y = xe^{-x}$.

14. $y^{(4)} + 2y''' - 2y' - y = (2x+1)e^x$.

15. $y''' + 3y'' - 9y' - 27y = xe^{-x}$. 16. $y''' - 3y'' + 3y' - y = 21e^x$.

17. $y''' + 2y'' - 4y' - 8y = (x-4)e^{-x}$.

18. $y^{IV} + 3y''' + 3y'' + y' = x^2 + x - 1$.

19. $y''' - 6y'' + 12y' - 8y = (x+1)e^{2x}$.

20. $y^{IV} - 3y''' - y'' + 3y' = x^2 + 5x + 9$.

21. $y^{IV} - 7y''' + 6y'' = 7x + 1$.   22. $y''' + 3y'' - 4y' - 12y = xe^{2x}$.

23. $y^{IV} - 2y''' + 2y'' = e^x \sin x$.   24. $y^{IV} - 3y'' - 4y = 9\sin x$.

25. $y^{IV} + 2y''' + 2y'' + 2y' + y = xe^{-x}$.

26. $y^{IV} - y''' - 3y'' + y' + 2y = (3x+1)e^x$.

27. $y''' - 2y'' + 16y' - 32y = (x-1)e^{2x}$.

28. $y^{IV} - 2y''' + 2y'' - 2y' + y = 14xe^x$.

29. $y''' + 9y'' + 27y' + 27y = 27e^{-x}$.

30. $y^{IV} - 3y''' + 3y'' - 3y' + 2y = 5(x+1)e^{2x}$.

2.4. O'ZGARMASLARNI VARIATSIYALASH USULI

O'zgarmas koeffitsientli chiziqli bir jinsli bo'lmagan

$$y^{(n)} + a_1 y^{(n-1)} + \ldots + a_{n-1}y' + a_n y = f(x), \quad a_j = const \ (j = 1, \ldots, n), \qquad (1)$$

tenglamaning o'ng tomoni biror kesmada uzluksiz bo'lgan ixtiyoriy funksiya bo'lsin. Bunday tenglama *o'zgarmaslarni variatsiyalash usuli* bilan yechiladi.

Bir jinsli bo'lmagan (1) tenglamaga mos quyidagi

$$y^{(n)} + a_1 y^{(n-1)} + \ldots + a_{n-1}y' + a_n y = 0 \qquad (2)$$

bir jinsli tenglamaning

$$y=C_1y_1(x)+C_2y_2(x)+...+C_ny_n(x),\quad C_j=const\ (j=1,...,n),\qquad (3)$$

umumiy yechimi topilgan bo'lsin. U holda (1) tenglamaning yechimi

$$y=C_1(x)y_1(x)+C_2(x)y_2(x)+...+C_n(x)y_n(x)\qquad (4)$$

ko'rinishda izlanadi. (4) funksiyani (1) tenglamaga qo'yib, $C_j'(x)$ funksiyalarga nisbatan chiziqli algebraik tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases} C_1'(x)y_1(x)+C_2'(x)y_2(x)+...+C_n'(x)y_n(x)=0, \\ C_1'(x)y_1'(x)+C_2'(x)y_2'(x)+...+C_n'(x)y_n'(x)=0, \\ \quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ C_1'(x)y_1^{(n-2)}(x)+C_2'(x)y_2^{(n-2)}(x)+...+C_n'(x)y_n^{(n-2)}(x)=0, \\ C_1'(x)y_1^{(n-1)}(x)+C_2'(x)y_2^{(n-1)}(x)+...+C_n'(x)y_n^{(n-1)}(x)=f(x). \end{cases}\qquad (5)$$

(5) sistemaning yechimi $C_j'(x)=\varphi_j(x)\ (j=1,...,n)$ bo'lsin. Bularni integrallab $C_j(x)\ (j=1,...,n)$ funksiyalarni topamiz.

O'zgarmaslarni variatsiyalash usuli yordamida quyidagi tenglamalarning umumiy yechimini toping (**117-118**).

117. $y''-y=\left(4x^2+1\right)x^{-3/2}.\qquad (6)$

◀Avvalo

$$y''-y=0\qquad (7)$$

bir jinsli tenglamaning umumiy yechimini yozamiz:

$$y=C_1e^x+C_2e^{-x}.\qquad (8)$$

Bizning holda $y_1(x)=e^x$, $y_2(x)=e^{-x}$. (8) umumiy yechimda o'zgarmaslarni variatsiyalaymiz, ya`ni uni

$$y=C_1(x)e^x+C_2(x)e^{-x}\qquad (9)$$

ko'rinishda yozib olamiz. $C_1(x)$ va $C_2(x)$ funksiyalarni aniqlash uchun (5) sistemani tuzamiz ($n=2$):

$$\begin{cases} C_1'(x)e^x + C_2'(x)e^{-x} = 0, \\ C_1'(x)(e^x)' + C_2'(x)(e^{-x})' = \dfrac{4x^2+1}{x\sqrt{x}}. \end{cases}$$

Bu sistemani yechib, $C_1(x)$ va $C_2(x)$ funksiyalarga nisbatan

$$C_1'(x) = \frac{4x^2+1}{2x\sqrt{x}}\, e^{-x},\; C_2'(x) = -\frac{4x^2+1}{2x\sqrt{x}}\, e^x$$

differensial tenglamalarga ega bo'lamiz. Bu yerdan topamiz:

$$C_1(x) = \int \frac{4x^2+1}{2x\sqrt{x}}\, e^{-x}dx = -e^{-x}\left(2\sqrt{x} + \frac{1}{\sqrt{x}}\right) + C_1,$$

$$C_2(x) = -\int \frac{4x^2+1}{2x\sqrt{x}} e^x dx = -e^x\left(2\sqrt{x} - \frac{1}{\sqrt{x}}\right) + C_2.$$

Olingan natijalarni (9) formulaga qo'yib, (6) tenglamaning umumiy yechimiga ega bo'lamiz: $y = C_1e^x + C_2e^{-x} - 4\sqrt{x}$.▶

118. $y''' + y' = \dfrac{\sin x}{\cos^2 x}.$

◀Berilgan tenglama bir jinsli qismining umumiy yechimini yozamiz:

$$y = C_1 + C_2\cos x + C_3\sin x.$$

Bizning holda $y_1(x) = 1,\ y_2(x) = \cos x,\ \ y_3(x) = \sin x$. Umumiy yechimdagi o'zgarmaslarni variatsiyalaymiz, ya`ni uni

$$y = C_1(x) + C_2(x)\cos x + C_3(x)\sin x$$

ko'rinishda yozib olamiz. $C_1'(x), C_2'(x)$ va $C_3'(x)$ funksiyalarni aniqlash uchun (5) ga mos algebraik sistemani tuzamiz ($n = 3$):

$$\begin{cases} C_1'(x)\cdot 1 + C_2'(x)\cos x + C_3'(x)\sin x = 0, \\ C_1'(x)\cdot 0 - C_2'(x)\sin x + C_3'(x)\cos x = 0, \\ C_1'(x)\cdot 0 - C_2'(x)\cos x - C_3'(x)\sin x = \sin x\cos^{-2} x. \end{cases}$$

Bu sistemani yechib, topamiz:

$$C_1'(x)=\sin x(1+tg^2x),\; C_2'(x)=-tg\,x,\; C_3'(x)=-tg^2x\,.$$

Bu differensial tenglamalarni integrallaymiz:

$$C_1(x)=\int \sin x(1+tg^2x)\,dx=\frac{1}{\cos x}+C_1,$$

$$C_2(x)=-\int tg\,x\,dx=\ln|\cos x|+C_2, \qquad C_3(x)=-\int tg^2x\,dx=x-tg\,x+C_3.$$

Olingan natijalar yordamida berilgan tenglamaning umumiy yechimini yozamiz:

$$y=C_1+C_2\cos x+C_3\sin x+\frac{1}{\cos x}+\cos x\ln|\cos x|+\sin x(x-tg\,x)\,.$$ ►

INDIVIDUAL TOPSHIRIQLAR

M43. Differensial tenglamani o'zgarmaslarni variatsiyalash usulida yeching.

1. $y''-y=\dfrac{e^x}{e^x+1}.$
2. $y''+4y=\dfrac{1}{\cos 2x}.$
3. $y''-4y'+5y=\dfrac{e^{2x}}{\cos x}.$
4. $y'''+y'=\dfrac{\sin x}{\cos^2 x}.$
5. $y''+9y=\dfrac{1}{\sin 3x}.$
6. $y''+2y'+y=xe^x+\dfrac{1}{xe^x}.$
7. $y''+2y'+2y=\dfrac{e^{-x}}{\cos x}.$
8. $y''-2y'+2y=\dfrac{e^x}{\sin^2 x}.$
9. $y''+2y'+2y=e^{-x}ctgx.$
10. $y''-2y'+2y=\dfrac{e^x}{\sin x}.$
11. $y''-2y'+y=\dfrac{e^x}{x^2}.$
12. $y''+y=tgx.$
13. $y''+4y=ctg2x.$
14. $y''+y=ctgx.$
15. $y''-2y'+y=\dfrac{e^x}{x}.$
16. $y''+2y'+y=\dfrac{e^{-x}}{x}.$
17. $y''+y=\dfrac{1}{\cos x}.$
18. $y''+y=\dfrac{1}{\sin x}.$

19. $y'' + 4y = \dfrac{1}{\sin 2x}.$ 20. $y'' + 4y = tg 2x.$

21. $y'' + 4y' + 4y = \dfrac{e^{-2x}}{x^3}.$ 22. $y'' - 4y' + 4y = \dfrac{e^{2x}}{x^3}.$

23. $y'' + 2y' + y = 3e^{-x}\sqrt{x+1}.$ 24. $y'' + y = -ctg^2 x.$

25. $y'' - y' = e^{2x}\cos\left(e^x\right).$ 26. $y'' - y' = e^{2x}\sin\left(e^x\right).$

27. $y'' + y = tg^2 x.$ 28. $y'' + y = \dfrac{2}{\sin^2 x}.$

29. $y'' + 2y' + 5y = \dfrac{e^{-x}}{\sin 2x}.$ 30. $y'' + 9y = \dfrac{1}{\cos 3x}.$

M44. Differensial tenglamani o'zgarmaslarni variatsiyalash usulida yeching.

1. $y'' - 2y' + y = \dfrac{e^x}{2x+1}.$ 2. $y'' - 2y' + y = \dfrac{e^x}{x^2+1}.$

3. $y'' - 4y' + 3y = \ln\left(1 + e^{-x}\right).$ 4. $y'' + 4y = tg^2 2x.$

5. $y'' - 3y' + 2y = \dfrac{e^{2x}}{\left(1+e^x\right)^2}.$ 6. $y'' + y = \dfrac{x}{\cos^3 x}.$

7. $y'' - 4y' + 4y = e^{2x}\ln\left(x^2+1\right).$ 8. $y'' + 2y' + y = e^{-x}\ln\left(x^2+4\right).$

9. $y'' + 4y = \dfrac{1}{3+\cos^2 2x}.$ 10. $y'' + y = \dfrac{1}{\sin^3 x}.$

11. $y'' + y = \dfrac{1}{2+\sin^2 x}.$ 12. $y'' - y = \dfrac{2e^x}{e^x - 1}.$

13. $y'' - 2y' + 2y = e^x tgx.$ 14. $y'' - y' = e^{2x}\cos\left(e^x\right).$

15. $y'' - 2y' + 2y = \dfrac{e^x}{\sin x}.$ 16. $y'' - 3y' + 2y = \sin\left(e^{-x}\right).$

17. $y'' - 4y' + 5y = \dfrac{e^{2x}\sin x}{1+\sin^2 x}.$ 18. $y'' - 4y' + 4y = \dfrac{\sqrt{x}}{x+1}e^{2x}.$

19. $y'' - 4y' + 5y = \dfrac{e^{2x}}{\sqrt{1+\sin^2 x}}.$ 20. $y'' + 2y' + y = e^{-x} arctgx.$

21. $y''+2y'+y=\frac{e^{-x}}{x^2-1}.$ 22. $y''-3y'+2y=\frac{e^x}{e^x+1}.$

23. $y''+3y'+2y=\cos\left(e^x\right).$ 24. $y''+4y'+3y=arctg\left(e^x\right).$

25. $y''+y=\frac{\sin x}{\cos^2 x-2}.$ 26. $y''+2y'+y=e^{-x}\ln x.$

27. $y''+2y'+y=\sqrt[3]{x}e^{-x}.$ 28. $y''+9y=\frac{\cos 3x}{1+\sin^2 3x}.$

29. $y''-2y'+y=e^x\sqrt{1-x}.$ 30. $y''+5y'+6y=e^{-x}\ln\left(1+e^x\right).$

2.5. O'ZGARMAS KOEFFISIENTLIGA KELTIRILADIGAN TENGLAMALAR. EYLER TENGLAMASI

O'zgarmas koeffitsientliga keltiriladigan chiziqli differensial tenglamalarning barcha sinflari ma`lum emas. Albatta, tenglamani o'zgarmas koeffitsientliga keltirish uchun shunday almashtirish bajarish kerakki, natijada chiziqlilik buzilmay qolsin. Bunday almashtirishlar yoki noma`lum funksiyani $y=u(x)z$ deb, yoki erkli o'zgaruvchini $x=\chi(t)$ $(t=\psi(x))$ deb almashtirishdan iborat bo'lishi mumkin.

Erkli o'zgaruvchi x ni $t=\psi(x)$, $\psi'(x)\neq 0$ almashtirish natijasida

$$y^{(n)}+p_1(x)y^{(n-1)}+p_2(x)y^{(n-2)}+...+p_{n-1}(x)y'+p_n(x)y=f(x)$$

tenglama o'zgarmas koeffitsientliga kelishi uchun

$$t=\psi(x)=A\int\sqrt[n]{p_n(x)}dx \tag{1}$$

formulaning o'rinli bo'lishi zarur, bu yerda A – o'zgarmas son.

O'zgarmas koeffitsientliga keltiriladigan chiziqli differensial tenglamalarga misollar keltiramiz.

Avval quyidagi

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+n^2y=0$$

*Chebishev tenglamasi*ni ko'raylik. Agar $x \neq \pm 1$ bo'lsa, uni yana quyidagicha

$$\frac{d^2 y}{dx^2} - \frac{x}{1-x^2}\frac{dy}{dx} + \frac{n^2}{1-x^2} y = 0$$

yozish mumkin. Bunda $p_1(x) = -\frac{x}{1-x^2}$, $p_2(x) = \frac{n^2}{1-x^2}$.

Aniqlik uchun $|x| < 1$ bo'lsin. Bu holda (1) formulaga ko'ra

$$t = \psi(x) = A\int\sqrt{\frac{n^2}{1-x^2}}dx = An\int\frac{dx}{\sqrt{1-x^2}} = An\arcsin x + C, \ |x| < 1.$$

Soddalik uchun $A = 1$, $C = 0$ deylik, bunda $t = \psi(x) = n \cdot \arcsin x$. Ikkinchi tartibli chiziqli bir jinsli

$$\frac{d^2 y}{dx^2} + p_1(x)\frac{dy}{dx} + p_2(x) y = 0$$

tenglama uchun $t = \psi(x)$ almashtirish natijasida hosil bo'ladigan

$$\frac{d^2 y}{dt^2} + Q_1(x)\frac{dy}{dt} + Q_2(x) y = 0$$

tenglama koeffitsientlari quyidagi

$$Q_1(x) = \frac{\psi''(x) + p_1(x)\psi'(x)}{(\psi'(x))^2}, \ Q_2(x) = \frac{p_2(x)}{(\psi'(x))^2}$$

formulalar bilan yoziladi. Buni bevosita hisoblab chiqish mumkin. Ko'rilayotgan holda:

$$\psi'(x) = \frac{n}{\sqrt{1-x^2}}, \ \psi''(x) = \frac{nx}{(1-x^2)^{3/2}}.$$

Shuning uchun, $Q_1(x) = 0$, $Q_2(x) = 1$.

Demak, $t = n \cdot \arcsin x$ almashtirish natijasida Chebishev tenglamasi

$$\frac{d^2y}{d\tau^2}+y=0$$

ko'rinishga keladi. Bu tenglamaning fundamental sistemasi $y_1(t)=\cos t$, $y_2(t)=\sin t$ bo'lib, $t=n\arcsin x$ bo'yicha eski erkli o'zgaruvchiga qaytsak,

$$y_1(x)=\cos(n\arcsin x),\quad y_2(x)=\sin(n\arcsin x)$$

bo'ladi. Amalda ko'proq $A=-1$, $C=0$ deb olinadi. Bunda $\psi(x)=n\arccos x$ kelib chiqadi. Shuning uchun fundamental sistemani

$$y_1(x)=\cos(n\arccos x),\quad y_2(x)=\sin(n\arccos x)$$

deb yozish mumkin. Chebishev tenglamasining umumiy yechimi

$$y(x)=C_1\cos(n\arccos x)+C_2\sin(n\arccos x),\ |x|<1$$

kabi yoziladi.

Ma`lumki, $\cos(\arccos x)=x$ va $\cos n\varphi$ funksiya n butun bo'lganda $\cos\varphi$ ning $n-$tartibli ko'phadi ko'rinishida yoziladi. Shuning uchun $\cos(n\arccos x)$ funksiya n butun bo'lsa, x ga nisbatan $n-$tartibli ko'phad bo'ladi. Bu ko'phad *Chebishev ko'phadi* deyiladi va $T_n(x)=\cos(n\arccos x)$ tarzda belgilanadi.

$|x|>1$ bo'lgan holda ham yuqoridagi mulohazalarni takrorlab, Chebishev tenglamasining umumiy yechimini topish qiyin emas:

$$y(x)=C_1ch(narchx)+C_2sh(narchx),\ |x|>1,$$

bu yerda $archx=\pm\ln\left|x+\sqrt{x^2-1}\right|$, $|x|\geq 1$ - teskari giperbolik funksiya.

Yana shuni ham ta`kidlash kerakki, noma`lum funksiyani $y=u(x)z$ almashtirish natijasida o'zgarmas koeffitsientliga keladigan tenglamalar uchun (1) turdagi zaruriy shart mavjud emas. Shuning uchun (1) tenglik bajarilmaganda faqat tanlash yo'li bilan turli almashtirishlar bajarib, berilgan tenglama tekshirib ko'riladi.

Quyidagi

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0,\ x>0$$

tenglama *Bessel tenglamasi* deb yuritiladi. Agar $n=\frac{1}{2}$ bo'lsa, $y=\frac{z}{\sqrt{x}}$ almashtirish bu tenglamani $z''+z=0$ ko'rinishga olib keladi. Uning fundamental sistemasi $z_1=\cos x,\quad z_2=\sin x$ bo'lib, eski noma`lum funksiyaga qaytganda $y_1=\frac{\cos x}{\sqrt{x}},\ y_2=\frac{\sin x}{\sqrt{x}}$ bo'ladi. Demak, $n=\frac{1}{2}$ bo'lganda Bessel tenglamasi o'zgarmas koeffitsientli tenglamaga o'tadi va uning umumiy yechimi

$$y=C_1\frac{\cos x}{\sqrt{x}}+C_2\frac{\sin x}{\sqrt{x}}$$

ko'rinishda yoziladi.

Izoh. Bu yerda Bessel tenglamasining umumiy yechimini n ning bitta xususiy qiymatida topish imkoniga ega bo'ldik, xolos. Keyingi bobda bu tenglamani boshqa usullar yordamida umumiy holda tadqiq etamiz.

Endi o'zgarmas koeffitsientliga keladigan tenglamalarning *Eyler tenglamasi* deb ataluvchi sinfini o'rganamiz.

Ushbu

$$x^n\frac{d^ny}{dx^n}+a_1x^{n-1}\frac{d^{n-1}y}{dx^{n-1}}+...+a_{n-1}x\frac{dy}{dx}+a_ny=0, \tag{2}$$

(bunda $a_j=const,\ i=1,2,...,n$) tenglama *Eylerning bir jinsli tenglamasi* deyiladi. Bu tenglama (1) formula bo'yicha $x>0$ da $t=\ln x,\ \ x=e^t$ almashtirish yordamida o'zgarmas koeffitsientli chiziqli differensial tenglamaga keltiriladi ($x<0$ da $x=-e^t$ deb olinadi). Hosil bo'lgan o'zgarmas koeffitsientli tenglama uchun xarakteristik tenglama quyidagi ko'rinishda bo'ladi:

$$\lambda(\lambda-1)(\lambda-2)...(\lambda-n+1)+...+a_{n-2}\lambda(\lambda-1)+a_{n-1}\lambda+a_n=0.$$

Bu tenglama λ ga nisbatan $n-$tartibli bo'lib, u *Eyler tenglamasining xarakteristik tenglamasi* deyiladi. Agar $x^{\lambda}=e^{\lambda\ln x}$ ekanini e`tiborga olsak, xarakteristik tenglamaning ildizlariga qarab Eyler tenglamasining umumiy yechimini yozish qiyin emas.

Ushbu

$$(ax+b)^n y^{(n)}+a_1(ax+b)^{n-1}y^{(n-1)}+...+a_{n-1}(ax+b)y'+a_n y=0 \quad (3)$$

tenglama ham $ax+b=e^t$ ($ax+b>0$) almashtirish yordamida o'zgarmas koeffitsientli chiziqli differensial tenglamaga keltiriladi ($ax+b<0$ da $ax+b=-e^t$ deb olinadi). Hosil bo'lgan o'zgarmas koeffitsientli tenglama uchun xarakteristik tenglama

$$a^n\lambda(\lambda-1)(\lambda-2)...(\lambda-n+1)+...+a_{n-2}\,a^{n-1}\lambda(\lambda-1)+a_{n-1}\,a\lambda+a_n=0$$

ko'rinishda bo'ladi.

Ushbu

$$x^n\frac{d^n y}{dx^n}+a_1x^{n-1}\frac{d^{n-1}y}{dx^{n-1}}+...+a_{n-1}x\frac{dy}{dx}+a_n y=F(x) \quad (4)$$

tenglama *bir jinsli bo'lmagan Eyler tenglamasi* deyiladi. Yuqorida bayon qilingan usul bilan, ya`ni erkli o'zgaruvchini $t=\ln x,\ x=e^t$ almashtirish yordamida bu bir jinsli bo'lmagan tenglama ham o'zgarmas koeffitsientli bir jinsli bo'lmagan tenglamaga keltiriladi. Farqi shundaki, o'ng tomondagi $F(x)$ funksiya argumentida x o'rniga e^t qo'yiladi.

Tenglamalarni yeching (**119-121**).

119. $x^3y'''+x^2y''-2xy'+2y=0.$

◄Xarakteristik tenglamani yozamiz:

$$\lambda(\lambda-1)(\lambda-2)+\lambda(\lambda-1)-2\lambda+2=0,\ \ (\lambda+1)(\lambda-1)(\lambda-2)=0,$$

Bundan $\lambda_1=-1,\ \lambda_2=1,\ \lambda_3=2$. Berilgan differensial tenglamaning fundamental sistemasi $x^{-1},\ x,\ x^2$ (e^{-t}, e^t, e^{2t}) ko'rinishda bo'ladi. Shuning uchun berilgan Eyler tenglamasining umumiy yechimi

$$y=C_1x^{-1}+C_2x+C_3x^2$$

ko'rinishda yoziladi.

120. $x^3y''' - x^2y'' + 2xy' - 2y = 4x^3.$ (5)

◀Mos bir jinsli tenglama

$$x^3y''' - x^2y'' + 2xy' - 2y = 0, \qquad (6)$$

xarakteristik tenglama esa $\lambda(\lambda-1)(\lambda-2) - \lambda(\lambda-1) + 2\lambda - 2 = 0$ ko'rinishda bo'ladi. Bundan $(\lambda-1)(\lambda^2 - 3\lambda + 2) = 0$ kelib chiqadi. Uning ildizlari $\lambda_1 = \lambda_2 = 1, \lambda_3 = 2.$ Demak, $\lambda = 1-$ ikki karrali ildiz. Bir jinsli (6) tenglamaning umumiy yechimi:

$$y_0 = (C_1 + C_2t)e^t + C_3e^{2t} = (C_1 + C_2\ln x)x + C_3x^2, \; x > 0.$$

Endi bir jinsli bo'lmagan (5) tenglamani qaraymiz. Unda $F(x) = 4x^3$ bo'lib, $F(e^t) = 4e^{3t}$ bo'ladi. (5) tenglamaning $F(e^t)$ o'ng tomoniga ko'ra aniqlanadigan $\gamma = 3$ soni xarakteristik tenglamaning ildizi emas $\left(\gamma \neq \lambda_{1,2,3}\right)$, shuning xususiy yechimni $y_1 = ae^{3t}$ ko'rinishda izlaymiz va $a = 1$ ni topamiz. Shunday qilib, xususiy yechim $y_1 = e^{3t}$, ya`ni $y_1 = x^3$ funksiyadan iborat.

Demak, bir jinsli bo'lmagan (5) Eyler tenglamasining umumiy yechimi

$$y = y_0 + y_1 = \left(C_1 + C_2\ln|x|\right)x + C_3x^2 + x^3$$

ko'rinishda bo'ladi.▶

121. $(2x+1)^2y'' - (x+0,5)y' + y = 6x.$ (7)

◀Bu tenglamada $2x + 1 = e^t$, ya`ni $x = \frac{1}{2}e^t - \frac{1}{2}$ deb olamiz. Mos bir jinsli tenglama

$$(2x+1)^2y'' - (x+0,5)y' + y = 0 \qquad (8)$$

ko'rinishda bo'lib, uning xarakteristik tenglamasi esa $2^2\lambda(\lambda-1) - \lambda + 1 = 0$ ko'rinishda yoziladi. Bundan $(\lambda-1)(4\lambda-1) = 0$

kelib chiqadi. Uning ildizlari $\lambda_1 = 1, \lambda_2 = 1/4$. Bir jinsli (8) tenglamaning umumiy yechimi:

$$y_0 = C_1 e^t + C_2 e^{t/4} = C_1(2x+1) + C_2 \sqrt[4]{2x+1} \quad (2x+1>0).$$

Endi bir jinsli bo'lmagan (7) tenglamani qaraymiz. Unda $F(x) = 6x$ bo'lib, $F\left(\frac{1}{2}e^t - \frac{1}{2}\right) = 3e^t - 3$ bo'ladi. (7) tenglamaning xususiy yechimini aniqmas koeffitsientlar usuli yordamida topish maqsadida uning o'ng tomonini $F = F_1 + F_2$ ko'rinishda yozib olamiz, bu yerda $F_1 = 3e^t$, $F_2 = -3$. $F_1 = 3e^t$ o'ng tomonga ko'ra aniqlanadigan $\gamma = 1$ soni xarakteristik tenglamaning bir karrali ildiziga teng ($\gamma = \lambda_1$), shuning uchun xususiy yechimni $y_1 = at\,e^t$ ko'rinishda izlaymiz va $a = 1$ ni topamiz. $F_2 = -3$ o'ng tomonga ko'ra xususiy yechimni $y_2 = b$ ko'rinishda izlaymiz va $b = -3$ ni topamiz. Shunday qilib, bir jinsli bo'lmagan (7) tenglamaning xususiy yechimi topiladi:

$$y_1 + y_2 = te^t - 3 = (2x+1)\ln(2x+1) - 3.$$

Demak, bir jinsli bo'lmagan (7) Eyler tenglamasining umumiy yechimi topamiz:

$$y = C_1(2x+1) + C_2\sqrt[4]{2x+1} + (2x+1)\ln(2x+1) - 3, \; 2x+1>0.$$

$2x+1<0$ bo'lgan hol xuddi shu kabi o'rganiladi.►

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M45. Eyler tenglamasini yeching.

1. $x^2y'' - 4xy' + 6y = 0.$
2. $x^2y'' - 5xy' + 8y = 0.$
3. $x^2y'' - xy' - 8y = 0.$
4. $x^2y'' - xy' - 35y = 0.$
5. $x^2y'' - 12xy' + 42y = 0.$
6. $x^2y'' - 8xy' + 20y = 0.$
7. $x^2y'' + 6xy' + 6y = 0.$
8. $x^2y'' - 6y = 0.$
9. $x^2y'' - 2xy' - 10y = 0.$
10. $x^2y'' + 10xy' + 20y = 0.$
11. $x^2y'' - xy' - 15y = 0.$
12. $x^2y'' - 15xy' + 63y = 0.$

13. $x^2y''-xy'-63y=0.$ 14. $x^2y''+13xy'+35y=0.$

15. $x^2y''+14xy'+42y=0.$ 16. $x^2y''-20y=0.$

17. $x^2y''+2xy'-20y=0.$ 18. $x^2y''+2xy'-6y=0.$

19. $x^2y''+3xy'-8y=0.$ 20. $x^2y''-42y=0.$

21. $x^2y''+2xy'-42y=0.$ 22. $x^2y''-7xy'+15y=0.$

23. $x^2y''-14xy'+56y=0.$ 24. $x^2y''-12xy'+40y=0.$

25. $x^2y''-10xy'+30y=0.$ 26. $x^2y''-30y=0.$

27. $x^2y''+2xy'-30y=0.$ 28. $x^2y''-2xy'-28y=0.$

29. $x^2y''+12xy'+28y=0.$ 30. $x^2y''-2xy'-21y=0.$

M46. Bir jinsli bo'lmagan Eyler tenglamasini yeching.

1. $x^2y''-2xy'+4y=2\cos(\ln x)+\ln^2 x.$
2. $x^2y''+3xy'+y=3x^{-1}\ln x+2x^2.$
3. $x^2y''+2xy'-6y=3x^2+2x^{-3}\ln x.$
4. $x^2y''-4xy'+6y=x^2\ln x+2x^2.$
5. $x^2y''-xy'+2y=x\sin(\ln x)+x\ln x.$
6. $x^2y''-3xy'+3y=2x\ln x+x^3.$
7. $x^2y''+xy'+4y=\sin(2\ln x)+3x^2.$
8. $x^2y''+xy'-y=x^{-1}\ln x+x\ln x.$
9. $x^2y''+5xy'+8y=x^{-2}\cos(2\ln x)+2x^2.$
10. $x^2y''-3xy'+4y=x^2\ln x+x^{-1}.$
11. $x^2y''-5xy'+13y=x^3\ln x+3x^2\sin(2\ln x).$
12. $x^2y''+2xy'-2y=\left(x+\frac{1}{x^2}\right)\ln x.$
13. $x^2y''+5xy'+3y=\left(x^{-1}+x^{-3}\right)\ln x.$
14. $x^2y''-3xy'+2y=x^2+\sqrt{x}\ln x.$
15. $x^2y''+3xy'+5y=\frac{\sin(2\ln x)}{x}+2x^2.$
16. $x^2y''-xy'+y=x\ln x+\frac{3}{x^2}.$

17. $4x^2y''+8xy'+y=\dfrac{(x+2)\ln x}{\sqrt{x}}.$

18. $x^2y''+3xy'-3y=x\ln x-\dfrac{2}{x^3}.$

19. $x^2y''+7xy'+10y=\dfrac{\sin(2\ln x)}{x^3}+3x.$

20. $2x^2y''+7xy'+2y=\dfrac{\ln x}{\sqrt{x}}-\dfrac{3}{x^2}.$

21. $x^2y''+xy'-4y=\left(x^2+\dfrac{1}{x^2}\right)\ln x.$

22. $2x^2y''+5xy'-2y=\sqrt{x}\ln x-\dfrac{2}{x^2}.$

23. $4x^2y''+4xy'-y=\left(\sqrt{x}-\dfrac{2}{\sqrt{x}}\right)\ln x.$

24. $2x^2y''-xy'-2y=x^2\ln x-\dfrac{3}{\sqrt{x}}.$

25. $x^2y''+6xy'+6y=\dfrac{\ln x}{x^2}+\sin(\ln x).$

26. $x^2y''-xy'+y=6x\ln x.$

27. $x^2y''-5xy'+8y=x^3 shx.$ 28. $x^2y''+xy'+y=2\sin(\ln x).$

29. $x^2y''-4xy'+6y=x^4-x^2.$ 30. $x^2y''-2xy'+2y=x^5\ln x.$

2.6. TURLI USULLAR BILAN YECHILADIGAN TENGLAMALAR. AMALIY MASALALAR

2.6.1. Kompleks koeffisientli yoki o'ng tomonida kompleks funksiya qatnashgan tenglamalar.

Qulay usulni qo'llab tenglamalarni yeching (**122-124**).

122. $y''+2y'+y=8\cos ix.$

◀Mos bir jinsli tenglamaning umumiy yechimi

$$y_0=(C_1+C_2x)e^{-x}, \quad (\lambda_1=\lambda_2=-1)$$

ko'rinishda bo'ladi.

Eyler formulasiga ko'ra, $\cos ix=(e^x+e^{-x})/2$ va xarakteristik tenglamaning $\lambda=-1$ ildizi ikki karrali bo'lganligi uchun bir jinsli bo'lmagan tenglamaning xususiy yechimini $y_1=a\,e^x$ va $y_2=bx^2e^{-x}$ ko'rinishlarda izlaymiz. Ularni mos ravishda

$$y''+2y'+y=4e^x \quad \text{va} \quad y''+2y'+y=4e^{-x}$$

tenglamalarga qo'yib, $a=1,\ b=2$ qiymatlarni olamiz. Shunday qilib, dastlabki tenglamaning umumiy yechimi

$$y=(C_1+C_2x)e^{-x}+e^x+2x^2e^{-x}$$

ko'rinishda bo'ladi.►

123. $y''+2iy=8e^x\sin x.$

◄Tenglamaga mos bir jinsli tenglamaning $\lambda^2+2i=0$ xarakteristik tenglamasi $\lambda_1=-1+i$, $\lambda_2=1-i$ ildizlarga ega. Shunga ko'ra, bir jinsli tenglamaning umumiy yechimi $y_0=C_1e^{(-1+i)x}+C_2e^{(1-i)x}$ formula bilan yoziladi.

Xususiy yechimni aniqmas koeffitsientlar usuli bilan topamiz.

Tenglamaning o'ng tomoni $8e^x\sin x=\frac{4}{i}e^{(1+i)x}-\frac{4}{i}e^{(1-i)x}$ ko'rinishda bo'lganligi uchun bir jinsli bo'lmagan tenglamaning xususiy yechimini $y_1=a\,e^{(1+i)x}$ va $y_2=bxe^{(1-i)x}$ ko'rinishlarda izlaymiz. Ularni mos ravishda

$$y''+2iy=\frac{4}{i}e^{(1+i)x}, \quad y''+2iy=-\frac{4}{i}e^{(1-i)x}$$

tenglamalarga qo'yib, $a=-1,\ b=-1+i$ qiymatlarni olamiz. Shunday qilib, dastlabki tenglamaning umumiy yechimi

$$y=C_1e^{(i-1)x}+[C_2+(i-1)x]e^{(1-i)x}-e^{(1+i)x}$$

ko’rinishda topiladi.►

124. $x^2 y'' - 2y = \dfrac{3x^2}{x+1}.$ (1)

◄Bu tenglama bir jinsli bo’lmagan Eyler tenglamasi ekanligi ravshan. $x = e^t$ almashtirish yordamida uni o’zgarmas koeffitsientli chiziqli differensial tenglamaga keltiramiz:

$$\frac{d^2 y}{dt^2} - \frac{dy}{dt} - 2y = \frac{3e^{2t}}{e^t + 1}. \qquad (2)$$

(2) tenglamani o’zgarmaslarni variatsiyalash usuli bilan yechamiz. (2) tenglamaga mos bir jinsli $y_t'' - y_t' - 2y = 0$ tenglamaning

$$y = C_1 e^{-t} + C_2 e^{2t} \qquad (3)$$

ko’rinishdagi umumiy yechimi formulasida o’zgarmaslarni variatsiya-laymiz, ya`ni (3) umumiy yechimni

$$y = C_1(t) e^{-t} + C_2(t) e^{2t} \qquad (4)$$

ko’rinishda ifodalab, uni (2) tenglamaga qo’yamiz. Buning natijasida hosil bo’lgan

$$C_1'(t)e^{-t} + C_2'(t)e^{2t} = 0, \;\; -C_1'(t)e^{-t} + 2C_2'(t)e^{2t} = \frac{3e^{2t}}{e^t + 1}$$

tenglamalar sistemasini yechib,

$$C_1'(t) = -\frac{e^{3t}}{e^t + 1}, \;\; C_2'(x) = \frac{1}{e^t + 1}$$

tenglamalarga ega bo’lamiz. Integrallab topamiz:

$$C_1(t) = C_1 + e^t - \frac{1}{2}e^{2t} - \ln(1 + e^t), \;\; C_2(t) = C_2 + t - \ln(1 + e^t).$$

$C_1(t)$ va $C_2(t)$ funksiyalarning ifodalarini (4) formulaga qo’yib, (2) tenglamaning umumiy yechimini olamiz:

$$y(t)=C_1e^{-t}+C_2e^{2t}+1-\frac{1}{2}e^t+te^{2t}-\left(e^{-t}+e^{2t}\right)\ln(1+e^t).$$

Endi $x=e^t$, ya`ni $t=\ln x$ ekanligini e`tiborga olib, dastlabki (1) tenglamaning umumiy yechimini hosil qilamiz:

$$y=\frac{1}{x}\ln\frac{C_1^*}{1+x}+x^2\ln\frac{C_2^*x}{1+x}-\frac{x}{2}+1,\ C_2^*=e^{C_2},\ C_1^*=e^{C_1}.►$$

125. Agar $y_1=x\sin x$ funksiya biror o’zgarmas koeffitsientli chiziqli bir jinsli differensial tenglamaning xususiy yechimi ekanligi ma`lum bo’lsa, shu tenglamani (imkon qadar eng past tartibli) tuzing.

◄ $y_1=x\sin x$ funksiyani ketma-ket differensiallaymiz:

$$y_1'=x\cos x+\sin x,\qquad y_1''=-x\sin x+2\cos x=-y+2\cos x,$$

$$y_1'''=-y_1'-2\sin x,\qquad y_1^{IV}=-y_1''-2\cos x.$$

Ikkinchi va to’rtinchi tartibli hosilalarning ifodalaridan $2\cos x$ ni yo’qotib, $y_1''+y_1^{IV}=-y-y''$, ya`ni $y_1^{IV}+2y''+y=0$ tenglamaga ega bo’lamiz.

$y_1=x\sin x$ xususiy yechim $\lambda^4+2\lambda^2+1=(\lambda^2+1)^2=0$ xarakteristik tenglamaning ikki karrali $\lambda=\pm i$ ildizlaridan hosil bo’lganligi tufayli bu yerda bundan past tartibli tenglama tuzib bo’lmaydi.►

2.6.2. Fizik masalalar. Garmonik, so’nuvchi, majburiy tebranishlar kabi mexanik hodisalarni o’rganish o’zgarmas koeffitsientli differensial tenglamalarni yechishga, ularning yechimlarini tahlil qilishga keltiriladi. Bunday jarayonlarni tavsiflovchi tenglamalarni tuzishda mexanika qonunlaridan foydalaniladi.

Shuningdek, elektr zanjiridagi tebranishlarga oid masalalar ham o’zgarmas koeffitsientli chiziqli differensial tenglamalar yordamida o’rganiladi. Bunday masalalarni yechishda elektr zanjirlari nazariyasining quyidagi qonunlaridan foydalanish mumkin:

Zanjirning har bir qismida barcha kirayotgan toklarning yig’indisi chiqayotgan toklar yig’indisiga teng;

Zanjirning ixtiyoriy yopiq konturidagi tok manbalari kuchlanishlarining algebraik yig'indisi shu konturning qolgan barcha qismlaridagi kuchlanishlar tushishlarining algebraik yig'indisiga teng;

R qarshilikda kuchlanishning pasayishi RI ga teng; L o'zinduksiyada kuchlanishning pasayishi $L\frac{df}{dt}$ ga teng; C sig'imli kondensatorda kuchlanishning pasayishi q/C ga teng, bu yerda kondensatorning t momentdagi zaryadi $q=q(t)$ ga teng; bu yerda $\frac{dq}{dt}=I$ deb olingan; uchala holda ham $I=I(t)$ - berilgan t vaqt momentida zanjirning qaralayotgan qismida oqayotgan tok kuchi. Bu formulalarda I amperlarda, R omlarda, L genrilarda, q kulonlarda, C faradlarda, t soniyalarda, kuchlanish voltlarda ifodalanadi.

126. Og'irligi P bo'lgan yuk vertikal prujinaga osilgan. Bu yuk prujinani tinch turgan holatdan l uzunlikka cho'zadi. Yuk a uzunlikka pastga tortilib, keyin qo'yib yuboriladi. Prujina massasini va havo qarshiligini hisobga olmay, yukning harakat qonunini toping (16-rasm).

◀ Ox o'qni yuk osilgan nuqta orqali pastga vertikal yo'naltiramiz. Koordinatalar boshi O ni yuk muvozanatda bo'lgan holatda, ya`ni yukning og'irligi prujinaning reaksiya kuchi bilan muvozanatlashgan nuqtada olamiz. Ixtiyoriy $A$ holatda yukka ikkita kuch ta`sir qiladi: $P=mg$ og'irlik kuchi va f prujinani o'z holiga qaytaruvchi taranglik kuchi.

Guk qonuniga ko'ra, prujinaning taranglik kuchi uning uzayishiga proporsionaldir.

O nuqtadagi taranglik kuchi P og'irlik kuchiga teng, unga mos uzayish esa l ga teng. Demak,

$$P=|kl|. \tag{1}$$

A nuqtadagi taranglik kuchi f ga, unga mos uzayish esa $l+x$ ga teng:

$$|f|=k(l+x). \tag{2}$$

(2) tenglikni (1) tenglikka hadma-had bo'lib,

$$\frac{|f|}{|P|} = \frac{l+x}{l}$$

munosabatni olamiz, bundan f kuchning absolyut qiymati

$$|f| = P + \frac{P}{l}x$$

ga tengligi kelib chiqadi.

Ma`lumki, $x$ musbat bo'lganda $f$ kuch manfiy tomonga ta`sir qiladi, uni manfiy kuch deb hisoblash kerak, shuning uchun

$$f = -P - \frac{P}{l}x.$$

Ikkala kuchlarning teng ta`sir etuvchisi

$$R = P + f,$$

ya`ni

$$R = -\frac{P}{l}x = -\frac{mg}{l}x$$

miqdorga teng.

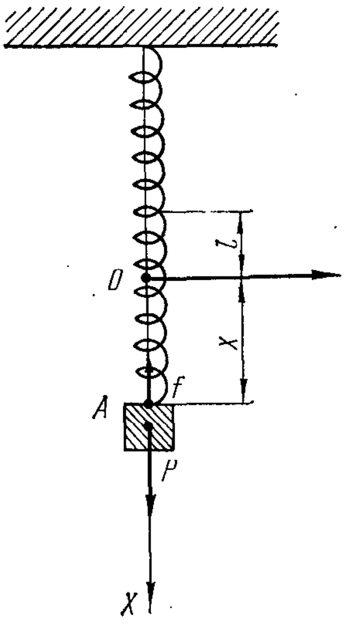
Dinamikaning ikkinchi qonuniga asosan harakatning differensial tenglamasi

$$m\frac{d^2x}{dt^2} = -\frac{mg}{l}x,$$

ya`ni

$$\frac{d^2x}{dt^2} + \frac{g}{l}x = 0$$

ko'rinishga ega.



16-rasm.

Bu ikkinchi tartibli chiziqli tenglama. Uni integrallab,

$$x = C_1 \cos\sqrt{\frac{g}{l}}\,t + C_2 \sin\sqrt{\frac{g}{l}}\,t \qquad (3)$$

umumiy yechimni olamiz.

Boshlang'ich shartlardan foydalanamiz. Yuk eng pastga tushganda

$$t = 0,\ x = a,\ \frac{dx}{dt} = 0$$

bo'ladi. Shu shartlardan $C_1 = a,\ C_2 = 0$ kelib chiqadi. Bu qiymatlarni (3) umumiy yechimga qo'yib, $T = 2\pi\sqrt{l/g}$ davrli garmonik tebranishning

$$x = a\cos\left(\sqrt{g/l}\cdot t\right)$$

qonunini topamiz.►

127. Tok manbaining kuchlanishi $E = V\sin\omega t$ qonun bo'yicha o'zgaradi. Tok manbai, R kuchlanish va C sig'im ketma-ket ulangan. Davriy rejimli zanjirning tok kuchini toping.

◄Ketma-ket ulash qonuniga ko'ra zanjirning ixtiyoriy qismidagi $I = I(t)$ tok kuchi bir xil. Kuchlanishning qarshilikdagi pasayishi RI ga, sig'imdagi pasayishi esa $\frac{q}{C}$ ga teng. Demak, $RI + \frac{q}{C} = V\sin\omega t$. Bu tenglikdan hosila olib va $\frac{dq}{dt} = I$ tenglikdan foydalanib,

$$R\frac{dI}{dt} + \frac{I}{C} = V\omega\sin\omega t \qquad (4)$$

tenglamani hosil qilamiz. Bu o'zgarmas koeffitsientli chiziqli tenglama. Bu tenglamaning davriy yechimini topamiz. Tenglamaning o'ng tomonidan kelib chiqib, yechimni

$$I = A_1\cos\omega t + B_1\sin\omega t \qquad (5)$$

ko'rinishda izlaymiz. (5) ni (4) ga qo'yib va o'xshash hadlar oldidagi koeffitsientlarni bir-biriga tenglab ikkita tenglamali sistemaga ega bo'lamiz. Bu sistemadan A_1 va B_1 ni topishimiz mumkin. Ammo elektrotexnikada A_1 va B_1 koeffitsientlarni emas, balki tok kuchining o'zgarish amplitudasini bilish muhimroq . Shuning uchun (5) ifodani

$$I = A\sin(\omega t - \varphi) \qquad (6)$$

shaklda yozib olamiz. (6) ni (4) ga qo'yib, ωt va φ burchaklarning trigonometrik funksiyalariga o'tib, avval $\sin\omega t$ oldidagi, so'ngra $\cos\omega t$ oldidagi koeffitsientlarni tenglab,

$$RA\omega\sin\varphi + \frac{A}{C}\cos\varphi = 0,\ RA\omega\cos\varphi - \frac{A}{C}\sin\varphi = V\omega$$

tenglamalar sistemasini olamiz. Bundan

$$tg\varphi = -\frac{1}{RC\omega},\ A = \frac{V}{\sqrt{R^2 + (\omega C)^{-2}}}$$

miqdorlarni topamiz.►

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M47. Quyidagi masalalarni differensial tenglamalar yordamida yeching.

1. Quyidagi xossalarga ega bo'lgan egri chiziqlarning tenglamalarini yozing: egri chiziqqa o'tkazilgan urinma, urinish nuqtasidan absissalar o'qiga tushirilgan perpendikulyar va absissalar o'qi bilan chegaralangan uchburchakning yuzi o'zgarmas b^2 ga teng.
2. Agar egri chiziqqa o'tkazilgan ixtiyoriy urinmaning absissalar o'qi bilan kesishish nuqtasi urinish nuqtasidan va koordinatalar boshidan bir xil uzoqlashgan bo'lsa, shu egri chiziqning tenglamasini yozing.
3. Quyidagi xossalarga ega bo'lgan egri chiziqlarning tenglamalarini yozing: egri chiziqqa o'tkazilgan urinma, koordinata o'qlari va urinish nuqtasidan absissalar o'qiga tushirilgan perpendikulyar bilan chegaralangan trapetsiyaning yuzi o'zgarmas $3a^2$ ga teng.
4. Quyidagi xossalarga ega bo'lgan egri chiziqlarning tenglamalarini yozing: egri chiziqqa o'tkazilgan urinma, absissalar o'qi va urinish nuqtasidan koordinatalar boshigacha bo'lgan kesma bilan chegaralangan uchburchakning yuzi o'zgarmas a^2 ga teng.

5.Agar egri chiziqqa o’tkazilgan ixtiyoriy urinmadan koordinatalar boshigacha bo’lgan masofa urinish nuqtasining absissasiga teng bo’lsa, shu egri chiziqning tenglamasini yozing.

6.Quyidagi xossalarga ega bo’lgan egri chiziqlarning tenglamalarini yozing: egri chiziqqa o’tkazilgan ixtiyoriy urinmaning absissalar o’qi bilan kesishish nuqtasining absissasi urinish nuqtasining absissasidan ikki marta kichik.

7.Quyidagi xossalarga ega bo’lgan egri chiziqlarning tenglamalarini yozing: egri chiziqqa o’tkazilgan urinma, urinish nuqtasidan absissalar o’qiga tushirilgan perpendikulyar va absissalar o’qi bilan chegaralangan uchburchak katetlarining yig’indisi o’zgarmas a soniga teng.

8.Quyidagi xossalarga ega bo’lgan egri chiziqlarning tenglamalarini yozing: egri chiziqqa o’tkazilgan ixtiyoriy urinmaning absissalar o’qi bilan kesishish nuqtasining absissasi urinish nuqtasi absissasining $2/3$ qismiga teng.

9.Quyidagi xossalarga ega bo’lgan egri chiziqlarning tenglamalarini yozing: egri chiziqqa ixtiyoriy nuqtasidan o’tkazilgan urinma va normalning absissalar o’qida ajratgan kesmasi o’zgarmas $2l$ soniga teng.

10. $A(2,4)$ nuqtadan o’tuvchi va quyidagi xossalarga ega bo’lgan egri chiziq tenglamasini tuzing: egri chiziqning ixtiyoriy nuqtasiga o’tkazilgan urinmaning absissalar o’qida ajratgan kesmasining uzunligi urinish nuqtasi absissasining kubiga teng.

11. $A(1,5)$ nuqtadan o’tuvchi va quyidagi xossalarga ega bo’lgan egri chiziq tenglamasini tuzing: egri chiziqning ixtiyoriy nuqtasiga o’tkazilgan urinmaning ordinatalar o’qida ajratgan kesmasining uzunligi urinish nuqtasi absissasining uchlanganiga teng.

12. $A(1,2)$ nuqtadan o’tuvchi va quyidagi xossalarga ega bo’lgan egri chiziq tenglamasini tuzing: egri chiziq ixtiyoriy nuqtasi ordinatasining absissasiga nisbati izlanayotgan egri chiziqqa shu nuqtada o’tkazilgan urinmaning burchak koeffisientiga proporsional. Proporsionallik koeffisienti 3 ga teng.

13. $A(2,-1)$ nuqtadan o’tuvchi va quyidagi xossalarga ega bo’lgan egri chiziq tenglamasini tuzing: egri chiziqning ixtiyoriy nuqtasida o’tkazilgan urinmaning burchak koeffisienti urinish nuqtasi ordinatasining kvadratiga proporsional. Proporsionallik koeffisienti 6 ga teng.

14. $A(1,2)$ nuqtadan o’tuvchi va quyidagi xossalarga ega bo’lgan egri chiziq tenglamasini tuzing: egri chiziqning ixtiyoriy nuqtasida o’tkazilgan

urinma burchak koeffisientining urinish nuqtasi koordinatalari yig'indisiga ko'paytmasi shu nuqtaning ikkilangan ordinatasiga teng.

15. $A(0,-2)$ nuqtadan o'tuvchi va quyidagi xossalarga ega bo'lgan egri chiziq tenglamasini tuzing: egri chiziqning ixtiyoriy nuqtasida o'tkazilgan urinmaning burchak koeffisienti shu nuqtaning uchlangan ordinatasiga teng.

16. Quyidagi xossalarga ega bo'lgan egri chiziqning tenglamasini yozing: koordinatalar boshidan urinmaga tushirilgan perpendikulyarning uzunligi urinish nuqtasining absissasiga teng.

17. Quyidagi xossalarga ega bo'lgan egri chiziqning tenglamasini yozing: egri chiziqning ixtiyoriy nuqtasida o'tkazilgan urinmaning burchak koeffisienti shu nuqtani koordinatalar boshi bilan tutashtiruvchi to'g'ri chiziqning burchak koeffisientidan n marta katta.

18. Quyidagi xossalarga ega bo'lgan egri chiziqning tenglamasini yozing: egri chiziqqa o'tkazilgan urinmaning koordinatalar o'qlari orasidagi kesmasi urinish nuqtasida teng ikkiga bo'linadi.

19. Quyidagi xossalarga ega bo'lgan egri chiziqning tenglamasini yozing: egri chiziq ixtiyoriy nuqtasida o'tkazilgan normalning ordinatalar o'qidan ajratgan kesmasining uzunligi shu nuqtadan koordinatalar boshigacha bo'lgan masofaga teng.

20. Quyidagi xossalarga ega bo'lgan egri chiziqning tenglamasini yozing: egri chiziq ixtiyoriy nuqtasida o'tkazilgan normalning Oy ordinatalar o'qidan ajratgan kesmasining uzunligi bilan shu nuqta absissasining ko'paytmasi shu nuqtadan koordinatalar boshigacha bo'lgan masofaning ikkilangan kvadratiga teng.

21. Quyidagi xossalarga ega bo'lgan egri chiziqning tenglamasini yozing: Oy ordinatalar o'qi, urinma va urinish nuqtasining radius-vektoridan hosil qilingan uchburchak teng yonli uchburchakdir.

22. $A(2,0)$ nuqtadan o'tuvchi va quyidagi xossalarga ega bo'lgan egri chiziq tenglamasini tuzing: egri chiziqning ixtiyoriy nuqtasida o'tkazilgan urinmaning urinish nuqtasi va Oy ordinatalar o'qi orasidagi kesmasining uzunligi o'zgarmas 2 ga teng.

23. Hamma urinmalari koordinatalar boshidan o'tadigan egri chiziq tenglamasini tuzing.

24. Quyidagi xossalarga ega bo'lgan egri chiziqning tenglamasini yozing: bu egri chiziqqa o'tkazilgan har qanday urinma $y=1$ to'g'ri chizig'ini kesib o'tgandagi nuqtaning absissasi urinish nuqtasining absissasidan ikki marta katta.

25. Quyidagi xossalarga ega bo'lgan egri chiziqning tenglamasini yozing: agar bu egri chiziqning ixtiyoriy nuqtasidan koordinata o'qlariga parallel to'g'ri chiziqlar o'tkazilsa va bu to'g'ri chiziqlar koordinata o'qlari bilan kesishguncha davom ettirilsa, u holda hosil bo'lgan to'g'ri to'rtburchakning yuzi egri chiziq yordamida shunday ikkita qismga bo'linadiki, qismlardan birining yuzi ikkinchisinikidan ikki marta katta bo'ladi.

26. Quyidagi xossalarga ega bo'lgan egri chiziqning tenglamasini yozing: bu egri chiziqqa o'tkazilgan har qanday urinmaning Oy o'qida ajratgan kesmasining uzunligi urinish nuqtasi koordinatalari yig'indisining $1/n$ qismiga teng.

27. Quyidagi xossalarga ega bo'lgan egri chiziqning tenglamasini yozing: egri chiziqning $M(x,y)$ nuqtasi normalining Ox o'qda ajratgan kesmasining uzunligi y^2/x ga teng.

28. Quyidagi xossalarga ega bo'lgan egri chiziqning tenglamasini yozing: egri chiziqqa o'tkazilgan urinmaning Oy o'qda ajratgan kesmasining uzunligi urinish nuqtasi absissasining kvadratiga teng.

29. Quyidagi xossalarga ega bo'lgan egri chiziqning tenglamasini yozing: egri chiziqning $M(x,y)$ nuqtasi normalining Oy o'qda ajratgan kesmasining uzunligi x^2/y ga teng.

30. Quyidagi xossalarga ega bo'lgan egri chiziqning tenglamasini yozing: ordinatasi 2 ga teng bo'lgan nuqtada egri chiziq Oy o'q bilan 45° tashkil etadi va egri chiziqqa o'tkazilgan ixtiyoriy urinmaning absissalar o'qida ajratgan kesmasining uzunligi urinish nuqtasi ordinatasining kvadratiga teng.

2 BOBNI TAKRORLASHGA DOIR ARALASH MASALALAR

Tenglamalarni yeching (**1-14**):

1. $y^3y''=1.$ **2.** $y''=2yy'.$ **3.** $y''(e^x+1)+y'=0.$

4. $2yy''=y^2+y'^2.$ **5.** $y''^2+y'=xy''.$ **6.** $y''-e^y=0.$

7. $2y'(y''+2)=xy''^2.$ **8.** $y'^2=(3y-2y')y''.$

9. $yy''-2yy'\ln y=y'^2.$ **10.** $xy''=y'+x\sin\dfrac{y'}{x}.$ **11.** $yy''+y=y'^2.$

12. $yy''' + 3y'y'' = 0.$ **13.** $yy'' = y'(y' + 1).$ **14.** $yy'' + y'^2 = 1.$

Tenglamalarning bir jinsliligidan foydalanib tartibini pasaytiring va tenglamalarni yeching (**15-20**):

15. $(x^2 + 1)(y'^2 - yy'') = xyy'.$ **16.** $xyy'' + xy'^2 = 2yy'.$

17. $x^2 yy'' = (y - xy')^2.$ **18.** $y(xy'' + y') = xy'^2(1 - x).$

19. $xyy'' = y'(y + y').$ **20.** $x^3 y'' = (y - xy')(y - xy' - x).$

Tenglamalarning berilgan boshlang'ich shartlarni qanoatlantiradigan yechimlarini toping (**21-22**):

21. $yy'' = 2xy'^2;\ y(2) = 2,\ y'(2) = 0{,}5.$

22. $x^2 y'' - 3xy' = \dfrac{6y^2}{x^2} - 4y;\ y(1) = 1,\ y'(1) = 4.$

23. Egiluvchan bir jinsli cho'zilmaydigan arqon ikki uchidan mahkamlangan bo'lib, o'zining og'irligi ostida osilib turadi. Arqonning muvozanat holatdagi shaklini aniqlang.

Tenglamalarni yeching (**24-33**):

24. $y'' + y' - 2y = 0.$ **25.** $y'' + 4y = 0.$

26. $y^{IV} - y = 0.$ **27.** $y^{VI} + 64y = 0.$

28. $y^{V} - 10y''' + 9y' = 0.$ **29.** $y''' - 3y'' + 3y' - y = 0.$

30. $y''' - 3y' + 2y = 0.$ **31.** $y'' - 2y' - 3y = e^{4x}.$

32. $y'' + 3y' - 4y = e^{-4x} + xe^{-x}.$ **33.** $y'' - 4y' + 8y = e^{2x} + \sin 2x.$

34-37 masalalarda har bir tenglamaning xususiy yechimi ko'rinishini aniqmas koeffitsientlar bilan yozing (koeffitsientlarning sonli qiymatlarini topish shart emas).

34. $y'' - 2y' + 2y = e^{x} + x\cos x.$

35. $y'' + 6y' + 10y = 3xe^{-3x} - 2e^{3x}\cos x.$

36. $y''' - 2y'' + 4y' - 8y = e^{2x}\sin 2x + 2x^2.$

37. $y'' - 6y' + 8y = 5xe^{2x} + 2e^{4x}\sin x.$

O'zgarmaslarni variatsiyalash usuli bilan yeching (38-40):

38. $y'' - 2y' + y = \dfrac{e^x}{x}.$ **39.** $y'' + y = \dfrac{1}{\sin x}.$

40. $y'' + 2y' + y = 3e^{-x}\sqrt{x+1}.$

Tenglamalarning berilgan boshlang'ich shartlarni qanoatlantiradigan yechimlarini toping:

41. $y'' + y = 4e^x;\ y(0) = 4,\ \ y'(0) = -3.$

42. $y'' + 2y' + 2y = xe^{-x};\ y(0) = y'(0) = 0.$

43. $y''' - 3y' - 2y = 9e^{2x};\ y(0) = 0,\ \ y'(0) = -3,\ y''(0) = 3.$

Eyler tenglamalarini yeching (**44-49**):

44. $x^2y'' - 4xy' + 6y = 0.$ **45.** $x^3y''' + xy' - y = 0.$

46. $x^2y'' - xy' + y = 8x^3.$ **47.** $x^3y'' - 2xy = 6\ln x.$

48. $x^2y'' - 6y = 5x^3 + 8x^2.$ **49.** $(x-2)^2y'' - 3(x-2)y' + 4y = x.$

Qulay usulni qo'llab tenglamalarni yeching (**50-53**):

50. $y'' - 2y' + y = xe^x\sin^2 ix.$ **51.** $y''' - 8i\,y = \cos 2x.$

52. $x^2y'' - xy' + y = \dfrac{\ln x}{x} + \dfrac{x}{\ln x}.$ **53.** $y'' + 2y' + y = xe^x + \dfrac{1}{xe^x}.$

54-56 masalalarda berilgan funksiya biror o'zgarmas koeffitsientli chiziqli bir jinsli differensial tenglamaning xususiy yechimi ekanligi ma`lum. Shu tenglamalarni (imkon qadar eng past tartibli) tuzing.

54. $y_1 = x^2e^x.$ **55.** $y_1 = xe^x,\ y_2 = e^{-x}.$ **56.** $y_1 = x,\ y_2 = \sin x.$

57. a va b parametrlarning qanday qiymatlarida $y'' + ay' + by = 0$ tenglama $x \to +\infty$ da nolga intiladigan kamida bitta $y \neq 0$ yechimga ega bo'ladi?

58. m massaning zarrasi Ox o'q bo'ylab harakatlanmoqda, bunda zarra $x=0$ nuqtadan $3mr_0$ kuch bilan qochmoqda va $x=1$ nuqtaga $4mr_1$ kuch bilan intilmoqda, bu yerda r_0 va r_1 - shu nuqtalargacha bo'lgan masofalar. Zarraning $x(0)=2, \dot{x}(0)=0$ boshlang'ich shartlarni qanoatlantiruvchi harakatini aniqlang.

59. V kuchlanish beradigan o'zgarmas tok manbai, R qarshilik, L o'zinduktsiya va kalit ketma-ket ulanib, elektr zanjir tuzilgan. Kalit $t=0$ da ulanadi. Tok kuchining vaqtga bog'lanishini toping ($t>0$).

60-65 masalalarda berilgan funksiyalar chiziqli bog'liq bo'ladimi?

60. $x+2,\ x-2.$ **61.** $\sin x, \cos x.$ **62.** $4-x,\ 2x+3,\ 6x+8.$

63. $x^2-x+3,\ 2x^2+x,\ 2x-4.$ **64.** $x, e^{-x}, xe^{-x}.$ **65.** $shx, chx, 2+e^x.$

66. Nechanchi tartibli chiziqli bir jinsli tenglama $(-1,1)$ intervalda ushbu $y_1=x^2-2x+2,\ y_2=(x-2)^2,\ y_3=x^2+x-1,\ y_4=1-x$ xususiy yechimlarga ega bo'lishi mumkin.

Xususiy yechimlari berilgan ko'rinishda bo'lgan chiziqli bir jinsli differensial tenglamalarni (mumkin qadar pastroq tartibli) tuzing (**67-69**):

67. x, e^x**.** **68.** $x^2-3x,\ 2x^2+9,\ 2x+3.$ **69.** $x, x^2, e^x.$

O'zgarmaslarni variatsiyalash usuli yordamida quyidagi o'zgaruvchi koeffitsientli tenglamalarning umumiy yechimini toping (**70-71**):

70. $y''+\frac{2}{x}y'-\frac{2}{x^2}y=3x^2.$ **71.** $xy'''-y''=(x-1)e^x.$

3-BOB

IKKINCHI TARTIBLI DIFFERENSIAL TENGLAMALAR

3.1. O'ZGARUVCHI KOEFFITSIENTLI CHIZIQLI DIFFERENSIAL TENGLAMALAR

3.1.1. Ostrogradskiy-Liuvill va Abel formulalari. Agar n-tartibli chiziqli bir jinsli differensial tenglamaning y_1 xususiy yechimi ma`lum bo'lsa, u holda tenglamaning chiziqliligini saqlagan holda uning tartibini pasaytirish mumkin. Buning uchun tenglamada $y = y_1 z$ deb olamiz va $z' = u$ o'rniga qo'yish yordamida tenglamaning tartibini pasaytiramiz.

Ikkinchi tartibli chiziqli bir jinsli

$$a_0(x)y'' + a_1(x)y' + a_2(x)y = 0 \qquad (1)$$

tenglamaning y_1 xususiy yechimi ma`lum bo'lganda, uning umumiy yechimini topish uchun yuqoridagi usul yordamida tenglamaning tartibini pasaytirish mumkin. Ammo ko'p hollarda quyidagi *Ostrogradskiy-Liuvill formulasi*dan foydalanib tenglamaning tartibini pasaytirish qulay bo'ladi:

$$\begin{vmatrix} y_1 & y \\ y_1' & y' \end{vmatrix} = C_1 \exp\left(-\int p_1(x)dx\right),$$

bu yerda $p_1(x) = \dfrac{a_1(x)}{a_0(x)}$, y_1 - berilgan (1) tenglamaning birorta xususiy yechimi, y esa uning izlanayotgan umumiy yechimi, C_1 - ixtiyoriy o'zgarmas.

Agar ikkinchi tartibli

$$y'' + p_1(x)y' + p_2(x)y = 0 \qquad (2)$$

tenglamaning y_1 xususiy yechimi berilgan bo'lsa, u holda uning umumiy yechimi

$$y = C_1 y_1(x) + C_2 y_1(x) \int \frac{\exp\left(-\int p_1(x)dx\right)}{y_1^2(x)} dx$$

Abel formulasi yordamida topiladi.

Tenglamalarni yeching (**128-130**).

128. $x y'' - (x+1)y' + y = 0;\ y_1(x) = e^x$.

◀ $y = e^x z$ va $z' = u$ o’rniga qo’yishlar yordamida tenglamaning tartibini pasaytiramiz:

$$xu' + (x-1)u = 0.$$

O’zgaruvchilari ajraladigan bu tenglamaning umumiy yechimi $u = C x e^{-x}$ ko’rinishga ega. Endi $z' = C x e^{-x}$ tenglamani integrallab, $z = C_1 (x+1) e^{-x} + C_2$, ya`ni $y = C_1(x+1) + C_2 e^x$ umumiy yechimga ega bo’lamiz.▶

129. $x^2(2x-1)y''' + (4x-3)x y'' - 2xy' + 2y = 0;\ y_1(x) = x,\ y_2(x) = \dfrac{1}{x}.$

◀ $y = x z(x)$ va $z'(x) = u(x)$ o’rniga qo’yishlar yordamida tenglamaning tartibini pasaytiramiz:

$$x^2(2x-1)u'' + 2x(5x-3)u' + 6(x-1)u = 0. \qquad (3)$$

Endi $y_2(x) = x z_2(x)$ va $z_2'(x) = u_2(x)$ munosabatlardan (3) tenglamaning $u_2(x) = \dfrac{1}{x^3}$ xususiy yechimini topamiz. Yana bir marta $u = \dfrac{w(x)}{x^3}$ va $w'(x) = v(x)$ o’rniga qo’yishlarni bajarsak, (3) tenglama $(1-2x)v' + 2v = 0$ ko’rinishni oladi.

O’zgaruvchilari ajraladigan bu tenglama $v = C_1(1-2x)$, ya`ni $w' = C_1(1-2x)$ ko’rinishdagi umumiy yechimga ega. Oxirgi tenglamadan $w(x) = C_1(x - x^2) + C_2$ kelib chiqadi. Shundan so’ng bir necha amallarni ketma-ket bajaramiz:

$$u(x) = C_1 x^{-3}(x - x^2) + C_2 x^{-3}, \quad z'(x) = C_1\left(\frac{1}{x^2} - \frac{1}{x}\right) + \frac{C_2}{x^3},$$

$$z(x) = C_1\left(-\frac{1}{x} - \ln|x|\right) - \frac{C_2}{2x^2} + C_3.$$

Endi $y = x z(x)$ tenglikni e`tiborga olib, berilgan tenglamaning umumiy yechimini yozamiz: $y = C_1\left(1 + x\ln|x|\right) + \dfrac{C_2}{x} + C_3 x.$ ▶

130. $(e^x + 1)\, y'' - 2y' - e^x y = 0;\ y_1(x) = e^x - 1.$

◀Ikkinchi yechimni topish uchun Ostrogradskiy-Liuvill formulasidan foydalanamiz:

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = C_1 \exp\left(-\int p_1(x)dx\right),\ p_1(x) = -\frac{2}{e^x+1}.$$

U holda y_2 ga nisbatan birinchi tartibli tenglama hosil bo'ladi:

$$y_1 y_2' - y_1' y_2 = \frac{C_1 e^{2x}}{(e^x+1)^2},\ \text{ya`ni}\ \left(\frac{y_2}{y_1}\right)' = \frac{C_1 e^{2x}}{y_1^2(e^x+1)^2}.$$

Bu tenglamani yechish qiyin emas:

$$\frac{y_2}{y_1} = C_1 \int \frac{e^{2x}}{(e^{2x}-1)^2} dx + C_2, \quad \frac{y_2}{e^x-1} = -\frac{C_1}{2}\frac{1}{e^{2x}-1} + C_2,$$

$$y_2 = \left(-\frac{C_1}{2}\frac{1}{e^{2x}-1} + C_2\right)(e^x-1),\ y_2 = \frac{C_1}{e^x-1} + C_2(e^x-1).$$

Topilgan y_2 yechim berilgan tenglamaning umumiy yechimi bo'ladi.▶

Quyidagi tenglamalarning xususiy yechimlarini biron-bir usulda topib, so'ngra ularning umumiy yechimlarini yozing (**131-132**).

131. $(2x+1)y'' + 4xy' - 4y = 0.$

◀Xususiy yechimni $y_1 = e^{ax}$ ko'rinishda izlab ko'ramiz. y_1 ni tenglamaga qo'yib va tenglamaning ikkala tomonini e^{ax} ga qisqartirib,

$$a^2(2x+1) + 4ax - 4 \equiv 0$$

ayniyatni olamiz. Ayniyatdagi a ning qiymati

$$2a^2 + 4a = 0,\quad a^2 - 4 = 0$$

sistemadan topiladi: $a = -2$. Shunga binoan, $y_1 = e^{-2x}$ - xususiy yechim.

Tenglamaning umumiy yechimini topish uchun Abel formulasidan foydalanamiz:

$$y \equiv C_1 e^{-2x} + C_2 e^{-2x} \int e^{4x} \exp\left(-\int \frac{4x}{2x+1} dx\right) dx = C_1 e^{-2x} + C_2 x.$$ ▶

132. $x(x+4)y'' - (2x+4)y' + 2y = 0.$

◀Xususiy yechimni $y_1 = e^{ax}$ ko'rinishda izlab ko'ramiz. y_1 ni tenglamaga qo'yib va tenglamaning ikkala tomonini e^{ax} ga qisqartirib,

$$a^2x(x+4)-a(2x+4)+2=0$$

ayniyatni olamiz. Bu ayniyat x ning darajalari oldidagi koeffitsientlar nolga teng bo'lganda, ya`ni

$$a^2=0,\ \ 4a^2-2a=0,\ \ -4a+2=0$$

tengliklar bajarilganda o'rinli. Lekin bu tenglamalar sistemasi yechimga ega emasligi ochiq ravshan. Bundan xususiy yechim $y_1=e^{ax}$ ko'rinishda emas, degan xulosaga kelamiz.

Endi xususiy yechimni

$$y_1=x^n+ax^{n-1}+\ldots \qquad (4)$$

ko'phad ko'rinishida izlaymiz. Birinchi navbatda ko'phadning darajasini, ya`ni n ning qiymatini aniqlash kerak. Buning uchun (4) ni berilgan tenglamaga qo'yib,

$$x(x+4)[n(n-1)x^{n-2}+a(n-1)(n-2)x^{n-3}+\ldots]-$$
$$-(2x+4)[nx^{n-1}+a(n-1)x^{n-2}+\ldots]+2[x^n+ax^{n-1}+\ldots]=0$$

hosil bo'lgan ko'phad ko'rinishidagi ayniyatda eng yuqori darajalar oldidagi koeffitsientlarni nolga tenglaymiz: $n(n-1)-2n+2=0$. Bundan $n=1$ yoki $n=2$ ekanligi ma`lum bo'ladi. Xususiy yechimni, masalan, birinchi darajali ko'phad $y_1=x+a$ ko'rinishida izlaymiz. $y_1=x+a$ funksiyani berilgan tenglamaga qo'yib, $y_1=x+2$ xususiy yechimga ega bo'lamiz.

Bu yerda ham avvalgi misoldagi kabi Abel formulasi yordamida umumiy yechimni topamiz: $y=C_1(x+2)+C_2x^2$.►

133. Ushbu ikkinchi tartibli chiziqli differensial tenglama

$$(3x^3+x)y''+2y'-6xy=4-12x^2 \qquad (5)$$

va uning ikkita $y_1=2x,\ y_2=(x+1)^2$ xususiy yechimlari berilgan. Bu tenglamaning umumiy yechimini toping.

◄Bir jinsli bo'lmagan (5) tenglamaning umumiy yechimi $y=y_0^*+y_1$ ko'rinishda bo'ladi, bu yerda y_0^* - ushbu

$$(3x^3+x)y''+2y'-6xy=0 \qquad (6)$$

bir jinsli tenglamaning umumiy yechimi.

Avvalo (6) tenglamaning birorta xususiy yechimini topamiz. Ma`lumki, bir jinsli bo'lmagan chiziqli tenglama ikkita xususiy yechimlarining ayirmasi mos bir jinsli tenglamaning xususiy yechimi bo'ladi:

$$y_0=y_2-y_1=x^2+1.$$

Endi bir jinsli (6) tenglamaning y_0^* umumiy yechimi Abel formulasi yordamida topiladi:

$$y_0^* \equiv C_1(x^2+1)+C_2(x^2+1)\int\frac{\exp\left(-\int\frac{2}{3x^3+x}dx\right)}{\left(x^2+1\right)^2}dx=C_1(x^2+1)+\frac{C_2}{x}.$$

Shunday qilib, berilgan bir jinsli bo'lmagan (5) tenglamaning umumiy yechimi $y=C_1(x^2+1)+\frac{C_2}{x}+2x$ ko'rinishda bo'ladi.▶

INDIVIDUAL TOPSHIRIQLAR

M48. Differensial tenglamaning bitta y_1 xususiy yechimi ma'lum bo'lsa, uning umumiy yechimini toping.

1. $x^2y''-\left(x^2+2\right)y=0,\ y_1=\left(1-\frac{1}{x}\right)e^x.$

2. $x^2y''-\left(4x^2+2\right)y=0,\ y_1=\left(2-\frac{1}{x}\right)e^{2x}.$

3. $x^2y''+x^2y'-2y=0,\ y_1=1-\frac{2}{x}.$

4. $x^2y''+x\left(2x+1\right)y'-4y=0,\ y_1=2-\frac{4}{x}+\frac{3}{x^2}.$

5. $x^2y''-x\left(x^2-1\right)y'-\left(x^2+1\right)y=0,\ y_1=1/x.$

6. $4x^2y''+4x^3y'+\left(x^2+6\right)\left(x^2-4\right)y=0,\ y_1=x^3e^{-x^2/4}.$

7. $x\left(ax+b\right)y''+2by'-2ay=0,\ y_1=1/x.$

8. $y''-(1+2tg^2x)y=0,\ y_1=1/\cos x.$

9. $x\left(7x+4\right)y''+8y'-14y=0,\ y_1=1/x.$

10. $x^2y''+6x^2y'-2y=0,\ y_1=1-\frac{1}{3x}.$

11. $y''+y'tgx+y\cos^2x=0,\ y_1=\cos\left(\sin x\right).$

12. $x^2y''+8x^2y'-2y=0,\ y_1=1-\frac{1}{4x}.$

13. $x(x+1)y''+2y'-2y=0,\ y_1=\frac{1}{x}.$

14. $x^2y''+3x^2y'-2y=0,\ y_1=1-\frac{2}{3x}.$

15. $y''+y'tgx-y\cos^2 x=0,\ y_1=e^{\sin x}.$

16. $x(2x+1)y''+2y'-4y=0,\ y_1=\frac{1}{x}.$

17. $x^2y''+5x^2y'-2y=0,\ y_1=1-\frac{2}{5x}.$

18. $y''-y'ctgx+y\sin^2 x=0,\ y_1=\cos(\cos x).$

19. $x(x+2)y''+4y'-2y=0,\ y_1=\frac{1}{x}.$

20. $x^2y''+7x^2y'-2y=0,\ y_1=1-\frac{2}{7x}.$

21. $y''\sin^2 x-2y=0,\ y_1=ctgx.$

22. $x(2-x)y''+4y'+2y=0,\ y_1=\frac{1}{x}.$

23. $x^2y''+9x^2y'-2y=0,\ y_1=1-\frac{2}{9x}.$

24. $(x\cos x-\sin x)y''+xy'\sin x-y\sin x=0,\ y_1=x.$

25. $x(2x-1)y''-2y'-4y=0,\ y_1=\frac{1}{x}.$

26. $x^2y''+2x^2y'-2y=0,\ y_1=1-\frac{1}{x}.$

27. $x^2(x-1)y''-x(5x-4)y'+3(3x-2)y=0,\ y_1=x^3.$

28. $x(2x+3)y''+6y'-4y=0,\ y_1=\frac{1}{x}.$

29. $x^2y''+4x^2y'-2y=0,\ y_1=1-\frac{1}{2x}.$

30. $x^2(x+1)y'' - x(2x+1)y' + (2x+1)y = 0,\ y_1 = x.$

3.2. DIFFERENSIAL TENGLAMALARNI DARAJALI QATORLAR YORDAMIDA YECHISH

3.2.1. Chiziqli tenglamalar yechishning darajali qatorlar usuli.

Ikkinchi tartibli tenglamani qaraymiz:

$$y'' + p(x)y' + q(x)y = 0. \tag{1}$$

Faraz qilaylik, (1) tenglamaning $p(x)$ va $q(x)$ koeffitsientlari $|x-x_0|<a$ intervalda analitik funksiyalar bo’lsin, ya`ni bu funksiyalar $|x-x_0|<a$ intervalda yaqinlashuvchi bo’lgan

$$p(x) = \sum_{k=0}^{\infty} p_k (x-x_0)^k, \quad q(x) = \sum_{k=0}^{\infty} q_k (x-x_0)^k \tag{2}$$

darajali qatorlarga yoyilsin.

Teorema. Agar $p(x)$ va $q(x)$ funksiyalar $|x-x_0|<a$ intervalda analitik funksiyalar bo’lsa, u holda (1) tenglamaning har qanday yechimi $|x-x_0|<a$ intervalda analitik bo’ladi, ya`ni bu yechim $|x-x_0|<a$ intervalda yaqinlashuvchi

$$y(x) = \sum_{k=0}^{\infty} c_k (x-x_0)^k$$

darajali qatorga yoyiladi.

Bu teorema (1) tenglamaning yechimini *darajali qator* ko’rinishida qurish (topish) imkonini beradi. Shu ishni amalga oshiramiz. Qisqalik uchun, $x_0 = 0$ deb olamiz. (1) tenglamaning yechimini x ning darajalari bo’yicha qator ko’rinishida izlaymiz:

$$y(x) = \sum_{k=0}^{\infty} c_k x^k, \tag{3}$$

bu yerda c_k- hozircha noma`lum va kelgusida aniqlanishi lozim bo’lgan koeffitsientlar.

(3) dan y' va y'' ni ketma-ket topib

$$y'(x) = \sum_{k=1}^{\infty} k\, c_k x^{k-1} = \sum_{k=0}^{\infty} (k+1) c_{k+1} x^k,$$

$$y''(x)=\sum_{k=2}^{\infty}k\,(k-1)\,c_k x^{k-2}=\sum_{k=0}^{\infty}(k+2)\,(k+1)\,c_{k+2}x^k\ ,$$

so'ngra y'', y' va y ning ifodalarini (1) tenglamaga qo'yamiz:

$$\sum_{k=0}^{\infty}(k+2)\,(k+1)\,c_{k+2}x^k+\sum_{k=0}^{\infty}p_k x^k\cdot\sum_{k=0}^{\infty}(k+1)\,c_{k+1}x^k+$$

$$+\sum_{k=0}^{\infty}q_k x^k\cdot\sum_{k=0}^{\infty}c_k x^k=0.$$

Shu yerda matematik analiz kursidan ma`lum bo'lgan darajali qatorlarni ko'paytirish formulasidan foydalanamiz:

$$\sum_{k=0}^{\infty}a_k x^k\cdot\sum_{k=0}^{\infty}b_k x^k=\sum_{k=0}^{\infty}\sum_{n=0}^{k}a_n x^n\cdot b_{k-n}x^{k-n}=\sum_{k=0}^{\infty}\left(\sum_{n=0}^{k}a_n b_{k-n}\right)x^k\ .$$

Hosil bo'lgan oxirgi tenglamada $x^0, x^1, x^2, x^3, \ldots,$ darajalar oldidagi koeffitsientlarni nolga tenglab, natijada $c_0, c_1, c_2, \ldots,$ noma`lum koeffitsientlarni aniqlash uchun rekurrent tenglamalar sistemasiga ega bo'lamiz:

$$q_0c_0+p_0c_1+1\cdot 2\ \ c_2=0,$$

$$q_1c_0+(q_0+p_1)c_1+2p_0\ \ c_2+2\cdot 3c_3=0,\ldots,$$

$$\sum_{i=1}^{k}\left[q_{k-i}c_i+(i+1)\,p_{k-i}c_{i+1}\right]+(k+1)(k+2)c_{k+2}=0,\ldots.\qquad(4)$$

c_0 va c_1 koeffitsientlarga ixtiyoriy qiymatlar berish mumkin (ulardan kamida bittasi noldan farqli bo'lishi lozim, aks holda $y\equiv 0$ yechim hosil bo'ladi). c_0 va c_1 koeffitsientlarga aniq qiymatlarni berib, (1) tenglamaning $y(0)=c_0,\ y'(0)=c_1$ boshlang'ich shartlarni qanoatlantiradigan yechimini izlaymiz. Birinchi tenglamadan c_2 ni, ikkinchi tenglamadan c_3 ni va h.k. aniqlaymiz.

Agar (1) tenglamada $p(x)$ va $q(x)$ - ratsional funksiyalar, ya`ni

$$p(x)=\frac{p_1(x)}{p_0(x)},\ \ q(x)=\frac{q_1(x)}{q_0(x)}$$

bo'lsa ($p_0(x), p_1(x), q_0(x), q_1(x)$- ko'phadlar), u holda $p_0(x)=0$ yoki $q_0(x)=0$ bo'lgandagi nuqtalar (1) tenglamaning *maxsus nuqtalari* deyiladi.

Ikkinchi tartibli

$$x^2y''+xp(x)y'+q(x)y=0 \qquad (5)$$

tenglamada $p(x)$ va $q(x)$ funksiyalar $|x|<a$ oraliqda analitik bo’lsin. Bu tenglama uchun $x=0$ nuqta maxsus nuqta bo’ladi, $p(x)$ va $q(x)$ funksiyalarni darajali qatorga yoyganda p_0 yoki q_0 koeffitsientlardan faqat bittasigina noldan farqli bo’ladi. Bu maxsus nuqtaning eng sodda misoli bo’lib, bunday nuqta *regulyar maxsus nuqta* (yoki *birinchi tur maxsus nuqta*) deb ataladi.

$x=x_0$ maxsus nuqtaning atrofida darajali qator ko’rinishidagi yechim mavjud bo’lmasligi ham mumkin. Bunday holda yechimni *umumlashgan darajali qator* ko’rinishida izlash kerak:

$$y=(x-x_0)^{\lambda}\sum_{k=0}^{\infty}c_k(x-x_0)^k, \qquad (6)$$

bu yerda λ va $c_0, c_1, c_2, \ldots, (c_0\neq 0)$ aniqlanishi lozim bo’lgan noma`lumlar.

(5) tenglamani $x>0$ da o’rganamiz. Bu tenglamaga (6) ifodani $x_0=0$ da qo’yib, topamiz:

$$\left[\lambda(\lambda-1)+p_0\lambda+q_0\right]c_0+\left\{\left[\lambda(\lambda+1)+p_0(\lambda+1)+q_0\right]c_1+\right.$$

$$\left.+(\lambda\ \ p_0+q_0)c_0\right\}x+\ldots+\left\{\left[(\lambda+n)(\lambda+n-1)+\ldots+p_0(\lambda+n)+q_0\right]c_k+\ldots\right.$$

$$\left.\ldots+(\lambda\ \ p_0+q_0)c_0\right\}x^k+\ldots=0$$

x ning darajalari oldidagi koeffitsientlarni bir-biriga tenglab, rekurrent tenglamalar sistemasini hosil qilamiz:

$$\begin{gathered} f_0(\lambda)c_0=0,\quad f_0(\lambda+1)c_1+f_1(\lambda)c_0=0,\ldots,\\ f_0(\lambda+k)c_k+f_1(\lambda+k-1)c_{k-1}+\\ +f_2(\lambda+k-2)c_{k-2}+\ldots+f_k(\lambda)c_0=0,\quad \ldots, \end{gathered} \qquad (7)$$

bu yerda

$$f_0(\lambda)=\lambda(\lambda-1)+p_0\lambda+q_0,\quad f_m(\lambda)=\lambda\ \ p_m+q_m,\ m\geq 1. \qquad (8)$$

Shartga ko’ra, $c_0\neq 0$ bo’lganligi uchun λ soni

$$\lambda(\lambda-1)+p_0\lambda+q_0=0$$

tenglamani qanoatlantirishi lozim. Bu tenglama *aniqlovchi tenglama* deyiladi.

λ_1 va λ_2 - aniqlovchi tenglamaning ildizlari bo’lsin. Agar $\lambda_1-\lambda_2$ ayirma butun son bo’lmasa, u holda har qanday butun $k>0$ da $f_0(\lambda_1+k)\neq 0$, $f_0(\lambda_2+k)\neq 0$ bo’ladi va, demak, yuqorida ko’rsatilgan usul bilan (1) tenglamaning o’zaro chiziqli bog’liq siz ikkita $y_1 = x^{\lambda_1}\sum_{k=0}^{\infty} c_k^{(1)} x^k$ va $y_2 = x^{\lambda_2}\sum_{k=0}^{\infty} c_k^{(2)} x^k$ yechimlarini qurish mumkin.

Agar $\lambda_1 - \lambda_2$ ayirma butun son bo’lsa, u holda yuqorida ko’rsatilgan usul bilan bitta $y_1(x)$ yechimni umumlashgan darajali qator ko’rinishida topish mumkin. Bu yechimni bilgan holda Liuvill-Ostrogradskiy formulasi yordamida $y_1(x)$ bilan chiziqli bog’liqsiz bo’lgan ikkinchi yechimni topish mumkin:

$$y_2(x) = y_1(x) \int \frac{\exp\left(-\int p(x)dx\right)}{y_1^2(x)}\, dx. \qquad (9)$$

Bu formuladan $y_2(x)$ yechimni

$$y_2(x) = Ay_1(x)\ln|x| + x^{\lambda_2}\sum_{k=0}^{\infty} c_k x^k$$

ko’rinishda izlash kerakligi kelib chiqadi (A soni nolga teng bo’lishi ham mumkin).

Quyidagi masalalarda berilgan tenglamalarning chiziqli bog’liqsiz yechimlarini darajali qatorlar ko’rinishida toping. Topilgan qatorning yig’indisini imkoni boricha elementar funksiyalar yordamida ifodalang (**134-136**).

134. $y'' - xy' - 2y = 0.$

◄ $p_0(x) \equiv 1,\ p_1(x) \equiv -x,\ p_2(x) \equiv -2$ funksiyalar $\forall x \in (-\infty,\ \infty)$ da analitik va $p_0(x) \neq 0$ bo’lganligi uchun $y = y(x),\ x \in (-\infty,\ \infty)$ analitik yechim mavjud. Bu yechimni (3) ko’rinishda izlaymiz va uni berilgan tenglamaga qo’yib, quyidagi ayniyatni olamiz:

$$\sum_{k=0}^{\infty}(k+1)(k+2)c_{k+2}x^k - \sum_{k=1}^{\infty} k\ \ c_k x^k - 2\sum_{k=0}^{\infty} c_k x^k = 0,$$

yoki

$$2c_2 - 2c_0 + \sum_{k=1}^{\infty}(k+2)\left[(k+1)\ \ c_{k+2} - \ c_k\right]\ x^k = 0.$$

Bu yerdan, x ning bir xil darajalari oldidagi koeffitsientlarni bir-biriga tenglashtirib, koeffitsientlarni aniqlash uchun rekurrent munosabatlarga ega bo’lamiz:

$$c_2 = c_0,\ c_{k+2} = \frac{1}{k+1}c_k,\ k = 1,\ 2,\ 3,\ ....$$

Aytaylik, $c_0 = 1, c_1 = 0$ bo’lsin, u holda

$$c_{2k+1} = 0,\ c_2 = 1, c_4 = \frac{1}{1\cdot 3}, c_6 = \frac{1}{1\cdot 3\cdot 5}, ...$$

Shunga asosan,

$$y_1(x) = 1 + \frac{x^2}{1} + \frac{x^4}{1\cdot 3} + \frac{x^6}{1\cdot 3\cdot 5} + ...$$

Shunga o’xshash, agar $c_0 = 0, c_1 = 1$ bo’lsa, u holda

$$c_{2k} = 0,\ c_3 = \frac{1}{2}, c_5 = \frac{1}{2\cdot 4}, c_7 = \frac{1}{2\cdot 4\cdot 6}, ...$$

va tegishli yechim

$$y_2(x) = x + \frac{x^3}{2} + \frac{x^5}{2\cdot 4} + \frac{x^7}{2\cdot 4\cdot 6} + ...$$

ko’rinishda bo’ladi.

Hosil bo’lgan darajali qatorlar $\forall x \in (-\infty,\ \infty)$ da yaqinlashuvchi ekanligini tekshirish qiyin emas. Bundan tashqari, $y_1(x) \equiv k\ \ y_2(x)$, $k = const$ ayniyat bajarilmaydi (masalan, $y_1(0) = 0$ tenglik $y_1(x)$ ning ta`rifiga zid), shuning uchun $y_1(x)$ va $y_2(x)$ yechimlar chiziqli bog’liqsiz bo’ladi. Shunday qilib, ular tenglama yechimlarining fundamental sistemasini tashkil etadi va berilgan tenglamaning umumiy yechimi

$$y(x) = C_1 y_1(x) + C_2 y_2(x),\ x \in (-\infty,\ \infty)$$

ko’rinishda topiladi.►

135. $(1 - x^2)y'' - 6xy' - 6y = 0.$

◄Ushbu

$$f(x, y, y') = \frac{6xy' + 6y}{1 - x^2},\ \ x \neq \pm 1$$

funksiya x, y, y' $(x \neq \pm 1)$ o’zgaruvchilar bo’yicha analitik bo’lganligi uchun bu tenglama $x \neq \pm 1$ bo’lganda analitik yechimlarga ega. Avvalo bu yechimlarni nol $(x = 0)$ ning biror atrofida topamiz, ya`ni yechimlarni

$$y(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + ...$$

ko’rinishda izlaymiz. Bu qatorni tenglamaga qo’ysak, quyidagi ayniyat hosil bo’ladi:

$$\sum_{k=0}^{\infty}(k+1)(k+2)c_{k+2}x^k - \sum_{k=2}^{\infty}k\ \ (k-1)c_k x^k - 6\sum_{k=1}^{\infty}k\ \ c_k x^k - 6\sum_{k=0}^{\infty}c_k x^k = 0,$$

yoki

$$2c_2-6c_0+(6c_3-12c_1)x+\sum_{k=2}^{\infty}(k+2)\left[(k+1)\ c_{k+2}-\ (k+3)c_k\right]\ x^k=0.$$

Bu yerdan, x ning bir xil darajalari oldidagi koeffitsientlarni bir-biriga tenglashtirib, koeffitsientlarni aniqlash uchun rekurrent munosabatlarga ega bo'lamiz:

$$c_2=3c_0,\ c_3=2c_1,\ c_{k+2}=\frac{k+3}{k+1}c_k,\ k=2,\ 3,\ ....$$

Aytaylik, $c_0=1, c_1=0$ bo'lsin, u holda

$$c_{2k}=2k+1,\ c_{2k+1}=0,\ k=1,\ 2, ...$$

Demak,

$$y_1(x)=\sum_{k=0}^{\infty}(2k+1)\ x^{2k}=\left(\sum_{k=0}^{\infty}\ x^{2k+1}\right)'=\left(\frac{x}{1-x^2}\right)',\ |x|<1,$$

ya`ni

$$y_1(x)=\frac{1+x^2}{\left(1-x^2\right)^2}.$$

Shunga o'xshash, agar $c_0=0, c_1=1$ bo'lsa, u holda

$$c_{2k}=0,\ c_{2k+1}=k+1,\ k=1,\ 2, ...$$

Shuning uchun

$$y_2(x)=\sum_{k=0}^{\infty}(k+1)\ x^{2k+1}=\left(\frac{1}{2}\ \sum_{k=0}^{\infty}\ x^{2k+2}\right)'=\frac{1}{2}\left(\frac{x^2}{1-x^2}\right)',\ |x|<1,$$

ya`ni

$$y_2(x)=\frac{x}{\left(1-x^2\right)^2}.$$

$y_1(x),\ y_2(x)$ funksiyalar $|x|>1$ da ham berilgan tenglamaning yechimlari bo'lishini tekshirib ko'rish qiyin emas.►

136. $y''+y\sin x=0.$

◄Bu tenglamani yechishda

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+...,-\infty<x<\infty$$

yoyilmadan foydalanamiz va yechimni

$$y(x)=c_0+c_1x+c_2x^2+c_3x^3+c_4x^4+...$$

ko'rinishda izlaymiz. Yuqoridagi mulohazalarni takrorlab, c_k $(k=0,\ 1,\ 2,...)$ koeffitsientlarni aniqlash uchun tenglamalar sistemasini tuzamiz:

$$c_2=0,\quad c_0+6c_3=0,\quad c_1+12c_4=0,$$

$$c_2-\frac{c_0}{6}+20c_5=0,\quad c_3-\frac{c_1}{6}+30c_6=0,....$$

Agar $c_0=1, c_1=0$ deb olsak, u holda

$$c_3=-\frac{1}{6},\ c_4=0,\ c_5=\frac{1}{120},\ c_6=0,\ ....$$

Demak,

$$y_1(x)=1-\frac{x^3}{6}+\frac{x^5}{120}+....$$

Shunga o'xshash, agar $c_0=0, c_1=1$ bo'lsa, u holda

$$c_3=0,\ c_4=-\frac{1}{12},\ c_5=0,\ c_6=\frac{1}{180},....$$

va tegishli yechim

$$y_2(x)=x-\frac{x^4}{12}+\frac{x^6}{180}+...$$

ko'rinishda bo'ladi.►

137. Ushbu $2x^2y''+(3x-2x^2)y'-(x+1)y=0$ tenglamalarning yechimlarini darajali (yoki umumlashgan darajali) qatorlar ko'rinishida toping.

◄Bu tenglamaning $p_0(x)\equiv 2x^2$ koeffitsienti $x=0$ da nolga teng bo'lganligi uchun berilgan tenglamaning

$$y=\sum_{k=0}^{\infty}c_kx^{k+\lambda} \tag{10}$$

umumlashgan darajali qator ko'rinishidagi kamida bitta notrivial yechimi mavjud. Qatorni tenglamaga qo'yib va x ning darajalari oldidagi koeffitsientlarni bir-biriga tenglashtirib, quyidagi sistemani olamiz:

$$\left(2\lambda^2+\lambda-1\right)c_0=0,\quad (2\lambda+2k-1)\left((\lambda+k+1)c_k-c_{k-1}\right)=0,$$

$$(k=1,\ 2,...).$$

Endi $c_0=1$ bo'lsin. Bu holda $2\lambda^2+\lambda-1=0$ aniqlovchi tenglamadan topamiz: $\lambda_1=-1,\ \lambda_2=1/2$. So'ngra, ushbu

$$c_k=\frac{c_{k-1}}{\lambda+k+1}\ (k=1,\ 2,...)$$

rekurrent formuladan (10) umumlashgan darajali qatorning koeffitsientlarini topamiz. Agar bu formulada $\lambda_1 = -1$ deb olsak, u holda $c_k = \dfrac{1}{k!}$ $(k = 0,\ 1,\ 2,...)$ bo'lib, tegishli yechim

$$y_1(x) \equiv \sum_{k=0}^{\infty} \frac{1}{k!} x^{k-1} = \frac{1}{x} + 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + ... = \frac{e^x}{x},\ x \neq 0$$

ko'rinishda bo'ladi. Xuddi shunga o'xshash, $\lambda_2 = 1/2$ da

$$c_1 = \frac{2}{5},\ c_2 = \frac{2^2}{5 \cdot 7},\ c_3 = \frac{2^3}{5 \cdot 7 \cdot 9},\ ...,$$

bo'ladi va ikkinchi chiziqli bog'liqsiz yechim

$$y_2(x) \equiv \sum_{k=0}^{\infty} c_k x^{k+0,5} = \sqrt{x}\left(1 + \frac{2x}{5} + \frac{(2x)^2}{5 \cdot 7} + \frac{(2x)^3}{5 \cdot 7 \cdot 9} + ...\right)$$

ko'rinishda topiladi.►

138. $y'' + y' + y = |\sin x|$ tenglamaning davriy yechimini trigonometrik qator ko'rinishida toping.

◄Tenglamaning o'ng tomonidagi $f(x) = |\sin x|$ funksiya juft va $\pi-$ davrli ekanligi ravshan, shuning uchun tenglamaning davriy xususiy yechimini

$$y = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos 2kx + b_k \sin 2kx \tag{11}$$

ko'rinishda izlaymiz. Avvalo Fur'ye qatorlari nazariyasi vositalari yordamida $\pi-$davrli $f(x) = |\sin x|$ funksiyani Fur'ye qatoriga yoyamiz:

$$|\sin x| = \frac{2}{\pi} - \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\cos 2kx}{4k^2 - 1}. \tag{12}$$

Endi (11) va (12) qatorlarni berilgan tenglamaga qo'yamiz, so'ngra $\cos 2kx$, $\sin 2kx$ funksiyalarning oldidagi koeffitsientlarni bir-biriga tenglashtirib, natijada

$$\frac{a_0}{2} = \frac{2}{\pi},\ 2ka_k + \left(4k^2 - 1\right)b_k = 0, \left(4k^2 - 1\right)a_k - 2kb_k = \frac{4}{\pi}\frac{1}{4k^2 - 1},$$
$$k = 1,\ 2,...$$

tenglamalar sistemasiga ega bo'lamiz. Bu tenglamalar sistemasini yechib, (11) qatorning koeffitsientlarini topamiz:

$$a_0 = \frac{4}{\pi},\ a_k = \frac{4}{\pi}\frac{1}{16k^4 - 4k^2 + 1},\ b_k = -\frac{4}{\pi}\frac{1}{4k^2 - 1} \cdot \frac{2k}{16k^4 - 4k^2 + 1}.$$

Shunday qilib, berilgan tenglamaning davriy yechimi

$$y(x)=\frac{2}{\pi}+\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{16k^4-4k^2+1}\left(\cos 2kx-\frac{2k}{4k^2-1}\sin 2kx\right)$$

ko'rinishda bo'ladi.►

3.2.2. Gipergeometrik tenglama. Ushbu tenglama

$$x(1-x)y''+[c-(a+b+1)x]y'-aby=0 \qquad (1)$$

gipergeometrik tenglama (yoki *Gauss tenglamasi*) deb ataladi, bu yerda a, b, c - haqiqiy sonlar.

$x=0$ va $x=1$ nuqtalar tenglamaning maxsus nuqtalaridir. $x=0$ nuqtaga mos kelgan aniqlovchi tenglama $\lambda(\lambda-1)+c\lambda=0$ ko'rinishga ega. Uning ildizlari $\lambda_1=0,\ \lambda_2=1-c$. Agar c parametr musbat bo'lmagan butun songa teng bo'lmasa, u holda gipergeometrik tenglamaning ikkita chiziqli erkli yechimini $|x|<1$ da yaqinlashuvchi umumlashgan darajali qator ko'rinishida topish mumkin.

Darhaqiqat, aniqlovchi tenglamaning $\lambda_1=0$ ildiziga

$$y_1(x)=\sum_{k=0}^{\infty}A_k x^k \qquad (2)$$

yechim mos keladi. Bu ifodani berilgan tenglamaga qo'yamiz. U holda

$$\sum_{k=0}^{\infty}x(1-x)(k+1)(k+2)A_{k+2}x^k$$

$$+\sum_{k=0}^{\infty}[c-(a+b+1)x](k+1)A_{k+1}x^k-\sum_{k=0}^{\infty}abA_k x^k=0.$$

Noma`lum $A_0, A_1, ..., A_k, ...$ o'zgarmaslarni topish uchun aniqmas koeffitsientlar usulidan foydalanamiz, bunga asosan x ning bir xil darajalari oldidagi koeffitsientlarni nolga tenglash kerak. x^k oldidagi umumiy koeffitsientlarni nolga tenglab, A_k koeffitsientlarni aniqlash uchun ushbu tenglamalarni olamiz:

$$-(k-1)kA_k+k(k+1)A_{k+1}-k(a+b+1)A_k+c(k+1)A_{k+1}-abA_k=0,$$

$$A_{k+1}=\frac{k(k-1)+k(a+b+1)+ab}{(k+1)(k+c)}A_k=\frac{(a+k)(b+k)}{(k+1)(c+k)}A_k,\ k=0,\ 1,\ 2,\dots.$$

$A_0=1$ deb olib, bu yerdan ketma-ket topamiz:

$$A_1=\frac{ab}{c},\ A_2=\frac{a(a+1)b(b+1)}{2!c(c+1)},\quad A_3=\frac{a(a+1)(a+2)b(b+1)(b+2)}{3!c(c+1)(c+2)},$$

$$A_k=\frac{a(a+1)(a+2)...(a+k-1)\ \ b(b+1)(b+2)...(b+k-1)}{k!c(c+1)(c+2)...(c+k-1)},....$$

Endi (1) gipergeometrik tenglamaning birinchi xususiy yechimi $y_1(x)$ ni $F(a,\ b,\ c;\ x)$ bilan belgilab, A_k koeffitsientlarning topilgan qiymatlarini (2) qatorga qo'yamiz. U holda

$$y_1(x) \equiv F(a,b,c;x) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{k!\ (c)_k} x^k. \qquad (3)$$

Bu yerda $(a)_k$ - *Poxgammer belgilashi*:

$(a)_0 = 1, (a)_k = a(a+1)(a+2)...(a+k-1), k = 1,\ 2,\ 3,\ ....$ Xususan, $(1)_k = k!$. Poxgammer belgilashi faktorialning umumlashmasidir.

(3) qator *gipergeometrik qator*, bu qatorning yig'indisi bo'lgan $F(a,\ b,\ c;x)$ funksiya esa *gipergeometrik funksiya* deyiladi. Gipergeometrik qatorning $|x|<1$ da yaqinlashuvchi ekanligini tekshirish qiyin emas.

Berilgan (1) tenglamaning $y_1(x)$ bilan chiziqli bog'liqsiz bo'lgan ikkinchi yechimini

$$y_2(x) = x^{1-c}\sum_{k=0}^{\infty} A_k \quad x^k$$

ko'rinishda topish mumkin. Ammo bu yerda boshqacharoq yo'l tutamiz.

(1) tenglamada $y = x^{1-c}z$ almashtirishni bajaramiz. U holda

$$y' = x^{1-c}z' + (1-c)x^{-c}z,\ \ y'' = x^{1-c}z'' + 2(1-c)x^{-c}z' - c(1-c)x^{-c-1}z.$$

So'ngra y, y', y'' larning ifodalarini dastlabki tenglamaga qo'yib va x^{1-c} ga qisqartirsak, ushbu

$$x(1-x)z'' + \{(2-c) - [(a-c+1) + (b-c+1) + 1]x\}z' -$$
$$-(a-c+1)(b-c+1)z = 0$$

tenglama hosil bo'ladi. Bu tenglama ham gipergeometrik tenglama, faqat parametrlari $a+1-c, b+1-c, 2-c$ bo'lgan gipergeometrik tenglamadir. Oxirgi tenglamaning xususiy yechimi $z = F(a+1-c,\ b+1-c, 2-c;\ x)$ gipergeometrik qator ko'rinishida topiladi. Shunday qilib,

$$y_2(x) = x^{1-c}F(a+1-c,\ b+1-c, 2-c;\ x).$$

Xullas, agar c butun son bo'lmasa, u holda $|x|<1$ da gipergeometrik tenglamaning barcha yechimlari

$$y(x) = C_1F(a,b,c;x) + C_2x^{1-c}F(a+1-c,\ b+1-c, 2-c;\ x) \qquad (4)$$

formula bilan ifodalanadi, bu yerda C_1 va C_2 ixtiyoriy o'zgarmaslar.

Ta`kidlash kerakki, agar $x^{1-c}$ ifoda $x<0$ da ma`noga ega bo'lmasa, u holda (4) formula faqat $0<x<1$ bo'lgandagina o'rinli bo'ladi.

Agar gipergeometrik funksiyaga simmetrik bo'lib kirgan a va b parametrlardan bittasi manfiy butun son $-n$ ga teng bo'lsa, u holda (3) qator uzilib qoladi va u $n-$darajali ko'phadga aylanadi.

Agar $a=-n_1, b=-n_2$, bunda $n_1>0, n_2>0$ - butun sonlar bo'lsa, u holda gipergeometrik qator ko'phadga aylanib, uning darajasi n_1, n_2 sonlardan kichigiga teng bo'ladi. (3) qatorni hadlab differensiallash natijasida

$$\frac{d}{dx}F(a,\ b,\ c;\ x)=\frac{ab}{c}F(a+1,\ b+1,\ c+1;\ x) \quad (5)$$

formulani hosil qilamiz. Bu formulaning umumlashmasi bo'lgan

$$\frac{d^n}{dx^n}F(a,\ b,\ c;\ x)=\frac{(a)_n(b)_n}{(c)_n}F(a+n,\ b+n,\ c+n;\ x)\ n=0,\ 1,\ 2,.... \quad (6)$$

formulani, masalan, matematik induktsiya usuli yordamida isbotlash mumkin.

Agar (3) qatorni avval x^a, x^b yoki x^{c-1} ga ko'paytirib, so'ngra hadlab differensiallasak, mos ravishda quyidagi formulalar kelib chiqadi:

$$\frac{d}{dx}\left[x^aF(a,\ b,\ c;\ x)\right]=ax^{a-1}F(a+1,\ b,\ c;x), \quad (7)$$

$$\frac{d}{dx}\left[x^bF(a,\ b,\ c;\ x)\right]=bx^{b-1}F(a,\ b+1,\ c;x), \quad (8)$$

$$\frac{d}{dx}\left[x^{c-1}F(a,\ b,\ c;\ x)\right]=(c-1)x^{c-2}F(a,\ b,\ c-1;x). \quad (9)$$

Gipergeometrik funksiya uchta a, b va c parametrlarga bog'liq va bu parametrlarning xususiy qiymatlarida gipergeometrik funksiyadan ko'pgina elementar funksiyalarni hosil qilish mumkin. Masalan, $|x|<1$ da quyidagi tengliklar o'rinli:

$$1)\ F(1,\ b,\ b,\ x)=\frac{1}{1-x}; \qquad 2)\ F(a,\ b,\ a,\ x)=(1-x)^{-b};$$

$$3)\ xF(1,\ 1,\ 2,\ -x)=\ln(1+x).$$

Haqiqatan ham, gipergeometrik funksiyaning ta`rifidan foydalanib, yozamiz:

$$1)\,F(1,b,b,x)=\sum_{k=0}^{\infty}\frac{(1)_k(b)_k}{k!\ (b)_k}x^k=\sum_{k=0}^{\infty}x^k=1+x+x^2+...+x^k+...=\frac{1}{1-x};$$

$$2)\ F(a,b,a,x)=\sum_{k=0}^{\infty}\frac{(a)_k(b)_k}{k!\ (a)_k}x^k=\sum_{k=0}^{\infty}\frac{(b)_k}{k!}x^k=(1-x)^{-b};$$

$$3)\ xF(1,1,2,-x)=x\sum_{k=0}^{\infty}\frac{(1)_k(1)_k}{k!\ (2)_k}(-x)^k=x\sum_{k=0}^{\infty}\frac{(-1)^k(1)_k}{(2)_k}x^k=$$

$$=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+...+\frac{(-1)^{k-1}x^k}{k}+...=\ln(1+x).$$

139. Gipergeometrik tenglamaning $x=1$ nuqta atrofidagi ikkita chiziqli bog'liqsiz yechimlarini toping, bunda $a+b+1-c$ ni musbat bo'lmagan butun songa teng emas deb hisoblang.

◀Erkli o'zgaruvchini $x=1-t$ almashtirish $x=1$ nuqtani $t=0$ nuqtaga, gipergeometrik tenglamani esa

$$t(1-t)\frac{d^2y}{dt^2}+[-c+(a+b+1)(1-t)x]\frac{dy}{dt}-aby=0$$

tenglamaga o'tkazadi. Oxirgi tenglamani

$$t(1-t)\frac{d^2y}{dt^2}+[(a+b+1-c)-(a+b+1)t]\frac{dy}{dt}-aby=0$$

ko'rinishda yozib olamiz.

Bu tenglama parametrlari a,b va $a+b+1-c$ bo'lgan gipergeometrik tenglamadir. Shartga ko'ra, $a+b+1-c$ son musbat bo'lmagan butun songa teng emas, shuning uchun quyidagi funksiyalar oxirgi almashtirilgan tenglamaning $t=0$ maxsus nuqta atrofidagi chiziqli bog'liqsiz yechimlari bo'ladi:

$$y_1(t)=F(a,\ b,\ a+b+1-c;\ t),$$

$$y_2(t)=t^{c-a-b}F(c-b,\ c-a,c+1-\ a-b;\ t).$$

Shunga binoan, dastlabki gipergeometrik tenglamaning $x=1$ nuqta atrofidagi chiziqli erkli yechimlari

$$y_1(x)=F(a,\ b,\ a+b+1-c;\ 1-x),$$

$$y_2(t)=(1-x)^{c-a-b}F(c-b,\ c-a,c+1-\ a-b;\ 1-x)$$

ko'rinishda bo'ladi.▶

140. Tenglamani yeching: $x(1-x)y''+(3-7x)y'-8y=0,\ |x|<1.$

◀Bu Gauss gipergeometrik tenglamasining xususiy holidir: $a=2$, $b=4,\ c=3$. (4) formulaga binoan berilgan tenglama

$$y(x)=C_1F(2,4,3,x)+C_2x^{-2}$$

ko'rinishdagi umumiy yechimga ega. $F(2,\ 4,\ 3,\ x)$ gipergeometrik funksiyani elementar funksiyalar orqali ifodalashga urinib ko'ramiz. Shu maqsadda (8) formulani

$$F(a,b+1,c;x)=\frac{1}{b}x^{1-b}\frac{d}{dx}\left[x^{b}F(a,b,c;x)\right]$$

ko'rinishda yozib olamiz va uni $F(2,\ 4,3;x)$ funksiyaga qo'llaymiz:

$$F(2,4,3;x)=\frac{1}{3}x^{-2}\frac{d}{dx}\left[x^{3}F(2,3,3;x)\right]..$$

So'ngra $F(a,\ b,\ b;x)=(1-x)^{-a}$ formulani e`tiborga olib, differensiallash amalini bajaramiz:

$$F(2,\ 4,\ 3;x)=\frac{1}{3}x^{-2}\frac{d}{dx}\left[x^{3}(1-x)^{-2}\right]=\frac{1}{3}(3-x)(1-x)^{-3}.$$

Shunday qilib, berilgan gipergeometrik tenglamaning umumiy yechimi

$$y(x)=C_1(3-x)(1-x)^{-3}+C_2x^{-2}$$

ko'rinishda topiladi.►

141. Tenglamani yeching: $x(x-1)y''-xy'+y=0$.

◄Bu Gauss gipergeometrik tenglamasining xususiy holidir: $a=b=-1,\ c=0$. Bu holda aniqlovchi tenglamaning ildizlari: $\lambda_1=0$ va $\lambda_2=1-c=1$. Berilgan tenglamaning bitta yechimini darajali qator ko'rinishida topishimiz mumkin. $y_1(x)=x$ tenglamaning yechimi ekanligini ko'rish qiyin emas. Endi $\lambda_2-\lambda_1$ ayirma butun son bo'lganligi uchun $y_1(x)$ bilan chiziqli bog'liqsiz bo'lgan yechim darajali qator ko'rinishida mavjud bo'lmasligi ham mumkin. $y_2(x)$ ni topish uchun (9) Liuvill-Ostrogradskiy formulasidan foydalanamiz:

$$y_2(x)=x\int\frac{\exp\left(-\int\frac{dx}{1-x}\right)}{x^2}dx=x\left(-\frac{1}{x}-\ln|x|\right)=-1-x\ln|x|.$$

Shunday qilib, berilgan tenglamaning ikkita chiziqli bog'liqsiz yechimlari topildi, demak, berilgan tenglamaning umumiy yechimi

$$y=C_1x+C_2\left(1+x\ln|x|\right)$$

ko'rinishda bo'ladi, bu yerda C_1 va C_2 - ixtiyoriy o'zgarmaslar.

142. Ushbu

$$P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$$

ifoda bilan aniqlanadigan funksiyalar *Lejandr ko'phadlari* deyiladi. Bu funksiyalarning har biri $(1-x^2)y''-2xy'+n(n+1)y=0$ ($n-$natural son) ko'rinishdagi *Lejandr tenglamasi*ning yechimi bo'ladi. Gipergeometrik funksiya va Lejandr ko'phadlari o'rtasidagi bog'lanishni toping.

◀Avvalo Lejandr tenglamasini gipergeometrik tenglamaga keltirish mumkinligini ko'rsatamiz. Buning uchun $x=1-2t$ deb olamiz. U holda

$$t=\frac{1-x}{2},\qquad \frac{dy}{dx}=-\frac{1}{2}\frac{dy}{dt},\quad \frac{d^2y}{dx^2}=\frac{1}{4}\frac{d^2y}{dt^2}.$$

Bu ifodalarni Lejandr tenglamasiga qo'yib, topamiz:

$$\frac{1}{4}\left(1-(1-2t)^2\right)\frac{d^2y}{dt^2}+(1-2t)\frac{dy}{dt}+n(n+1)y=0,$$

ya`ni

$$t\left(1-t\right)\frac{d^2y}{dt^2}+(1-2t)\frac{dy}{dt}+n(n+1)y=0.$$

Bu parametrlari $a=n+1, b=-n, c=1$ bo'lgan gipergeometrik tenglamadir. Bu tenglamaning yechimlaridan bittasi $y_1=F(n+1,-n,1;t)$ gipergeometrik funksiyadir.

Shunday qilib, berilgan Lejandr tenglamasining yechimlaridan bittasi $y_1=F\left(n+1,-n,1;\ \frac{1-x}{2}\right)$ funksiyadir. $F\left(n+1,\ -n,\ 1;\ \frac{1-x}{2}\right)=P_n(x)$ ekanligini ko'rsatamiz. Haqiqatan ham, $F(n+1,-n,1;t)$ funksiya t bo'yicha n darajali ko'phad bo'lib, t^n ning oldidagi koeffitsient

$$\frac{(n+1)(n+2)...(2n-1)\ 2n\ \ (-n)(-n+1)...(-1)}{n!\cdot 1\cdot 2\cdot ...\cdot n}=(-1)^n\frac{(2n)!}{(n!)^2}$$

ga teng. Gipergeometrik funksiyaning hosilasiga oid (5) formuladan foydalanib, topamiz:

$$\frac{d^n}{dt^n}F(n+1,-n,1;t)=(-1)^n\frac{(2n)!}{n!}.$$

Qisqartirishlardan so'ng

$$F(n+1,-n,1,t)=\frac{(-1)^n}{n!}\frac{d^n}{dt^n}\left(t^n(1-t)^n\right),$$

ya`ni

$$F\left(n+1,-n,1;\ \frac{1-x}{2}\right)=\frac{(-1)^n}{n!2^n}\frac{d^n}{dx^n}\left(x^2-1\right)^n,$$

ifodaga ega bo'lamiz, ya`ni $P_n(x)$ Lejandr ko'phadlari parametrlari

$a=n+1,\ b=-n, c=1$ va argumenti $(1-x)/2$ bo’lgan gipergeometrik funksiyadir.▶

143. $(1-x^2)y''-2xy'+6y=0$ Lejandr tenglamasining $y(0)=0$, $y'(0)=-2$ shartlarni qanoatlantiradigan yechimini toping.

◀Bu tenglamaning yechimlaridan bittasi $P_2(x)=\frac{1}{2}(3x^2-1)$ Lejandr ko’phadidir. Shuning uchun $c\,P_2(x)$ funksiyalar ixtiyoriy c da tenglamaning yechimlari bo’ladi. Ammo bu yechimlardan birortasi ham masalaning shartlarini qanoatlantirmaydi. $x=0$ nuqta Lejandr tenglamasi uchun oddiy nuqta bo’lganligi uchun izlanayotgan yechimni x bo’yicha darajali qator ko’rinishida qidiramiz. Boshlang’ich shartlarni e`tiborga olib,

$$y=-2x+\sum_{k=2}^{\infty}c_k x^k$$

deb olamiz. Bu ifodani dastlabki tenglamaga qo’yib, topamiz:

$$(1-x^2)\sum_{k=2}^{\infty}k(k-1)\ c_k x^{k-2}-2x\left(-2+\sum_{k=2}^{\infty}k\ \ c_k x^{k-1}\right)+6\left(-2x+\sum_{k=2}^{\infty}\ c_k x^k\right)=0.$$

Bu yerdan c_k larni aniqlash uchun quyidagi tenglamalarni olamiz:

$$c_2=0, (k+2)(k+1)c_{k+2}=(k+3)(k-2)c_k,\ k=1,\ 2,\ ..., c_1=-2.$$

Bu tenglamalardan topamiz:

$$c_{2m}=0,\ c_{2m+1}=\frac{(2m+2)(2m-3)}{(2m+1)\cdot 2m}c_{2m-1},\ m=1,\ 2,\ ....$$

Shunga binoan, izlanayotgan yechim

$$y=\sum_{m=0}^{\infty}\frac{1}{2}\left[\frac{3}{2m-1}-\frac{1}{2m+1}\right]x^{2m+1}$$

formula bilan yoziladi. Bu yechimni

$$y=-\frac{3}{2}x+\frac{1}{2}(3x^2-1)\sum_{m=1}^{\infty}\frac{x^{2m-1}}{m}=\frac{1}{4}(3x^2-1)\ln\frac{1+x}{1-x}-\frac{3}{2}x$$

ko’rinishda yozib olish qiyin emas. Shu topilgan

$$y=Q_2(x)=\frac{1}{4}(3x^2-1)\ln\frac{1+x}{1-x}-\frac{3}{2}x$$

yechim *ikkinchi tur Lejandr funksiyasi* deyiladi.▶

3.2.3. Bessel tenglamasi. Ba’zi differensial tenglamalar biror almashtirish yordamida Bessel tenglamasiga keltiriladi.

144. $x^2y''+xy'+(x^2-\nu^2)y=0$ *Bessel tenglamasi*ni yeching, bunda ν ni butun son emas deb hisoblang.

◀Tenglamada x ning o'rniga $-x$ olinsa ham u o'zgarmaydi, shuning uchun x ning nomanfiy qiymatlarini qarash yetarli. $x=0$ yagona maxsus nuqtadir. Bu nuqtaga mos kelgan aniqlovchi tenglama

$$\lambda(\lambda-1)+\lambda-\nu^2=0,\ \lambda^2-\nu^2=0$$

ko'rinishda bo'ladi. Agar $\nu\neq 0$ bo'lsa, u holda aniqlovchi tenglama ikkita ildizga ega: $\lambda_1=\nu$ va $\lambda_1=-\nu$. Berilgan tenglamaning yechimini umumlashgan darajali qator ko'rinishida izlaymiz:

$$y=\sum_{k=0}^{\infty}c_kx^{k+\lambda},\ c_0\neq 0.$$

Bu qatorni berilgan tenglamaga qo'yib, bir necha zarur amallarni bajargandan so'ng

$$\sum_{k=0}^{\infty}\left[(\lambda+k)(\lambda+k-1)+\lambda+k-\nu^2\right]c_kx^k+\sum_{k=0}^{\infty}c_kx^{k+2}=0,$$

ya`ni

$$\sum_{k=0}^{\infty}\left[(\lambda+k)^2-\nu^2\right]c_kx^k+\sum_{k=0}^{\infty}c_kx^{k+2}=0$$

tenglikka ega bo'lamiz. Bu tenglik bajarilishi uchun c_k koeffitsientlar

$$(\lambda^2-\nu^2)c_0=0,\ [(\lambda+1)^2-\nu^2]c_1=0,\quad [(\lambda+k)^2-\nu^2]c_k+c_{k-2}=0,$$
$$k=2,\ 3,\ ...$$

tenglamalarni qanoatlantirishi lozim. Aniqlovchi tenglamaning $\lambda=\nu$ ildiziga mos kelgan yechimni topamiz. $\lambda=\nu$ ildizni oxirgi tenglamaga qo'yib ko'ramizki, c_0 sifatida noldan farqli har qanday sonni olish mumkin, $c_1=0,\ k=2,\ 3,\ ...$ da

$$c_k=-\frac{c_{k-2}}{k(2\nu+k)}.$$

Bu yerdan

$$c_{2k-1}=0,\ \ c_{2k}=(-1)^k\frac{c_0}{2^{2k}\cdot k!(\nu+1)(\nu+2)...(\nu+k)},\ k=1,\ 2,\ 3,\ ...$$

ekanligi kelib chiqadi. Shunday qilib, hamma c_k koeffitsientlar topildi, demak,

$$y_1(x)=\sum_{k=0}^{\infty}(-1)^k\frac{c_0}{2^{2k}\cdot k!(\nu+1)(\nu+2)...(\nu+k)}x^{2k+\nu}.$$

Bu qator ixtiyoriy chekli $[0,\ a]$ kesmada tekis yaqinlashadi (buni, masalan, Dalamber alomati yordamida tekshirish mumkin), shunga ko'ra, $y_1(x)$ funksiya ixtiyoriy c_0 da Bessel tenglamasining yechimi bo'ladi. c_0 sifatida

$$c_0 = \frac{1}{2^{\nu}\Gamma(\nu+1)}$$

sonni olish qulaydir, bu yerda $\Gamma(\alpha)$ - gamma-funksiya, ya`ni

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1}e^{-x}dx,\ \alpha > 0.$$

Endi $\Gamma(\nu+1) = \nu\ \Gamma(\nu)$ ekanligini e`tiborga olib, $y_1(x)$ ni yozamiz:

$$y_1(x) = \sum_{k=0}^{\infty}(-1)^k \frac{1}{k!\Gamma(\nu+k+1)}\left(\frac{x}{2}\right)^{2k+\nu}.$$

Ushbu

$$J_{\nu}(x) = \sum_{k=0}^{\infty}(-1)^k \frac{1}{k!\Gamma(\nu+k+1)}\left(\frac{x}{2}\right)^{2k+\nu}$$

funksiya $\nu-$*tartibli birinchi tur Bessel funksiyasi* deyiladi.

Bessel tenglamasining $y_1(x)$ bilan chiziqli bog'liqsiz bo'lgan ikkinchi yechimini

$$y_2(x) = \sum_{k=0}^{\infty} c_k x^{k-\nu}$$

ko'rinishda izlaymiz. $\lambda = -\nu$ bo'lganda c_k koeffitsientlarni aniqlash uchun

$$(\lambda^2 - \nu^2)c_0 = 0,\ (1-2\nu)c_1 = 0,\ [(-\nu+k)^2 - \nu^2]c_k + c_{k-2} = 0,$$
$$k = 2,\ 3,\ ...$$

tenglamalarga ega bo'lamiz. Bu yerda $c_0 \neq 0, c_1 = 0$ deb olib, topamiz:

$$c_{2k-1} = 0,\ c_{2k} = (-1)^k \frac{c_0}{2^{2k}\cdot k!(-\nu+1)(-\nu+2)...(-\nu+k)},$$
$$k = 1,\ 2,\ 3,\ ....$$

Shunday qilib,

$$y_2(x) = \sum_{k=0}^{\infty}(-1)^k \frac{c_0}{2^{2k}\cdot k!(-\nu+1)(-\nu+2)...(-\nu+k)}x^{2k-\nu}.$$

c_0 sifatida $c_0 = \dfrac{1}{2^{-\nu}\Gamma(-\nu+1)}$ sonni olib, $y_2(x)$ funksiyani yozamiz:

$$y_2(x)=\sum_{k=0}^{\infty}(-1)^k\frac{1}{k!\Gamma(-\nu+k+1)}\left(\frac{x}{2}\right)^{2k-\nu}.$$

Ushbu

$$J_{-\nu}(x)=\sum_{k=0}^{\infty}(-1)^k\frac{1}{k!\Gamma(-\nu+k+1)}\left(\frac{x}{2}\right)^{2k-\nu}$$

funksiya *manfiy indeksli birinchi tur Bessel funksiyasi* deyiladi.

Xullas, agar ν butun son bo'lmasa, u holda dastlab berilgan Bessel tenglamasining umumiy yechimi $J_\nu(x)$ va $J_{-\nu}(x)$ Bessel funksiyalarining chiziqli kombinatsiyasidan iborat bo'ladi: $y=C_1J_\nu(x)+C_2J_{-\nu}(x)$.►

145. $x^2y''+xy'+\left(x^2-\frac{1}{16}\right)y=0$ tenglamani yeching.

◄Bu Bessel tenglamasi $(\nu=1/4)$. Shuning uchun berilgan tenglamaning umumiy yechimi $y=C_1J_{1/4}(x)+C_2J_{-1/4}(x)$ ko'rinishda bo'ladi.►

146. $x^2y''+xy'+\left(9x^2-\frac{1}{9}\right)y=0$ tenglamani yeching.

◄Bu yerda $3x=t$ deb olamiz. U holda

$$\frac{dy}{dx}=3\frac{dy}{dt},\quad \frac{d^2y}{dx^2}=9\frac{d^2y}{dt^2}$$

va berilgan tenglama

$$t^2\frac{d^2y}{dt^2}+t\frac{dy}{dt}+\left(t^2-\frac{1}{9}\right)y=0$$

ko'rinishdagi Bessel tenglamasiga o'tadi. Bu yerdan

$$y=C_1J_{1/3}(t)+C_2J_{-1/3}(t),\quad y=C_1J_{1/3}(3x)+C_2J_{-1/3}(3x)$$

umumiy yechimni olamiz.►

147. $xy''+y'+xy=0$ tenglamani yeching.

◄Bu tenglama Bessel tenglamasining xususiy holidir ($\nu=0$). Uning yechimlaridan bittasi $y_1(x)=J_0(x)$ ko'rinishdagi nolinchi tartibli Bessel funksiyasidir. **3.2.1**-bandda ta`kidlanganidek (yana **141**-misolga ham qarang), agar aniqlovchi tenglama ildizlarining ayirmasi butun songa teng bo'lsa, u holda $y_1(x)$ bilan chiziqli bog'liqsiz bo'lgan ikkinchi yechimni umumlashgan darajali qator va $y_1(x)\ln|x|$ funksiyaning yig'indisi ko'rinishida izlash kerak. Biz qarayotgan holda $\nu-$butun son, shuning uchun $y_2(x)$ yechimni

$$y_2(x) = J_0(x)\ln x + \sum_{k=0}^{\infty} c_k x^k$$

ko'rinishda izlaymiz (aniqlik uchun $x > 0$ deb olindi, $x < 0$ bo'lgan hol ham shu kabi o'rganiladi). $y_2(x)$ yechimni tenglamaga qo'yib va x ning bir xil darajalari oldidagi koeffitsientlarni bir-biriga tenglashtirib, c_k noma`lumlarni aniqlash uchun quyidagi tenglamalarga ega bo'lamiz:

$$c_1 = 0,\ (2k+1)^2 c_{2k+1} + c_{2k-1} = 0,\ k = 1,\ 2,\ 3,\ ...,$$

$$4(k+1)^2 c_{2(k+1)} + c_{2k} = \frac{(-1)^k}{(k+1)!k!\ 2^{2k}},\ k = 0,\ 1,\ 2,\ 3,\ ....$$

Agar $c_0 = 0$ deb olsak, u holda barcha toq o'rindagi koeffitsientlar nolga teng bo'ladi, juft o'rindagilari esa

$$c_1 = 0,\ c_{2k} = (-1)^{k+1} \frac{1}{2^2 \cdot 4^2 ... (2k)^2} \left(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{k}\right),\ k = 1,\ 2,\ 3,\ ...$$

formulalardan topiladi. Shunday qilib,

$$y_2(x) = J_0(x)\ln x + \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{k}}{2^2 \cdot 4^2 ... (2k)^2} x^{2k}. ▶$$

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M49. Differensial tenglamani darajali qatorlar yordamida yeching.

1. $y'' + 4xy' + \left(4x^2 + 2\right)y = 0.$
2. $y'' - 4xy' + \left(4x^2 - 2\right)y = 0.$
3. $y'' - 4xy' + \left(4x^2 - 3\right)y = e^{x^2}.$
4. $y'' + 2xy' + x^2 y = 0.$
5. $y'' - 2xy' + x^2 y = 0.$
6. $y'' + 8xy' + 16x^2 y = 0.$
7. $y'' - 8xy' + 16x^2 y = 0.$
8. $xy'' - (x+1)y' + y = 0.$
9. $xy'' - (x+1)y' - 2(x-1)y = 0.$
10. $xy'' + 2(x-1)y' + \left(x-2\right)y = 0.$
11. $xy'' - 2(1 + xtgx)y' + 2ytgx = 0.$
12. $xy'' + xy' - y = 3x^2.$
13. $xy'' - 2(x+1)y' + \left(x+2\right)y = 0.$
14. $x^2 y'' - 2xy' + \left(x^2 + 2\right)y = 0.$
15. $x^2 y'' - 2x\left(x+1\right)y' + 2\left(x+1\right)y = 0.$
16. $xy'' - 2(x+2)y' + \left(x+4\right)y = 0.$

17. $x^2y'' - 2xy' + 2\left(2x^2+1\right)y = 0.$ 18. $\left(x^2+1\right)y'' - 2xy' + 2y = 0.$

19. $xy'' - 2(2x+1)y' + 4\left(x+1\right)y = 0.$

20. $x^2y'' - 2xy' + \left(9x^2+2\right)y = 0.$

21. $\left(x^2-1\right)y'' - 2xy' + 2y = 0.$

22. $xy'' - 2(3x+2)y' + 3\left(3x+4\right)y = 0.$

23. $x^2y'' - 2xy' + 2\left(8x^2+1\right)y = 0.$

24. $\left(x^2+3x+4\right)y'' + \left(x^2+x+1\right)y' - \left(2x+3\right)y = 0.$

25. $xy'' - 4(x+1)y' + 4\left(x+2\right)y = 0.$

26. $x^2y'' - 2xy' + \left(25x^2+2\right)y = 0.$

27. $x^2y'' - xy' + y = 3x^3.$

28. $xy'' + 2(2x-1)y' + 4\left(x-1\right)y = 0.$

29. $xy'' - 2(x+3)y' + \left(x+6\right)y = 0.$

30. $xy'' - 2(3x+1)y' + 3\left(3x+2\right)y = 0.$

3.3. CHEGARAVIY MASALALAR. XOS QIYMAT VA XOS FUNKSIYALAR

Biz yuqorida oddiy differensial tenglamalar uchun Koshi masalasiga oid misollarni ham o’rgandik. Bu masalaning geometrik ma`nosi berilgan nuqtadan o’tadigan integral egri chiziqni izlashdan iborat edi. Shu integral egri chiziq yana boshqa shartlarni ham qanoatlantiradimi yoki yo’qmi? degan tabiiy savol tug’iladi.

3.3.1. Chegaraviy masalalar. Oddiy differensial tenglamalar uchun Koshi masalasidan farqli o’laroq chegaraviy masalada izlanayotgan funksiyaning qiymati (yoki funksiya va uning hosilasi chiziqli kombinatsiyasining qiymati) bitta nuqtada emas, balki yechim topilishi lozim bo’lgan kesmaning ikkita nuqtasida beriladi.

Ushbu

$$a_0(x)y'' + a_1(x)y' + a_2(x)y = f(x), \quad a \le x \le b \qquad (1)$$

$$\alpha\, y(a) + \beta\, y'(a) = A, \gamma\, y(b) + \delta\, y'(b) = B \qquad (2)$$

chegaraviy masalani yechish uchun avvalo (1) tenglamaning umumiy yechimini topish, so'ngra umumiy yechim formulasidagi ixtiyoriy o'zgarmaslarni (2) chegaraviy shartlar bajariladigan qilib tanlash kerak. Chegaraviy masalaning Koshi masalasidan yana bitta farqi shundaki, chegaraviy masala har doim ham yechimga ega emas, yechimga ega bo'lgan taqdirda esa yechim yagona bo'lmasligi ham mumkin.

Tenglamalarning berilgan chegaraviy shartlarni qanoatlantiradigan yechimlarini toping (**148-152**).

148. $y'' - y = 2x;\ y(0) = 0,\ y(1) = -1.$

◀Berilgan tenglamaning umumiy yechimini topib

$$y(x) = C_1e^x + C_2e^{-x} - 2x,$$

uni chegaraviy shartlarga qo'yamiz:

$$y(0) \equiv C_1 + C_2 = 0,\ y(1) \equiv C_1 e + C_2 e^{-1} - 2 = -1.$$

So'ngra C_1 va C_2 o'zgarmaslarga nisbatan hosil bo'lgan

$$C_1 + C_2 = 0, C_1e + C_2e^{-1} = 1$$

tenglamalar sistemasini yechib, $C_1 = \dfrac{1}{2sh1}$ va $C_2 = -\dfrac{1}{2sh1}$ qiymatlarni olamiz. Shunday qilib, qo'yilgan chegaraviy masalaning yechimi

$$y = \frac{sh\,x}{sh1} - 2x,\ 0 \le x \le 1$$

ko'rinishda topiladi.►

149. $y'' - y = 0;\ y(0) = 3,\ y(1) - y'(1) = 1.$

◀Berilgan tenglamaning umumiy yechimini topib

$$y(x) = C_1 + C_2e^x,$$

uni chegaraviy shartlarga qo'yamiz:

$$C_1 + C_2 = 3, C_1 + C_2e - C_2e = 1$$

va $C_1 = 1$ va $C_2 = 2$ qiymatlarni olamiz. Shunday qilib, qo'yilgan chegaraviy masalaning yechimi $y = 1 + 2e^x, 0 \le x \le 1$ ko'rinishda topiladi.►

150. $y'' - 2y' - 3y = 0;\ y(0) = 1,\ x \to +\infty$ da $y(x) = 0$.

◀Berilgan tenglamaning umumiy yechimini yozamiz: $y(x) = C_1e^{-x} + C_2e^{3x}$. So'ngra,

$$\lim_{x\to+\infty} y(x) = \lim_{x\to+\infty}\left(C_1e^{-x} + C_2e^{3x}\right) = 0$$

shartdan $C_2 = 0$ va $y(0) = 1$ shartdan esa $C_1 = 1$ qiymatlarni olamiz.

Shunday qilib, qo’yilgan chegaraviy masalaning yechimi $y = e^{-x}$, $0 \le x < +\infty$ ko’rinishda topiladi.►

151. $x^2 y'' - 2xy' + 2y = 0;\ x \to 0$ da $y(x) = o(x),\ y(1) = 3.$

◄Bu Eyler tenglamasi. Uning umumiy yechimi $y(x) = C_1 x + C_2 x^2$ ko’rinishda bo’ladi. Masalaning $x \to 0$ da $y(x) = o(x)$ bo’lsin degan sharti $\lim\limits_{x\to 0}\dfrac{y(x)}{x} = 0$ tenglik bajarilishi kerakligini anglatadi. Shuning uchun $\lim\limits_{x\to 0}(C_1 + C_2 x) = 0$ tenglikdan $C_1 = 0$ qiymatni olamiz. Ikkinchi $y(1) = 3$ shartdan esa $C_2 = 3$ kelib chiqadi. Shunday qilib, qo’yilgan chegaraviy masalaning yechimi $y = 3x^2,\ 0 \le x \le 1$ ko’rinishda topiladi.►

152. $y'' + \pi^2 y = 1;\ y(0) = 0,\ y(1) = 0.$

◄Berilgan tenglamaning $y(x) = 1 + C_1 \cos \pi x + C_2 \sin \pi x$ umumiy yechimini chegaraviy shartlarga qo’yib, $C_1 + 1 = 0,\quad -C_1 + 1 = 0$ tenglamalar sistemasiga ega bo’lamiz. Bu sistema yechimga emasligi ochiq ravshan. Demak, qo’yilgan chegaraviy masala ham yechimga ega emas.►

3.3.2. Grin funksiyasi. Ushbu bir jinsli chegaraviy masalani qaraymiz:

$$a_0(x)y'' + a_1(x)y' + a_2(x)y = f(x),\ a \le x \le b, \tag{1}$$

$$\alpha y(a) + \beta y'(a) = 0,\ \gamma y(b) + \delta y'(b) = 0. \tag{2}$$

(1)-(2) chegaraviy masalaning *Grin funksiyasi* deb shunday $G(x,s)$ funksiyaga aytiladiki, bu funksiya $\{(x,s): a \le x \le b,\ a \le s \le b\}$ sohada aniqlangan bo’lib, har bir $s \in [a,b]$ da x o’zgaruvchining funksiyasi sifatida quyidagi xossalarga ega:

1) $x \ne s$ bo’lganda $G(x,s)$ funksiya

$$a_0(x)y'' + a_1(x)y' + a_2(x)y = 0 \tag{3}$$

bir jinsli tenglamani qanoatlantiradi;

2) $x = a$ va $x = b$ bo’lganda $G(x,s)$ funksiya (2) chegaraviy shartlarni qanoatlantiradi;

3) $x = s$ nuqtada $G(x,s)$ funksiya x bo’yicha uzluksiz, uning x bo’yicha hosilasi esa birinchi tur uzilishga ega, ya`ni

$$G(s+0,s) = G(s-0,s),\ G'_x(s+0,s) = G'_x(s-0,s) + 1/a_0(x). \tag{4}$$

(1)-(2) chegaraviy masalaning Grin funksiyasini qurish uchun avvalo (3) bir jinsli tenglamaning umumiy yechimini topish kerak. Bu umumiy yechimdan ikkita $y_1(x)$ va $y_2(x)$ yechimlarni (ular aynan nolga teng emas) shunday ajratish kerakki, $y_1(x)$ funksiya (2) shartlardan birinchisini, $y_2(x)$ funksiya esa (2) shartlardan ikkinchisini qanoatlantirsin.

Agar $y_1(x)$ funksiya bir vaqtda ikkala chegaraviy shartlarni qanoatlantirmasa, ya`ni (2) shartlardan birinchisini qanoatlantirib, ikkinchisini qanoatlantirmasa, u holda $G(x,s)$ Grin funksiyasi mavjud bo'ladi va uni

$$G(x,s)=\begin{cases} a(s)\,y_1(x), & a\le x\le s,\\ b(s)\,y_2(x), & s\le x\le b\end{cases} \tag{5}$$

ko'rinishda izlash mumkin. Bu yerdagi $a(s)$ va $b(s)$ funksiyalarni shunday tanlash kerakki, natijada (5) funksiya (4) shartlarni qanoatlantirsin, ya`ni

$$\begin{cases} b(s)y_2(s)-a(s)y_1(s)=0,\\ b(s)y_2'(s)-a(s)y_1'(s)=1/a_0(x).\end{cases} \tag{6}$$

Agar $G(x,s)$ Grin funksiyasi ma`lum bo'lsa, u holda (1), (2) chegaraviy masalaning yechimi

$$y(x)=\int_a^b G(x,s)f(s)ds \tag{7}$$

formula yordamida oshkor ko'rinishda topiladi.

Chiziqli chegaraviy masalalarni yechishda ko'pincha quyidagi usul foydalidir. Agar (1) chiziqli differensial tenglama $y(x)$ va uning hosilalariga qo'yilgan ikkita $x=a$ va $x=b$ nuqtalarda chegaraviy shartlar bilan berilgan bo'lsa, u holda chegaraviy masalaning yechimi

$$y=y_0(x)+mu(x)+nv(x) \tag{8}$$

ko'rinishda bo'ladi, bu yerda $y_0(x)$, $u(x), v(x)$ funksiyalar mos ravishda quyidagi uchta Koshi masalalaridan topiladi:

$$a_0(x)y''+a_1(x)y'+a_2(x)y=f(x),\ y(a)=y'(a)=0;$$
$$a_0(x)y''+a_1(x)y'+a_2(x)y=0,\ y(a)=1,\ y'(a)=0;$$
$$a_0(x)y''+a_1(x)y'+a_2(x)y=0;\ y(a)=0,\ y'(a)=1.$$

Berilgan chegaraviy shartlarni (8) ga qo'yib, m va n koeffitsientlarni aniqlab beradigan ikkita tenglamali sistemaga ega bo'lamiz.

Grin funksiyasini quring (**153-154**).

153. $y''+y=f(x);\ y'(0)=0,\ y(\pi)=0.$

◀Avval $y''+y=0$ bir jinsli tenglamaning $y(x)=C_1\cos x+C_2\sin x$ umumiy yechimini topamiz. So'ngra $y'(0)=0$ shartni qanoatlantiradigan $y_1(x)$ funksiyani ajratamiz:

$$y_1'(x)=-C_1\sin x+C_2\cos x,\ \ y_1'(0)\equiv C_2=0.$$

Bu tenglik ixtiyoriy C_1 da bajariladi, shuning uchun $C_1=1$ deb olamiz. Demak, $y_1(x)=\cos x$. Bu funksiya $y(\pi)=0$ shartni qanoatlantirmasligi lozim. Haqiqatan ham shunday ekanligi ravshan.

Xuddi shu kabi $y_2(x)=\sin x$ funksiyani topamiz. Bu yechim $y(\pi)=0$ shartni qanoatlantirishini va $y'(0)=0$ shartni qanoatlantirmasligini tekshirish qiyin emas.

Endi (6) tenglamalar sistemasini tuzamiz:

$$b(s)\sin s-a(s)\cos s=0,\ b(s)\cos s+a(s)\sin s=1.$$

Bu sistemani yechib, $a(s)=\sin s,\ b(s)=\cos s$ funksiyalarga ega bo'lamiz. Shunday qilib, qo'yilgan masalaning Grin funksiyasi

$$G(x,s)=\begin{cases}\sin s\cos x, & 0\le x\le s,\\ \sin x\cos s, & s\le x\le\pi\end{cases}$$

ko'rinishda bo'ladi.▶

154. $y''+4y'+3y=f(x);\ y(0)=0,\ x\to+\infty$ da $y=O(e^{-2x})$.

◀Bir jinsli tenglamaning $y(x)=C_1e^{-3x}+C_2e^{-x}$ umumiy yechimidan foydalanib, $y_1(x)=e^{-3x}-e^{-x}$ funksiyani topish qiyin emas.

$x\to+\infty$ da $y=O(e^{-2x})$ bo'lgan $y_2(x)$ funksiyani aniqlash uchun

$$\left|C_1e^{-3x}+C_2e^{-x}\right|\le Ce^{-2x},\ C>0$$

tengsizlik bajarilishi kerak. Bu tengsizlik $C_2=0$ da o'rinli bo'lganligi uchun $y_2(x)$ funksiya $y_2(x)=e^{-3x}$ ko'rinishda topiladi.

Endi $G(x,s)$ Grin funksiyasini tuzish uchun

$$\begin{cases}b(s)e^{-3s}-a(s)\left(e^{-3s}-e^{-s}\right)=0\\ -3b(s)e^{-3s}-a(s)\left(-3e^{-3s}+e^{-s}\right)=1\end{cases}$$

tenglamalar sistemasini yechamiz:

$$a(s)=\frac{1}{2}e^{s},\ b(s)=\frac{1}{2}(e^{s}-e^{3s}).$$

Shunday qilib, qo'yilgan chegaraviy masalaning Grin funksiyasi

$$G(x,s)=\begin{cases}\frac{1}{2}e^{s}(e^{-3x}-e^{-x}), & 0\le x\le s,\\ \frac{1}{2}e^{-3x}(e^{s}-e^{3s}), & s\le x<+\infty\end{cases}$$

ko'rinishda topiladi.►

155. Grin funksiyasini qurib, chegaraviy masalani yeching:

$xy''+y'=2x;$ $x\to 0$ da $y(x)$ chegaralangan, $y(1)=y'(1).$

◄Bir jinsli $xy''+y'=0$ tenglamaning umumiy yechimini topish qiyin emas: $y=C_1\ln x+C_2,\ x>0.$

Shartga ko'ra, $y_1(x)$ funksiya $x\to 0$ da chegaralangan bo'lishi kerak. Umumiy yechimdan shunday xossaga ega bo'lgan yechimni topamiz: $y_1(x)=1$ $(C_1=0,\ C_2=1)$. Bu funksiya, haqiqatan ham, $x\to 0$ da chegaralangan va $y(1)=y'(1)$ shartni qanoatlantirmaydi.

$y_2(x)$ funksiya ham shunday tarzda tuziladi: $y_2(x)=1+\ln x.$ Haqiqatan ham, $y_2(1)=y_2'(1)$ va $y_2(x)$ funksiya $x\to 0$ da chegaralanmagan.

Endi $G(x,s)$ Grin funksiyasini tuzish uchun

$$\begin{cases}b(s)(1+\ln s)-a(s)\cdot 1=0,\\ b(s)\cdot\frac{1}{s}-a(s)\cdot 0=\frac{1}{s}\end{cases}$$

tenglamalar sistemasini yechamiz: $a(s)=1+\ln s,\ b(s)=1.$

Qo'yilgan chegaraviy masalaning Grin funksiyasi

$$G(x,s)=\begin{cases}1+\ln s, & 0<x\le s,\\ 1+\ln x, & s\le x\le 1\end{cases}$$

ko'rinishda yoziladi.

Agar $G(x,s)$ Grin funksiyasi ma`lum bo'lsa, u holda qo'yilgan chegaraviy masalaning yechimi (7) formula yordamida oshkor topiladi. Biz o'rganayotgan masalada $f(x)=2x,\ a=0,\ b=1.$ Grin funksiyasining simmetriklik xossasi $[G(x,s)=G(s,x)]$ dan foydalanib, topilgan Grin funksiyasini

$$G(x,s)=\begin{cases}1+\ln x, & 0<s\le x,\\ 1+\ln s, & x\le s\le 1\end{cases}$$

ko'rinishda yozib olamiz va uni tegishli formulaga qo'yib, chegaraviy masalaning $y(x)$ yechimini topamiz:

$$y(x)=\int_0^x(1+\ln x)\cdot 2s\,ds+\int_x^1(1+\ln s)\cdot 2s\,ds,$$

ya`ni $2y(x)=x^2+1.$

Bu funksiya qo'yilgan chegaraviy masalaning yechimi ekanligini bevosita tekshirish ham tasdiqlaydi.►

156. Chegaraviy masalani yeching:

$$xy''-y'=\frac{3}{x^2};\ y(1)=y'(1),\quad 3y(2)-2y'(2)=3.$$

◄Shu chegaraviy masala misolida chegaraviy masalalarni Koshi masalalariga keltirishni ko'rsatamiz. Ko'rilayotgan chegaraviy masalaning yechimini $y=y_0(x)+mu(x)+nv(x)$ ko'rinishda izlaymiz, bu yerda $y_0(x),\ u(x), v(x)$ funksiyalar mos ravishda quyidagi uchta Koshi masalalaridan topiladi:

$$xy''-y'=\frac{3}{x^2};\ y(1)=y'(1)=0;$$
$$xu''-u'=0;\ u(1)=1, u'(1)=0;$$
$$xv''-v'=0;\ v(1)=0, v'(1)=1.$$

Bu Koshi masalalarining har birini yechib, topamiz:

$$y_0(x)=\frac{1}{2}(x^2-3)+\frac{1}{x},\ u(x)=1,\ v(x)=\frac{1}{2}(x^2-1).$$

Endi

$$y(x)=\frac{1}{2}(x^2-3)+\frac{1}{x}+m+\frac{n}{2}(x^2-1)$$

ifodada qatnashayotgan m va n koeffitsientlarni aniqlaymiz. Buning uchun bu ifodani $y(1)=y'(1),\quad 3y(2)-2y'(2)=3$ chegaraviy shartlarga qo'yib, $m-n=0,\ 6m+n=7$ tenglamalar sistemasiga ega bo'lamiz. Bundan $m=n=1$ topiladi.

Shunday qilib, qo'yilgan chegaraviy masalaning yechimi

$$y(x)=\frac{1}{2}(x^2-3)+\frac{1}{x}+1+\frac{1}{2}(x^2-1),$$

ya`ni $y(x) = x^2 + x^{-1} - 1$ ko'rinishda bo'ladi.►

3.3.3. Xos qiymat va xos funksiyalar. Shturm-Liuvill masalasi. Agar shunday $\lambda = \lambda_0$ son topilsaki, λ ning shu qiymatida quyidagi

$$a_0(x)y'' + a_1(x)y' + a_2(x)y = \lambda y, \; a \le x \le b, \tag{1}$$

$$\alpha y(a) + \beta y'(a) = 0, \gamma y(b) + \delta y'(b) = 0 \tag{2}$$

chegaraviy masala aynan nolga teng bo'lmagan $y(x) \ne 0$ yechimga ega bo'lsa, u holda bu $\lambda = \lambda_0$ son (1) va (2) chegaraviy masalaning *xos qiymati* deyiladi, bu xos qiymatga mos kelgan $y(x) \ne 0$ yechim esa *xos funksiya* deyiladi.

O'ziga qo'shma bo'lgan ushbu

$$\frac{d}{dx}\left(p(x)\frac{dy}{dx} \right) + q(x)y = \lambda y$$

ikkinchi tartibli differensial tenglamaning (2) chegaraviy shartlarni qanoatlantiradigan va aynan nolga teng bo'lmagan yechimini topish masalasi *Shturm-Liuvill masalasi* deyiladi.

Chegaraviy masalalarning xos qiymat va xos funksiyalarini toping (**157-158**).

157. $y'' = \lambda y; \; y(0) = 0, \; y(l) = 0, \; l > 0.$

◄Bu yerda bir necha holni qaraymiz.

1) $\lambda = 0$ bo'lsin. Bu holda $y'' = 0$ tenglamaning umumiy yechimi $y = C_1 x + C_2$ ko'rinishda bo'lib, chegaraviy shartlarni faqat $y \equiv 0$ trivial yechimgina qanoatlantiradi, ya`ni $\lambda = 0$ hos qiymat emas.

2) $\lambda > 0$ bo'lsin. Bu holda $y'' = \lambda y$ tenglama umumiy yechimining ko'rinishi $y = C_1 e^{\sqrt{\lambda} x} + C_2 e^{-\sqrt{\lambda} x}$ bo'ladi. Bu yechimlar ichidan faqat $y \equiv 0$ trivial yechimgina $y(0) = 0$ va $y(l) = 0$ chegaraviy shartlarni qanoatlantiradi. Demak, $\lambda > 0$ bo'lgan holda ham xos qiymatlar yo'q.

3) $\lambda < 0$ bo'lsin. Bu holda $y'' = \lambda y$ tenglama

$$y = C_1 \sin\sqrt{-\lambda}\, x + C_2 \cos\sqrt{-\lambda}\, x \tag{3}$$

ko'rinishdagi umumiy yechimga ega. Bu ifodani chegaraviy shartlarga qo'yib,

$$C_2 = 0, \; C_1 \sin\left(\sqrt{-\lambda} l\right) = 0 \tag{4}$$

tenglamalar sistemasiga ega bo'lamiz. Xos qiymat va xos funksiyalarning ta`riflariga ko'ra, (3) umumiy yechim aynan nolga teng bo'lmagan funksiya bo'lishi kerak. Shuning uchun (4) tenglamalar sistemasida

$C_1 \neq 0$ bo'lishi kerak (agar $C_1 = 0$ bo'lsa, $y \equiv 0$ bo'lib qolar edi). Shunday qilib, $\sin\sqrt{-\lambda}\, l = 0$, ya`ni $\sqrt{-\lambda}\, l = k\pi$, $k = 1, 2, ...$ bo'lsa, berilgan chegaraviy masala noldan farqli yechimlarga ega bo'ladi. Oxirgi tengliklardan olingan $\lambda_k = -\left(\frac{k\pi}{l}\right)^2$, $k = 1, 2, ...$ qiymatlar berilgan chegaraviy masalaning xos qiymatlaridir. Bu xos qiymatlarga mos kelgan xos funksiyalar $y_k = \sin\frac{k\pi x}{l}$, $k = 1, 2, ...$ ko'rinishda bo'ladi.►

158. $x^2 y'' - 3xy' + 4y = \lambda y;\ y(1) = 0,\ y(e) = 0.$

◄Berilgan tenglamani

$$x^2 y'' - 3xy' + (4 - \lambda) y = 0$$

ko'rinishda yozib olib, bu tenglama Eyler tenglamasi ekanligini ko'rish qiyin emas. Eyler tenglamasining yechimini $y = x^k$ ko'rinishda izlab, k ni aniqlash uchun $k(k-1) - 3k + 4 - \lambda = 0$, ya`ni $(k-2)^2 - \lambda = 0$ xarakteristik tenglamani tuzamiz. Bu xarakteristik tenglamani yechish $\lambda$ ning ishorasi bilan bog`liq masala ekanligi tushunarli.

$\lambda = 0$ bo'lsin. Bu holda $k_1 = k_2 = 2$, ya`ni ikki karrali ildiz bo'lib, Eyler tenglamasining umumiy yechimi $y = x^2 (C_1 + C_2 \ln x)$ ko'rinishda bo'ladi. Bu yechimni $y(1) = 0$ va $y(e) = 0$ chegaraviy shartlarga qo'yib $C_1 = 0, C_2 = 0$, ya`ni $y \equiv 0$ trivial yechimni olamiz. Demak, $\lambda = 0$ - xos qiymat emas.

$\lambda > 0$ bo'lsin. Bu holda $k_{1,2} = 2 \pm \sqrt{\lambda}$ bo'lib, Eyler tenglamasining umumiy yechimi $y = x^2 (C_1 x^{-\sqrt{\lambda}} + C_2 x^{\sqrt{\lambda}})$ ko'rinishda bo'ladi. Bu yechimni $y(1) = 0$ va $y(e) = 0$ chegaraviy shartlarga qo'yib $C_1 = 0, C_2 = 0$, ya`ni $y \equiv 0$ trivial yechimni olamiz. Demak, $\lambda > 0$ holda xos qiymatlar yo'q.

$\lambda < 0$ bo'lsin. Bu holda $k_{1,2} = 2 \pm i\sqrt{-\lambda}$ bo'lib, Eyler tenglamasining umumiy yechimi

$$y = x^2 [C_1 \cos(\sqrt{-\lambda} \ln x) + C_2 \sin(\sqrt{-\lambda} \ln x)]$$

ko'rinishda bo'ladi. Bu ifodani $y(1) = 0$ va $y(e) = 0$ chegaraviy shartlarga qo'yib, $C_1 = 0$, $C_2 \sin\sqrt{-\lambda} = 0$ tenglamalar sistemasiga ega bo'lamiz. Endi $C_2 \neq 0$ ekanligini e`tiborga olsak, $\sin\sqrt{-\lambda} = 0$, ya`ni $\sqrt{-\lambda} = k\pi$,

$\lambda = -(k\pi)^2$, $k = 1, 2, \ldots$ bo'ladi. Shunday qilib, $\lambda = \lambda_k = -(k\pi)^2$, $k = 1, 2, \ldots$ sonlar berilgan chegaraviy masalaning xos qiymatlaridir. Bularga mos xos funksiyalar $y_k(x) = x^2 \sin(k\pi \ln x)$, $k = 1, 2, \ldots$ ko'rinishda bo'ladi.▶

INDIVIDUAL TOPSHIRIQLAR

M50. Quyidagi tenglamalarning ko'rsatilgan chegaraviy shartlarni qanoatlantiradigan yechimlarini toping:

1. $y'' - y' = 0;\ y(0) = -1,\ y'(1) - y(1) = 2.$
2. $y'' + y = 1;\ y(0) = 0,\ y(\pi) = 0.$
3. $y'' - y' = 0;\ y(0) = 3,\ y(1) - y'(1) = 1.$
4. $y'' + y = 0;\ y(0) = 0,\ y(a) = y_0.$
5. $x^2 y'' - 2y = 0;\ y(1) = 1,\ y'(+\infty) = 0.$
6. $x^2 y'' - 2y = 0;\ y(0) = 0,\ y'(1) = 1.$
7. $y'' - y' - 2y = 0;\ y'(0) = 2,\ y(+\infty) = 0.$
8. $y'' - 2i\,y = 0;\ y(0) = -1,\ y(+\infty) = 0.$
9. $x^2 y'' + 5xy' + 3y = 0;\ y'(1) = 3,$ $x \to +\infty$ da $y(x) = O(x^{-2})$.
10. $y'' + y = 1;\ y(0) = 0,\ y'(\pi/2) = 1.$
11. $y'' - 2y' - 3y = 0;\ y(0) = 1,\ y(+\infty) = 0.$
12. $y'' - 2y' - 3y = 0;\ y(0) = 1,\ y'(+\infty) = 2.$
13. a ning qanday qiymatlarida $y'' + a\,y = 1,\ y(0) = 0,\ y(1) = 0$ chegaraviy masala yechimga ega emas?

Chegaraviy masalalarning har biri uchun Grin funksiyasini quring.

14. $y'' = f(x);\ y(0) = 0,\ y(1) = 0.$
15. $y'' - y = f(x);\ y'(0) = 0,\ y'(2) + y(2) = 0.$
16. $x^2 y'' + 2xy' = f(x);\ y(1) = 0,\ y'(3) = 0.$

17. $y'' + y' = f(x);\ y'(0) = 0,\ \ y(+\infty) = 0.$

18. $xy'' + y' = f(x);\ y(1) = 0,\ \ x \to +\infty$ da $y(x)$ chegaralangan.

19. $x^2 y'' + 2xy' - 2y = f(x);\ \ y(0)$ chegaralangan, $y(1) = 0.$

20. $y'' - y = f(x);\ \ x \to \pm\infty$ da $y(x)$ chegaralangan.

21. $x^2 y'' - 2y = f(x);\ \ x \to 0$ da va $x \to +\infty$ da $y(x)$ chegaralangan.

22. $y'' = f(x);\ y(-1) = 0,\ y(1) = 0.$

Grin funksiyasi yordamida chegaraviy masalani yeching.

23. $y'' = f(x);\ y(a) = y(b) = 0.$

24. $xy'' + y' = 2x;\ y(1) = y'(1),\ \ x \to 0$ da $y(x)$ chegaralangan.

25. a ning qanday qiymatlarida $y'' + a\,y = f(x),\ y(0) = 0,\ y(1) = 0$ chegaraviy masala uchun Grin funksiyasi mavjud bo'ladi?

Xos qiymatlar va xos funksiyalarni toping.

26. $y'' = \lambda y;\ y'(0) = 0,\ y'(\pi) = 0.$

27. $y'' = \lambda y;\ y(0) = 0,\ y'(\pi) = 0.$ 28. $x^2 y'' = \lambda y;\ y(1) = y(e) = 0.$

29. $y'' = \lambda y;\ y'(0) = 0,\ y'(l) = 0.$ 30. $x^2 y'' = \lambda y;\ y(1) = y(a) = 0.$

3-BOBNI TAKRORLASHGA DOIR ARALASH MASALALAR

Quyidagi masalalarda berilgan tenglamalarning xususiy yechimlarini bilgan holda ularning umumiy yechimlarini toping. Xususiy yechim berilmagan masalalarda xususiy yechimni tanlash yo'li bilan, masalan, $y_1 = e^{ax}$ ko'rsatkichli funksiya yoki $y_1 = x^n + ax^{n-1} + bx^{n-2} + \ldots$ algebraik ko'phad ko'rinishida izlab toping.

1. $xy'' - (2x+1)y' + (x+1)y = 0.$

2. $y'' - 2(1 + tg^2 x)\,y = 0;\ y_1 = tg\,x.$

3. $xy'' - (x+1)y' - 2(x-1)y = 0.$

4. $y'' + 4xy' + (4x^2 + 2)y = 0;\ y_1 = \exp(ax^2).$

5. $(x^2 + 1)y'' - 2y = 0.$

6. $(x^2 - 2x + 3)y''' - (x^2 + 1)y'' + 2xy' - 2y = 0;\ y_1 = x,\ y_2 = e^x.$

7. Bir jinsli bo'lmagan ikkinchi tartibli chiziqli tenglamaning ikkita xususiy yechimlarini bilgan holda uning umumiy yechimini toping:

$$(x^2 - 1)y'' + 4xy' + 2y = 6x;\ y_1 = x,\ y_2 = \frac{x^2 + x + 1}{x + 1}.$$

Quyidagi tenglamalarda $y = a(x)z$ o'rniga qo'yish yordamida birinchi tartibli hosilali hadni yo'qoting:

8. $x^2y'' - 4xy' + (6 - x^2)y = 0.$ **9.** $(1 + x^2)y'' + 4xy' + 2y = 0.$

10-12 tenglamalarda $t = \varphi(x)$ erkli o'zgaruvchini almashtirish yordamida birinchi tartibli hosilali hadni yo'qoting.

10. $xy'' - y' - 4x^3y = 0.$

11. $y'' - y' + e^{4x}y = 0.$ **12.** $2xy'' + y' - 2y = 0.$

13-15 masalalarda Liuvill almashtirishi va tasdiqlardan foydalanib, berilgan tenglamalar yechimlarining $x \to +\infty$ dagi asimptotik ko'rinishini o'rganing.

13. $y'' - x^2y = 0.$ **14.** $y'' + e^{2x}y = 0.$ **15.** $y'' - 2(x-1)y' + x^2y = 0.$

4-BOB
DIFFERENSIAL TENGLAMALAR SISTEMASI

4.1. O'ZGARMAS KOEFFITSIENTLI BIR JINSLI CHIZIQLI SISTEMALAR

$x \in R^n$ - vektor va A- o'zgarmas $n \times n$ kvadrat matritsa berilgan bo'lsin:

$$x = \begin{pmatrix} x_1 \\ \dots \\ x_n \end{pmatrix}, \quad A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{pmatrix}.$$

O'zgarmas koeffitsientli bir jinsli differensial tenglamalarning

$$\dot{x} = Ax \tag{1}$$

sistemasini o'rganamiz, bu yerda $\dot{x}$ yozuv $\dfrac{dx}{dt}$ hosilani bildiradi.

(1) sistemani o'rganishda $A(\lambda) = A - \lambda E$ matritsa va uning determinanti muhim ahamiyatga ega, bu yerda E birlik matritsa. Kelgusida $\det(A - \lambda E)$ determinatni *xarakteristik ko'phad* deb ataymiz.

Yuqoridagi (1) sistemani bir necha usulda yechish mumkin.

1-usul. Bu usul *noma`lumlarni yo'qotish usuli* bo'lib, bunda (1) sistemani, umuman aytganda, yuqoriroq tartibli bitta noma`lum funksiyali tenglamaga keltirish mumkin. Bu usul bilan uncha murakkab bo'lmagan sistemalarnigina yechish mumkin.

2-usul. Bu usulda avvalo

$$\det(A - \lambda E) = \begin{vmatrix} a_{11} - \lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} - \lambda & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} - \lambda \end{vmatrix} = 0 \tag{2}$$

xarakteristik tenglamaning ildizlarini topish kerak.

Xarakteristik tenglamaning har bir oddiy λ_j ildiziga $C_j v^j e^{\lambda_j t}$ yechim mos keladi, bu yerda C_j - ixtiyoriy o'zgarmas son, v^j esa A matritsaning shu λ_j ildizga mos kelgan xos vektori.

Agar λ ildiz k karrali bo'lib, unga k ta chiziqli erkli $v^1,...,v^k$ xos vektorlar mos kelsa, u holda bu λ ildizga mos kelgan yechim $C_1 v^1 e^{\lambda t} + ... + C_k v^k e^{\lambda t}$ ko'rinishda bo'ladi.

Agar k karrali λ ildiz uchun m ta chiziqli erkli xos vektorlar mavjud bo'lib, $m < k$ bo'lsa, u holda bu λ ga mos kelgan yechimni $(k-m)$– darajali ko'phadning $e^{\lambda t}$ funksiyaga ko'paytmasi, ya`ni

$$x_1 = (a + bt + ... + dt^{k-m})e^{\lambda t}, ..., x_n = (p + qt + ... + st^{k-m})e^{\lambda t} \qquad (3)$$

ko'rinishda izlash mumkin. $a, b, ..., s$ koeffitsientlarni topish uchun (3) yechimni (1) sistemaga qo'yish kerak. Tenglamalarning chap va o'ng tomonlaridagi o'xshash hadlarning koeffitsientlarini bir-biriga tenglab, $a, b, ..., s$ noma`lumlarga nisbatan chiziqli algebraik tenglamalar sistemasini hosil qilamiz. Topilgan $a, b, ..., s$ koeffitsientlarning har biri ko'pi bilan k ta ixtiyoriy o'zgarmaslarga bog'liq bo'lishi kerak.

Ko'rsatilgan ko'rinishdagi har bir λ uchun yechimlarni topib va ularni qo'shib, (1) sistemaning umumiy yechimini olamiz.

Agar (2) xarakteristik tenglamaning ildizi kompleks son bo'lsa, u holda yuqorida bayon qilingan usullar yordamida olinadigan yechimlar kompleks funksiyalar bilan yoziladi. Agar bunda (1) sistemaning koeffitsientlari haqiqiy sonlar bo'lsa, u holda yechimni faqat haqiqiy funksiyalar orqali ifodalash mumkin bo'ladi. Buning uchun $\lambda = \alpha + \beta i$ $(\beta \neq 0)$ ildizga mos kelgan kompleks yechimning haqiqiy va mavhum qismlari chiziqli bog'liqsiz yechimlar ekanligidan foydalanish kerak.

Tenglamalar sistemalarini yeching (**159-161**).

159. $\dot{x} = 2x + y, \ \dot{y} = 3x + 4y.$

◄Sistemaning birinchi tenglamasini y noma`lumga nisbatan yechib va sistemaning ikkinchi tenglamasiga qo'yib, topamiz:

$$y=\dot{x}-2x,\ (\dot{x}-2x)'=3x+4(\dot{x}-2x),\ \ \ddot{x}-6\dot{x}+5x=0.$$

$\lambda^2-6\lambda+5=0$ xarakteristik tenglamaning ildizlari $\lambda_1=1$ va $\lambda_2=5$ ga teng. Shunday qilib, birinchi tenglamaning umumiy yechimi

$$x=C_1e^t+C_2e^{5t}.$$

x ning topilgan qiymatini sistemaning birinchi tenglamasiga qo'yib, y ni topamiz: $y=\left(C_1e^t+C_2e^{5t}\right)'-2\left(C_1e^t+C_2e^{5t}\right)=-C_1e^t+3C_2e^{5t}$.►

160. $\dot{x}+x+5y=0,\ \dot{y}-x-y=0.$

◄Sistemaning ikkinchi tenglamasini x noma`lumga nisbatan yechib va sistemaning birinchi tenglamasiga qo'yib, $y$ noma`lumga nisbatan tenglamani olamiz: $\ddot{y}+4y=0$. Bu ikkinchi tartibli tenglamaning $y=C_1\cos 2t+C_2\sin 2t$ ko'rinishdagi umumiy yechimini sistemaning ikkinchi tenglamasiga qo'yib,

$$x=\left(2C_2-C_1\right)\cos 2t-\left(2C_1+C_2\right)\sin 2t$$

yechimni olamiz.►

161. $\dot{x}=x-2y-z,\ \dot{y}=y-x+z,\ \dot{z}=x-z.$

◄Berilgan sistemaga mos A va $A(\lambda)$ matritsalarni yozamiz:

$$A=\begin{pmatrix}1 & -2 & -1\\ -1 & 1 & 1\\ 1 & 0 & -1\end{pmatrix},\ A(\lambda)=\begin{pmatrix}1-\lambda & -2 & -1\\ -1 & 1-\lambda & 1\\ 1 & 0 & -1-\lambda\end{pmatrix}.$$

Endi $\det A(\lambda)=0$, ya`ni

$$\begin{vmatrix}1-\lambda & -2 & -1\\ -1 & 1-\lambda & 1\\ 1 & 0 & -1-\lambda\end{vmatrix}=0$$

xarakteristik tenglamani yozamiz. Bu yerda

$$-(1-\lambda)^2(1+\lambda)-2+(1-\lambda)+2(1+\lambda)=0;\ \ \lambda(\lambda-2)(\lambda+1)=0$$

kubik tenglamaning ildizlari $\lambda_1 = 0,\ \lambda_2 = 2,\ \lambda_3 = -1$ bo'lib, ular oddiy (bir karrali) ildizlardir.

Har bir λ_j ildizga mos kelgan yechimni topamiz.

$\lambda_1 = 0$ bo'lsin. Bu ildizga mos kelgan yechimni $x_1 = \alpha,\ y_1 = \beta,$ $z_1 = \gamma$ ko'rinishda izlaymiz. α, β va γ noma`lumlar

$$A(0)\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} = 0, \text{ ya`ni } \begin{pmatrix}1 & -2 & -1\\ -1 & 1 & 1\\ 1 & 0 & -1\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} = 0$$

tenglamalar sistemasidan topiladi. Ushbu

$$\begin{cases}\alpha - 2\beta - \gamma = 0,\\ -\alpha + \beta + \gamma = 0,\\ \alpha - \gamma = 0\end{cases}$$

tenglamalar sistemasining notrivial yechimi $\alpha = 1,\ \beta = 0, \gamma = 1$ ekanligini ko'rish qiyin emas. Shunday qilib, $\lambda = 0$ ildizga mos kelgan yechim $x_1 = 1,\ y_1 = 0,\ z_1 = 1$ ko'rinishda topiladi.

$\lambda_2 = 2$ bo'lsin. Bu ildizga mos kelgan yechimni $x_2 = \alpha e^{2t},\ y_2 = \beta e^{2t},$ $z_2 = \gamma e^{2t}$ ko'rinishda izlaymiz.

$$A(2)\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} = 0, \text{ ya`ni } \begin{cases}\alpha + 2\beta + \gamma = 0,\\ -\alpha - \beta + \gamma = 0,\\ \alpha - 3\gamma = 0\end{cases}$$

tenglamalar sistemasini yechib ($\alpha = 3,\ \beta = -2,\ \gamma = 1$), $\lambda_2 = 2$ ildizga mos yechimni yozamiz: $x_2 = 3e^{2t},\ y_2 = -2e^{2t},\ \ z_2 = e^{2t}$.

$\lambda_3 = -1$ bo'lsin. Bu holda ham yuqoridagi kabi

$$A(-1)\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} = 0, \text{ ya`ni } \begin{cases}2\alpha - 2\beta - \gamma = 0,\\ -\alpha + 2\beta + \gamma = 0,\\ \alpha = 0\end{cases}$$

tenglamalar sistemasini yechib ($\alpha = 0,\ \beta = 1,\ \gamma = -2$), $x_3 = 0,\ y_3 = e^{-t}$, $z_3 = -2e^{-t}$ yechimni olamiz.

Shunday qilib, berilgan sistemaning umumiy yechimi

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = C_1 \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + C_2 \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} + C_3 \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix},$$

ya`ni $x = C_1 + 3C_2e^{2t},\ y = -2C_2e^{2t} + C_3e^{-t},\ z = C_1 + C_2e^{2t} - 2C_3e^{-t}$ ko'rinishda yoziladi.►

Tenglamalar sistemalarini yeching (**162-164**)[Qavslar ichida xarakteristik tenglamaning ildizlari ko'rsatilgan].

162. $\dot{x} = x - y - z,\ \dot{y} = x + y,\ \dot{z} = 3x + z$ $(\lambda_1 = 1,\ \lambda_{2,3} = 1 \pm 2i)$.

◄Avvalo $A(\lambda) = \begin{pmatrix} 1-\lambda & -1 & -1 \\ 1 & 1-\lambda & 0 \\ 3 & 0 & 1-\lambda \end{pmatrix}$ matritsani tuzamiz.

$\lambda_1 = 1$ bo'lgan holda bu ildizga mos kelgan yechim oldingi misoldagi kabi topiladi: $x_1 = 0,\ y_1 = e^t,\ z_1 = -e^t$.

Berilgan tenglamalar sistemasining $\lambda_2 = 1 + 2i$ ildizga mos keladigan $x = \alpha e^{\lambda_2 t},\ y = \beta e^{\lambda_2 t},\ z = \gamma e^{\lambda_2 t}$ kompleks yechimini topamiz. $\alpha,\ \beta$ va γ kompleks sonlarni

$$A(1+2i)\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = 0, \text{ ya`ni } \begin{cases} -2i\alpha - \beta - \gamma = 0 \\ \alpha - 2i\beta = 0 \\ 3\alpha - 2i\gamma = 0 \end{cases}$$

tenglamalar sistemasidan topamiz: $\alpha = 2i,\ \beta = 1,\ \gamma = 3.$ Shuning uchun berilgan tenglamalar sistemasining

$$x = 2i\,e^{(1+2i)t},\ y = e^{(1+2i)t},\ z = 3e^{(1+2i)t}$$

yechimini

$$x = 2i\,e^t\left(\cos 2t + i\sin 2t\right) = 2e^t\left(-\sin 2t + i\cos 2t\right),$$

$$y = e^t\left(\cos 2t + i\sin 2t\right),\;\; z = 3e^t\left(\cos 2t + i\sin 2t\right)$$

ko’rinishda yozib olamiz.

Ma`lumki, olingan yechimning haqiqiy va mavhum qismlari alohida-alohida berilgan tenglamalarning yechimlari bo’ladi.

Shunga ko’ra, berilgan sistemaning ikkita haqiqiy yechimlarini olamiz:

$$x_2 = \operatorname{Re} x = -2e^t \sin 2t,\; y_2 = \operatorname{Re} y = e^t\cos 2t,\quad z_2 = \operatorname{Re} z = 3e^t\cos 2t;$$

$$x_3 = \operatorname{Im} x = 2e^t \cos 2t,\; y_3 = \operatorname{Im} y = e^t\sin 2t,\quad z_3 = \operatorname{Im} z = 3e^t\sin 2t.$$

Topilgan yechimlarning chiziqli bog’liqsiz ekanligiga ishonch hosil qilish qiyin emas.

Shunday qilib, dastlabki berilgan tenglamalar sistemasining umumiy yechimi

$$x = -2C_2e^t\sin 2t + 2C_3e^t\cos 2t,$$

$$y = C_1e^t + C_2e^t\cos 2t + C_3e^t\sin 2t,$$

$$z = -C_1e^t + 3C_2e^t\cos 2t + 3C_3e^t\sin 2t$$

ko’rinishda topiladi.►.

163. $\dot{x} = 2x - y - z,\; \dot{y} = 3x - 2y - 3z,\; \dot{z} = 2z - x + y$ $\left(\lambda_1 = 0,\; \lambda_2 = \lambda_3 = 1\right)$.

◄Avvalo $A(\lambda) = \begin{pmatrix} 2-\lambda & -1 & -1 \\ 3 & -2-\lambda & -3 \\ -1 & 1 & 2-\lambda \end{pmatrix}$ matritsani tuzamiz.

$\lambda_1 = 0$ oddiy ildizga mos keladigan yechimni topish qiyin emas:

$$x_1 = 1,\;\; y_1 = 3,\;\; z_1 = -1.$$

$\lambda_2 = \lambda_3 = 1$ karrali ildiz bo’lgan holga alohida to’xtalamiz.

Berilgan sistemaning tartibi $n=3$ ga teng. Xarakteristik tenglama $\lambda_2=\lambda_3=1$ ildizining karraliligi $k=2$ teng.

$$A(1)=\begin{pmatrix} 1 & -1 & -1 \\ 3 & -3 & -3 \\ -1 & 1 & 1 \end{pmatrix}$$

matritsaning rangi $r=1$ ga teng. Chiziqli bog'liqsiz xos vektorlar soni $m=n-r=2$ ga teng.

λ ildizning karraliligi chiziqli bog'liqsiz xos vektorlar soniga teng $(k=m)$ bo'lganligi uchun bu ildizga mos kelgan yechimni $k-m=0-$ darajali ko'phadning $e^{\lambda t}$ funksiyaga ko'paytmasi, ya`ni $x=ae^t$, $y=be^t$, $z=ce^t$ ko'rinishda izlaymiz.

Bu yechimni berilgan sistemaga qo'yib, o'xshash hadlar oldidagi koeffitsientlarini bir-biriga tenglab,

$$a=2a-b-c, b=3a-2b-3c, c=2c-a+b$$

chiziqli tenglamalar sistemasiga ega bo'lamiz. Bu sistema $a=C_2$, $b=C_3$, $c=C_2-C_3$ yechimga ega, bu yerda C_2 va C_3 $-$ixtiyoriy o'zgarmaslar. Shunga binoan, $\lambda_2=\lambda_3=1$ ildizga mos kelgan yechim

$$x_{2,3}=C_2e^t,\ y_{2,3}=C_3e^t,\ z_{2,3}=\left(C_2-C_3\right)e^t$$

ko'rinishda bo'ladi, bunga yuqorida topilgan $\lambda_1=0$ ildizga mos kelgan yechimni ham qo'shib, berilgan sistemaning umumiy yechimini yozamiz:

$$x=C_1+C_2e^t,\ y=3C_1+C_3e^t,\ z=-C_1+\left(C_2-C_3\right)e^t. ►$$

164. $\dot{x}=y-2z-x, \dot{y}=4x+y, \dot{z}=2x+y-z$ $(\lambda_1=1, \lambda_2=\lambda_3=-1)$.

◄Avvalo $A(\lambda)$ matritsani yozamiz:

$$A(\lambda)=\begin{pmatrix} -1-\lambda & 1 & -2 \\ 4 & 1-\lambda & 0 \\ 2 & 1 & -1-\lambda \end{pmatrix}.$$

$\lambda_1 = 1$ oddiy ildizga mos keladigan yechimni topish qiyin emas:

$$x_1 = 0, \quad y_1 = 2e^t, \quad z_1 = e^t.$$

Endi $\lambda_2 = \lambda_3 = -1$ bo'lsin. Avvalgi misoldagi kabi mulohazalar yordamida topamiz: $n = 3, \ r = 2, \ k = 2, \ m = n - r = 1, \quad k - m = 1.$

Demak, $\lambda_2 = \lambda_3 = -1$ karrali ildizga mos kelgan yechimni $k - m = 1$ – darajali ko'phadning $e^{\lambda t}$ ga ko'paytmasi ko'rinishida izlashimiz kerak:

$$x = (a + bt)e^{-t}, \ y = (c + dt)e^{-t}, \ z = (f + gt)e^{-t}.$$

Izlanayotgan yechimning bu qiymatini berilgan sistemaga qo'yib, o'xshash hadlar oldidagi koeffitsientlarni bir-biriga tenglashtirib,

$$b = c - 2f, d = 2g, d = 4a + 2c, d = -2b, g = 2a + c$$

chiziqli algebraik tenglamalar sistemasiga ega bo'lamiz. Bu sistemaning yechimi

$$a = C_2, \ b = C_3, \ d = -2C_3, \ c = -C_3 - 2C_2, \quad f = -C_2 - C_3, \ g = -C_3.$$

ko'rinishda topiladi, bu yerda $C_1, \ C_2$ va C_3 –ixtiyoriy o'zgarmaslar.

Olingan ma`lumotlar asosida berilgan sistemaning umumiy yechimini yozamiz:

$$x = (C_2 + C_3 t)e^{-t}, \ y = 2C_1 e^t - (2C_2 + C_3 + 2C_3 t)e^{-t},$$

$$z = C_1 e^t - (C_2 + C_3 + C_3 t)e^{-t}. ►$$

Normal ko'rinishga keltirilmagan sistemalarni yeching (**165-167**).

165. $\ddot{x} = 2x - 3y, \ \ddot{y} = x - 2y.$

◄Sistemaning ikkinchi tenglamasidan $x = \ddot{y} + 2y$ ni topib, uni birinchi tenglamaga qo'yamiz: $y^{(IV)} - y = 0$.

Bu yuqori tartibli o'zgarmas koeffitsientli tenglamaning $\lambda^4 - 1 = 0$ xarakteristik tenglamasi $\lambda_{1,2} = \pm 1, \ \lambda_{3,4} = \pm i$ ildizlarga ega bo'lganligi uchun uning umumiy yechimi $y = C_1 e^t + C_2 e^{-t} + C_3 \cos t + C_4 \sin t$

ko'rinishda bo'ladi. Endi $x=\ddot{y}+2y$ munosabatdan foydalanamiz: $x=3C_1e^t+3C_2e^{-t}+C_3\cos t+C_4\sin t$.►

166. $2\dot{x}-5\dot{y}=4y-x, 3\dot{x}-4\dot{y}=2x-y.$

◄Bu yerda

$$A=\begin{pmatrix}-1 & 4\\ 2 & -1\end{pmatrix},\ A(\lambda)=\begin{pmatrix}-1-2\lambda & 4+5\lambda\\ 2-3\lambda & -1+4\lambda\end{pmatrix}.$$

Endi $\det A(\lambda)=0$ xarakteristik tenglamani yechamiz:

$$(-1-2\lambda)(-1+4\lambda)-(4+5\lambda)(2-3\lambda)=0;\ -7\lambda^2+7=0,\ \lambda_{1,2}=\pm1.$$

$\lambda_1=1$ bo'lsin. Bu ildizga mos kelgan yechimni $x_1=\alpha e^t$, $y_1=\beta e^t$ ko'rinishda izlaymiz. α va β koeffitsientlar $A(1)\begin{pmatrix}\alpha\\ \beta\end{pmatrix}\equiv\begin{pmatrix}-3 & 9\\ -1 & 3\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix}=0$, ya`ni $-\alpha+3\beta=0$ tenglamadan topiladi: $\alpha=3,\ \beta=1.$Shunday qilib, $x_1=3e^t$, $y_1=e^t$. Xuddi shunday mulohazalar yordamida $\lambda_2=-1$ ga mos yechimni ham topamiz:

$$x_2=e^{-t},\quad y_2=e^{-t}.$$

Topilgan yechimlar chiziqli bog'liqsiz bo'lganligi uchun berilgan sistemaning umumiy yechimini

$$x=3C_1e^t+C_2e^{-t},\ y=C_1e^t+C_2e^{-t}$$

ko'rinishda olamiz.►

167. $\ddot{x}+5\dot{x}+2\dot{y}+y=0, 3\ddot{x}+5x+\dot{y}+3y=0.$

◄Bu yerda

$$A=\begin{pmatrix}0 & -1\\ -5 & -3\end{pmatrix},\ A(\lambda)=\begin{pmatrix}-\lambda^2-5\lambda & -2\lambda-1\\ -3\lambda^2-5 & -\lambda-3\end{pmatrix}.$$

Endi $\det A(\lambda)\equiv(\lambda-1)(\lambda^2-1)=0$ xarakteristik tenglamani yechib, $\lambda_1=\lambda_2=1,\ \lambda_3=-1$ xos qiymatlarga ega bo'lamiz.

$\lambda_1 = \lambda_2 = 1$ bo'lsin. Karrali ildiz bo'lgan holda **163-**misoldagi kabi mulohazalarni yuritib,

$$n = 2,\ k = 2,\ r = \text{rang}\,A(1) = 1,\ m = n - r = 1,\ k - m = 1$$

ma`lumotlarni olamiz.

Demak, $\lambda_1 = \lambda_2 = 1$ karrali ildizga mos kelgan yechimni $k - m = 1-$ darajali ko'phadning $e^{\lambda t}$ ga ko'paytmasi ko'rinishida izlashimiz kerak:

$$x = (at + b)e^t,\ y = (ct + d)e^t.$$

Izlanayotgan yechimning bu qiymatini berilgan sistemaga qo'yib, o'xshash hadlar oldidagi koeffitsientlarni bir-biriga tenglashtirib, koeffitsientlar uchun

$$a = C_2,\ b = C_1,\ c = -2C_2,\ d = -2C_1 - C_2$$

qiymatlarni olamiz, bu yerda C_1 va C_2 $-$ixtiyoriy o'zgarmaslar.

Olingan ma`lumotlar asosida berilgan sistemaning $\lambda_1 = \lambda_2 = 1$ karrali ildizga mos keladigan umumiy yechimini yozamiz:

$$x_{1,2} = (C_1 + C_2 t)e^t,\ y_{1,2} = (-2C_1 - C_2 - 2C_2 t)e^t.$$

$\lambda_3 = -1$ bo'lsin. Bu holda yechimni $x_3 = \alpha e^{-t}, y_3 = \beta e^{-t}$ ko'rinishda izlab, topamiz: $x_3 = e^{-t}, y_3 = -4e^{-t}$ $(\alpha = 1, \beta = -4)$.

Topilgan yechimlar chiziqli bog'liqsiz bo'lganligi uchun berilgan sistemaning umumiy yechimini

$$x = (C_1 + C_2 t)e^t + C_3 e^{-t},\ y = (-2C_1 - C_2 - 2C_2 t)e^t - 4C_3 e^{-t}$$

ko'rinishda olamiz.►

INDIVIDUAL TOPSHIRIQLAR

M51. Differensial tenglamalar sistemasini yeching.

1. $\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = 3x + 4y. \end{cases}$
2. $\begin{cases} \dot{x} = x - y, \\ \dot{y} = -4x + y. \end{cases}$
3. $\begin{cases} \dot{x} = -x + 8y, \\ \dot{y} = x + y. \end{cases}$

4. $\begin{cases} \dot{x} = 7x - 3y, \\ \dot{y} = -x + 9y. \end{cases}$ 5. $\begin{cases} \dot{x} = x - y, \\ \dot{y} = -4x + 4y. \end{cases}$ 6. $\begin{cases} \dot{x} = -2x + y, \\ \dot{y} = -3x + 2y. \end{cases}$

7. $\begin{cases} \dot{x} = 6x - y, \\ \dot{y} = 3x + 2y. \end{cases}$ 8. $\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = -6x - 3y. \end{cases}$ 9. $\begin{cases} \dot{x} = 2x - 5y, \\ \dot{y} = -2x - 7y. \end{cases}$

10. $\begin{cases} \dot{x} = 3x + 2y, \\ \dot{y} = x + 4y. \end{cases}$ 11. $\begin{cases} \dot{x} = 5x + 5y. \\ \dot{y} = 2x - 4y. \end{cases}$ 12. $\begin{cases} \dot{x} = 4x + 2y, \\ \dot{y} = 4x + 6y. \end{cases}$

13. $\begin{cases} \dot{x} = 8x - 3y, \\ \dot{y} = 2x + y. \end{cases}$ 14. $\begin{cases} \dot{x} = 3x + y, \\ \dot{y} = x + 3y. \end{cases}$ 15. $\begin{cases} \dot{x} = 2x + 3y, \\ \dot{y} = 5x + 4y. \end{cases}$

16. $\begin{cases} \dot{x} = x + 2y, \\ \dot{y} = 3x + 6y. \end{cases}$ 17. $\begin{cases} \dot{x} = 5x + 4y, \\ \dot{y} = 4x + 5y. \end{cases}$ 18. $\begin{cases} \dot{x} = x + 2y, \\ \dot{y} = 4x + 3y. \end{cases}$

19. $\begin{cases} \dot{x} = x + 4y, \\ \dot{y} = x + y. \end{cases}$ 20. $\begin{cases} \dot{x} = 3x - 2y, \\ \dot{y} = 2x + 8y. \end{cases}$ 21. $\begin{cases} \dot{x} = 8x + 2y, \\ \dot{y} = -2x + 3y. \end{cases}$

22. $\begin{cases} \dot{x} = 7x + 3y, \\ \dot{y} = x + 5y. \end{cases}$ 23. $\begin{cases} \dot{x} = 4x - y, \\ \dot{y} = -x + 4y. \end{cases}$ 24. $\begin{cases} \dot{x} = 2x + 8y, \\ \dot{y} = x + 4y. \end{cases}$

25. $\begin{cases} \dot{x} = 5x + 8y, \\ \dot{y} = 3x + 3y. \end{cases}$ 26. $\begin{cases} \dot{x} = 3x + y, \\ \dot{y} = 8x + y. \end{cases}$ 27. $\begin{cases} \dot{x} = x - 5y, \\ \dot{y} = -x - 3y. \end{cases}$

28. $\begin{cases} \dot{x} = -5x + 2y, \\ \dot{y} = x - 6y. \end{cases}$ 29. $\begin{cases} \dot{x} = 6x + 3y, \\ \dot{y} = -8x - 5y. \end{cases}$ 30. $\begin{cases} \dot{x} = 4x - 8y, \\ \dot{y} = -8x + 4y. \end{cases}$

M52. Differensial tenglamalar sistemasini yeching.

1. $\begin{cases} x' = 6x + 2y, \\ y' = 5x - 3y. \end{cases}$ 2. $\begin{cases} x' = 3x + 5y, \\ y' = 2x - 6y. \end{cases}$ 3. $\begin{cases} x' = 3x + y, \\ y' = 9x - 5y. \end{cases}$

4. $\begin{cases} x' = 5x + 3y, \\ y' = 3x - 3y. \end{cases}$ 5. $\begin{cases} x' = 13x - 9y, \\ y' = 4x - 7y. \end{cases}$ 6. $\begin{cases} x' = x + 5y, \\ y' = 2x - 8y. \end{cases}$

7. $\begin{cases} x' = 8x - 5y, \\ y' = -2x - y. \end{cases}$ 8. $\begin{cases} x' = -6x + 2y, \\ y' = 11x + 3y. \end{cases}$ 9. $\begin{cases} x' = 3x - 11y, \\ y' = -2x - 6y. \end{cases}$

10. $\begin{cases} x' = 5x + 4y, \\ y' = 6x - 5y. \end{cases}$ 11. $\begin{cases} x' = 6x + 3y, \\ y' = 8x - 4y. \end{cases}$ 12. $\begin{cases} x' = 7x - 2y, \\ y' = -12x - 3y. \end{cases}$

13. $\begin{cases} x' = 5x + 13y, \\ y' = 2x - 6y. \end{cases}$ 14. $\begin{cases} x' = 6x + 2y, \\ y' = 13x - 5y. \end{cases}$ 15. $\begin{cases} x' = 6x + 7y, \\ y' = 8x - 4y. \end{cases}$

16. $\begin{cases} x' = 4x + 8y, \\ y' = 7x - 6y. \end{cases}$ 17. $\begin{cases} x' = -5x + 13y, \\ y' = 5x + 3y. \end{cases}$ 18. $\begin{cases} x' = 4x + 3y, \\ y' = 3x - 4y. \end{cases}$

19. $\begin{cases} x' = 7x + 5y, \\ y' = 9x - 5y. \end{cases}$ 20. $\begin{cases} x' = 8x + 3y, \\ y' = 15x - 4y. \end{cases}$ 21. $\begin{cases} x' = -9x + 9y, \\ y' = 5x + 3y. \end{cases}$

22. $\begin{cases} x' = 10x + 8y, \\ y' = 4x - 4y. \end{cases}$ 23. $\begin{cases} x' = 9x + 4y, \\ y' = 8x - 5y. \end{cases}$ 24. $\begin{cases} x' = 6x - 2y, \\ y' = 4x - 3y. \end{cases}$

25. $\begin{cases} x' = -5x + 6y, \\ y' = -4x + 9y. \end{cases}$ 26. $\begin{cases} x' = 7x - 2y, \\ y' = 7x - 8y. \end{cases}$ 27. $\begin{cases} x' = 10x + 13y, \\ y' = -2x - 5y. \end{cases}$

28. $\begin{cases} x' = 11x - 5y, \\ y' = 9x - 7y. \end{cases}$ 29. $\begin{cases} x' = -12x + 15y, \\ y' = -3x + 6y. \end{cases}$ 30. $\begin{cases} x' = 6x - 4y, \\ y' = 3x - 7y. \end{cases}$

M53. Differensial tenglamalar sistemasini yeching

1. $\begin{cases} x' = x + 5y, \\ y' = -5x - 9y. \end{cases}$ 2. $\begin{cases} x' = -8x + 3y, \\ y' = -12x + 4y. \end{cases}$ 3. $\begin{cases} x' = 13x - 9y, \\ y' = 9x - 5y. \end{cases}$

4. $\begin{cases} x' = 5x - 6y, \\ y' = 6x - 7y. \end{cases}$ 5. $\begin{cases} x' = 7x - 8y, \\ y' = 8x - 9y. \end{cases}$ 6. $\begin{cases} x' = 13x - 5y, \\ y' = 20x - 7y. \end{cases}$

7. $\begin{cases} x' = 8x - 5y, \\ y' = 5x - 2y. \end{cases}$ 8. $\begin{cases} x' = 6x + 7y, \\ y' = -7x - 8y. \end{cases}$ 9. $\begin{cases} x' = 7x + 9y, \\ y' = -4x - 5y. \end{cases}$

10. $\begin{cases} x' = 10x + 7y, \\ y' = -7x - 4y. \end{cases}$ 11. $\begin{cases} x' = -7x + 9y, \\ y' = -9x + 11y. \end{cases}$ 12. $\begin{cases} x' = 9x - 7y, \\ y' = 7x - 5y. \end{cases}$

13. $\begin{cases} x' = 5x - 16y, \\ y' = 4x - 11y. \end{cases}$ 14. $\begin{cases} x' = -5x + 4y, \\ y' = -4x + 3y. \end{cases}$ 15. $\begin{cases} x' = -7x + 12y, \\ y' = -3x + 5y. \end{cases}$

16. $\begin{cases} x' = 4x + y, \\ y' = -x + 2y. \end{cases}$ 17. $\begin{cases} x' = 7x + 4y, \\ y' = -x + 3y. \end{cases}$ 18. $\begin{cases} x' = -5x - 3y, \\ y' = 3x + y. \end{cases}$

19. $\begin{cases} x' = -2x - y, \\ y' = x - 4y. \end{cases}$ 20. $\begin{cases} x' = 7x + 9y, \\ y' = -x + y. \end{cases}$ 21. $\begin{cases} x' = 7x - y, \\ y' = x + 5y. \end{cases}$

22. $\begin{cases} x' = 8x + 4y, \\ y' = -x + 4y. \end{cases}$ 23. $\begin{cases} x' = -9x + 9y, \\ y' = -x - 3y. \end{cases}$ 24. $\begin{cases} x' = 4x - 3y, \\ y' = 3x - 2y. \end{cases}$

25. $\begin{cases} x' = -3x - 2y, \\ y' = 2x + y. \end{cases}$ 26. $\begin{cases} x' = x - y, \\ y' = x + 3y. \end{cases}$ 27. $\begin{cases} x' = 3x - y, \\ y' = 4x - y. \end{cases}$

28. $\begin{cases} x' = 4x - y, \\ y' = x + 2y. \end{cases}$ 29. $\begin{cases} x' = 3x + y, \\ y' = -x + y. \end{cases}$ 30. $\begin{cases} x' = 2x + y, \\ y' = -x. \end{cases}$

M54. Differensial tenglamalar sistemasini yeching.

1. $\begin{cases} x' = -9x + 10y, \\ y' = -10x + 7y. \end{cases}$ 2. $\begin{cases} x' = 6x - 5y, \\ y' = 10x - 4y. \end{cases}$ 3. $\begin{cases} x' = 5x - 4y, \\ y' = 5x - 3y. \end{cases}$

4. $\begin{cases} x' = 4x - 5y, \\ y' = 2x - 2y. \end{cases}$ 5. $\begin{cases} x' = 3x - 9y, \\ y' = 2x - 3y. \end{cases}$ 6. $\begin{cases} x' = -3x + 5y, \\ y' = -5x + 5y. \end{cases}$

7. $\begin{cases} x' = 3x - 2y, \\ y' = 4x - y. \end{cases}$ 8. $\begin{cases} x' = 5x - y, \\ y' = 13x - y. \end{cases}$ 9. $\begin{cases} x' = 3x - 5y, \\ y' = 5x - 3y. \end{cases}$

10. $\begin{cases} x' = 7x - 9y, \\ y' = 5x - 5y. \end{cases}$ 11. $\begin{cases} x' = 6x - 13y, \\ y' = 4x - 2y. \end{cases}$ 12. $\begin{cases} x' = 6x - y, \\ y' = 61x - 4y. \end{cases}$

13. $\begin{cases} x' = 11x - 2y, \\ y' = 17x + 5y. \end{cases}$ 14. $\begin{cases} x' = 7x + 3y, \\ y' = -6x + 13y. \end{cases}$ 15. $\begin{cases} x' = 9x - 2y, \\ y' = 4x + 5y. \end{cases}$

16. $\begin{cases} x' = x - y, \\ y' = 2x + 3y. \end{cases}$ 17. $\begin{cases} x' = 2x + y, \\ y' = -5x + 4y. \end{cases}$ 18. $\begin{cases} x' = 2x - 4y, \\ y' = x + 2y. \end{cases}$

19. $\begin{cases} x' = 5x - y, \\ y' = 17x - 3y. \end{cases}$ 20. $\begin{cases} x' = 3x + 13y, \\ y' = -x - y. \end{cases}$ 21. $\begin{cases} x' = -3x - 2y, \\ y' = 4x + y. \end{cases}$

22. $\begin{cases} x' = -5x - 5y, \\ y' = 5x + y. \end{cases}$ 23. $\begin{cases} x' = -4x + 2y, \\ y' = -5x - 2y. \end{cases}$ 24. $\begin{cases} x' = 5x - y, \\ y' = 2x + 3y. \end{cases}$

25. $\begin{cases} x' = -7x - 3y, \\ y' = 6x - y. \end{cases}$ 26. $\begin{cases} x' = 2x - y, \\ y' = x + 2y. \end{cases}$ 27. $\begin{cases} x' = x - 2y, \\ y' = x - y. \end{cases}$

28. $\begin{cases} x' = 4x - 5y, \\ y' = x. \end{cases}$ 29. $\begin{cases} x' = x + y, \\ y' = -2x + 3y. \end{cases}$ 30. $\begin{cases} x' = x + 4y, \\ y' = -3x + y. \end{cases}$

M55. Differensial tenglamalar sistemasini yeching

1. $\begin{cases} x' = 2x - y + z, \\ y' = x + 2y - z, \\ z' = x - y + 2z \end{cases}$ $(\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 3)$.

2. $\begin{cases} x' = 3x + 2y - 2z, \\ y' = y - 3z, \\ z' = 3y + z \end{cases}$ $(\lambda_1 = 3,\ \lambda_{2,3} = 1 \pm 3i)$.

3. $\begin{cases} x' = 5x + 2y - 2z, \\ y' = x + 4y - z, \\ z' = 3x + 3y \end{cases}$ $(\lambda_1 = \lambda_2 = \lambda_3 = 3)$.

4. $\begin{cases} x' = x - 2y, \\ y' = x + 3y + z, \\ z' = -2x - 2y - z \end{cases}$ $(\lambda_1 = \lambda_2 = \lambda_3 = 1)$.

5. $\begin{cases} x' = -3x + 4y - 2z, \\ y' = x + z, \\ z' = 6x - 6y + 5z \end{cases}$ $(\lambda_1 = -1,\ \lambda_2 = 1,\ \lambda_3 = 2)$.

6. $\begin{cases} x' = 2x + y - z, \\ y' = -x + 2y + z, \\ z' = -x + y + 2z \end{cases}$ $(\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 3)$.

7. $\begin{cases} x' = 5x - 2y - 2z \\ y' = 2x + y - 2z, \\ z' = 2x - 2y + z \end{cases}$ $(\lambda_1 = 1,\ \lambda_2 = \lambda_3 = 3)$.

8. $\begin{cases} x' = 4x - 4y + 2z \\ y' = 2x - 2y + z, \\ z' = -4x + 4y - 2z. \end{cases}$ $(\lambda_1 = \lambda_2 = \lambda_3 = 0)$.

9. $\begin{cases} x' = -x - 2y + 2z \\ y' = -2x - y + 2z, \\ z' = -3x - 2y + 3z \end{cases}$ $(\lambda_1 = 1,\ \lambda_{2,3} = \pm i)$.

10. $\begin{cases} x' = y + z, \\ y' = x + z, \\ z' = x + y \end{cases}$ $(\lambda_1 = \lambda_2 = -1,\ \lambda_3 = 2)$.

11. $\begin{cases} x' = 6x - 12y - z, \\ y' = x - 3y - z, \\ z' = -4x + 12y + 3z \end{cases}$ $(\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 3)$.

12. $\begin{cases} x' = 4x - 5y + 7z, \\ y' = x - 4y + 9z, \\ z' = -4x + 5z \end{cases}$ $(\lambda_1 = 1,\ \lambda_{2,3} = 2 \pm 3i)$.

13. $\begin{cases} x' = 4x - 3y + 3z \\ y' = 9x - 4y + 3z, \\ z' = 7x - 3y + 2z \end{cases}$

$(\lambda_1 = -1,\ \lambda_2 = 1,\ \lambda_3 = 2)$.

14. $\begin{cases} x' = 4x - y + 4z \\ y' = x + y + z, \\ z' = x - y + 3z \end{cases}$

$(\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 5)$.

15. $\begin{cases} x' = -3x + 2y + 2z, \\ y' = -3x - y + z, \\ z' = -x + 2y \end{cases}$

$(\lambda_1 = -2,\ \lambda_{2,3} = -1 \pm 2i)$.

16. $\begin{cases} x' = -9x - 3y + 4z \\ y' = -20x - 4y + 10z, \\ z' = -32x - 9y + 15z \end{cases}$

$(\lambda_1 = -1,\ \lambda_2 = 1,\ \lambda_3 = 2)$.

17. $\begin{cases} x' = 13x + 23y - 5z \\ y' = -6x - 10y + 3z, \\ z' = 4x + 10y + z. \end{cases}$

$(\lambda_1 = -1,\ \lambda_2 = 2,\ \lambda_3 = 3)$.

18. $\begin{cases} x' = -4x - 3y + 4z \\ y' = 6x + 5y - 4z, \\ z' = -5x - 3y + 5z \end{cases}$

$(\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 3)$.

19. $\begin{cases} x' = x - 3y + 3z \\ y' = -2x - 6y + 13z, \\ z' = -x - 4y + 8z \end{cases}$

$(\lambda_1 = \lambda_2 = \lambda_3 = 1)$.

20. $\begin{cases} x' = x - 4y - 2z \\ y' = -4x + 13y + 10z, \\ z' = 5x - 16y - 12z \end{cases}$

$(\lambda_1 = -1,\ \lambda_2 = 1,\ \lambda_3 = 2)$.

21. $\begin{cases} x' = -2x - 2y - 4z \\ y' = -2x + y - 2z, \\ z' = 5x + 2y + 7z \end{cases}$

$(\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 3)$.

22. $\begin{cases} x' = x - y - z \\ y' = x + y, \\ z' = 3x + z \end{cases}$

$(\lambda_1 = 1,\ \lambda_{2,3} = 1 \pm 2i)$.

23. $\begin{cases} x' = -3x + 18y - 18z \\ y' = -7x + 24y - 22z, \\ z' = -4x + 12y - 10z \end{cases}$

$(\lambda_1 = 2,\ \lambda_2 = 3,\ \lambda_3 = 6)$.

24. $\begin{cases} x' = -7x - 48y + 4z \\ y' = 4x + 21y - 4z, \\ z' = 8x + 32y - 11z. \end{cases}$

$(\lambda_1 = -3,\ \lambda_2 = 1,\ \lambda_3 = 5)$.

25. $\begin{cases} x' = 2x - y - z, \\ y' = 3x - 2y - 3z, \\ z' = -x + y + 2z \end{cases}$

$(\lambda_1 = 0,\ \lambda_2 = \lambda_3 = 1)$.

26. $\begin{cases} x' = 2x + y + z \\ y' = x + 2y + z, \\ z' = x + y + 2z, \end{cases}$

$(\lambda_1 = \lambda_2 = 1,\ \lambda_3 = 4)$.

27. $\begin{cases} x' = -13x - 16y - 14z \\ y' = -8x - 15y - 11z, \\ z' = 18x + 28y + 22z \end{cases}$

$(\lambda_1 = -3,\ \lambda_2 = -2,\ \lambda_3 = -1).$

28. $\begin{cases} x' = x + 2y + z, \\ y' = 2x - 5y + 2z, \\ z' = x + 2y + z \end{cases}$

$(\lambda_1 = 0,\ \lambda_2 = -6,\ \lambda_3 = 3).$

29. $\begin{cases} x' = x - y + z, \\ y' = x + y - z, \\ z' = 2x - y \end{cases}$

$(\lambda_1 = -1,\ \lambda_2 = 1,\ \lambda_3 = 2).$

30. $\begin{cases} x' = 2x - 5y - 3z \\ y' = -x - 2y - 3z, \\ z' = 3x + 15y + 12z, \end{cases}$

$(\lambda_1 = \lambda_2 = 3,\ \lambda_3 = 6).$

4.2. O'ZGARMAS KOEFFITSIENTLI BIR JINSLI BO'LMAGAN CHIZIQLI SISTEMALAR

Bir jinsli bo'lmagan har qanday sistemaning umumiy yechimi unga mos bir jinsli sistemaning umumiy yechimi va bir jinsli bo'lmagan sistemaning birorta xususiy yechimining yig'indisidan iborat.

Bir jinsli bo'lmagan sistemaning xususiy yechimini bir nechta usullar bilan topish mumkin.

Aniqmas koeffitsientlar usuli. O'zgarmas koeffitsientli bir jinsli bo'lmagan chiziqli sistema berilgan bo'lsin:

$$\frac{dx_k}{dt} = \sum_{j=1}^{n} a_{kj}\, x_j + f_k(t),\ k = \overline{1,n} \qquad (1)$$

bu yerda a_{kj} - o'zgarmas sonlar ($k,\ j = \overline{1,n}$).

Agar (1) sistemaning o'ng tomoni $b_0 + b_1 t + b_2 t^2 +,,, + b_m t^m$, e^{at}, $\cos\beta t$, $\sin\beta t$ funksiyalarning yig'indilari va ko'paytmalaridan iborat bo'lsa, u holda bu sistemaning xususiy yechimini aniqmas koeffitsientlar usuli bilan topish mumkin. Buning uchun o'zgarmas koeffitsientli bitta chiziqli tenglama uchun ko'rsatilgan qoidalardan (**2.3**-bandga qarang) farqli qoidadan foydalanish kerak. Agar $f_k(t) = P_{m_k}(t)e^{\gamma t}$ ($P_m(t)$ – m-darajali ko'phad) bo'lsa, u holda (1) sistemaning xususiy yechimini $t^s Q_m(t)e^{\gamma t}$ ko'rinishda emas, balki

$$x_k = Q_{m+s}^k(t)e^{\gamma t}, \ k = \overline{1,n} \tag{2}$$

ko'rinishda izlash kerak, bu yerda $Q_{m+s}^k(t)$ - koeffitsientlari noma`lum bo'lgan $m+s-$darajali ko'phad, $m = max\{m_k\}$; agar $\gamma$ soni xarakteristik tenglamaning ildizi bo'lmasa, u holda $s=0$ deb olinadi, aks holda, ya`ni agar λ soni xarakteristik tenglamaning ildizi bo'lsa, u holda s soni shu ildizning karraliligiga teng. Ko'phadlarning koeffitsientlarini aniqlash uchun (2) ifodalarni (1) sistemaga qo'yib, chap va o'ng tomonlardagi o'xshash hadlar oldidagi koeffitsientlarni bir-biriga tenglashtirish kerak. $f_k(t)$ funksiyalar $e^{\alpha t}\cos\beta t$ va $e^{\alpha t}\sin\beta t$ funksiyalarni o'z ichiga olib, $\gamma = \alpha + \beta i$ son xarakteristik tenglamaning ildizi bo'lgan holda ham xuddi yuqoridagi kabi yo'l tutiladi.

Bir jinsli bo'lmagan sistemalarni yeching (**168-171**).

168. $\dot{x} = y - 5\cos t, \ \dot{y} = 2x + y.$

◀Bu sistemani noma'lumlarni yo'qotish yo'li bilan yechsa ham bo'ladi. Sistemadagi birinchi tenglamani

$$y = \dot{x} + 5\cos t \tag{3}$$

ko'rinishida yozib olib, uni ikkinchi tenglamaga qo'yamiz. Natijada bitta noma'lumli yuqori tartibli

$$\ddot{x} - \dot{x} - 2x = 5\cos t + 5\sin t \tag{4}$$

tenglamani olamiz. Unga mos $\ddot{x} - \dot{x} - 2x = 0$ bir jinsli tenglamaning umumiy yechimi ($\lambda_1 = -1, \lambda_2 = 2-$xarakteristik tenglamaning ildizlari) $x_0 = C_1e^{-t} + C_2e^{2t}$ ko'rinishda bo'ladi. Bir jinsli bo'lmagan (4) tenglamaning xususiy yechimini $x_1 = a\cos t + b\sin t$ ko'rinishda izlab, $x_1 = -\cos t - 2\sin t$ xususiy yechimni olamiz. Demak,

$$x \equiv x_0 + x_1 = C_1 e^{-t} + C_2e^{2t} - \cos t - 2\sin t. \tag{5}$$

Endi (5) yechimni (3) munosabatga qo'yib,

$$y = -C_1e^{-t} + 2C_2e^{2t} + 3\cos t + \sin t \tag{6}$$

yechimni olamiz. (5) va (6) ifodalar birgalikda berilgan sistemaning umumiy yechimini tashkil etadi.►

169. $\dot{x} = 2y - x + 1,\ \dot{y} = 3y - 2x.$

◄Bu sistemaga mos bir jinsli sistema xarakteristik tenglamasining ildizlarini topamiz:

$$\begin{vmatrix} -1-\lambda & 2 \\ -2 & 3-\lambda \end{vmatrix} = 0, \quad \lambda^2 - 2\lambda + 1 = 0, \quad \lambda_1 = \lambda_2 = 1.$$

Bir jinsli sistemaning umumiy yechimi

$$x_0 = (C_1 + 2C_2 t)e^t, \quad y_0 = (C_1 + C_2 + 2C_2 t)e^t.$$

Endi bir jinsli bo'lmagan sistemaning xususiy yechimini topamiz. Bizda

$$f_1(t) = 1,\ P_{m_1}(t) = 1,\ m_1 = 0,\ \gamma = 0,\ s = 0,$$

$$f_2(t) = 0\ \ P_{m_2}(t) = 0,\ m_2 = 0,\ \gamma = 0,\ s = 0.$$

Demak, $m \equiv \max(m_1, m_2) = 0$ bo'lganligi uchun, xususiy yechimni $x_1 = a,\ y_1 = b$ ko'rinishda izlab, $a = -3,\ b = -2$ ni topamiz. Shunday qilib, berilgan sistemaning umumiy yechimi

$$x = (C_1 + 2C_2 t)e^t - 3, \quad y = \left(C_1 + C_2 + 2C_2 t\right)e^t - 2$$

ko'rinishda topiladi.►

170. $\dot{x} = 2x - 4y,\ \dot{y} = x - 3y + 3e^t.$

◄Tegishli xarakteristik tenglamaning ildizlari $\lambda_1 = 1,\ \lambda_2 = -2$ bo'lib, bir jinsli sistemaning umumiy yechimi $x_0 = 4C_1e^t + C_2e^{-2t}$, $y_0 = C_1e^t + C_2e^{-2t}$ ko'rinishda bo'ladi. Endi bir jinsli bo'lmagan sistemaning xususiy yechimini topamiz. Bizda

$$f_1(t) = 0,\ P_{m_1}(t) = 0,\ m_1 = 0;$$

$$f_2(t) = 3e^t,\ P_{m_1}(t) = 3,\ m_2 = 0,\ \gamma = 1,\ s = 1.$$

Shunga ko'ra, $m = \max\{m_1, m_2\} = 0$ va $m + s = 1$. Shuning uchun xususiy yechimni birinchi darajali ko'phadning e^t ga ko'paytmasi ko'rinishida izlaymiz:

$$x_1 = (at + b)e^t, \; y_1 = (ct + d)e^t.$$

x_1 va y_1 xususiy yechimlarning ifodalarini dastlabki berilgan sistemaga qo'yamiz:

$$at + b + a = 2at + 2b - 4ct - 4d,$$
$$ct + d + c = at + b - 3ct - 3d + 3.$$

O'xshash hadlarning oldidagi koeffitsientlarni bir-biriga tenglashtirib, $a = -4, \; b = 0, \; c = -1, \; d = 1$ koeffitsentlarni va, demak,

$$x_1 = -4te^t, \; y_1 = (-t + 1)e^t$$

xususiy yechimni topamiz .

Natijada berilgan bir jinsli bo'lmagan sistemaning umumiy yechimi

$$x = 4C_1e^t + C_2e^{-2t} - 4te^t, \quad y = C_1e^t + C_2e^{-2t} - (t-1)e^t.$$

ko'rinishda topiladi.►

171. $\dot{x} = 2x + y + 2e^t, \dot{y} = x + 2y - 3e^{4t}.$

◄Xarakteristik tenglamaning ildizlari $\lambda_1 = 1$ va $\lambda_2 = 3$ bo'lganligi uchun tegishli bir jinsli sistemaning umumiy yechimi

$$x_0 = C_1e^t + C_2e^{3t}, \; y_0 = -C_1e^t + C_2e^{3t}$$

ko'rinishda bo'ladi .

Endi bir jinsli bo'lmagan sistemaning ikkita sistemaga ajratamiz:

$$\dot{x} = 2x + y + 2e^t, \dot{y} = x + 2y; \tag{7}$$

$$\dot{x} = 2x + y, \dot{y} = x + 2y - 3e^{4t}. \tag{8}$$

(7) sistemada $f_1(t) = 2e^t$ bo'lib $m = 0, \; s = 1$ bo'lganligi uchun sistemaning xususiy yechimini

$$x_1=(at+b)e^t,\ \ y_1=(ct+d)e^t$$

ko'rinishda izlaymiz va uni topamiz: $x_1=te^t$, $y_1=-(t+1)e^t$.

(8) sistemada $g_1(t)=-3e^{4t}$ bo'lib, $m=0, s=0$ bo'lganligi uchun bu sistemaning xususiy yechimini $x_2=ae^{4t}$, $y_2=be^{4t}$ ko'rinishda izlaymiz va uni $x_2=-e^{4t}$, $y_2=-2e^{4t}$ ko'rinishda topamiz .

Olingan natijalar asosida berilgan sistemaning umumiy yechimini yozamiz:

$$x=C_1e^t+C_2e^{3t}+te^t-e^{4t},\ y=-C_1e^t+C_2e^{3t}-(t+1)e^t-2e^{4t}.$$ ►

INDIVIDUAL TOPSHIRIQLAR

M56. Bir jinsli bo'lmagan tenglamalar sistemasini yeching.

1. $\begin{cases} x'=x+3y+2t, \\ y'=2x+2y+1. \end{cases}$

2. $\begin{cases} x'=-4x+y+e^t, \\ y'=-5x+2y+5e^t. \end{cases}$

3. $\begin{cases} x'=x+2y+2e^t, \\ y'=2x+y+6e^{3t}. \end{cases}$

4. $\begin{cases} x'=2x-5y+3, \\ y'=5x-6y+1. \end{cases}$

5. $\begin{cases} x'=x-y+t^2, \\ y'=-4x+4y. \end{cases}$

6. $\begin{cases} x'=-2x+y+3t, \\ y'=-3x+2y. \end{cases}$

7. $\begin{cases} x'=6x-y+\sin t, \\ y'=3x+2y. \end{cases}$

8. $\begin{cases} x'=2x+y+te^t, \\ y'=-6x-3y. \end{cases}$

9. $\begin{cases} x'=y, \\ y'=x+\sin t. \end{cases}$

10. $\begin{cases} x'=3x+2y+t^3, \\ y'=x+4y. \end{cases}$

11. $\begin{cases} x'=-2x, \\ y'=y+te^{2t}. \end{cases}$

12. $\begin{cases} x'=4x+2y+\cos t, \\ y'=4x+6y. \end{cases}$

13. $\begin{cases} x'=8x-3y, \\ y'=2x+y+e^{2t}. \end{cases}$

14. $\begin{cases} x'=3x+y+2\sin 2t, \\ y'=x+3y. \end{cases}$

15. $\begin{cases} x' = 2x + 3y, \\ y' = 5x + 4y + t\sin t. \end{cases}$

16. $\begin{cases} x' = x + 2y + t^2 e^t, \\ y' = 3x + 6y. \end{cases}$

17. $\begin{cases} x' = 5x + 4y + e^{3t}, \\ y' = 4x + 5y. \end{cases}$

18. $\begin{cases} x' = x + 2y + 2t + 1, \\ y' = 4x + 3y. \end{cases}$

19. $\begin{cases} x' = x + 4y, \\ y' = x + y + t\cos 2t. \end{cases}$

20. $\begin{cases} x' = 3x - 2y + e^{-t}, \\ y' = 2x + 8y. \end{cases}$

21. $\begin{cases} x' = 3x - 2y + 5\sin t, \\ y' = 2x + 8y. \end{cases}$

22. $\begin{cases} x' = 7x + 3y, \\ y' = x + 5y + t\cos 2t. \end{cases}$

23. $\begin{cases} x' = 4x - y + 4te^t, \\ y' = -x + 4y. \end{cases}$

24. $\begin{cases} x' = 2x + 8y, \\ y' = x + 4y + 6. \end{cases}$

25. $\begin{cases} x' = 5x + 8y, \\ y' = 3x + 3y + \sin 3t. \end{cases}$

26. $\begin{cases} x' = 3x + y, \\ y' = 8x + y + 2e^{5t}. \end{cases}$

27. $\begin{cases} x' = x - 5y + t^2 + 3t, \\ y' = -x - 3y. \end{cases}$

28. $\begin{cases} x' = -5x + 2y, \\ y' = x - 6y + 21\sin 3t. \end{cases}$

29. $\begin{cases} x' = x - y, \\ y' = -4x + y + 3e^{2t}. \end{cases}$

30. $\begin{cases} x' = 2x + y + 26\sin t, \\ y' = 3x + 4y. \end{cases}$

M57. Bir jinsli bo’lmagan differensial tenglamalar sistemasini yeching.

1. $\begin{cases} x' = 4x + 3y + t, \\ y' = -2x - y. \end{cases}$

2. $\begin{cases} x' = x - 2y + e^t, \\ y' = x + 4y + 1. \end{cases}$

3. $\begin{cases} x' = x - y + \cos 3t, \\ y' = x - 3y. \end{cases}$

4. $\begin{cases} x' = x - y + t^2 - 1, \\ y' = 5x + 5y - 2. \end{cases}$

5. $\begin{cases} x' = x - 3y + \sin t, \\ y' = x + 5y - \cos t. \end{cases}$

6. $\begin{cases} x' = -3x - y + 2t, \\ y' = -x + 4y - t. \end{cases}$

7. $\begin{cases} x' = -6x - 4y + t, \\ y' = 3x + 2y - 5. \end{cases}$

8. $\begin{cases} x' = 2x + y + t - 2, \\ y' = 3x + 4y + 3. \end{cases}$

9. $\begin{cases} x' = 2x + 2y - \sin 2t, \\ y' = x + 3y. \end{cases}$

10. $\begin{cases} x' = -2x - 5y + e^t, \\ y' = x + 4y + 2. \end{cases}$

11. $\begin{cases} x' = 4x - 5y + e^{2t}, \\ y' = 2x - 2y. \end{cases}$

12. $\begin{cases} x' = x - y - e^{-t}, \\ y' = 2x + 3y + 2e^{-t}. \end{cases}$

13. $\begin{cases} x' = 2x + 5y, \\ y' = 2y + \sin t. \end{cases}$

14. $\begin{cases} x' = -x + 4y, \\ y' = x + 2y + e^t. \end{cases}$

15. $\begin{cases} x' = 4x - y + e^t, \\ y' = x + 2y + 2e^t. \end{cases}$

16. $\begin{cases} x' = -2x + 5y + 3, \\ y' = -x + 2y - 4. \end{cases}$

17. $\begin{cases} x' = -2x - 2y - \cos t, \\ y' = 4x + 2y + 2. \end{cases}$

18. $\begin{cases} x' = x + 2y - 2t, \\ y' = -5x - y + 3t. \end{cases}$

19. $\begin{cases} x' = 2x - 4y + e^{-2t}, \\ y' = 5x - 2y + 4. \end{cases}$

20. $\begin{cases} x' = 6x + 8y + e^{2t}, \\ y' = -2x - 2y - 3e^{2t}. \end{cases}$

21. $\begin{cases} x' = x - y - t - 2, \\ y' = 2x + 3y + 2t. \end{cases}$

22. $\begin{cases} x' = -3x + 4y + e^t, \\ y' = -5x + 5y + 3e^t. \end{cases}$

23. $\begin{cases} x' = 2x + 3y + \sin t, \\ y' = -3x + 2y + \cos t. \end{cases}$

24. $\begin{cases} x' = -6x - y + e^{-t}, \\ y' = 17x + 2y - e^{-t}. \end{cases}$

25. $\begin{cases} x' = -5x + 5y + t^2, \\ y' = -5x + y + t - 4. \end{cases}$

26. $\begin{cases} x' = 2x + 2y + t^2, \\ y' = -2x - 3y. \end{cases}$

27. $\begin{cases} x' = -x - 2y + e^t, \\ y' = x - 3y + t. \end{cases}$

28. $\begin{cases} x' = -3x - 2y + e^{-t}, \\ y' = 2x + y. \end{cases}$

29. $\begin{cases} x' = -3x - y - e^{-2t}, \\ y' = x - y + 1. \end{cases}$

30. $\begin{cases} x' = 3x + 17y + t^2, \\ y' = -2x - 3y - t. \end{cases}$

M58. Bir jinsli bo’lmagan differensial tenglamalar sistemasini yeching.

1. $\begin{cases} x' = x - 2y + 7\sin t, \\ y' = -2x + 2y - 2z + 11\cos t, \\ z' = -2y + 3z + \sin t + 2\cos t \end{cases}$

$(\lambda_1 = -1,\ \lambda_2 = 2,\ \lambda_3 = 5)$.

2. $\begin{cases} x' = 7x + y + z + te^t, \\ y' = x + 7y + z - 9e^t, \\ z' = x + y + 7z + (t+1)e^t \end{cases}$

$(\lambda_{1,2} = 6,\ \lambda_3 = 9)$.

3. $\begin{cases} x' = 3x + 2y + 5\sin 4t + 11\cos 4t, \\ y' = 2x + 4y - 2z + 3\cos 4t, \\ z' = -2y + 5z + 3\sin t \end{cases}$

$(\lambda_1 = 1,\ \lambda_2 = 4,\ \lambda_3 = 7)$.

4. $\begin{cases} x' = x + 2y + 2z + e^{2t}, \\ y' = 2x + y + 2z + 13e^{2t}, \\ z' = 2x + 2y + z - 12e^{2t} \end{cases}$

$(\lambda_1 = \lambda_2 = -1,\ \lambda_3 = 5)$.

5. $\begin{cases} x' = 5x - 2y - 2z, \\ y' = -2x + 6y + 4\sin 2t, \\ z' = -2x + 4z + 3\cos 2t \end{cases}$

$(\lambda_1 = 2,\ \lambda_2 = 5,\ \lambda_3 = 8)$.

6. $\begin{cases} x' = 17x - 2y - 2z + 5e^{9t}, \\ y' = -2x + 14y - 4z + 3e^{9t}, \\ z' = -2x - 4y + 14z \end{cases}$

$(\lambda_1 = 9,\ \lambda_{2,3} = 18)$

7. $\begin{cases} x' = 7x - 4y + 7\sin 5t, \\ y' = -4x + 5y + 4z, \\ z' = 4y + 3z + 5\cos 5t \end{cases}$

$(\lambda_1 = -1,\ \lambda_2 = 5,\ \lambda_3 = 11)$.

8. $\begin{cases} x' = 8x + 4y - z + 21e^{2t}, \\ y' = 4x - 7y + 4z + 19e^{2t}, \\ z' = -x + 4y + 8z + te^{2t} \end{cases}$

$(\lambda_1 = -9,\ \lambda_{2,3} = 9)$.

9. $\begin{cases} x' = 3x - 2z + 5\sin t + \cos t, \\ y' = y - 2z + 9\sin t - \cos t, \\ z' = -2x - 2y + 2z \end{cases}$

$(\lambda_1 = -1,\ \lambda_2 = 2,\ \lambda_3 = 5)$.

10. $\begin{cases} x' = 5x - 2z + te^{4t}, \\ y' = 3y + 2z + 15e^{4t}, \\ z' = -2x + 2y + 4z \end{cases}$

$(\lambda_1 = 1,\ \lambda_2 = 4,\ \lambda_3 = 7)$.

11. $\begin{cases} x' = 4x - 2y + 23\sin 2t, \\ y' = -2x + 5y - 2z, \\ z' = -2y + 6z + 3\cos 2t \end{cases}$

$(\lambda_1 = 2,\ \lambda_2 = 5,\ \lambda_3 = 8)$.

12. $\begin{cases} x' = 3x + 4z + t\sin t, \\ y' = 7y - 4z + 7\cos t, \\ z' = 4x - 4y + 5z \end{cases}$

$(\lambda_1 = -1,\ \lambda_2 = 5,\ \lambda_3 = 11)$.

13. $\begin{cases} x' = 2x - 2y - 2z + 11e^{-t}\sin t, \\ y' = -2x + 3y + 8e^{-t}\cos t, \\ z' = -2x + z + e^{-t}(\cos t + 6\sin t) \end{cases}$

$(\lambda_1 = -1,\ \lambda_2 = 2,\ \lambda_3 = 5)$.

14. $\begin{cases} x' = 4x - 2y + 2z + te^{2t}, \\ y' = -2x + 5y + 25e^{2t}, \\ z' = 2x + 3z \end{cases}$

$(\lambda_1 = 1,\ \lambda_2 = 4,\ \lambda_3 = 7)$.

15. $\begin{cases} x' = 6x - 2z + 5e^{2t}, \\ y' = 4y - 2z - 19e^{2t}, \\ z' = -2x - 2y + 5z \end{cases}$

$(\lambda_1 = 2,\ \lambda_2 = 5,\ \lambda_3 = 8)$.

16. $\begin{cases} x' = 5x + 4y - 4z, \\ y' = 4x + 3y + 5te^{t}, \\ z' = -4x + 7z + 19e^{t} \end{cases}$

$(\lambda_1 = -1,\ \lambda_2 = 5,\ \lambda_3 = 11)$.

17. $\begin{cases} x' = 5x - 2y + 9\cos 3t, \\ y' = -2x + 6y + 2z, \\ z' = 2y + 7z + 11\sin 3t \end{cases}$

$(\lambda_1 = 3,\ \lambda_2 = 6,\ \lambda_3 = 9)$.

18. $\begin{cases} x' = -6x - 2y + 13z + 14e^{-t}, \\ y' = -3x + y + 3z + 22e^{-t}, \\ z' = -4x - y + 8z \end{cases}$

$(\lambda_1 = \lambda_2 = \lambda_3 = 1)$.

19. $\begin{cases} x' = 2x - 10y + 2z + 3\sin t, \\ y' = -10x + 5y + 8z + 4\cos t, \\ z' = 2x + 8y + 11z + 22\sin t \end{cases}$

$(\lambda_{1,2} = \pm 9,\ \lambda_3 = 18)$.

20. $\begin{cases} x' = 8x - 4y - z + 5\sin t, \\ y' = 13x - 6y - 2z + 3\cos t, \\ z' = 3x - 3y + z \end{cases}$

$(\lambda_1 = \lambda_2 = \lambda_3 = 1)$.

21. $\begin{cases} x' = x - 3y + z + 8e^{3t}, \\ y' = -3x + y - z + 19e^{3t}, \\ z' = x - y + 5z \end{cases}$

$(\lambda_1 = -2,\ \lambda_2 = 3,\ \lambda_3 = 6)$.

22. $\begin{cases} x' = 7x + 2z + te^{3t}, \\ y' = 5y - 2z + (t+4)e^{3t}, \\ z' = 2x - 2y + 6z \end{cases}$

$(\lambda_1 = 3,\ \lambda_2 = 6,\ \lambda_3 = 9)$.

23. $\begin{cases} x' = x - 2y + 2z + 19\cos t, \\ y' = x + 4y - 2z + 6\sin t, \\ z' = x + 5y - 3z + 11\cos t \end{cases}$

$(\lambda_{1,2} = \pm 1,\ \lambda_3 = 2)$.

24. $\begin{cases} x' = 6x - 2y + 2z, \\ y' = -2x + 5y + 5\cos 6t, \\ z' = 2x + 7z + 9\sin 6t \end{cases}$

$(\lambda_1 = 3,\ \lambda_2 = 6,\ \lambda_3 = 9)$.

25. $\begin{cases} x' = x - 3y - z + te^{2t}, \\ y' = -3x + y + z + 7e^{2t}, \\ z' = -x + y + 5z + 11te^{2t} \end{cases}$

$\left(\lambda_1 = -2,\ \lambda_2 = 3,\ \lambda_3 = 6\right).$

26. $\begin{cases} x' = 4x + 2y + 2z, \\ y' = -2x + y + 17\cos 2t, \\ z' = 3y + 4z - 4\sin 2t, \end{cases}$

$\left(\lambda_1 = 1,\ \lambda_{2,3} = 4 \pm 2i\right).$

27. $\begin{cases} x' = -x + y - 2z, \\ y' = 4x + y + 12te^{t}, \\ z' = 2x + y - z \end{cases}$

$\left(\lambda_{1,2} = -1,\ \lambda_3 = 1\right).$

28. $\begin{cases} x' = 2x + y + z, \\ y' = 5y + 3z + 5\cos 5t, \\ z' = -4x + 5z - 41\sin 5t \end{cases}$

$\left(\lambda_1 = 2,\ \lambda_{2,3} = 5 \pm 2i\right).$

29. $\begin{cases} x' = 7x - 7y + 6z + 11e^{3t}, \\ y' = 8x - 7y + 4z, \\ z' = 4x - 3y + z + 5e^{3t} \end{cases}$

$\left(\lambda_{1,2} = -1,\ \lambda_3 = 3\right).$

30. $\begin{cases} x' = y + 2z + 5e^{-t}\cos t, \\ y' = x + 2z + 3e^{-t}\sin t, \\ z' = -x - y - 3z \end{cases}$

$\left(\lambda_{1,2,3} = -1\right).$

4.3. O'ZGARMASLARNI VARIATSIYALASH USULI

4.2-bandda keltirilgan tushunchalar va belgilashlardan foydalanamiz. Faqat shuni qo'shimcha qilish mumkinki,

$$\frac{dx_k}{dt} = \sum_{j=1}^{n} a_{kj}(t)x_j + f_k(t),\ k = \overline{1,n} \tag{1}$$

ko'rinishdagi tenglamalar sistemasi yoki

$$\dot{x} = A(t)x + f \tag{1'}$$

vektor tenglama *bir jinsli bo'lmagan o'zgaruvchi koeffitsientli chiziqli differensial tenglamalar sistemasi* deyiladi. Bunga mos kelgan bir jinsli sistema

$$\frac{dx_k}{dt} = \sum_{j=1}^{n} a_{kj}(t)x_j,\ k = \overline{1,n}, \tag{2}$$

yoki

$$\dot{x} = A(t)x \tag{2'}$$

vektor ko’rinishida bo’ladi. O’zgarmaslarni variatsiyalash usulini ikkita bosqichda amalga oshiriladi.

1-bosqich. (2) ko’rinishdagi bir jinsli tenglamalar sistemasining

$$x_k = C_1 x_{k1}(t) + C_2 x_{k2}(t) + ... + C_n x_{kn}(t), \; k = \overline{1,n} \tag{3}$$

umumiy yechimi topib olinadi.

2-bosqich. (1) sistemaning umumiy yechimi

$$x_k = C_1(t) x_{k1}(t) + C_2(t) x_{k2}(t) + ... + C_n(t) x_{kn}(t), \; k = \overline{1,n} \tag{4}$$

ko’rinishda izlanadi. (4) formuladagi $C_i(t)$ funksiyalarni topish uchun

$$C_1'(t) x_{k1}(t) + C_2'(t) x_{k2}(t) + ... + C_n'(t) x_{kn}(t) = f_k(t), \quad k = \overline{1,n}$$

tenglamalar sistemasini yechish kerak.

Xuddi shu kabi, (1') va (2') vektor tenglamalarning umumiy yechimlari, mos ravishda,

$$x(t) = X(t)C, \tag{3'}$$

$$x(t) = X(t)C + \overline{x}(t) \tag{4'}$$

ko’rinishlarda bo’ladi, bu yerda $X(t)$ – (2) tenglamaning fundamental matritsasi, C – ixtiyoriy o’zgarmas vektor, $\overline{x}(t)$ esa (1') tenglamaning biron bir xususiy yechimi (vektor-funksiya).

Agar (2') tenglamaning fundamental yechimi ma`lum bo’lsa, u holda (1') tenglamaning $\overline{x}(t)$ xususiy yechimini ixtiyoriy o’zgarmas C vektorni variatsiyalash usulida topish mumkin. Bu vektor

$$X(t)C'(t) = f(t) \tag{5}$$

tenglamani qanoatlantiradi. $\det X(t) = W(t) \neq 0$ bo’lgani uchun har doim $\left(X(t)\right)^{-1}$ teskari matritsa mavjud va $C'(t) = X^{-1}(t) f(t)$ bo’ladi. Bundan

$$C(t) = \int X^{-1}(t) f(t) dt + C_0, \qquad (6)$$

bu yerda C_0 − ixtiyoriy o'zgarmas vektor. (6) ni (3') ga qo'yib, topamiz:

$$x(t) = X(t)C_0 + X(t)\int X^{-1}(t) f(t) dt. \qquad (7)$$

Endi (4') va (7) munosabatlardan

$$\overline{x}(t) = X(t)\int X^{-1}(t) f(t) dt$$

ekanligini ko'rish qiyin emas .

172. Tenglamalar sistemasini yeching:

$$\dot{x} = y + tg^2 t - 1, \dot{y} = -x + tg\, t.$$

◀ Berilgan tenglamalar sistemasiga mos

$$\dot{x} = y, \ \dot{y} = -x \qquad (8)$$

bir jinsli sistemaning umumiy yechimini

$$\begin{pmatrix} x \\ y \end{pmatrix} = X(t)\begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

ko'rinishda topish qiyin emas, bu yerda

$$X(t) = \begin{pmatrix} \cos t \ \sin t \\ -\sin t \ \cos t \end{pmatrix},$$

$X(t)-$ (8) bir jinsli sistemaning fundamental matritsasi.

Endi $X(t)$ fundamental matritsaga teskari bo'lgan

$$X^{-1}(t) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

matritsani topamiz. Shartga ko'ra, $X(t)X^{-1}(t) = E$, ya`ni

$$a\cos t + c\sin t = 1, \qquad -a\sin t + c\cos t = 0,$$

$$b\cos t + d\sin t = 0, \ -b\sin t + d\cos t = 1$$

bo'lishi kerak. Bu oxirgi sistemani yechib, $X^{-1}(t)$ matritsani yozamiz:

$$X^{-1}(t)=\begin{pmatrix}\cos t & -\sin t\\ \sin t & \cos t\end{pmatrix}.$$

Dastlabki berilgan bir jinsli bo'lmagan sistemaning $\overline{x}$ xususiy yechimini topish maqsadida

$$X^{-1}(t)f(t)\equiv\begin{pmatrix}\cos t & -\sin t\\ \sin t & \cos t\end{pmatrix}\begin{pmatrix}tg^2t-1\\ tg\ t\end{pmatrix}=\begin{pmatrix}-\cos t\\ \sin t-tg^2t\end{pmatrix}$$

ifodani integrallaymiz:

$$\int X^{-1}(t)f(t)dt=\begin{pmatrix}-\sin t\\ \cos t+\dfrac{1}{\cos t}\end{pmatrix}$$

va xususiy yechimni yozamiz:

$$\overline{x}(t)\equiv X(t)\int X^{-1}(t)f(t)dt=\begin{pmatrix}tgt\\ 2\end{pmatrix}.$$

Shunday qilib, berilgan bir jinsli bo'lmagan sistemaning umumiy yechimi

$$\begin{pmatrix}x(t)\\ y(t)\end{pmatrix}=\begin{pmatrix}\cos t & \sin t\\ -\sin t & \cos t\end{pmatrix}\begin{pmatrix}C_1\\ C_2\end{pmatrix}+\begin{pmatrix}tg\ t\\ 2\end{pmatrix},$$

ya`ni

$$x(t)=C_1\cos t+C_2\sin t+tgt,\ \ y(t)=-C_1\sin t+C_2\cos t+2$$

ko'rinishda topiladi.►

173. Tenglamalar sistemasini yeching:

$$\dot{x}=-4x-2y+\frac{2}{e^t-1},\ \ \dot{y}=6x+3y-\frac{3}{e^t-1}. \qquad (9)$$

◄ (9) tenglamalar sistemasiga mos

$$\dot{x}=-4x-2y,\ \dot{y}=6x+3y \qquad (10)$$

bir jinsli sistemaning umumiy yechimini

$$x(t) = C_1 + 2C_2 e^{-t}, \ \ y(t) = -2C_1 - 3C_2 e^{-t} \tag{11}$$

ko'rinishda topish qiyin emas.

Bir jinsli bo'lmagan (9) sistemaning umumiy yechimini topish uchun (10) sistemaning (11) umumiy yechimida o'zgarmaslarni variatsiyalaymiz, ya`ni (9) sistemaning umumiy yechimini

$$x(t) = C_1(t) + 2C_2(t)e^{-t}, \ \ y(t) = -2C_1(t) - 3C_2(t)e^{-t} \tag{12}$$

ko'rinishda izlaymiz. (12) ifodalarni (9) sistemaga qo'yib,

$$C_1'(t) + 2C_2'(t)e^{-t} = \frac{2}{e^t - 1}, \ -2C_1'(t) - 3C_2'(t)e^{-t} = -\frac{3}{e^t - 1} \tag{13}$$

tenglamalar sistemasini olamiz. Bu sistemani $C_1'(t)$ va $C_2'(t)$ noma`lumlarga nisbatan yechib, topamiz:

$$C_1'(t) = 0, \ \ C_2'(t) = \frac{e^t}{e^t - 1},$$

ya`ni

$$C_1(t) = C_1, \ \ C_2(t) = \ln\left|e^t - 1\right| + C_2.$$

$C_1(t)$ va $C_2(t)$ funksiyalarning ifodalarini (12) ga qo'yib, bir jinsli bo'lmagan (9) sistemaning umumiy yechimini topamiz:

$$x = C_1 + 2C_2 e^{-t} + 2e^{-t}\ln\left|e^t - 1\right|,$$
$$y = -2C_1 - 3C_2 e^{-t} - 3e^{-t}\ln\left|e^t - 1\right|. \blacktriangleright$$

INDIVIDUAL TOPSHIRIQLAR

M59. Bir jinsli bo'lmagan differensial tenglamalar sistemasini o'zgarmaslarni variatsiyalash usulida yeching.

1. $\begin{cases} x' = x + y + ctgt, \\ y' = -2x - y + \sec t. \end{cases}$

2. $\begin{cases} x' = x - 2y + \csc t, \\ y' = x - y + tgt. \end{cases}$

3. $\begin{cases} x' = x - y + \dfrac{1}{1+e^t}, \\ y' = y - 4x + \dfrac{1}{1+e^t}. \end{cases}$

4. $\begin{cases} x' = x + 3y + \dfrac{1}{e^{2t}+1}, \\ y' = x + y - \dfrac{2}{e^{2t}+1}. \end{cases}$

5. $\begin{cases} x' = y + tht, \\ y' = 2x + y + ctht. \end{cases}$

6. $\begin{cases} x' = x - y + \sec t, \\ y' = 2x - y + \csc t. \end{cases}$

7. $\begin{cases} x' = 4x + y + \dfrac{e^{3t}}{1+e^t}, \\ y' = -2x + y - \dfrac{1}{1+e^t}. \end{cases}$

8. $\begin{cases} x' = -x + 2y + \dfrac{e^t}{t}, \\ y' = -2x + 3y + \sqrt{t}e^t. \end{cases}$

9. $\begin{cases} x' = 2x - 2y + e^t \sec t, \\ y' = x + e^{-t} \csc t. \end{cases}$

10. $\begin{cases} x' = 2x - 3y + \csc t, \\ y' = x - 2y - \sec t. \end{cases}$

11. $\begin{cases} x' = 2x - y + \dfrac{e^{3t}}{t}, \\ y' = -2x + y - \dfrac{e^{-3t}}{t}. \end{cases}$

12. $\begin{cases} x' = -2x + y + \dfrac{e^t}{t}, \\ y' = -x + \dfrac{e^{-t}}{t+1}. \end{cases}$

13. $\begin{cases} x' = 2x - 4y + tg2t, \\ y' = 2x - 2y + \csc 2t. \end{cases}$

14. $\begin{cases} x' = x - y + \sec 2t, \\ y' = 5x - y - ctg2t. \end{cases}$

15. $\begin{cases} x' = 2x - y + \dfrac{e^{2t}}{e^t+1}, \\ y' = -x + 2y + \dfrac{e^{2t}}{e^t-1}. \end{cases}$

16. $\begin{cases} x' = 3x - 2y + \dfrac{e^t}{1+\sqrt{t}}, \\ y' = 2x - y + \dfrac{e^t}{\sqrt{t}}. \end{cases}$

17. $\begin{cases} x' = 3x - 2y + e^t tg2t, \\ y' = 4x - y - e^t \csc 2t. \end{cases}$

18. $\begin{cases} x' = x + 2y + \sec t, \\ y' = -x - y - \cos t \cdot ctgt. \end{cases}$

19. $\begin{cases} x' = 3x - y + \dfrac{e^t}{e^{2t}-1}, \\ y' = 2x + 2y + \dfrac{1}{e^t+1}. \end{cases}$

20. $\begin{cases} x' = -2x - 4y + \dfrac{t}{\sqrt{t+1}}, \\ y' = x + 2y - \dfrac{2}{t}. \end{cases}$

21. $\begin{cases} x' = x + 2y + \dfrac{e^{-t}}{t^2+1}, \\ y' = -2x - 3y - \dfrac{e^{-t}}{t}. \end{cases}$ 22. $\begin{cases} x' = -x - 2y + \dfrac{e^{2t}}{e^t-1}, \\ y' = 3x + 4y + \dfrac{e^t}{e^t+1}. \end{cases}$

23. $\begin{cases} x' = 2x - 3y + 2e^{2t}\sec 3t, \\ y' = 3x + 2y + e^{2t} ctg 3t. \end{cases}$ 24. $\begin{cases} x' = 4x - y + \dfrac{e^{3t}}{\sqrt{t^2+1}}, \\ y' = x + 2y - \dfrac{e^{3t}}{t}. \end{cases}$

25. $\begin{cases} x' = 2x - y - e^{2t} tgt, \\ y' = x + 2y + e^{2t} tgt. \end{cases}$ 26. $\begin{cases} x' = x + 2y + 8 + e^{-t}\ln t, \\ y' = 3x + 2y + 12 - e^{-t}\ln t. \end{cases}$

27. $\begin{cases} x' = x - y + \sec t, \\ y' = 2x - y. \end{cases}$ 28. $\begin{cases} x' = y + tg^2 t - 1, \\ y' = -x + tgt. \end{cases}$

29. $\begin{cases} x' = 3x + y + t^{-1} - 4\ln t, \\ y' = -x + y + t^{-1}. \end{cases}$ 30. $\begin{cases} x' = 3x - 2y, \\ y' = 2x - y + 15e^t\sqrt{t}. \end{cases}$

Belgilashlar:

$\csc x = \dfrac{1}{\sin x}, \ \sec x = \dfrac{1}{\cos x}$ - kosekans va sekans funksiyalar;

$th\, x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}, \ cth\, x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$ - tangens giperbolik va kotangens giperbolik funksiyalar.

4-BOBNI TAKRORLASHGA DOIR ARALASH MASALALAR

Tenglamalar sistemalarini yeching:

1. $\begin{cases} y' = \dfrac{y^2}{z-x}, \\ z' = y + 1. \end{cases}$ **2.** $\begin{cases} y' = \dfrac{z}{x}, \\ z' = \dfrac{z(y+2x-1)}{x(y-1)}. \end{cases}$

3. $\begin{cases} \dfrac{dx}{dt} = x + y, \\ 2x\dfrac{dy}{dt} = y^2 - x^2 + 1. \end{cases}$

4. $\dfrac{dx}{xz} = \dfrac{dy}{yz} = \dfrac{dz}{-xy}.$

5. $\dfrac{dx}{z} = \dfrac{dy}{u} = \dfrac{dz}{x} = \dfrac{du}{y}.$

6. $\dfrac{dx}{x(y+z)} = \dfrac{dy}{z(z-y)} = \dfrac{dz}{y(y-z)}.$

7. Quyidagi $\phi = C$ munosabatlardan qaysilari

$$\frac{dx}{dt} = \frac{x^2 - t}{y}, \quad \frac{dy}{dt} = -x$$

tenglamalar sistemasining birinchi integrali bo'lishi yoki bo'lmasligini aniqlang:

a) $\varphi = t^2 + 2xy,$ b) $\varphi = x^2 - ty$.

8. $\dfrac{dx}{x} = \dfrac{dy}{y} = \dfrac{dz}{z}$ sistemaning $\dfrac{x+y}{z+x} = C_1,$ $\dfrac{z-y}{x+y} = C_2$ ko'rinishdagi birinchi integrallarini bog'liqsizlikka tekshiring.

9-24 masalalarda berilgan tenglamalar sistemalarini yeching (Hisoblashni soddalashtirish maqsadida ba`zi masalalarda xarakteristik tenglamaning ildizlari ko'rsatilgan).

9. $\begin{cases} \dfrac{dx}{dt} = 2y, \\ \dfrac{dy}{dt} = 2x. \end{cases}$

10. $\begin{cases} \dfrac{dx}{dt} = 2x + 5y, \\ \dfrac{dy}{dt} = -x + 4y. \end{cases}$

11. $\begin{cases} \dot{x} = x - y, \\ \dot{y} = y - 4x. \end{cases}$

12. $\begin{cases} \dot{x} = x + y, \\ \dot{y} = 3y - 2x. \end{cases}$

13. $\begin{cases} \dot{x} + x - 8y = 0, \\ \dot{y} - x - y = 0. \end{cases}$

14. $\begin{cases} \dot{x} = x - 3y, \\ \dot{y} = 3x + y. \end{cases}$

15. $\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = 4y - x. \end{cases}$

16. $\begin{cases} \dot{x} = 3x - y, \\ \dot{y} = 4x - y. \end{cases}$

17. $\begin{cases} \dot{x} = 2y - 3x, \\ \dot{y} = y - 2x. \end{cases}$

18. $\begin{cases} \dot{x} - 5x - 3y = 0, \\ \dot{y} + 3x + y = 0. \end{cases}$

19. $\begin{cases} \dot{x} = 3x - y + z, \\ \dot{y} = x + y + z, \\ \dot{z} = 4x - y + 4z \end{cases}$

$(\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 5)$.

20. $\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = x + 3y - z, \\ \dot{z} = 2y + 3z - x \end{cases}$

$(\lambda_1 = 2,\ \lambda_{2,3} = 3 \pm i)$.

21. $\begin{cases} \dot{x} = 2x + 2z - y, \\ \dot{y} = x + 2z, \\ \dot{z} = y - 2x - z \end{cases}$

$(\lambda_1 = 1, \lambda_{2,3} = \pm i)$.

22. $\begin{cases} \dot{x} = 3x - 2y - z, \\ \dot{y} = 3x - 4y - 3z, \\ \dot{z} = 2x - 4y \end{cases}$

$(\lambda_1 = \lambda_2 = 2, \lambda_3 = -5)$.

23. $\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = 2y + 4z, \\ \dot{z} = x - z \end{cases}$

$(\lambda_1 = \lambda_2 = 0, \lambda_3 = 3)$.

24. $\begin{cases} \dot{x} = 2x - y - z, \\ \dot{y} = 2x - y - 2z, \\ \dot{z} = 2z - x + y \end{cases}$

$(\lambda_1 = \lambda_2 = \lambda_3 = 1)$.

25-28 masalalarda normal ko’rinishga keltirilmagan sistemalarni yeching.

25. $\begin{cases} \ddot{x} = 2y, \\ \ddot{y} = -2x. \end{cases}$

26. $\begin{cases} \ddot{x} - 2\ddot{y} + \dot{y} + x - 3y = 0, \\ 4\ddot{y} - 2\ddot{x} - \dot{x} - 2x + 5y = 0. \end{cases}$

27. $\begin{cases} \ddot{x} - 2\dot{y} + 2x = 0, \\ 3\dot{x} + \ddot{y} - 8y = 0. \end{cases}$

28. $\begin{cases} 2\ddot{x} + 2\dot{x} + x + 3\ddot{y} + \dot{y} + y = 0, \\ \ddot{x} + 4\dot{x} - x + 3\ddot{y} + 2\dot{y} - y = 0. \end{cases}$

29-36 masalalarda vektor yozuvda berilgan sistemalarni yeching.

29. $\dot{x} = Ax,\ A = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

30. $\dot{x} = Ax,\ A = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$.

31. $\dot{x} = Ax,\ A = \begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix}$.

32. $\dot{x} = Ax,\ A = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$.

33. $\dot{x} = Ax,\ A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 3 & -1 & -2 \end{pmatrix}.$ **34.** $\dot{x} = Ax,\ A = \begin{pmatrix} 3 & -3 & 1 \\ 3 & -2 & 2 \\ -1 & 2 & 0 \end{pmatrix}.$

35. $\dot{x} = Ax,\ A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{pmatrix}.$ **36.** $\dot{x} = Ax,\ A = \begin{pmatrix} 4 & 2 & -2 \\ 1 & 3 & -1 \\ 3 & 3 & -1 \end{pmatrix}.$

37-39 masalalarda berilgan A matritsaning e^A eksponensial matritsasini toping:

37. $A = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}.$ **38.** $A = \begin{pmatrix} -2 & -4 \\ 1 & 2 \end{pmatrix}.$ **39.** $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$

40. Agar

$$A = \begin{pmatrix} 1 & 0 & 3 \\ -1 & 2 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

matritsa berilgan bo'lsa, e^A matritsani hisoblamasdan $\det e^A$ ni toping.

41. Fundamental matritsasi

$$\begin{pmatrix} e^t \cos t & e^t \sin t \\ -\sin t & \cos t \end{pmatrix}$$

ko'rinishda bo'lgan chiziqli bir jinsli differensial tenglamalar sistemasini tuzing.

42-45 masalalarda bir jinsli bo'lmagan chiziqli sistemalarni yeching.

42. $\begin{cases} \dot{x} = 2x - 4y + 4e^{-2t}, \\ \dot{y} = 2x - 2y. \end{cases}$ **43.** $\begin{cases} \dot{x} = 2x + y + e^t, \\ \dot{y} = -2x + 2t. \end{cases}$

44. $\begin{cases} \dot{x} = x - y + 2\sin t, \\ \dot{y} = 2x - y. \end{cases}$ **45.** $\begin{cases} \dot{x} = 2x - y, \\ \dot{y} = 2y - x - 5e^t \sin t. \end{cases}$

46-47 masalalarda berilgan sistemalarni o’zgarmaslarni variatsiyalash usulida yeching.

46. $\begin{cases} \dot{x} = 2y - x, \\ \dot{y} = 4y - 3x + \dfrac{e^{3t}}{e^{2t}+1}. \end{cases}$ **47.** $\begin{cases} \dot{x} = x - y + \dfrac{1}{\cos t}, \\ \dot{y} = 2x - y. \end{cases}$

48. Massasi m bo’lgan jism xOy tekislikda $(0,0)$ nuqtaga a^2mr kuch bilan tortilib harakatlanmoqda, bu yerda r - shu nuqtagacha bo’lgan masofa. $x(0) = d$, $y(0) = 0, \dot{x}(0) = 0,$ $\dot{y}(0) = v$ boshlang’ich shartlarda jismning harakatini va bu harakatning traektoriyasini toping.

5-BOB
BIRINCHI TARTIBLI XUSUSIY HOSILALI DIFFERENSIAL TENGLAMALAR

5.1. CHIZIQLI DIFFERENSIAL TENGLAMALAR

5.1.1. Bir jinsli chiziqli tenglamalar. Ushbu

$$X_1(x_1, x_2, ..., x_n)\frac{\partial u}{\partial x_1} + X_2(x_1, x_2, ..., x_n)\frac{\partial u}{\partial x_2} + \\ ... + X_n(x_1, x_2, ..., x_n)\frac{\partial u}{\partial x_n} = 0 \quad (1)$$

tenglamani o’rganamiz. (1) tenglama *birinchi tartibli xususiy hosilali bir jinsli chiziqli tenglama* deyiladi. (1) tenglamaning $X_1, X_2, ..., X_n$ koeffitsientlari berilgan $\left(x_1^0, x_2^0, ..., x_n^0\right)$ nuqtaning biror atrofida aniqlangan, o’zlarining birinchi tartibli hosilalari bilan uzluksiz hamda bir vaqtda nolga aylanmaydi, deb faraz qilamiz. Masalan, aniqlik uchun $X_n\left(x_1^0, x_2^0, ..., x_n^0\right) \neq 0$ deb hisoblashimiz mumkin.

(1) tenglama bilan bir qatorda ushbu

$$\frac{dx_1}{X_1(x_1, x_2, ..., x_n)} = \frac{dx_2}{X_2(x_1, x_2, ..., x_n)} = ... = \frac{dx_n}{X_n(x_1, x_2, ..., x_n)} \quad (2)$$

simmetrik shakldagi oddiy differensial tenglamalar sistemasini tekshiramiz. Bu sistema *birinchi tartibli xususiy hosilali chiziqli bir jinsli tenglamaga mos simmetrik ko’rinishdagi oddiy differensial tenglamalar sistemasi* deyiladi. $X_1, X_2, ..., X_n$ koeffitsientlarga nisbatan yuqorida qo’yilgan shartlarga asosan (2) sistema $(n-1)$ ta bog’liqsiz birinchi integrallarga ega:

$$\psi_1(x_1, x_2, ..., x_n) = C_1,\ ..., \psi_{n-1}(x_1, x_2, ..., x_n) = C_{n-1}. \tag{3}$$

Buning to'g'riligi (2) sistemaning ushbu $(n-1)$ ta

$$\frac{dx_1}{dx_n} = \frac{X_1}{X_n},\ \ \frac{dx_2}{dx_n} = \frac{X_2}{X_n},\ ..., \frac{dx_{n-1}}{dx_n} = \frac{X_{n-1}}{X_n} \tag{4}$$

tenglamalarning normal sistemasiga teng kuchliligidan kelib chiqadi Birinchi integrallarning (3) sistemasi $x_1, x_2, ..., x_n$ o'zgaruvchilarning fazosida $(n-1)$ ta parametrli chiziqlar oilasini aniqlaydi. Bu chiziqlar (1) tenglamaning *xarakteristikalari* deyiladi.

(2) sistema ixtiyoriy $\psi(x_1, x_2, ..., x_n) = C$ birinchi integralining chap qismi xususiy hosilali (1) tenglamaning yechimidan iborat bo'ladi. Bundan tashqari, (1) tenglamani qanoatlantiradigan ixtiyoriy $\psi(x_1, x_2, ..., x_n)$ funksiyani o'zgarmas songa tenglashtirilsa, (2) sistemaning birinchi integrali hosil bo'ladi.

Aytaylik, (2) sistemaning chiziqli bog'liqsiz integrallari

$$\psi_1(x_1, x_2, ..., x_n),\ \psi_2(x_1, x_2, ..., x_n),\ ..., \psi_{n-1}(x_1, x_2, ..., x_n) \tag{5}$$

ko'rinishda topilgan bo'lsin. U holda

$$u = \Phi(\psi_1, \psi_2, ..., \psi_{n-1}) \tag{6}$$

funksiya (shu jumladan $\Phi = const$ ham) (1) tenglamaning yechimi bo'ladi, bu yerda $\Phi -$ ixtiyoriy funksiya bo'lib, $\psi_1, \psi_2, ..., \psi_{n-1}$ bo'yicha uzluksiz hosilalarga ega. (6) ifoda (4) tenglamaning *umumiy yechimi* deyiladi.

Ma`lumki, oddiy differensial tenglamalarning umumiy yechimida ixtiyoriy o'zgarmaslar qatnashadi. Xususiy hosilali (1) tenglamaning (6) umumiy yechimi esa ixtiyoriy o'zgarmaslarni emas, balki ixtiyoriy funksiyalarni o'z ichiga olganligiga e`tibor berish kerak.

Shunday qilib, (1) tenglamaning umumiy yechimini topish masalasi (1) tenglamaga mos simmetrik shakldagi (2) oddiy differensial tenglamalar sistemasining $(n-1)$ ta bog'liqsiz integrallarini topish masalasiga teng kuchli ekan.

Ikkita erkli o'zgaruvchili holni qaraylik. Bu holda, noma`lum funksiyani z, erkli o'zgaruvchilarni x va y bilan belgilab, (1) tenglamaning o'rniga

$$X(x,y)\frac{\partial z}{\partial x}+Y(x,y)\frac{\partial z}{\partial y}=0 \tag{7}$$

tenglamaga ega bo'lamiz. Oddiy differensial tenglamalarning simmetrik shakldagi (2) sistemasi bu holda bitta

$$\frac{dx}{X(x,y)}=\frac{dy}{Y(x,y)} \tag{8}$$

differensial tenglamaga aylanadi. Agar $\psi(x,y)$ funksiya shu tenglamaning integrali bo'lsa, u holda

$$z=\Phi[\psi(x,y)] \tag{9}$$

funksiya (bunda $\Phi(\psi)$ funksiya ψ ning ixtiyoriy uzluksiz differensiallanuvchi funksiyasi) (7) tenglamaning umumiy yechimi bo'ladi.

Agar x, y va z uch o'lchovli fazodagi nuqtaning to'g'ri burchakli koordinatalari sifatida qaralsa, u holda (7) tenglamaning $z=z(x,y)$ yechimiga biror sirt mos keladi. Bu sirt (7) tenglamaning *integral sirti* deyiladi.

(1) chiziqli bir jinsli tenglama uchun Koshi masalasi quyidagicha qo'yiladi. (1) tenglamaning hamma yechimlari ichidan

$$u\Big|_{x=x_n^{(0)}}=\varphi(x_1,x_2,\ ...,x_{n-1}) \tag{10}$$

boshlang'ich shartni qanoatlantiradigan

$$u=f(x_1,\ x_2,\ ...,\ x_n) \tag{11}$$

yechimni topish kerak, bu yerda φ - berilgan uzluksiz differensialla-nuvchi funksiya.

Izlanayotgan funksiya ikki o'zgaruvchili bo'lgan holda, ya`ni (7) tenglama uchun Koshi masalasi $x=x_0$ da

$$z=\varphi(y) \tag{12}$$

boshlang'ich shartni qanoatlantiradigan

$$z=f(x,y) \tag{13}$$

yechimni topishdan iboratdir, bu yerda φ- berilgan funksiya. Geometrik nuqtai nazardan bu shuni anglatadiki, (7) tenglamadan topiladigan barcha integral sirtlar ichidan yOz tekislikka parallel bo'lgan $x=x_0$ tekislikda yotuvchi (12) egri chiziq orqali o'tuvchi (13) integral sirtni topish kerak.

Xususiy hosilali tenglama uchun qo'yilgan Koshi masalasining oddiy differensial tenglama uchun qo'yilgan Koshi masalasidan farqi shundaki, oddiy tenglama uchun Koshi masalasi berilgan nuqta orqali o'tuvchi integral egri chiziqni topishdan iborat bo'lgani holda, xususiy hosilali (7) tenglama uchun Koshi masalasi berilgan egri chiziq orqali o'tuvchi integral sirtni topishdan iboratdir.

(1) tenglamaning umumiy yechimi

$$u=\Phi(\psi_1,\psi_2,...,\psi_{n-1})$$

ko'rinishdagi (6) formula bilan berilishi ma`lum. Bu formulani (10) boshlang'ich shart bilan taqqoslab,

$$\Phi(\psi_1,\psi_2,...,\psi_{n-1})\Big|_{x=x_n^{(0)}}=\varphi(x_1,x_2,...,x_{n-1}) \tag{14}$$

shartni qanoatlantiradigan Φ funksiyani topish kerakligini ko'ramiz.

Agar

$$\psi_1(x_1,x_2,...,x_{n-1},x_n^{(0)})=\overline{\psi}_1,\quad ...,\ \psi_{n-1}(x_1,x_2,...,x_{n-1},x_n^{(0)})=\overline{\psi}_{n-1} \tag{15}$$

belgilashlar kiritsak, u holda (14) tenglik quyidagicha yoziladi:

$$\Phi\left(\overline{\psi}_1,\overline{\psi}_2,...,\overline{\psi}_{n-1}\right)=\varphi(x_1,x_2,...,x_{n-1}). \tag{16}$$

Agar $X_n\left(x_1^{(0)},x_2^{(0)},...,x_n^{(0)}\right)\neq 0$ deb faraz qilsak, u holda (15) sistema $x_1,x_2,...,x_{n-1}$ ga nisbatan (hech bo'lmaganda $\left(x_1^{(0)},x_2^{(0)},...,x_n^{(0)}\right)$ nuqtaning atrofida) yechiladi. (15) sistemani $x_1, x_2, ..., x_{n-1}$ ga nisbatan yechib, topamiz:

$$x_1 = \omega_1\left(\overline{\psi}_1, \overline{\psi}_2, ..., \overline{\psi}_{n-1}\right), \quad ..., \quad x_{n-1} = \omega_{n-1}\left(\overline{\psi}_1, \overline{\psi}_2, ..., \overline{\psi}_{n-1}\right).$$

Endi Φ funksiya sifatida

$$\Phi(\psi_1, \psi_2, ..., \psi_{n-1}) = \varphi\left[\omega_1(\psi_1, \psi_2, ..., \psi_{n-1}), ..., \omega_n(\psi_1, \psi_2, ..., \psi_{n-1})\right]$$

funksiyani olsak, (16) shart bajariladi.

Shunday qilib,

$$u = \varphi\left[\omega_1(\psi_1, \psi_2, ..., \psi_{n-1}), ..., \omega_n(\psi_1, \psi_2, ..., \psi_{n-1})\right]$$

funksiya Koshi masalasining yechimini beradi. Bu yerdagi φ funksiya (10) boshlang'ich shartda qatnashayotgan funksiyadir.

174. Quyida $z = z(x, y)$ funksiyalar berilgan:

$$1)\ z = x + \ln(y - e^x);\ \ 2)\ z = ye^x + e^{2x};\ \ 3)\ z = \sin\left(ye^x - e^{2x}\right).$$

Bu funksiyalar $y > e^x$ sohada $\frac{\partial z}{\partial x} + \left(2e^x - y\right)\frac{\partial z}{\partial y} = 0$ tenglamaning yechimi bo'lishi yoki bo'lmasligini aniqlang.

◄ $z = z(x, y)$ funksiyaning xususiy hosilalarini hisoblaymiz.

$$\frac{\partial z}{\partial x} = 1 - \frac{e^x}{y - e^x}, \qquad \frac{\partial z}{\partial y} = \frac{1}{y - e^x}.$$

Shu tengliklarga ko'ra topamiz:

$$\frac{\partial z}{\partial x} + \left(2e^x - y\right)\frac{\partial z}{\partial y} = 1 - \frac{e^x}{y - e^x} + \left(2e^x - y\right)\cdot\frac{1}{y - e^x} \equiv 0,$$

ya`ni $z = x + \ln(y - e^x)$ funksiya berilgan tenglamaning yechimi.

2) Xususiy hosilalarni hisoblaymiz:

$$\frac{\partial z}{\partial x} = ye^x + 2e^{2x}, \qquad \frac{\partial z}{\partial y} = e^x.$$

Bu ifodalarni tenglamaga qo'yamiz:

$$\frac{\partial z}{\partial x}+\left(2e^x-y\right)\frac{\partial z}{\partial y}=ye^x+2e^{2x}+\left(2e^x-y\right)\cdot e^x=4e^{2x}\neq 0.$$

Shunga ko'ra, $z=ye^x+e^{2x}$ funksiya berilgan tenglamaning yechimi emas.

3) $z=\sin\left(ye^x-e^{2x}\right)$ funksiyaning xususiy hosilalarini topamiz:

$$\frac{\partial z}{\partial x}=\cos\left(ye^x-e^{2x}\right)\cdot\left(ye^x-2e^{2x}\right),\ \ \frac{\partial z}{\partial y}=\cos\left(ye^x-e^{2x}\right)\cdot e^x.$$

Bu yerdan

$$\frac{\partial z}{\partial x}+\left(2e^x-y\right)\frac{\partial z}{\partial y}=\cos\left(ye^x-e^{2x}\right)\left(ye^x-2e^{2x}+(2e^x-y)e^x\right)\equiv 0.$$

Demak, $z=\sin\left(ye^x-e^{2x}\right)$ funksiya berilgan tenglamaning yechimi.►

Tenglamalarning umumiy yechimini toping (**175-178**).

175. $y\frac{\partial z}{\partial x}-x\frac{\partial z}{\partial y}=0$.

◄ Bu tenglamaga mos oddiy differensial tenglamalar sistemasi bu holda bitta tenglamadan iborat:

$$\frac{dx}{y}=-\frac{dy}{x}.$$

Bu tenglamaning birinchi integrali $x^2+y^2=C$. Demak, berilgan tenglamaning umumiy yechimi $z=\varphi\left(x^2+y^2\right)$ (bunda $\varphi-$ixtiyoriy funksiya) bo'lib, aylanish o'qi Oz koordinatalar o'qidan iborat bo'lgan aylanma sirtlardir.►

176. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$. (17)

◀Bu tenglamaga mos oddiy differensial tenglamalar sistemasini tuzamiz:

$$\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}.$$

So’ngra $\frac{dx}{x}=\frac{dy}{y}$ va $\frac{dx}{x}=\frac{dz}{z}$ tenglamalarni yechib, bu yerdan ikkita birinchi integrallarni topamiz: $\frac{y}{x}=C_1$, $\frac{z}{x}=C_2$. Bu birinchi integrallarni bog’liqsizlikka tekshiramiz. Buning uchun $\Phi_1(x,y,z)=\frac{y}{x}$ va $\Phi_2(x,y,z)=\frac{z}{x}$ belgilashlarni kiritib,

$$\frac{D(\Phi_1,\Phi_2)}{D(x,y,z)}=\begin{pmatrix} \frac{\partial\Phi_1}{\partial x} & \frac{\partial\Phi_1}{\partial y} & \frac{\partial\Phi_1}{\partial z} \\ \frac{\partial\Phi_2}{\partial x} & \frac{\partial\Phi_2}{\partial y} & \frac{\partial\Phi_2}{\partial z} \end{pmatrix}=\begin{pmatrix} -\frac{y}{x^2} & \frac{1}{x} & 0 \\ -\frac{z}{x^2} & 0 & \frac{1}{x} \end{pmatrix}$$

matritsani tuzamiz. Matritsaning rangi 2 ga teng bo’lganligi uchun $\frac{y}{x}=C_1$ va $\frac{z}{x}=C_2$ birinchi integrallar bog’liqsiz bo’ladi.

Shunday qilib, berilgan (17) tenglamaning umumiy yechimi

$$u=f\left(\frac{y}{x},\frac{z}{x}\right) \tag{18}$$

ko’rinishda topiladi, bu yerda f – ixtiyoriy ikki o’zgaruvchili uzluksiz differensiallanuvchi funksiya.

Endi (18) funksiyalar (17) tenglamani qanoatlantirishini ko’rsatamiz.

f funksiyadan birinchi argument $\frac{y}{x}$ bo’yicha olingan hosilani f_1 bilan, ikkinchi argument $\frac{z}{x}$ bo’yicha olingan hosilani esa f_2 bilan belgilaymiz.

u ning xususiy hosilalarini hisoblaymiz:

$$\frac{\partial u}{\partial x} = f_1 \cdot \frac{\partial}{\partial x}\left(\frac{y}{x}\right) + f_2 \cdot \frac{\partial}{\partial x}\left(\frac{z}{x}\right) = -\frac{y}{x^2} f_1 - \frac{z}{x^2} f_2 \ ;$$

$$\frac{\partial u}{\partial y} = f_1 \cdot \frac{\partial}{\partial y}\left(\frac{y}{x}\right) + f_2 \cdot \frac{\partial}{\partial y}\left(\frac{z}{x}\right) = \frac{1}{x} f_1 \ ;$$

$$\frac{\partial u}{\partial z} = f_1 \cdot \frac{\partial}{\partial z}\left(\frac{y}{x}\right) + f_2 \cdot \frac{\partial}{\partial z}\left(\frac{z}{x}\right) = \frac{1}{x} f_2$$

va ularni (17) tenglamaga qo'yib, ayniyatga kelamiz:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = x \cdot \left(-\frac{y}{x^2} f_1 - \frac{z}{x^2} f \right) + y \cdot \frac{1}{x} f_1 + z \cdot \frac{1}{x} f_2 \equiv 0.$$

Olingan ayniyat (18) funksiyalar (17) tenglamaning umumiy yechimi ekanligini ko'rsatadi.►

Izoh. Bundan keyin berilgan tenglamaning yechimi (integrali) ni topish bilan cheklanib, birinchi integrallarning bog'liqsizlikka tekshirish va yechimning tenglamani qanoatlantirishini tekshirishni o'quvchiga havola qilinadi.

177. $2x\frac{\partial u}{\partial x} + (y-x)\frac{\partial u}{\partial y} + (z-x)\frac{\partial u}{\partial z} = 0$.

◄Bu tenglamaga mos oddiy differensial tenglamalar sistemasini tuzamiz:

$$\frac{dx}{2x} = \frac{dy}{y-x} = \frac{dz}{z-x}.$$

Bu sistemadan ikkita integrallanuvchi kombinatsiyalarni hosil qilib

$$\frac{d(x+y)}{x+y} = \frac{dx}{2x}, \quad \frac{d(x+z)}{x+z} = \frac{dx}{2x},$$

bu yerdan ikkita bog'liqsiz birinchi integrallarni topamiz:

$$\frac{(x+y)^2}{x} = C_1, \quad \frac{(x+z)^2}{x} = C_2.$$

Shunday qilib, $u = f\left(\frac{(x+y)^2}{x}, \frac{(x+z)^2}{x}\right)$ - berilgan tenglamaning umumiy yechimi.►

178. $x_1\frac{\partial u}{\partial x_1} + x_2\frac{\partial u}{\partial x_2} + \ldots + x_n\frac{\partial u}{\partial x_n} = 0$.

◄Bu tenglamaga mos oddiy differensial tenglamalar sistemasi quyidagicha:

$$\frac{dx_1}{x_1} = \frac{dx_2}{x_2} = \ldots = \frac{dx_n}{x_n}.$$

Bu sistemaning chiziqli bog'liqsiz integrallari

$$\frac{x_1}{x_n} = C_1, \quad \frac{x_2}{x_n} = C_2, \quad \ldots, \quad \frac{x_{n-1}}{x_n} = C_{n-1} \ (x_n \neq 0).$$

Shunga ko'ra, berilgan tenglamaning umumiy yechimi

$$u = \Phi\left(\frac{x_1}{x_n}, \frac{x_2}{x_n}, \ldots, \frac{x_{n-1}}{x_n}\right).$$

ko'rinishda bo'ladi, bu yerda $\Phi -$ $\frac{x_1}{x_n}, \frac{x_2}{x_n}, \ldots, \frac{x_{n-1}}{x_n}$ nisbatlarning ixtiyoriy uzluksiz differensiallanuvchi funksiyasi, ya`ni $u - n$ ta $x_1, x_2, \ldots, x_n$ erkli o'zgaruvchilarning ixtiyoriy nolinchi darajali bir jinsli funksiyasidir (bir jinsli funksiyalar haqida **1.3**-bandga qarang). Masalan,

$$u_1 = \frac{x_1}{x_n}, \quad u_2 = \frac{x_2}{x_n}, \quad \ldots, \quad u_{n-1} = \frac{x_{n-1}}{x_n}, \quad u_n = \frac{x_1}{x_n} + \frac{x_2}{x_n} + \ldots + \frac{x_{n-1}}{x_n},$$

$u_{n+1} = \left(\frac{x_1}{x_n}\right)^2$, $u_{n+2} = \sin\frac{x_1}{x_n}$ va h.k. funksiyalar berilgan tenglamaning yechimi bo'lishini tekshirib ko'rish qiyin emas.►

Quyidagi tenglamalarning ko'rsatilgan boshlang'ich shartlarni qanoatlantiradigan yechimlarini toping (**179-180**).

179. $x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=0;\ y=1$ da $z=2x.$

◀Avvalo umumiy yechimni topishimiz lozim. Buning uchun

$$\frac{dx}{x}=-\frac{dy}{y}$$

tenglamadan $xy=C$ birinchi integralni topamiz. Shunga ko'ra, umumiy yechim $z=\varphi(xy)$ bo'ladi. Endi boshlang'ich shartdan foydalanib, φ funksiyani topamiz:

$$\begin{cases} y=1, \\ z=2x, \\ xy=C, \end{cases} \Rightarrow \begin{cases} y=1, \\ z=2x, \\ x=C, \end{cases} \Rightarrow z=2C \Rightarrow z=2xy\,.$$

Shunday qilib, izlanayotgan yechim $z=2xy$ ko'rinishda bo'ladi.▶

180. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+xy\frac{\partial u}{\partial z}=0;\ z=0$ da $u=x^2+y^2.$

◀Berilgan tenglamaga teng kuchli bo'lgan

$$\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{xy}$$

sistemadan $u=\varphi\left(\frac{x}{y},xy-2z\right)$ umumiy yechimni topish qiyin emas. Avvalgi misoldagi kabi, boshlang'ich shartlarni qanoatlantiradigan φ funksiyani topamiz:

$$\begin{cases} z=0, \\ u=x^2+y^2, \\ \frac{y}{x}=C_1, \\ xy-2z=C_2, \end{cases} \Rightarrow \begin{cases} z=0, \\ u=x^2+y^2, \\ \frac{y}{x}=C_1, \\ xy=C_2, \end{cases} \Rightarrow \begin{cases} z=0, \\ u=x^2+y^2, \\ x^2=\frac{C_2}{C_1}, \\ y^2=C_1C_2, \end{cases} \Rightarrow u=\frac{C_2}{C_1}+C_1C_2.$$

Endi C_1 va C_2 o'zgarmaslarning o'rniga ularning ifodalarini qo'yib, berilgan tenglamaning ko'rsatilgan boshlang'ich shartlarni qanoatlantiradigan yechimini olamiz: $u=(xy-2z)\left(\frac{x}{y}+\frac{y}{x}\right)$.►

5.1.2. Bir jinsli bo'lmagan chiziqli tenglamalar. Bir jinsli bo'lmagan ushbu

$$X_1(x)\frac{\partial u}{\partial x_1}+X_2(x)\frac{\partial u}{\partial x_2}+...+X_n(x)\frac{\partial u}{\partial x_n}=b(x),\ x=\left(x_1,x_2,...,x_n\right) \tag{1}$$

chiziqli tenglamani yechish uchun unga mos oddiy differensial tenglamalar sistemasini

$$\frac{dx_1}{X_1(x)}=\frac{dx_2}{X_2(x)}=...=\frac{dx_n}{X_n(x)}=\frac{du}{b(x)} \tag{2}$$

yozib olish va shu sistemaning n ta

$$\psi_1(x_1,x_2,...,x_n,u)=C_1,\ ...,\psi_n(x_1,x_2,...,x_n,u)=C_n \tag{3}$$

bog'liqsiz birinchi integrallarini topish kerak.

(1) sistemaning umumiy yechimi

$$F(\psi_1,...,\psi_n)=0 \tag{4}$$

oshkormas ko'rinishda yoziladi, bu yerda F – ixtiyoriy differensialla-nuvchi funksiya.

Xususan, agar u noma`lum funksiya (3) birinchi integrallardan faqat bittasida, masalan, oxirgisida qatnashsa, u holda umumiy yechim

$$\psi_n(x_1,x_2,...,x_n,u)=f(\psi_1,...,\psi_{n-1}) \tag{5}$$

ko'rinishda yozilishi ham mumkin, bu yerda f – ixtiyoriy differensiallanuvchi funksiya. (5) tenglikni u ga nisbatan yechib, (1) tenglamaning umumiy yechimini oshkor ko'rinishda topamiz.

Bir jinsli bo'lmagan tenglama uchun Koshi masalasi bir jinsli tenglamadagi kabi qo'yiladi. (1) tenglamaning hamma yechimlari ichidan

$$u\big|_{x=x_n^{(0)}}=\varphi(x_1,x_2,...,x_{n-1}) \qquad (6)$$

boshlang'ich shartni qanoatlantiradigan

$$u=f(x_1,\ x_2,\ ...,\ x_n) \qquad (7)$$

yechimni topish kerak, bu yerda φ - berilgan uzluksiz differensialla-nuvchi funksiya.

Izlanayotgan funksiya ikki o'zgaruvchili bo'lgan holda, ya`ni

$$a_1(x,y,z)\frac{\partial z}{\partial x}+a_2(x,y,z)\frac{\partial z}{\partial y}=b(x,y,z) \qquad (8)$$

tenglamani qanoatlantirib, berilgan

$$x=u(t),\ y=v(t),\ z=w(t) \qquad (9)$$

chiziq orqali o'tuvchi $z=z(x,y)$ sirtni aniqlash uchun quyidagi

$$\frac{dx}{a_1}=\frac{dy}{a_2}=\frac{dz}{b} \qquad (10)$$

sistemaning

$$\varphi_1(x,y,z)=C_1,\ \varphi_2(x,y,z)=C_2 \qquad (11)$$

ikkita bog'liqsiz birinchi integrallarini topamiz, so'ngra topilgan birinchi integrallarda x, y, z larning o'rniga ularning (9) ifodalarini qo'yamiz. Natijada

$$\Phi_1(t)=C_1,\ \Phi_2(t)=C_2 \qquad (12)$$

ko'rinishdagi ikkita tenglamani olamiz. Ulardan t parametrni yo'qotib, $F(C_1,C_2)=0$ munosabatga ega bo'lamiz. Bu yerda C_1 va C_2 o'zgarmaslarning o'rniga (11) birinchi integrallarning chap tomonlarini qo'yib, izlanayotgan yechimni olamiz.

Tenglamalarning umumiy yechimini toping (**181-184**).

181. $2x\frac{\partial z}{\partial x}+(y-x)\frac{\partial z}{\partial y}-x^2=0.$

◀ Avvalo

$$\frac{dx}{2x}=\frac{dy}{y-x}=\frac{dz}{x^2}$$

sistemani tuzamiz. So'ngra $\frac{dx}{2x}=\frac{dz}{x^2}$ tenglikdan $x^2-4z=C_1$ birinchi integralni topamiz. Keyingi birinchi integralni topish uchun

$$\frac{dx}{2x}=\frac{dy}{y-x}=\frac{dx+dy}{2x+y-x}=\frac{d(x+y)}{x+y}$$

integrallanuvchi kombinatsiyani tuzamiz va $(x+y)^2=C_2x$ birinchi integralga ega bo'lamiz.

Shunday qilib, $F\left(x^2-4z,\,(x+y)^2/x\right)=0$ ko'rinishdagi funksiyalar berilgan tenglamaning yechimlaridir, bu yerda F – ixtiyoriy differensiallanuvchi funksiya.

z o'zgaruvchi ikkala birinchi integrallardan faqat bittasiga kirganligi uchun berilgan tenglamaning yechimini

$$z=\frac{1}{4}x^2+f\left(\frac{(x+y)^2}{x}\right)$$

ko'rinishda ham yozish mumkin, bu yerda f – ixtiyoriy differensiallanuvchi funksiya.▶

182. $yz\frac{\partial z}{\partial x}-xz\frac{\partial z}{\partial y}=e^z.$

◀ Dastlab

$$\frac{dx}{yz}=-\frac{dy}{xz}=\frac{dz}{e^z}$$

sistemani tuzamiz. So'ngra $\frac{dx}{yz}=-\frac{dy}{xz}$ tenglikdan $x^2+y^2=C_1$ birinchi integralni topamiz. Navbatdagi birinchi integralni topish uchun

$$\frac{dx}{yz} = -\frac{dy}{xz} = \frac{ydx - xdy}{y^2 z + x^2 z} = \frac{dz}{e^z}$$

integrallanuvchi kombinatsiyani tuzamiz va

$$arctg\frac{x}{y} + (z+1)\ e^{-z} = C_2$$

ko’rinishdagi birinchi integralga ega bo’lamiz.

Demak, agar F – ixtiyoriy differensiallanuvchi funksiya bo’lsa, u holda $F\left(x^2 + y^2, arctg\frac{x}{y} + (z+1)e^{-z}\right) = 0$ ko’rinishdagi funksiyalar berilgan tenglamaning yechimlari bo’ladi.►

183. $(xz + y)\frac{\partial z}{\partial x} + (x + yz)\frac{\partial z}{\partial y} = 1 - z^2.$

◄Oldingi misollardagi kabi,

$$\frac{dx}{xz + y} = -\frac{dy}{x + yz} = \frac{dz}{1 - z^2}$$

sistemani tuzamiz. So’ngra

$$\frac{dx + dy}{xz + y + x + yz} = \frac{dz}{1 - z^2}, \qquad \frac{dx - dy}{xz + y - x - yz} = \frac{dz}{1 - z^2}$$

tenglamalardan mos ravishda

$$(x + y)(z - 1) = C_1,\ \ (x - y)(z + 1) = C_2$$

bog’liqsiz birinchi integrallarni topamiz.

Shunday qilib, berilgan tenglamaning umumiy yechimi

$$F\big((x + y)(z - 1),\ (x - y)(z + 1)\big) = 0$$

ko’rinishda bo’ladi.►

184. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + (z + u)\frac{\partial u}{\partial z} = xy.$

◄ Bu yerda ham

$$\frac{dx}{x}=\frac{dy}{y},\quad \frac{ydx+xdy}{2xy}=\frac{du}{xy},\quad \frac{dx}{x}=\frac{d(z+u-xy)}{z+u-xy}$$

tenglamalardan mos ravishda

$$\frac{x}{y}=C_1,\quad xy-2u=C_2,\ \frac{z+u-xy}{x}=C_3$$

bog'liqsiz birinchi integrallarni topamiz.

Shunday qilib, berilgan tenglamaning umumiy yechimi

$$F\left(\frac{x}{y}, xy-2u, \frac{z+u-xy}{x}\right)=0$$

ko'rinishda bo'ladi.►

Quyidagi masalalarda berilgan tenglamani qanoatlantirib, berilgan egri chiziq orqali o'tuvchi sirtni toping (**185-187**).

185. $x\frac{\partial z}{\partial x}-2y\frac{\partial z}{\partial y}=x^2+y^2;\ y=1,\ z=x^2.$

◄Avvalo

$$\frac{dx}{x}=-\frac{dy}{2y}=\frac{dz}{x^2+y^2}$$

sistemaning birinchi integrallarini topamiz. Ushbu

$$\frac{dx}{x}=-\frac{dy}{2y}$$

tenglamadan $x^2y=C_1$ birinchi integralni topish qiyin emas. Keyingi birinchi integralni topish uchun $y=C_1x^{-2}$ munosabatni

$$\frac{dx}{x}=\frac{dz}{x^2+y^2}$$

tenglamaga qo'yamiz. Buning natijasida hosil bo'lgan $\left(x+C_1^2x^{-5}\right)dx=dz$

tenglamani integrallab va C_1 o'zgarmasning ifodasidan foydalanib,

$$2x^2 - y^2 - 4z = C_2$$

birinchi integralni olamiz va berilgan tenglamaning umumiy yechimini yozamiz: $z = \frac{x^2}{2} - \frac{1}{4} + \varphi(x^2 y).$

Endi ko'rsatilgan boshlang'ich shartlarni qanoatlantiradigan φ funksiyani topamiz:

$$\begin{cases} y = 1, \\ z = x^2, \\ x^2 y = C_1, \\ 2x^2 - y^2 - 4z = C_2, \end{cases} \Rightarrow \begin{cases} y = 1, \\ z = x^2, \\ x^2 = C_1, \\ -2x^2 - 1 = C_2, \end{cases} \Rightarrow \quad 2C_1 + 1 = -C_2 \; .$$

So'ngra C_1 va C_2 o'zgarmaslarning o'rniga ularning ifodalarini qo'yib, izlanayotgan sirt tenglamasini oshkormas $2x^2(y+1) = y^2 + 4z - 1,$ yoki oshkor $z = \frac{x^2}{2} - \frac{y^2}{4} + \frac{x^2 y}{2} + \frac{1}{4}$ ko'rinishda olamiz.►

186. $z\frac{\partial z}{\partial x} - xy\frac{\partial z}{\partial y} = 2xz;\; x + y = 2,\; yz = 1.$

◄Berilgan tenglamaning birinchi integrallarini topish qiyin emas: $x^2 - z = C_1, y^2 z = C_2.$ Endi $x + y = 2,$ $yz = 1,$ $x^2 - z = C_1, y^2 z = C_2$ munosabatlardan x, y, z o'zgarmaslarni yo'qotib, berilgan egri chiziq ustidagi integrallar orasidagi bog'lanishni topamiz: $\left(2 - C_2\right)^2 - C_2^{-1} - C_1 = 0.$

C_1 va C_2 o'zgarmaslarning o'rniga ularning ifodalarini qo'yib, berilgan tenglamaning ko'rsatilgan boshlang'ich shartlarni qanoatlantiradigan yechimini olamiz: $\left((2 - y^2 z)^2 - x^2 + z\right)y^2 z = 1.$►

187. $\dfrac{\partial z}{\partial x}=\dfrac{z}{x},\ \dfrac{\partial z}{\partial y}=\dfrac{2z}{y}.$

◀Avval birinchi tenglamaning umumiy yechimini topamiz. Buning uchun uni

$$\frac{dx}{1}=\frac{dy}{0}=\frac{xdz}{z}$$

ko'rinishda yozib olib, $y=C_1,\ \dfrac{z}{x}=C_2$ birinchi integrallarni topamiz. Shunga binoan, $\Phi\left(y,z/x\right)=0,$ yoki $z=x\varphi(y)$. So'ngra, topilgan z ni ikkinchi tenglamaga qo'yib, φ funksiyaga nisbatan differensial tenglamaga ega bo'lamiz:

$$x\varphi'(y)=\frac{2}{y}x\varphi(y),\ \ \text{yoki}\ \ \frac{\varphi'(y)}{\varphi(y)}=\frac{2}{y}\ \ (xy\neq 0).$$

Oxirgi tenglamani integrallab, topamiz: $\varphi(y)=Cy^2$.

Shunday qilib, $z=Cxy^2\ (xy\neq 0)$ funksiyalar berilgan sistemaning yechimi bo'ladi.▶

188. $z^2=Cxy$ oila sirtlarini to'g'ri burchak ostida kesib o'tadigan sirtlarning umumiy tenglamasini toping.

◀$F(x,y,z)=0$ - izlanayotgan sirt tenglamasi bo'lsin. Ma`lumki, sirtga o'tkazilgan N normalning vektori $N=\left(\dfrac{\partial F}{\partial x},\dfrac{\partial F}{\partial y},\dfrac{\partial F}{\partial z}\right)$ formuladan topiladi. Ortogonallik shartidan foydalanib,

$$-Cy\frac{\partial F}{\partial x}-Cx\frac{\partial F}{\partial y}+2z\frac{\partial F}{\partial z}=0$$

tenglamani olamiz. Bu tenglamadan va berilgan sirtning $z^2=Cxy$ tenglamasidan C parametrni yo'qotib, berilgan oila sirtlarining hammasiga ortogonal bo'lgan sirtlarni aniqlash uchun quyidagi tenglamaga ega bo'lamiz:

$$\frac{z}{x}\frac{\partial F}{\partial x}+\frac{z}{y}\frac{\partial F}{\partial y}-2\frac{\partial F}{\partial z}=0.$$

Bu tenglamaning $2y^2+z^2=C_1$, $2x^2+z^2=C_2$, $x^2-y^2=C_3$ birinchi integrallarini topib, so'ngra izlanayotgan sirtlarning umumiy tenglamasini yozamiz: $F\left(2y^2+z^2,2x^2+z^2,x^2-y^2\right)=0.$ ▶

5.1.3. Pfaff tenglamasi. Ushbu

$$P(x,y,z)dx+Q(x,y,z)dy+R(x,y,z)dz=0 \tag{1}$$

ko'rinishdagi tenglama *Pfaff tenglamasi* deyiladi. Vektor maydon nazariyasidan ma`lum bo'lgan $\vec{F}=\left(P(x,y,z),Q(x,y,z),R(x,y,z)\right)$ maydonning vektor chiziqlariga ortogonal bo'lgan $u(x,y,z)=C$ sirtlar oilasini topish haqidagi masala Pfaff tenglamasini yechishga keltiriladi. Bunda dx, dy, dz- izlanayotgan sirtlarga o'tkazilgan urinma tekislikda yotgan vektorning koordinatalari.

Agar $\vec{F}$ maydon *potentsial maydon* bo'lsa, ya`ni $P=\frac{\partial u}{\partial x}$, $Q=\frac{\partial u}{\partial y}$, $R=\frac{\partial u}{\partial z}$ bo'lsa, u holda izlanayotgan u sirt egri chiziqli integral yordamida topiladi:

$$u(x,y,z)=\int_{(x_0,y_0,z_0)}^{(x,y,z)} Pdx+Qdy+Rdz. \tag{2}$$

Agar $\vec{F}$ potensial maydon bo'lmasa, u holda ayrim hollarda shunday skalyar $\mu=\mu(x,y,z)$ ko'paytuvchini tanlab olish mumkinki, $\vec{F}$ ni $\mu=\mu(x,y,z)$ ga ko'paytirilgandan so'ng potensial maydon hosil bo'ladi. Shunga binoan,

$$\mu P=\frac{\partial u}{\partial x},\ \mu Q=\frac{\partial u}{\partial y},\ \mu R=\frac{\partial u}{\partial z}.$$

Vektor chiziqlarga ortogonal bo'lgan sirtlar oilasi mavjud bo'lishining zarur va etarli sharti $\left(\vec{F},rot\vec{F}\right)=0$ tenglikning bajarilishidir, bu yerda

$rot\vec{F}=\left(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z},\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x},\ \frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)$- $\vec{F}$ maydonning uyurmasi (vektor). Agar bu shart bajarilsa, u holda (1) tenglamani nafaqat integrallovchi ko'paytuvchi yordamida, balki quyidagi usul bilan ham integrallash mumkin. Bu usulda (1) tenglamada erkli o'zgaruvchilardan bittasini, masalan, z ni o'zgarmas hisoblab, ushbu

$$P(x,y,z)dx+Q(x,y,z)dy=0 \tag{3}$$

oddiy differensial tenglama integrallanadi, bunda z o'zgaruvchi parametr vazifasini bajaradi. Endi (3) tenglamaning integrali

$$u(x,y,z)=C \tag{4}$$

ko'rinishda topilgan bo'lsin. Bu yerdagi ixtiyoriy C o'zgarmas z ning funksiyasi bo'lishi mumkin. Bu $C(z)$ funksiyani shunday tanlash kerakki, natijada (4) integral (1) tenglamani qanoatlantirsin.

Agar $\left(\vec{F},\ rot\vec{F}\right)=0$ bo'lsa, u holda *Pfaff tenglamasi bitta qadamda integrallanadi*, deb aytiladi. Agar $\left(\vec{F},\ rot\vec{F}\right)\neq 0$ bo'lsa, u holda *Pfaff tenglamasi ikkita qadamda integrallanadi*, ya`ni F maydonning vektor chiziqlariga ortogonal bo'lgan sirtlar emas, balki shunday xossaga ega bo'lgan va berilgan $u(x,y,z)=0$ sirtda yotgan egri chiziqlar izlanadi. (1) va $u(x,y,z)=0$ tenglamalarda o'zgaruvchilardan bittasini yo'qotib, birinchi tartibli oddiy differensial tenglama hosil qilinadi.

Pfaff tenglamalarini bitta qadamda (agar imkoni bo'lsa) integrallang (**189-192**).

189. $2yzdx+2xzdy-xydz=0.$

◄Bu yerda $\vec{F}=\left(2yz,2xz,-xy\right)$. Uyurmaning ta`rifiga ko'ra, $rot\vec{F}=\left(-3x,3y,0\right)$ va $\left(\vec{F},\ rot\vec{F}\right)=0$ bo'lganligi uchun berilgan tenglama bitta qadamda integrallanadi. Bunga ko'ra, shunday $\mu=\mu(x,y,z)$ ko'paytuvchini topish mumkinki, $\mu\vec{F}$ maydon potensial maydon bo'ladi. Shunday qilib, μ ko'paytuvchini topish uchun

$$\begin{cases} \frac{\partial}{\partial y}(-xy\mu) = \frac{\partial}{\partial z}(2xz\mu), \\ \frac{\partial}{\partial z}(2yz\mu) = \frac{\partial}{\partial x}(-xy\mu), \\ \frac{\partial}{\partial x}(2xz\mu) = \frac{\partial}{\partial y}(2yz\mu), \end{cases} \quad \text{ya`ni} \quad \begin{cases} y\frac{\partial\mu}{\partial y} + 2z\frac{\partial\mu}{\partial z} + 3\mu = 0, \\ y\frac{\partial\mu}{\partial x} + 2z\frac{\partial\mu}{\partial z} + 3\mu = 0, \\ x\frac{\partial\mu}{\partial x} - y\frac{\partial\mu}{\partial y} = 0 \end{cases}$$

tenglamalar sistemasiga ega bo'lamiz. Sistemaning birinchi va ikkinchi tenglamalaridan $\frac{\partial\mu}{\partial x} = \frac{\partial\mu}{\partial y}$ tenglikning o'rinli ekanligi kelib chiqadi, bu tenglik esa o'z navbatida μ ko'paytuvchining x o'zgaruvchiga ham, y o'zgaruvchiga ham bog'liq emasligini ko'rsatadi. Shunga binoan, μ ko'paytuvchini $\mu = \mu(z)$ ko'rinishda izlaymiz. Bu holda μ ko'paytuvchini

$$2z\frac{\partial\mu}{\partial z} + 3\mu = 0$$

tenglamadan topamiz: $\mu = Cz^{-3/2}$. Berilgan tenglamani $z^{-3/2}$ funksiyaga (C ni birga teng deb oldik) hadma-had ko'paytirib,

$$yz^{-1/2}dx + xz^{-1/2}dy - \frac{1}{2}xyz^{-3/2}dz = 0$$

tenglamani olamiz. Bu tenglamaning chap tomoni $u = xyz^{-1/2}$ funksiya-ning to'liq differensiali bo'lganligi uchun berilgan tenglamaning izlanayotgan integrali $x^2y^2 = Cz$ ko'rinishda topiladi.►

190. $(z + xy)dx - (z + y^2)dy + ydz = 0.$

◄Bu yerda $\left(\vec{F},\ rot\vec{F}\right) = z + xy - y^2 \neq 0$ bo'lganligi uchun berilgan tenglamani bitta qadamda integrallash mumkin emas. Demak, $z = y^2 - xy$ funksiyaning berilgan tenglamaning ildizi bo'lishi yoki bo'lmasligini tekshirish kerak. Buning uchun $dz = 2ydy - xdy - ydx$ ni hisoblab hamda z va dz qiymatlarini qo'yib, ayniyatga kelamiz. Shunga binoan,

$z = y^2 - xy$ sirt $\vec{F} = \left(z + xy, -z - y^2, y\right)$ maydonga ortogonal bo'lgan yagona sirtdir.►

191. $(x - y)dx + zdy - xdz = 0.$

◄Bu yerda $\left(\vec{F},\ rot\vec{F}\right) = z - 2x + y$ va $z = 2x - y$ funksiya berilgan tenglamaning yechimi bo'lmaganligi uchun bu tenglama bitta qadamda integrallanishi mumkin emas. Tenglamani integrallash uchun, masalan, $z = x + y$ deb olamiz. Bu holda $dx - dy = 0$ tenglama hosil bo'ladi, uning yechimi $y = x + C$ ko'rinishda bo'ladi. Shunga ko'ra, $x = t,$ $y = t + C,$ $z = 2t + C$ bir parametrli chiziqlar oilasi berilgan tenglamani qanoatlantiradi.►

192. $(yz - 2xy)dx + (z^2 - x^2 + xz)dy + (xy + 2z + 2yz)dz = 0.$

◄ $rot\ \vec{F} = 0$ ekanligini hisoblab topish mumkin. Demak, $\vec{F}$ potensial maydon bo'ladi. Shunga binoan, berilgan tenglamaning chap tomoni ushbu

$$u(x, y, z) = \int_{(0,0,0)}^{(x,y,z)} F_x dx + F_y dy + F_z dz = z^2(y + 1) + xyz - x^2 y$$

funksiyaning to'liq differensiali bo'ladi. Shuning uchun izlanayotgan integral $z^2(y + 1) + xyz - x^2 y = C$ ko'rinishda bo'ladi.►

Vektor maydonning vektor chiziqlariga ortogonal bo'lgan sirtlarni toping (**193-194**).

193. $\vec{F} = 3y(x^2 y - z)\vec{i} + x(2x^2 y - 3z)\vec{j} - 3xy\vec{k}.$

◄Agar $d\vec{r} = (dx, dy, dz)$- izlanayotgan sirtga o'tkazilgan urinma tekislikda yotgan vektor bo'lsa, u holda shartga ko'ra, $\left(\vec{F}, d\vec{r}\right) = 0$, yoki

$$3y(x^2 y - z)dx + x(2x^2 y - 3z)dy - 3xydz = 0$$

bo'lishi lozim. $rot\vec{F} = 0$ bo'lganligi uchun $\vec{F}$ maydon potentsial maydon bo'ladi. Shunga binoan,

$$u(x,y,z)=\int\limits_{(0,0,0)}^{(x,y,z)} dz = x^3y^2-3xyz.$$

Shunday qilib, $x^3y^2-3xyz=C$ - izlanayotgan sirt.►

194. $\vec{F}=(2x-y)\vec{i}+(3y-z)\vec{j}+(x-2y)\vec{k}.$

◄Bu yerda $\left(\vec{F}, rot\vec{F}\right)=z-x-4y$ bo'lib, $z=x+4y$ funksiya $\left(\vec{F}, d\vec{r}\right)=0$ tenglamani qanoatlantirmaydi. Shunga binoan, berilgan $\vec{F}$ vektor maydonga ortogonal bo'lgan birorta ham silliq sirt mavjud emas.►

5.2. CHIZIQLI BO'LMAGAN TENGLAMALAR

5.2.1. To'liq integral. Avval izlanayotgan funksiya ikkita erkli o'zgaruvchiga bog'liq bo'lgan holni o'rganamiz. Uchta o'zgaruvchili birinchi tartibli xususiy hosilali tenglamalar

$$F(x,\ y,\ z,\ p,\ q)=0 \qquad (1)$$

ko'rinishda bo'ladi, bu yerda $p=\dfrac{\partial z}{\partial x},\ q=\dfrac{\partial z}{\partial y}.$

Birinchi tartibli xususiy hosilali tenglamaning ikkita ixtiyoriy o'zgarmaslarga bog'liq yechimi uning *to'liq integrali* deyiladi. To'liq integral oshkormas ko'rinishda quyidagicha yoziladi:

$$\Phi(x,\ y,\ z,\ a,\ b)=0. \qquad (2)$$

Boshqacha aytganda, to'liq integral uchta o'zgaruvchi va ikkita ixtiyoriy o'zgarmas orasidagi shunday munosabatki, undan va uni erkli o'zgaruvchilar bo'yicha differensiallash natijasida hosil bo'ladigan munosabatlardan o'zgarmaslarni chiqarib tashlash natijasida berilgan tenglama hosil bo'ladi. Shunga asosan, (1) tenglama ushbu

$$\Phi(x,\ y,\ z,\ a,\ b)=0,\ \ \frac{\partial\Phi}{\partial x}+\frac{\partial\Phi}{\partial z}p=0,\ \ \frac{\partial\Phi}{\partial y}+\frac{\partial\Phi}{\partial z}q=0 \qquad (3)$$

sistemadan a va b larni chiqarish natijasida hosil bo'lgan tenglamaga teng kuchlidir. Birinchi tartibli xususiy hosilali tenglamalarning hamma yechimlarini to'liq integraldan o'zgarmaslarni variatsiyalash usuli bilan hosil qilish mumkinligi Lagranj tomonidan ko'rsatilgan.

Faraz qilaylik, a va b lar x, y o'zgaruvchilarning biror funksiyalari bo'lsin. z ning x, y o'zgaruvchilar bo'yicha hosilalar, ya`ni p va q lar ushbu

$$\begin{cases} \frac{\partial\Phi}{\partial x}+\frac{\partial\Phi}{\partial z}p+\frac{\partial\Phi}{\partial a}\frac{\partial a}{\partial x}+\frac{\partial\Phi}{\partial b}\frac{\partial b}{\partial x}=0, \\ \frac{\partial\Phi}{\partial y}+\frac{\partial\Phi}{\partial z}q+\frac{\partial\Phi}{\partial a}\frac{\partial a}{\partial y}+\frac{\partial\Phi}{\partial b}\frac{\partial b}{\partial y}=0 \end{cases} \tag{4}$$

munosabatlardan hisoblanadi. (3) va (4) formulalarni taqqoslab, quyidagi tenglamalarni hosil qilamiz:

$$\frac{\partial\Phi}{\partial a}\frac{\partial a}{\partial x}+\frac{\partial\Phi}{\partial b}\frac{\partial b}{\partial x}=0,\ \frac{\partial\Phi}{\partial a}\frac{\partial a}{\partial y}+\frac{\partial\Phi}{\partial b}\frac{\partial b}{\partial y}=0. \tag{5}$$

Bu tenglamalardan a va b ni aniqlash kerak. Bu yerda uchta hol bo'lishi mumkin:

1) Agar

$$\frac{\partial\Phi}{\partial a}=0,\quad \frac{\partial\Phi}{\partial b}=0 \tag{6}$$

tengliklar bajarilsa, (4) tenglamalar qanoatlantiriladi. (6) tenglamalarni a va b ga nisbatan yechish natijasida hosil bo'lgan x va y ning funksiyalarini, ya`ni a va b ning qiymatlarini (2) ga qo'ysak, hosil bo'lgan ifoda (1) tenglamaning ixtiyoriy o'zgarmaslarga ham, ixtiyoriy funksiyalarga ham bog'liq bo'lmagan yechimidan iborat bo'ladi. Bunday yechim *maxsus integral* deyiladi.

2) Endi $\frac{\partial a}{\partial x}=\frac{\partial a}{\partial y}=\frac{\partial b}{\partial x}=\frac{\partial b}{\partial y}=0$ bo'lsin. Bu holda $a=const$, $b=const$ bo'lib, biz to'liq integralga qaytgan bo'lamiz.

3) Umumiy holda, (5) ni ikki noma`lumli ikkita chiziqli algebraik tenglamalar sistemasi deb qarasak, uning yechimga ega bo'lishi uchun ushbu

$$\frac{D(a,b)}{D(x,y)} = 0 \tag{7}$$

shartning bajarilishi zarurligi kelib chiqadi. (7) tenglik a va b o'rtasida funksional bog'liqlik mavjudligini ko'rsatadi. Agar $\frac{\partial a}{\partial x} \neq 0$ yoki $\frac{\partial a}{\partial y} \neq 0$ bo'lsa, u holda bu bog'liqlikni

$$b = \omega(a) \tag{8}$$

ko'rinishda yozish mumkin, bu yerda ω - ixtiyoriy funksiya. (8) ga asosan, (5) sistema quyidagi bitta munosabatga keladi:

$$\frac{\partial \Phi}{\partial a} + \frac{\partial \Phi}{\partial b} \cdot \omega'(a) = 0.$$

Agar bu tenglikdan a ni x va y ning funksiyasi sifatida topish mumkin bo'lsa, u holda (8) tenglamadan b erkli o'zgaruvchilarning funksiyasi sifatida topamiz. a va b ning topilgan qiymatlarini (2) ga qo'yib, (1) tenglamaning yechimini hosil qilamiz. Differensiallanuvchi $\omega(a)$ funksiyani ixtiyoriy tanlab olingandagi yechimlarning bunday to'plami (1) tenglamaning *umumiy integrali* deyiladi. Ixtiyoriy $\omega(a)$ funksiyaning har bir tanlab olinishiga, umuman aytganda, umumiy integralga kiruvchi biror *xususiy yechim* mos keladi. Shu ma`noda, umumiy yechim ixtiyoriy funksiyaga bog'liq bo'ladi deb aytishimiz mumkin.

Birinchi tartibli xususiy hosilali tenglamaning to'liq umumiy va maxsus integrallarini soddagina geometrik talqin qilish mumkin. Xususiy hosilali tenglamaning yechimi (x, y, z) koordinatalar fazosida sirtni aniqlaydi, bu sirt *integral sirt* deb ataladi. Beshta (x, y, z, p, q) miqdorlar to'plami e*lement* deyiladi, bunda x, y, z biror nuqtaning koordinatalari, p va q esa shu nuqtadan o'tuvchi tekislikning burchak koeffitsientlari. Shunga asosan (1) tenglamaning yechimini topish masalasi quyidagicha qo'yilishi mumkin: Shunday sirt topilsinki, bu sirtning nuqtalari va urinma tekisliklarning burchak koeffitsientlaridan tashkil topgan elementlar (1) munosabatni qanoatlantirsin. (2) to'liq integral ikkita parametrga bog'liq bo'lgan sirtlar oilasidan iboratdir.

Dastlabki (1) tenglama izlanayotgan sirtlarga o'tkazilgan normallarning yo'nalishigagina shartlar qo'yganligi uchun normallari integral sirtlarning normallari bilan ustma-ust tushgan har qanday sirt integral sirt bo'ladi. Shunga binoan, integral sirtlar ikki yoki bir parametrli oilasining o'ramalari ham integral sirtlar bo'ladi, chunki, haqiqatan ham, o'ramaga o'tkazilgan normal oilaning shu nuqtadan o'tadigan integral sirtlarining normali bilan ustma-ust tushadi.

Ikki parametrli sirtlar oilasining o'rama sirti

$$\Phi(x,y,z,a,b)=0\,,\ \frac{\partial\Phi}{\partial a}=0,\ \frac{\partial\Phi}{\partial b}=0 \qquad (9)$$

tenglamalardan topiladi.

b ni a parametrning ixtiyoriy differensiallanuvchi funksiyasi deb qarab, integral sirtlarning ikki parametrli $\Phi(x,y,z,a,b)=0$ oilasidan ixtiyoriy tarzda bir parametrli oilani ajratib olib va $\Phi(x,y,z,a,b(a))=0$ bir parametrli oilaning o'ramasini topib, yana integral sirtni hosil qilamiz. Bu *bir parametrli sirtlar oilasining o'rama sirti*

$$\Phi(x,y,z,a,b(a))=0\,,\ \frac{\partial\Phi}{\partial a}+\frac{\partial\Phi}{\partial b}\cdot\omega'(a)=0 \qquad (10)$$

tenglamalardan topiladi.

Shunday qilib, to'liq integralni bilgan holda ixtiyoriy funksiyaga bog'liq bo'lgan integralni qurish mumkin.

To'liq integralni topish ko'p hollarda uncha qiyin emas, masalan:

1) Agar (1) tenglama $F(p,\ q)=0$ yoki $p=\varphi(q)$ ko'rinishda bo'lsa, u holda, $q=a$ deb olib (a- ixtiyoriy o'zgarmas),

$$p=\varphi(a),\ dz=pdx+qdy=\varphi(a)dx+ady$$

munosabatlarga ega bo'lamiz, bu yerdan

$$z=\varphi(a)x+ay+b$$

to'liq integral bo'ladi.

2) Agar (1) tenglama $\varphi_1(x,\ p)=\varphi_2(y,\ q)$ ko'rinishga keltirilgan bo'lsa, u holda $\varphi_1(x,\ p)=\varphi_2(y,\ q)=a$ deb olib (a- ixtiyoriy o'zgarmas) va tenglamani $p,\ q$ ga nisbatan yechib (agar imkoni bo'lsa), topamiz:

$$p=\psi_1(x,a),\ \ p=\psi_2(y,a),$$

$$dz=pdx+qdy=\psi_1(x,a)dx+\psi_2(y,a)dy.$$

Bu yerdan

$$z=\int\psi_1(x,a)dx+\int\psi_2(y,a)dy$$

to'liq integralni topamiz.

3) Agar (1) tenglama $F(z,p,q)=0$ ko'rinishga ega bo'lsa, u holda, $z=z(u),\ \ u=ax+y$ deb olib,

$$F\left(x,a,\frac{dz}{du},\frac{dz}{du}\right)=0$$

tenglamani olamiz. Bu oddiy differensial tenglamani integrallab,

$z=\Phi(u,a,b)$ (b - ixtiyoriy o'zgarmas), yoki

$$z=\Phi(ax+y,a,b)$$

to'liq integralni olamiz.

4) Agar (1) tenglama Klero tenglamasini eslatadigan quyidagi

$$z=px+qy+\varphi(p,\ q)$$

ko'rinishda bo'lsa, u holda, bevosita o'rniga qo'yish yordamida, to'liq integral $z=ax+by+\varphi(a,b)$ ko'rinishda bo'lishini ko'rish qiyin emas.

Tenglamalarning to'liq integrallarini toping (**195-198**).

195. $p=\sin q.$

◄ $q=a$ deb olib (a- ixtiyoriy o'zgarmas),

$$p=\sin a,\ dz=\sin a dx+a dy$$

munosabatlarga ega bo’lamiz, bu yerdan

$$z=x\sin a+ay+b$$

to’liq integralni olamiz.►

196. $pq=x^2y^4$.

◄Berilgan tenglamani

$$\frac{p}{x^2}=\frac{y^4}{q}=a \quad (a\text{- ixtiyoriy o’zgarmas})$$

ko’rinishda yozib olamiz. Bu yerdan

$$p=ax^2,\ q=\frac{y^4}{a},\ dz=ax^2dx+\frac{y^4}{a}dy$$

munosabatlarga ega bo’lamiz. Oxirgi tenglamani integrallab,

$$z=\frac{ax^3}{3}+\frac{y^5}{5a}+b$$

to’liq integralni topamiz.►

197. $z^3=p^2q.$

◄ Agar $z=z(u), u=ax+y$ deb olib, so’ngra, p va q ning

$$p=a\cdot\frac{dz}{du},\ q=\frac{dz}{du}$$

ifodalarini berilgan tenglamaga qo’ysak,

$$z^3=a^2\left(\frac{dz}{du}\right)^3 \text{ yoki } \frac{dz}{du}=a_1z,\quad a_1=a^{-2/3}$$

tenglamani olamiz. Oxirgi oddiy differensial tenglamani integrallab, $z=b\exp(a_1u)$ (b-ixtiyoriy o’zgarmas), yoki

$z = b\exp\left(\frac{x}{\sqrt{a_1}} + a_1 y\right)$ to’liq integralni olamiz.►

198. $z = px + qy + p^3 + q^2$.

◄Bu tenglamaning to’liq integrali $z = ax + by + a^3 + b^2$ ko’rinishda bo’ladi.►

5.2.2. Lagranj-Sharpi usuli. Chiziqli bo’lmagan ushbu

$$F(x,\ y,\ z,\ p,\ q) = 0 \tag{1}$$

tenglamaning ko’rinishi murakkabroq bo’lgan hollarda to’liq integral topishning umumiyroq usullaridan foydalanamiz. Shunday usullardan biri *Lagranj-Sharpi usuli*dir.

Bu usulga ko’ra, (1) tenglamaga mos kelgan shunday bir

$$U(x,\ y,\ z,\ p,\ q) = a \tag{2}$$

tenglama tanlab olinadiki, (1) va (2) tenglamalardan tuzilgan sistemani yechishdan topilgan $p = p(x,y,z,a)$, $q = q(x,y,z,a)$ funksiyalardan bitta qadamda integrallana-digan

$$dz = p(x,y,z,a)dx + q(x,y,z,a)dx \tag{3}$$

Pfaff tenglamasini tuzish mumkin bo’ladi. Bu holda Pfaff tenglamasining $\Phi(x,y,z,a,b) = 0$ integrali (b- ixtiyoriy o’zgarmas) (1) tenglamaning to’liq integrali bo’ladi. U funksiya (3) tenglamaning bitta qadamda integrallanuvchanlik shartidan topiladi:

$\left(\vec{F}, rot\vec{F}\right) = 0,$ bu yerda $\vec{F} = p(x,y,z,a)\vec{i} + q(x,y,z,a)\vec{j} - \vec{k}$, ya`ni, yoyilgan holda

$$p\frac{\partial q}{\partial z} - q\frac{\partial p}{\partial z} - \frac{\partial p}{\partial y} + \frac{\partial q}{\partial x} = 0. \tag{4}$$

Bu shartdagi $\frac{\partial q}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}, \frac{\partial q}{\partial z}$ hosilalarni topish uchun

$$F(x,y,z,p,q) = 0,\ U(x,y,z,p,q) = a \tag{5}$$

sistemadagi ayniyatlarni differensiallash kerak.

x bo'yicha differensiallab, topamiz:

$$\frac{\partial F}{\partial x}+\frac{\partial F}{\partial p}\frac{\partial p}{\partial x}+\frac{\partial F}{\partial q}\frac{\partial q}{\partial x}=0,\quad \frac{\partial U}{\partial x}+\frac{\partial U}{\partial p}\frac{\partial p}{\partial x}+\frac{\partial U}{\partial q}\frac{\partial q}{\partial x}=0,$$

bu yerdan

$$\frac{\partial q}{\partial x}=-\frac{\frac{\partial F}{\partial p}\frac{\partial U}{\partial x}-\frac{\partial F}{\partial x}\frac{\partial U}{\partial p}}{\frac{\partial F}{\partial p}\frac{\partial U}{\partial q}-\frac{\partial F}{\partial q}\frac{\partial U}{\partial p}}$$

xususiy hosilaning qiymatini aniqlaymiz. Xuddi shunga o'xshash, (1), (2) sistemani y bo'yicha differensiallab, $\frac{\partial p}{\partial y}$ ni topamiz:

$$\frac{\partial p}{\partial y}=-\frac{\frac{\partial F}{\partial y}\frac{\partial U}{\partial p}-\frac{\partial F}{\partial p}\frac{\partial U}{\partial y}}{\frac{\partial F}{\partial p}\frac{\partial U}{\partial q}-\frac{\partial F}{\partial q}\frac{\partial U}{\partial p}}.$$

Nihoyat, (1), (2) sistemani z bo'yicha differensiallab, hosil bo'lgan sistemadan $\frac{\partial p}{\partial z}$ va $\frac{\partial q}{\partial z}$ hosilalarni topamiz:

$$\frac{\partial q}{\partial z}=-\frac{\frac{\partial F}{\partial p}\frac{\partial U}{\partial z}-\frac{\partial F}{\partial z}\frac{\partial U}{\partial p}}{\frac{\partial F}{\partial p}\frac{\partial U}{\partial q}-\frac{\partial F}{\partial q}\frac{\partial U}{\partial p}},\quad \frac{\partial p}{\partial z}=-\frac{\frac{\partial F}{\partial z}\frac{\partial U}{\partial p}-\frac{\partial F}{\partial p}\frac{\partial U}{\partial z}}{\frac{\partial F}{\partial p}\frac{\partial U}{\partial q}-\frac{\partial F}{\partial q}\frac{\partial U}{\partial p}}.$$

Topilgan hosilalarni (4) shartga qo'yib, bir necha amallarni bajarib, U ni aniqlash uchun bir jinsli chiziqli tenglamani olamiz:

$$\frac{\partial F}{\partial p}\frac{\partial U}{\partial x}+\frac{\partial F}{\partial q}\frac{\partial U}{\partial y}+\left(p\frac{\partial F}{\partial p}+q\frac{\partial F}{\partial q}\right)\frac{\partial U}{\partial z}-$$

$$-\left(\frac{\partial F}{\partial x}+p\frac{\partial F}{\partial z}\right)\frac{\partial U}{\partial p}-\left(\frac{\partial F}{\partial y}+q\frac{\partial F}{\partial z}\right)\frac{\partial U}{\partial q}=0. \qquad (6)$$

Endi

$$\frac{dx}{\frac{\partial F}{\partial p}}=\frac{dy}{\frac{\partial F}{\partial q}}=\frac{dz}{p\frac{\partial F}{\partial p}+q\frac{\partial F}{\partial q}}=-\frac{dp}{\frac{\partial F}{\partial x}+p\frac{\partial F}{\partial z}}=-\frac{dq}{\frac{\partial F}{\partial y}+q\frac{\partial F}{\partial z}} \qquad (7)$$

tengliklardan (7) sistemaning kamida bitta $U_1(x,y,z,p,q)=a$ birinchi integrali topiladi. Agar F va U_1 funksiyalar p va q ga nisbatan bog'liqsiz bo'lsa, ya`ni $\frac{D(F,U_1)}{D(p,q)}\neq 0$ bo'lsa, u holda topilgan $U_1(x,y,z,p,q)=a$ birinchi integral aynan (6) tenglamaning yechimi bo'ladi.

Shunga binoan,

$$F(x,y,z,p,q)=0,\ U_1(x,y,z,p,q)=a$$

tenglamalar sistemasidan $p=p(x,y,z,a)$ va $q=q(x,y,z,a)$ funksiyalarni aniqlab va (3) tenglamaga qo'yib, bitta qadamda integrallanadigan Pfaff tenglamasini olamiz, o'z navbatida bu tenglamani ham yechib dastlabki (1) tenglamaning $\Phi(x,y,z,a,b)=0$ to'liq integraliga ega bo'lamiz.

(1) tenglamaning $\Phi(x,y,z,a,b)=0$ to'liq integralini bilgan holda, umuman aytganda, asosiy boshlang'ich shartli masalani yoki

$$x=x(t),\ y=y(t),\ z=z(t) \qquad (8)$$

biror berilgan egri chiziq orqali o'tuvchi integral sirtni topish haqidagi umumiyroq masalani yechish mumkin.

$b=b(a)$ funksiyani shunday topish kerakki, ushbu

$$\Phi(x,y,z,a,b(a))=0 \qquad (9)$$

bir parametrli chiziqlar oilasining (9) va

$$\frac{\partial\Phi}{\partial a}+\frac{\partial\Phi}{\partial b}\cdot b'(a)=0 \qquad (10)$$

tenglamalardan topiladigan o'ramasi berilgan (8) egri chiziq orqali o'tsin. Ammo bu tenglamalardan $b=b(a)$ funksiyani topish ancha murakkab. Bu funksiyani

$$\begin{cases}\Phi(x(t),y(t),z(t),a,b(a))=0,\\ \dfrac{\partial\Phi}{\partial x}x'(t)+\dfrac{\partial\Phi}{\partial y}y'(t)+\dfrac{\partial\Phi}{\partial z}z'(t)=0\text{'}\end{cases} \qquad (11)$$

tenglamalardan, yoki qisqa yozuvda $\left(\vec{N},\ \vec{t}\right)=0$ tenglamadan aniqlash qulayroq, $\vec{t}$ - berilgan (8) egri chiziqqa o'tkazilgan urinmaning vektori, $\vec{N}$ esa $\Phi=0$ sirtga, demakki, izlanayotgan o'ramaga tegishli nuqtalarda o'tkazilgan normal vektori.

Tenglamalarning to'liq integralini toping (**199-200**).

199. $yzp^2-q=0$.

◀Berilgan tenglamaga mos kelgan (7) sistemani yozamiz:

$$\frac{dx}{2pyz}=\frac{dy}{-1}=\frac{dz}{2p^2yz-q}=-\frac{dp}{yp^3}=-\frac{dq}{zp^2+yp^2q}.$$

Endi dastlabki tenglamadagi $q=yzp^2$ tenglikdan foydalanib, sistemadagi uchinchi nisbatning mahrajini soddalashtiramiz va

$$\frac{dz}{p^2yz}=-\frac{dp}{p^3y}$$

integrallanuvchi kombinatsiyani olamiz, bu yerdan $p=a/z$ hosil bo'ladi. So'ngra, $yzp^2-q=0$ va $p=a/z$ tenglamalardan $q=a^2y/z$ ni topamiz. p va q larning topilgan qiymatlaridan foydalanib,

$$dz=\frac{a}{z}dx+\frac{a^2y}{z}dy$$

munosabatni olamiz. Oxirgi tenglamani $2z$ ga ko'paytirib, so'ngra esa integrallab, dastlabki tenglamaning $z^2=2ax+a^2y^2+b$ to'liq integralini topamiz.▶

200. $z = px + qy + p^3q^3$.

◀Berilgan tenglamaga mos kelgan (7) sistemani yozamiz:

$$\frac{dx}{x+3p^2q^3} = \frac{dy}{y+3p^3q^2} = \frac{dz}{px+qy+6p^3q^3} = \frac{dp}{0} = \frac{dq}{0}.$$

Bu tenglamalar sistemasidan $p = a,\ q = b$ integrallarni olamiz. Birinchi tenglamadan topamiz: $dz = pdx + qdy$, yoki $dz = adx + bdy$. Shunga ko'ra, $z = ax + by + C$. Ammo $z = px + qy + p^3q^3$ bo'lganligi uchun $C = a^3b^3$ bo'ladi. Shunday qilib, to'liq integral $z = ax + by + a^3b^3$ ko'rinishda bo'ladi.▶

201. $z = px + qy - \frac{q^2}{2}$ tenglamaning $x = 0,\ 2z = y^2$ parabola orqali o'tuvchi integral sirtini toping.

◀Berilgan tenglamaning to'liq integrali $z = ax + by - \frac{b^2}{2}$ ko'rinishda bo'ladi. Berilgan egri chiziq(parabola)ning tenglamasini parametrik shakldа yozish mumkin: $x = 0,\ y = t,\ 2z = t^2$.

$b = b(a)$ funksiyani topish uchun (11) tenglamalar sistemasini tuzamiz. Biz qarayotgan holda bu tenglamalar $2bt - b^2 - t^2 = 0,\ b - t = 0$ ko'rinishda bo'ladi. Bu yerdan $b = t$, ya`ni $t = y$ bo'lganligi uchun $b = y,\ a$ esa ixtiyoriy o'zgarmasligicha qoladi. Shunday qilib, berilgan egri chiziq orqali o'tadigan integral sirt $z = ax + \frac{y^2}{2}$ ko'rinishda bo'ladi.▶

5.2.3. Koshi usuli. Koshining umumlashgan masalasi quyidagicha qo'yiladi:

$$F(x,\ y,\ z,\ p,\ q) = 0 \qquad (1)$$

tenglamaning berilgan

$$x_0 = x_0(s),\ y_0 = y_0(s),\ z_0 = z_0(s)$$

egri chiziqdan o'tuvchi $z = z(x, y)$ integral sirti topilsin.

Qo'yilgan masalani yechish uchun avvalo

$$\begin{cases} F(x_0(s), y_0(s), z_0(s), p_0(s), q_0(s)) = 0, \\ p_0(s)x_0'(s) + q_0(s)y_0'(s) - z_0'(s) = 0 \end{cases} \tag{2}$$

tenglamalardan $p_0 = p_0(s),\ q_0 = q_0(s)$ funksiyalarni aniqlaymiz. So'ngra topilgan $p_0,\ q_0$ funksiyalardan foydalanib, ushbu

$$\frac{dx}{F_p} = \frac{dy}{F_q} = \frac{dz}{pF_p + qF_q} = -\frac{dp}{F_x + pF_z} = -\frac{dq}{F_y + qF_z} = dt \tag{3}$$

tenglamalar sistemasining $t = 0$ da

$$x = x_0(s),\ y = y_0(s),\ z = z_0(s),\ p = p_0(s),\ q = q_0(s) \tag{4}$$

boshlang'ich shartlarni qanoatlantiradigan yechimini topamiz. (3), (4) masalaning yechimi bo'lgan $x = x(t,s), y = y(t,s), z = z(t,s)$ funksiyalar aynan o'sha izlanayotgan integral sirtning parametrik tenglamasi bo'ladi.

Koshi usuli ushbu

$$F(x_1, x_2, ..., x_n, z, p_1, p_2, ..., p_n) = 0,\ p_k = \frac{\partial z}{\partial x_k},\ k = \overline{1,\ n} \tag{5}$$

ko'rinishdagi tenglamaga ham umumlashtirilishi mumkin. (5) tenglamaning berilgan $(n-1)-$ o'lchovli

$$x_{k0} = x_{k0}(s_1, s_2, ..., s_{n-1}),\ z_0 = z_0(s_1, s_2, ..., s_{n-1}),\ k = \overline{1,\ n} \tag{6}$$

sirt orqali o'tuvchi $n-$o'lchovli $z = z(x_1,\ x_2,\ ..., x_n)$ integral sirtini topish talab etilgan bo'lsin.

(5), (6) masala quyidagicha yechiladi. Avvalo ushbu

$$F(x_{10}, x_{20}, ..., x_{n0}, z_0, p_{10}, p_{20}, ..., p_{n0}) = 0, \tag{7}$$

$$\frac{\partial z_0}{\partial s_j} - \sum_{k=1}^{n} p_{k0} \frac{\partial x_{k0}}{\partial s_j} = 0,\ j = \overline{1, n-1} \tag{8}$$

tenglamalardan $p_{k0}(s_1, s_2, ..., s_{n-1})$ funksiyalar aniqlab olinadi. So'ngra

$$\frac{dx_1}{\frac{\partial F}{\partial p_1}} = \frac{dx_2}{\frac{\partial F}{\partial p_2}} = ... = \frac{dx_n}{\frac{\partial F}{\partial p_n}} = \frac{dz}{\sum_{k=1}^{n} p_k \frac{\partial F}{\partial p_k}} =$$

$$= -\frac{dp_1}{\frac{\partial F}{\partial x_1} + p_1 \frac{\partial F}{\partial z}} = -\frac{dp_2}{\frac{\partial F}{\partial x_2} + p_2 \frac{\partial F}{\partial z}} = ... = -\frac{dp_n}{\frac{\partial F}{\partial x_n} + p_n \frac{\partial F}{\partial z}} = dt$$

yordamchi tenglamalar sistemasining quyidagi

$$x_k\big|_{t=0} = x_{k0}(s_1, s_2, ..., s_{n-1}), \;\; z\big|_{t=0} = z_0(s_1, s_2, ..., s_{n-1}),$$

$$p_k\big|_{t=0} = p_{k0}(s_1, s_2, ..., s_{n-1}), \quad k = \overline{1, n}$$

boshlang'ich shartlarni qanoatlantiradigan yechimini topamiz. Natijada izlanayotgan integral sirtning parametrik tenglamasini hosil qilamiz:

$$x_k = x_k(t, s_1, s_2, ..., s_{n-1}), \;\; z = z(t, s_1, s_2, ..., s_{n-1}), \; k = \overline{1, n}.$$

Koshi usulidan foydalanib quyidagi masalalarni yeching (**202-204**).

202. $2z = p^2 + q^2, \; x = 0, z = y.$

◀Masalani Koshi usuli bilan yechamiz. Buning uchun

$$F(x, y, z, p, q) \equiv z - \frac{1}{2}p^2 - \frac{1}{2}q^2$$

deb belgilab olamiz va berilgan egri chiziqni parametrik ko'rinishda yozamiz: $x_0 = 0, \;\; y_0 = s, \; z_0 = s$. Avvalo

$$s - \frac{1}{2}p_0^2(s) - \frac{1}{2}q_0^2(s) = 0, \;\; p_0(s) \cdot 0 + q_0(s) \cdot 1 - 1 = 0$$

tenglamalardan $p_0 = p_0(s), \; q_0 = q_0(s)$ funksiyalarni aniqlaymiz:

$$p_0 = \pm\sqrt{2s-1}, \quad q_0 = 1.$$

Shu topilgan funksiyalardan foydalanib ushbu tenglamalar sistemasini tuzamiz:

$$\frac{dx}{-p}=\frac{dy}{-q}=\frac{dz}{-p^2-q^2}=-\frac{dp}{p}=-\frac{dq}{q}=dt.$$

So’ngra bu sistemaning umumiy yechimini topamiz:

$$p=C_1e^{-t},\ q=C_2e^{-t}, x=C_1e^{-t}+C_3,\ y=C_2e^{-t}+C_4,$$
$$z=\frac{1}{2}\left(C_1^2+C_2^2\right)e^{-2t}+C_5.$$

Endi $t=0$ da $x=0, y=s,\ z=s,\ p=\pm\sqrt{2s-1},\ q=1$ bo’lishi kerakligidan foydalanib, integrallash o’zgarmaslarini aniqlash uchun quyidagi sistemaga ega bo’lamiz:

$$C_1+C_3=0,\ C_2+C_4=s,\ \frac{1}{2}\left(C_1^2+C_2^2\right)+C_5=s, C_1=\pm\sqrt{2s-1}, C_2=1.$$

Bu sistemaning yechimi

$$C_1=\pm\sqrt{2s-1}, C_2=1, C_3=\mp\sqrt{2s-1}, C_4=s-1, C_5=0$$

ekanligini ko’rish qiyin emas. Shunga binoan, qo’yilgan masalaning boshlang’ich shartlarini qanoatlantiradigan integral sirt quyidagi

$$x=\pm\sqrt{2s-1}\left(e^{-t}-1\right), y=e^{-t}+s-1, z=se^{-2t}$$

ko’rinishda bo’ladi.►

203. $z=p_1^2+p_2^2-p_3^2;\ x_{10}=s_1+s_2,\ x_{20}=s_1-s_2,\ x_{30}=0,$ $z_0=1-s_1+s_2.$

◄Avvalo (7)-(8) tenglamalar sistemasini tuzamiz:

$$z_0-p_{10}^2-p_{20}^2+p_{30}^2=0, -1-p_{10}-p_{20}=0,\ 1-p_{10}+p_{20}=0$$

va bu yerdan p_{10}, p_{20}, p_{30} funksiyalarni aniqlaymiz:

$$p_{10}=0,\ p_{20}=-1,\ p_{30}=\pm\sqrt{s_1-s_2}.$$

So’ngra yordamchi differensial tenglamalar sistemasini tuzamiz:

$$\frac{dx_1}{2p_1}=\frac{dx_2}{2p_2}=\frac{dx_3}{-2p_3}=\frac{dz}{2(p_1^2+p_2^2-p_3^2)}=-\frac{dp_1}{-p_1}=-\frac{dp_2}{-p_2}=-\frac{dp_3}{-p_3}=dt.$$

Bu sistemaning oxirgi uchta tenglamalaridan topamiz:

$$p_1 = C_1 e^t, \;\; p_2 = C_2 e^t, \; p_3 = C_3 e^t.$$

Endi p_1, p_2, p_3 hosilalarning topilgan qiymatlaridan foydalanib, sistemaning dastlabki uchta tenglamasini yechamiz:

$$x_1 = 2C_1 e^t + C_4, \; x_2 = 2C_2 e^t + C_5, \; x_3 = -2C_3 e^t + C_6,$$
$$z = \left(C_1^2 + C_2^2 - C_3^2\right) e^{2t} + C_7.$$

Integrallash o'zgarmaslarining qiymatlarini topish uchun masalaning boshlang'ich shartlaridan foydalanamiz:

$$x_1\big|_{t=0} = s_1 + s_2, \; x_2\big|_{t=0} = s_1 - s_2, \; x_3\big|_{t=0} = 0,$$

$$p_1\big|_{t=0} = 0, \;\; p_2\big|_{t=0} = -1, \; p_3\big|_{t=0} = \pm\sqrt{s_1 - s_2}, \; z\big|_{t=0} = 1 - s_1 + s_2.$$

Natijada o'zgarmaslarning qiymatlarini topamiz:

$$C_1 = 0, \; C_2 = -1, \; C_3 = \pm\sqrt{s_1 - s_2}, \; C_4 = s_1 + s_2,$$

$$C_5 = s_1 - s_2 + 2, \; C_6 = \pm 2\sqrt{s_1 - s_2}, \; C_7 = 0.$$

Shunga binoan, izlanayotgan sirtning parametrik tenglamasi quyidagi

$$x_1 = s_1 + s_2, \; x_2 = -2e^t + 2 + s_1 - s_2, \;\; x_3 = \mp 2\sqrt{s_1 - s_2} \cdot e^t \pm$$

$$\pm 2\sqrt{s_1 - s_2} = \mp 2\sqrt{s_1 - s_2}\,(e^t - 1), \qquad z = (1 + s_2 - s_1)e^{2t}$$

ko'rinishda yoziladi.►

204. $z = p_1^2 + p_2^2 + p_3^2 + p_4^2; \; x_{10} = 1, \; x_{20} = s_1, \; x_{30} = s_1 + s_2,$
$x_{40} = s_1 + s_2 + s_3, \;\; z_0 = s_1^2 + s_2^2 + s_3^2.$

◄Qo'yilgan masala avvalgi masalaga o'xshash yechiladi, ya`ni quyidagicha yo'l tutamiz.

a) Quyidagi tenglamalar sistemasini yechamiz:

$$p_{10}^2 + p_{20}^2 + p_{30}^2 + p_{40}^2 - z = 0, \; 2s_1 - p_{20} - p_{30} - p_{40} = 0,$$

$$2s_2 - p_{30} - p_{40} = 0, \; 2s_3 - p_{40} = 0.$$

Bu yerdan topamiz:

$$p_{40} = 2s_3,\ \ p_{30} = 2(s_2 - s_3),\ p_{20} = 2(s_1 - s_2),$$

$$p_{10} = \pm\sqrt{s_1^2 + s_2^2 + s_3^2 - 4(s_1 - s_2)^2 - 4(s_2 - s_3)^2 - 4s_3^2} =$$
$$= \pm\sqrt{8s_1 s_2 + 8s_2 s_3 - 3s_1^2 - 7s_2^2 - 7s_3^2}\,.$$

b) Yordamchi sistemani tuzamiz:

$$\frac{dx_1}{2p_1} = \frac{dx_2}{2p_2} = \frac{dx_3}{2p_3} = \frac{dx_4}{2p_4} = \frac{dz}{2(p_1^2 + p_2^2 + p_3^2 + p_4^2)} =$$

$$= -\frac{dp_1}{-p_1} = -\frac{dp_2}{-p_2} = -\frac{dp_3}{-p_3} = -\frac{dp_3}{-p_4} = dt$$

Oxirgi to'rtta tenglamadan $p_k = C_k e^t,\ k = \overline{1,4}$ tenglamalar, dastlabki to'rtta tenglamadan esa

$$x_k = 2C_k e^t + D_k,\ k = \overline{1,4};\ z = (C_1^2 + C_2^2 + C_3^2 + C_4^2)e^{2t} + D_5$$

tenglamalar kelib chiqadi.

c) Masalaning boshlang'ich shartlaridan foydalanib, integrallash o'zgarmaslarining qiymatlarini topamiz:

$$C_1 = \pm\sqrt{s_1^2 + s_2^2 + s_3^2 - 4(s_1 - s_2)^2 - 4(s_2 - s_3)^2 - 4s_3^2}\,,$$

$$C_2 = 2(s_1 - s_2),\ C_3 = 2(s_2 - s_3),\ \ C_4 = 2s_3,$$

$$D_1 = 1 \mp 2\sqrt{s_1^2 + s_2^2 + s_3^2 - 4(s_1 - s_2)^2 - 4(s_2 - s_3)^2 - 4s_3^2}\,,$$

$$D_2 = -3s_1 + 4s_2,\ \ D_3 = s_1 - 3s_2 + 4s_3,\ D_4 = s_1 + s_2 - 3s_3,\ D_5 = 0.$$

d) Izlanayotgan sirtning parametrik tenglamasini yozamiz:

$$x_1 = \pm 2\sqrt{s_1^2 + s_2^2 + s_3^2 - 4(s_1 - s_2)^2 - 4(s_2 - s_3)^2 - 4s_3^2}\,(e^t - 1) + 1,$$

$$x_2 = 4(s_1 - s_2)e^t - 3s_1 + 4s_2,\ \ x_3 = 4(s_2 - s_3)e^t + s_1 - 3s_2 + 4s_3,$$

$$x_4 = 4s_3 e^t + s_1 + s_2 - 3s_3,\ \ z = (s_1^2 + s_2^2 + s_3^2)e^{2t}\,. \blacktriangleright$$

5-BOBNI TAKRORLASHGA DOIR ARALASH MASALALAR

Quyidagi tenglamalarning umumiy yechimini toping:

1. $(x+2y)\dfrac{\partial z}{\partial x}-y\dfrac{\partial z}{\partial y}=0.$ **2.** $\dfrac{\partial z}{\partial x}+(2e^{x}-y)\dfrac{\partial z}{\partial y}=0.$

3. $(x-z)\dfrac{\partial u}{\partial x}+(y-z)\dfrac{\partial u}{\partial y}+2z\dfrac{\partial u}{\partial z}=0.$ **4.** $y\dfrac{\partial z}{\partial x}+x\dfrac{\partial z}{\partial y}=x-y.$

5. $x\dfrac{\partial z}{\partial x}+2y\dfrac{\partial z}{\partial y}=x^{2}y+z.$ **6.** $2y^{4}\dfrac{\partial z}{\partial x}-xy\dfrac{\partial z}{\partial y}=x\sqrt{z^{2}+1}.$

7. $\sin^{2}x\dfrac{\partial z}{\partial x}+tgz\dfrac{\partial z}{\partial y}=\cos^{2}z.$ **8.** $(u-x)\dfrac{\partial u}{\partial x}+(u-y)\dfrac{\partial u}{\partial y}-z\dfrac{\partial u}{\partial z}=x+y.$

Quyidagi Koshi masalalarini yeching:

9. $2\sqrt{x}\dfrac{\partial z}{\partial x}-y\dfrac{\partial z}{\partial y}=0;\quad x=1,\ z=y^{2}.$

10. $x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=z-x^{2}-y^{2};\ z\big|_{y=-2}=x-x^{2}.$

Quyidagi differensial masalalarni yeching:

11. $tgx\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=z;\ y=x,\ z=x^{3}.$

12. $(y-z)\dfrac{\partial z}{\partial x}+(z-x)\dfrac{\partial z}{\partial y}=x-y;\ z=y=-x.$

13. $y^{2}\dfrac{\partial z}{\partial x}+yz\dfrac{\partial z}{\partial y}+z^{2}=0;\ x-y=0,\ x-yz=1.$

14. $(y+2z^{2})\dfrac{\partial z}{\partial x}-2x^{2}z\dfrac{\partial z}{\partial y}=x^{2};\ x=z,\ y=x^{2}.$

15. $x=y,\ z=1$ to’g’ri chiziq orqali o’tib, $x^{2}+y^{2}+z^{2}=Cy$ sirtlarga ortogonal bo’lgan sirtni toping.

16. $y = Cx$ tekisliklar oilasiga ortogonal traektoriyalarni toping.

Tenglamalar sistemalarini yeching:

17. $\frac{\partial z}{\partial x} = y - z, \frac{\partial z}{\partial y} = xz.$ **18**. $\frac{\partial z}{\partial x} = 2yz - z^2, \frac{\partial z}{\partial y} = xz.$

Pfaff tenglamalarini yeching:

19. $3yzdx + 2xzdy + xydz = 0$.

20. $(y + 3z^2)dx + (x + y)dy + 6xzdz = 0.$

Quyidagi tenglamalarning to'liq integrallarini toping:

21. $pq = x^2 y^2.$ **22.** $px + qy + z = 0.$

23. $z = px + qy + p^3.$ **24.** $px - qy = x + y.$

25. $z = p^2 + q^2.$ **26.** $pq = 9z^2.$

27. $z^2 = p^2 + zpq.$ **28**. $yzp^2 - q = 0.$

Quyidagi masalalarda berilgan egri chiziq orqali o'tuvchi integral sirtni toping:

29. $z = pq + 1; y = 2$, $z = 2x + 1.$ **30.** $2z = pq - 3xy; x = 5$, $z = 15y.$

31. $4z = p^2 + q^2; x = 0$, $z = y^2.$ **32.** $px + qy - pq = 0; x = 0$, $z = y.$

Koshi usuli yordamida berilgan egri chiziq orqali o'tuvchi integral sirtni toping:

33. $z = pq; x_0 = s, y_0 = s^2, z_0 = s^3.$

34. $p^2 + q^2 = 1; x_0 = \cos s, y_0 = \sin s, z_0 = s/2.$

35. $z = p_1 x_1 + p_2 x_2 + p_3 x_3 + p_1^2 + p_2^2 + p_3^2;$ $x_{10} = 1, x_{20} = s_1,$ $x_{30} = s_1 + s_2,$ $z_0 = s_1^2.$

36. $p_1^2 + p_2^2 + p_3^2 + p_4^2 = 1;$ $x_{10} = 1,$ $x_{20} = s_1,$ $x_{30} = s_1 + s_2,$ $x_{40} = s_1 + s_2 + s_3,$ $z_0 = s_1^2 + s_2^2 + s_3^2.$

6-BOB
DIFFERENSIAL TENGLAMALARNI TAQRIBIY YECHISH USULLARI

6.1. YECHIMNING BOSHLANG'ICH SHART VA PARAMETRLARGA BOG'LIQLIGI

6.1.1. Taqribiy yechimning xatoligini baholash. Ushbu

$$\frac{dx}{dt} = f(t,x),\ \ x\big|_{t=0} = x(0) \tag{1}$$

Koshi masalasini o'rganamiz, bu yerda $x = (x_1, x_2, ..., x_n),$- argument, $f = (f_1, f_2, ..., f_n)$ esa berilgan funksiya-vektor.

$y = y(t)$ vektor-funksiya (1) Koshi masalasining taqribiy yechimi bo'lsin. Bu yerda va bundan keyin f vektor-funksiyani t, x o'zgaruvch-lar bo'yicha uzluksiz va x o'zgaruvchi bo'yicha

$$\| f(t,\ y) - f(t,x)\| \le K\|\ y - x\|,\ K = const \tag{2}$$

Lipshits shartini qanoatlantiradi, deb hisoblaymiz, bu yerda $\|\cdot\|$ bilan vektorning normasi belgilangan. Quyidagi normalardan foydalaniladi:

$$\|x(t)\| = \sqrt{\sum_{k=1}^{n}|x_k(t)|^2}\ ,\ \ \|x(t)\| = \max_k\ |x_k(t)|,\ \ \|x(t)\| = \sum_{k=1}^{n}|x_k(t)|.$$

Aytaylik, (1) Koshi masalasining $y(t)$ taqribiy yechimi quyidagi

$$\left\|\frac{dy}{dt} - f(t,\ y)\right\| \le \varepsilon,\ \ \| y(0) - x(0)\| \le \delta \tag{3}$$

tengsizliklarni qanoatlantirsin. U holda yechimning xatoligi

$$\| x(t)-y(t)\| \le \delta e^{K|t|}+\frac{\varepsilon}{K}\left(e^{K|t|}-1\right) \qquad (4)$$

tengsizlik bilan baholanadi.

Agar $f(t,x)$ funksiya o'zining aniqlanish sohasida x o'zgaruvchi bo'yicha qavariq bo'lib, $\left|\partial f_i/\partial x_j\right|\le C$ tengsizlikni qanoatlantirsa, u holda Lipshits o'zgarmasi sifatida $K=n\cdot C$ ni olish mumkin. Ma`lumki, agar funksiya biror intervalda differensiallanuvchi bo'lsa, u holda bu funksiyaning shu intervalda (qat`iy) qavariq bo'lishi uchun uning hosilasi (o'suvchi bo'lishi) kamaymasligi zarur va yetarli.

205-208-masalalarda taqribiy yechimning ko'rsatilgan oraliqdagi xatoligini baholang (taqribiy yechim to'lqin bilan belgilangan).

205. $y'=\frac{x}{4}-\frac{1}{1+y^2}$, $y(0)=1$; $\tilde{y}=1-\frac{x}{2}$, $|x|\le\frac{1}{2}$.

◀Tenglamaning o'ng tomoni x, y o'zgaruvchilar bo'yicha uzluksiz va y bo'yicha

$$\frac{\partial f}{\partial y}=\frac{2y}{(1+y^2)^2}$$

uzluksiz hosilaga ega, binobarin,

$$\left|\frac{\partial f}{\partial y}\right|=\frac{2|y|}{1+|y|^2}\cdot\frac{1}{1+|y|^2}\le\frac{2|y|}{1+|y|^2}\le 1$$

tengsizlikning bajarilishi ravshan. Demak, Lipshits o'zgarmasi sifatida bir sonini olish mumkin ($K=1$). So'ngra, (3) formulalarga binoan topamiz:

$$\left|\tilde{y}'-\frac{x}{4}+\frac{1}{1+\tilde{y}^2}\right|=\left|-\frac{1}{2}-\frac{x}{4}+\frac{1}{1+\left(1-x/2\right)^2}\right|=\left|\frac{1}{2}+\frac{x}{4}-\frac{4}{8-4x+x^2}\right|=$$

$$=\left|\frac{x^2(x-2)}{4(8-4x+x^2)}\right|=\left|\frac{x^2}{4}\right|\cdot\left|\frac{x-2}{8-4x+x^2}\right|\le\frac{1}{16}\max_{|x|\le\frac{1}{2}}\left|\frac{x-2}{8-4x+x^2}\right|=\frac{1}{64},$$

$$\tilde{y}(0)=y(0).$$

Shuning uchun $\varepsilon=1/64, \delta=0.$ Endi yechimning xatoligini topamiz:

$$\|y(x)-\tilde{y}(x)\|=|y(x)-\tilde{y}(x)|\le\frac{1}{64}\left(e^{|x|}-1\right)\le\frac{1}{64}\left(e^{\frac{1}{2}}-1\right)<0{,}011.\blacktriangleright$$

206. $\dot{x}_1=x_1-x_2,\ \dot{x}_2=t^2x_1,$

$$x_1(0)=1,\ x_2(0)=0;\ \tilde{x}_1=1+t+\frac{1}{2}t^2,\ \tilde{x}_2=\frac{1}{3}t^3,\ |t|\le 0{,}1.$$

◀ $\|x\|=|x_1|+|x_2|$ deb olamiz. U holda

$$\left\|\frac{d\tilde{x}}{dt}-f(t,\ \tilde{x})\right\|=\left|\frac{d\tilde{x}_1}{dt}-f_1(t,\ \tilde{x}_1,\ \tilde{x}_2)\right|+\left|\frac{d\tilde{x}_2}{dt}-f_2(t,\ \tilde{x}_1,\tilde{x}_2)\right|,$$

bu yerda

$$f_1(t,\tilde{x}_1,\tilde{x}_1)=\tilde{x}_1-\tilde{x}_2,\ f_2(t,\tilde{x}_1,\tilde{x}_2)=t^2\tilde{x}_1.$$

Shunga ko’ra,

$$\left\|\frac{d\tilde{x}}{dt}-f(t,\ \tilde{x})\right\|=\left|1+t-\left(1+t+\frac{1}{2}t^2-\frac{1}{3}t^3\right)\right|+\left|t^2-t^2\left(1+t+\frac{1}{2}t^2\right)\right|=$$

$$=t^2\left|\frac{1}{3}t-\frac{1}{2}\right|+t^2\left|t+\frac{1}{2}t^2\right|\le 0{,}005\left(\left|\frac{2}{3}t-1\right|+\left|2t+t^2\right|\right)<0{,}0064,$$

$$\varepsilon=0{,}0064,\ \ \delta=0.$$

f_1 va f_2 funksiyalarning xususiy hosilalari uchun

$$\frac{\partial f_1}{\partial x_1}=1,\ \frac{\partial f_1}{\partial x_2}=-1,\ \frac{\partial f_2}{\partial x_1}=t,\ \frac{\partial f_1}{\partial x_2}=0$$

tengliklar o’rinli bo’lganligi uchun Lipshits o’zgarmasi $K=2$ ga teng. Shunga asoslanib yechimning xatoligini topish qiyin emas:

$$\|x(t)-\tilde{x}(t)\|\le 0{,}0032\left(e^{2|t|}-1\right)\le 0{,}0032\left(e^{0{,}2}-1\right)<0{,}00071.\blacktriangleright$$

207. $y'' - x^4 y = 0,\ y(0)=1,\ y'(0)=0;\ \tilde{y} = \exp\left(x^6/30\right),\ |x| \le 0,5.$

◀Ikkinchi tartibli tenglamadan birinchi tartibli tenglamalar sistemasiga o'tamiz:

$$x = t,\ y = x_1,\ y' = x_2,\ x_1' = x_2,\ x_2' = t^4 x_1,$$

$$x_1(0)=1,\ x_2(0)=0;\ \tilde{x}_1 = \exp\left(t^6/30\right),$$

$$\tilde{x}_2' = \tilde{y}' = \frac{1}{5} t^5 \exp\left(t^6/30\right),\ |t| \le 0,5$$

va $\|x\| = |x_1| + |x_2|$ deb olamiz. U holda

$$\left\| \frac{d\tilde{x}}{dt} - f(t,\tilde{x}) \right\| = \left| \frac{d\tilde{x}_1}{dt} - f_1(t,\tilde{x}_1,\tilde{x}_2) \right| + \left| \frac{d\tilde{x}_2}{dt} - f_2(t,\tilde{x}_1,\tilde{x}_2) \right|,$$

bu yerda $f_1(t,\tilde{x}_1,\tilde{x}_1) = \tilde{x}_2,\ f_2(t,\tilde{x}_1,\tilde{x}_2) = t^4 \tilde{x}_1.$ So'ngra,

$$\tilde{x}_1' = \frac{1}{5} t^5 \exp\left(t^6/30\right),\ \tilde{x}_2' = \left(t^4 + \frac{1}{25} t^{10}\right) \exp\left(t^6/30\right)$$

ekanligidan

$$\left\| \frac{d\tilde{x}}{dt} - f(t,\tilde{x}) \right\| = \left| \frac{t^{10}}{25} \exp\left(\frac{t^6}{30}\right) \right| \le \max_{|t| \le 0,5} \frac{t^{10}}{25} \exp\left(\frac{t^6}{30}\right) =$$

$$= \frac{(0,5)^{10}}{25} \exp\left(\frac{(0,5)^6}{30}\right) < 0,00006$$

baho kelib chiqadi. Shunga ko'ra, $\varepsilon = 0,00006,\ \delta = 0.$

Hosil bo'lgan sistemaning o'ng tomonidagi funksiyalarni x_1 va x_2 o'zgaruvchilar bo'yicha qavariqlikka tekshiramiz:

$$\frac{\partial f_1}{\partial x_1} = 0,\ \frac{\partial f_1}{\partial x_2} = 1,\ \frac{\partial f_2}{\partial x_1} = t^4,\ \frac{\partial f_1}{\partial x_2} = 0\,.$$

Shunga binoan, Lipshits o'zgarmasi $K = 2\max\limits_{|t|\le 0,5}(1,t^4) = 2$ ga teng. Yuqoridagilarga asoslanib yechimning xatoligini topamiz:

$\|x(t)-\tilde{x}(t)\| \le 0,00003\ \left(e^{2|t|}-1\right) \le 0,00003\ (e-1) < 0,000052.$ ►

208. $y' = \dfrac{1}{y} + x,\ y(0)=1;\ \tilde{y} = 1+x,\ 0 \le x \le \dfrac{1}{4}.$

◄Avvalo ε va δ sonlarini topamiz. Yuqoridagi kabi hisoblaymiz:

$$\left|\tilde{y}' - \frac{1}{\tilde{y}} - x\right| = \left|1 - x - \frac{1}{1+x}\right| = \frac{x^2}{1+x} \le \frac{1}{20},\quad |y(0)-\tilde{y}(0)| = 0.$$

Shuning uchun $\varepsilon = 1/20$, $\delta = 0$.

Endi $y(x)$ yechim biror $R = \{(x,y): 0 \le x \le 0,25;\ |y-1| \le 0,25\}$ to'g'ri to'rtburchakda mavjud bo'lsin, deb faraz qilamiz ($\tilde{y}(x) \in R$) va Lipshits o'zgarmasini topamiz:

$$K \le \max_R \left|\frac{\partial f}{\partial y}\right| = \max_R \left|-\frac{1}{y^2}\right| = \frac{16}{9}.$$

Olingan natijalardan foydalanib, yechimning xatoligini topamiz:

$$\|y(x)-\tilde{y}(x)\| \le \frac{9}{320}\ \left(e^{K|t|}-1\right) \le \frac{9}{320}\left(e^{\frac{4}{9}}-1\right) < 0,015.$$ ►

6.1.2. Yechimlardan parametr bo'yicha hosila olish. Ushbu

$$\frac{dx_k}{dt} = f_k(t, x_1, x_2, \dots, x_n, \mu), \tag{1}$$

$$x_k(0) = a_k(\mu),\ k = \overline{1,n}, \tag{2}$$

masalada ($\mu-$parametr) f_k, a_k $-$ uzluksiz va uzluksiz hosilalarga ega bo'lgan funksiyalar bo'lsin. Bunday holda $(x_1, x_2, \dots, x_n)$ yechim μ

parametr bo’yicha uzluksiz hosilaga ega va uning $\frac{\partial x_k}{\partial \mu} = u_k, \ k = \overline{1,n}$ xususiy hosilalari quyidagi masalaning yechimlari bo’ladi:

$$\frac{du_k}{dt} = \sum_{j=1}^{n} \frac{\partial f_k}{\partial x_j} \cdot u_j + \frac{\partial f_k}{\partial \mu}, \tag{3}$$

$$u_k(0) = a'_k(\mu), k = \overline{1,n}. \tag{4}$$

(3) formulalardagi xususiy hosilalarni hisoblash uchun avval (1)-(2) masalaning $x_k = x_k(t), \ k = \overline{1,n}$ yechimlarini topish, so’ngra bu yechimlarni $\frac{\partial f_k}{\partial x_j}, \frac{\partial f_k}{\partial \mu}$ xususiy hosilalarga qo’yish kerak.

Agar $k \neq s$ bo’lganda $a_s(\mu) = \mu, a_k(\mu) = const$ bo’lib, f_k, $k = \overline{1,n}$ funksiyalar μ ga bog’liq bo’lmasa, u holda (3)-(4) dan quyidagilar kelib chiqadi:

$$\frac{du_k}{dt} = \sum_{j=1}^{n} \frac{\partial f_k}{\partial x_j} \cdot u_j, \ \ u_k(0) = 0, k \neq s, u_s(0) = 1, \tag{5}$$

bu yerda $u_k = \frac{\partial x_k}{\partial a_s}$.

Quyidagi masalalarda parametr yoki boshlang’ich shartlar bo’yicha hosilalarni toping (**209-212**).

209. $y' = 2x + \mu y^2, \ y(0) = \mu - 1; \quad \left.\frac{\partial y}{\partial \mu}\right|_{\mu=0}$ ni toping.

◀Berilgan masalani

$$y'_x(x,\mu) \equiv 2x + \mu y^2(x,\mu), \ y(0,\mu) = \mu - 1$$

ko’rinishda batafsil yozib olamiz, so’ngra bu ayniyatlardan μ parametr bo’yicha hosila olamiz:

$$\frac{du}{dx} = y^2(x,\mu) + 2\mu y(x,\mu)u, \quad u(0,\mu) = 1,$$

bu yerda $u = \dfrac{\partial y(x,\mu)}{\partial \mu}$. U holda $\mu = 0$ deb olib, $\left.\dfrac{\partial y}{\partial \mu}\right|_{\mu=0} = u(x,0)$ funksiyani aniqlash masalasiga ega bo'lamiz:

$$\frac{du(x,0)}{dx} = y^2(x,0), \; u(0,0) = 1. \tag{6}$$

Endi dastlab berilgan masalani $\mu = 0$ da yozamiz:

$$y'_x(x,0) = 2x, \;\; y(0,0) = -1.$$

Bu masalaning yechimi $y(x,0) = x^2 - 1$ bo'lganligi uchun (6) masala

$$\frac{du(x,0)}{dx} = \left(x^2 - 1\right)^2, \; u(0,0) = 1$$

ko'rinishni oladi va uning yechimi uzil-kesil topiladi:

$$u(x,0) = \left.\frac{\partial y}{\partial \mu}\right|_{\mu=0} = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + 1. \blacktriangleright$$

210. $\begin{cases} \dot{x} = 4ty^2, \\ \dot{y} = 1 + 5\mu x, \end{cases}$ $x(0) = y(0) = 0;$ $\left.\dfrac{\partial x}{\partial \mu}\right|_{\mu=0}$ ni toping.

◀Berilgan masaladagi har bir tenglikni μ parametr bo'yicha differensiallaymiz:

$$\begin{cases} \dfrac{\partial u(t,\mu)}{\partial t} = 8t\, y(t,\mu)v(t,\mu), \; u(0,\mu) = 0, \\ \dfrac{\partial v(t,\mu)}{\partial t} = 5x(t,\mu) + 5\mu u(t,\mu), \; v(0,\mu) = 0, \end{cases} \tag{7}$$

bu yerda

$$u(t,\mu) = \frac{\partial x(t,\mu)}{\partial \mu}, \; v(t,\mu) = \frac{\partial y(t,\mu)}{\partial \mu}.$$

Ma`lumki, x, y funksiyalar dastlabki masalaning yechimlaridir. Dastlabki masalada $\mu = 0$ deb olib va tegishli tenglamalarni integrallab, topamiz: $x(t,0) = t^4$, $y(t,0) = t.$ Topilgan funksiyalarni (7) ga qo'yib va u yerda $\mu = 0$ deb olib,

$$\begin{cases} \dfrac{du(t,0)}{dt} = 8t^2\, v(t,0),\ u(0,0) = 0, \\ \dfrac{dv(t,0)}{dt} = 5t^4,\ v(0,0) = 0 \end{cases} \tag{8}$$

masalaga ega bo'lamiz. (8) sistemaning ikkinchi tenglamasidan topamiz: $v(t,0) = t^5$. $v(t,0)$ ning ifodasini (8) sistemaning birinchi tenglamasiga qo'yib, so'ngra integrallab, $u(t,0) = t^8$ yechimni olamiz. Shunday qilib, $\left.\dfrac{\partial x}{\partial \mu}\right|_{\mu=0} = t^8.$ ►

211. $\begin{cases} \dot{x} = xy + t^2, & x(1) = x_0, \\ 2\dot{y} = -y^2, & y(1) = y_0; \end{cases}$ $\left.\dfrac{\partial x}{\partial y_0}\right|_{\substack{x_0=3 \\ y_0=2}}$ ni toping.

◄Berilgan masaladagi har bir tenglikni y_0 parametr bo'yicha differensiallaymiz:

$$\begin{cases} \dfrac{\partial u(t,x_0,y_0)}{\partial t} = x(t,x_0,y_0)v(t,x_0,y_0) + u(t,x_0,y_0)y(t,x_0,y_0), \\ u(1,x_0,y_0) = 0, \\ 2\dfrac{\partial v(t,x_0,y_0)}{\partial t} = -2y(t,x_0,y_0)v(t,x_0,y_0), \\ v(1,x_0,y_0) = 1 \end{cases} \tag{9}$$

bu yerda

$$u(t,x_0,y_0) = \frac{\partial x(t,x_0,y_0)}{\partial y_0},\ v(t,x_0,y_0) = \frac{\partial y(t,x_0,y_0)}{\partial y_0}.$$

Ma`lumki, x, y funksiyalar dastlabki masalaning yechimlaridir. Berilgan dastlabki masalada $x_0 = 3,\ y_0 = 2$ deb olib va tegishli tenglamalarni integrallab, topamiz:

$$x(t,3,2) = t^3 + 2t^2,\ y(t,3,2) = 2t^{-1}.$$

Topilgan funksiyalarni (9) ga qo'yib va u yerda $x_0 = 3,\ y_0 = 2$ deb olib,

$$\begin{cases} \dfrac{\partial u(t,3,2)}{\partial t} = \left(t^3 + 2t^2\right)v(t,3,2) + \dfrac{2}{t}u(t,3,2), \\ \dfrac{\partial v(t,3,2)}{\partial t} = -\dfrac{2}{t}v(t,3,2),\ u(1,3,2) = 0,\ v(1,3,2) = 1 \end{cases} \tag{10}$$

masalaga ega bo'lamiz. (10) sistemaning ikkinchi tenglamasidan topamiz: $v(t,3,2) = t^{-2}$. $v(t,3,2)$ funksiyani (10) sistemaning birinchi tenglamasiga qo'yib, so'ngra integrallab, $u(t,3,2) = t^2 \ln t - 2t + 2t^2$ yechimni olamiz. Shunday qilib,

$$\left.\frac{\partial x}{\partial y_0}\right|_{\substack{x_0=3 \\ y_0=2}} = t^2 \ln t - 2t + 2t^2\ .\blacktriangleright$$

212. $\ddot{x} = \dfrac{2}{t} - \dfrac{2}{x},\ x(1) = 1,\ \dot{x}(1) = b;\ \left.\dfrac{\partial x}{\partial b}\right|_{b=1}$ ni topish kerak.

◀Berilgan masalani

$$\frac{d^2x(t,b)}{dt^2} = \frac{2}{t} - \frac{2}{x(t,b)},\ x(1,b) = 1,\ \frac{dx(t,b)}{dt} = b \tag{11}$$

ko'rinishda yozib olib, undagi tengliklarni b parametr bo'yicha differensiallaymiz va so'ngra ularning har birida $b = 1$ deb olib, quyidagi masalaga ega bo'lamiz:

$$\frac{d^2u(t,1)}{dt^2} = \frac{2\ u(t,1)}{x^2(t,1)},\ u(1,1) = 0,\ \frac{du(t,1)}{dt} = 1, \tag{12}$$

bu yerda $u(t,b)=\dfrac{\partial x(t,b)}{\partial b}$. Ravshanki, $x=t$ funksiya (11) masalaning $b=1$ bo'lgandagi yechimidir. Shuni hisobga olsak, (12) masaladagi tenglama

$$\frac{d^2u(t,1)}{dt^2}=\frac{2\ u(t,1)}{t^2}$$

ko'rinishni oladi. Bu tenglama Eyler tenglamasidir. Uning (12) dagi shartlarni qanoatlantiradigan yechimini topamiz:

$$u(t,\ 1)=\frac{t^2}{3}-\frac{1}{3t}.$$

Shunday qilib, $\left.\dfrac{\partial x}{\partial b}\right|_{b=1}=\dfrac{t^2}{3}-\dfrac{1}{3t}$.►

213. $\ddot{x}+\sin x=0$ tenglamaning yechimini taqribiy topish maqsadida uning o'rniga $\ddot{x}+x=0$ tenglama olindi. Agar $|x|\le 0,25$ da $|x-\sin x|<0,003$ ekanligi ma`lum bo'lsa, $x(0)=0,25,\ \dot{x}(0)=0$ boshlang'ich shartlarni qanoatlantiradigan yechimda paydo bo'ladigan xatolikni $0\le t\le 2$ da baholang.

◄Faraz qilaylik,

$$\ddot{y}+\sin y=0,\ y(0)=0,25,\ \dot{y}(0)=0 \tag{13}$$

masalaning yechimi $y(t)$,

$$\ddot{x}+x=0,\ x(0)=0,25,\ \dot{x}(0)=0 \tag{14}$$

masalaning yechimi esa $x(t)$ bo'lsin. U holda (13) tengliklardan (14) tengliklarni hadma-had ayirib, $u(t)=x(t)-y(t)$ xatolikni topish masalasiga kelamiz:

$$\ddot{u}(t)+u(t)=\sin y-y,\quad u(0)=0,\ \dot{u}(0)=0.$$

Bu masalaning yechimi

$$u(t)=\int_0^t \left(\sin y(\tau)-y(\tau)\right)\sin(t-\tau)d\tau \qquad (15)$$

ko'rinishda bo'ladi.

(13) tenglamani $\dot{y}$ ga hadma-had ko'paytirib va integrallab, hamda boshlang'ich shartlarni e`tiborga olib, topamiz:

$$\dot{y}^2=2(\cos y-\cos 0,25).$$

Bu yerdan $|y|\le 0,25$ ekanligi kelib chiqadi. Shuning uchun, $|\sin y - y|\le 0,003$, va (15) dan kerakli natijani olamiz:

$$|u(t)|\le \int_0^t |\sin y(\tau)-y(\tau)||\sin(t-\tau)|d\tau \le$$

$$\le 0,003\int_0^t |\sin(t-\tau)|d\tau \le 0,003\int_0^2 d\tau = 0,006. \blacktriangleright$$

6.2. ANALITIK TAQRIBIY USULLAR

6.2.1. Darajali qatorlar usuli. Agar $f(x,y)$ funksiya (x_0,y_0) nuqtaning atrofida analitik bo'lsa, ya`ni $(x-x_0)$ va $(y-y_0)$ ayirmalarning darajalari bo'yicha absolyut yaqinlashuvchi qatorga yoyilsa, u holda $y'=f(x,\ y)$ tenglamaning $y(x_0)=y_0$ boshlang'ich shartni qanoatlantiradigan yechimi ham analitik funksiya bo'ladi, ya`ni x_0 nuqtaning atrofida darajali qatorga yoyiladi. Xuddi shunday tasdiq boshlang'ich shartlari $y(x_0)=y_0,\ y'(x_0)=y'_0,\ \ldots,\ \ y^{(n-1)}(x_0)=y_0^{(n-1)}$ bo'lgan $y^{(n)}=f\left(x,y,y',...,y^{(n-1)}\right)$ tenglama uchun ham o'rinlidir.

Quyidagi masalalarning har birida berilgan boshlang'ich shartlarni qanoatlantiradigan yechimni darajali qator ko'rinishida izlang. Qatorning dastlabki bir necha ($x^0,\ x^1,\ x^2,\ x^3, x^4$ darajalar oldidagi) koeffitsientlarini hisoblang (**214-217**).

214. $y' = x + \frac{1}{y};\ y(0) = 1.$

◀Aniqlanishiga ko'ra, izlanayotgan yechim $y \neq 0$ bo'lishi kerak. $f(x,y) = x + \frac{1}{y}$ funksiya $(0,1)$ nuqtaning atrofida x, y o'zgaruvchilar bo'yicha analitik funksiyadir, shuning uchun bu masalaning $y(x) = \sum_{k=0}^{\infty} c_k x^k$ ko'rinishdagi analitik yechimi mavjud. Berilgan tenglamani $yy' = xy + 1$ ko'rinishda yozib, so'ngra qatorni shu tenglamaga qo'yamiz:

$$\left(c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \ldots\right) \cdot \left(c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + \ldots\right) =$$

$$= x\left(c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \ldots\right) + 1.$$

x ning bir xil darajalari oldidagi koeffitsientlarni bir-biriga tenglashtirib, $c_k\ (k = 0,\ 1,\ 2, \ldots)$ sonlarga nisbatan tenglamalar sistemasiga ega bo'lamiz:

$c_0 c_1 = 1,$

$c_1^2 + 2c_0 c_2 = c_0,$

$3c_1 c_2 + 3c_0 c_3 = c_1,$

$4c_0 c_4 + 4c_1 c_3 + 2c_2^2 = c_2,$

...

$y(0) = 1$ shartga ko'ra, $c_0 = 1$. U holda hosil bo'lgan sistemaning tenglamalaridan ketma-ket topamiz:

$$c_1 = 1,\ c_2 = 0,\ c_3 = \frac{1}{3},\ c_4 = -\frac{1}{3}, \ldots$$

Shunday qilib, taqribiy yechim

$$y(x) \approx 1 + x + \frac{1}{3}x^3 - \frac{1}{3}x^4$$

ko'rinishda bo'ladi.►

215. $y' = 2x + \cos y;\ y(0) = 0.$

◄ $f(x,y) = 2x + \cos y$ funksiyani $(0,0)$ nuqtaning atrofida $x,\ y$ o'zgaruvchilar bo'yicha darajali qatorga yoyamiz:

$$f(x,y) = 2x + 1 - \frac{y^2}{2!} + \frac{y^4}{4!} + \ldots$$

So'ngra, $y(0) = 0$ boshlang'ich shartni e`tiborga olib, yechimni

$$y(x) = c_1x + c_2x^2 + c_3x^3 + c_4x^4 + \ldots .$$

qator ko'rinishida izlaymiz. Bu qatorni

$$y' = 2x + 1 - \frac{y^2}{2!} + \frac{y^4}{4!} + \ldots$$

tenglamaga qo'yib va x ning bir xil darajalari oldidagi koeffitsientlarni bir-biriga tenglashtirib, $c_k\ (k = 0,\ 1,\ 2,\ldots)$ sonlarga nisbatan tengamalar sistemasiga ega bo'lamiz:

$$c_1 = 1,\ 2c_2 = 2,\ 3c_3 = -\frac{1}{2}c_1^2,\ 4c_4 = -c_1c_2, \ldots .$$

Bu yerdan c_k koeffitsientlarni topamiz:

$$c_1 = 1,\ c_2 = 1,\ c_3 = -\frac{1}{6},\ c_4 = -\frac{1}{4}, \ldots,$$

Shunday qilib, qo'yilgan masalaning yechimi

$$y(x) = x + x^2 - \frac{1}{6}x^3 - \frac{1}{4}x^4 - \ldots$$

ko'rinishda bo'ladi.►

216. $y' = x^2 + y^3;\ y(1) = 1.$

◀ $f(x,y)=x^2+y^3$ funksiya $(1,1)$ nuqtaning atrofida x, y o'zgaruvchilar bo'yicha analitik bo'lganligi uchun yechimni

$$y(x)=c_0+c_1(x-1)+c_2(x-1)^2+c_3(x-1)^3+c_4(x-1)^4+\dots$$

qator ko'rinishida izlaymiz. Bu qatorni berilgan tenglamaga qo'yib va $(x-1)$ ning bir xil darajalari oldidagi koeffitsientlarni bir-biriga tenglashtirib, $c_k\ (k=0,\ 1,\ 2,\dots)$ sonlarga nisbatan tenglamalar sistemasiga ega bo'lamiz:

$$c_1=1+c_0^3,\ 2c_2=2+3c_0^2c_1,$$

$$3c_3=1+3c_0c_1^2+3c_0^2c_2,\ 4c_4=6c_0c_1c_2+3c_0^2c_3+c_1^3,\dots$$

$y(1)=1$ shartga ko'ra, $c_0=1.$ U holda hosil bo'lgan sistemaning tenglamalaridan c_k koeffitsientlarni ketma-ket topamiz:

$$c_1=2, c_2=4, c_3=\frac{25}{3}, c_4=\frac{81}{4},\dots$$

Demak, qo'yilgan masalaning yechimi

$$y(x)=1+2(x-1)+4(x-1)^2+\frac{25}{3}(x-1)^3+\frac{81}{4}(x-1)^4+\dots$$

ko'rinishga ega.▶

217. $y''=y'^2+xy;\ y(0)=4,\ y'(0)=-2.$

◀Avvalgi masalalardagi kabi bu masalada ham $y(x)$ taqribiy yechimni darajali qatorning xususiy yig'indisi ko'rinishida topishimiz mumkin edi. Ammo hozirgi holda boshqacha yo'l tutamiz. Izlanayotgan darajali qator Teylor qatori ekanligini bilgan holda berilgan tenglamaning o'ng tomonini x bo'yicha ketma-ket differensiallab kerakli hosilalarning $x=0$ nuqtadagi qiymatlarini hisoblaymiz. Shunga binoan, boshlang'ich shartlarni e`tiborga olib, topamiz:

$$y''(0)=y'^2(0)=4,$$

$$y'''(x)=\frac{d}{dx}\left(y'^2+xy\right)=2y'y''+xy'+y,\ \ y'''(0)=-12,$$

$$y^{(IV)}(x)=\frac{d}{dx}\left(2y'y''+xy'+y\right)=2y''^2+2y'y'''+xy''+2y',$$

$$y^{(IV)}(0)=76,\ \ldots.$$

Shunday qilib, Teylor formulasiga ko'ra, masalaning yechimi

$$y(x)=4-2x+2x^2-2x^3+\frac{19}{6}x^4+...$$

ko'rinishda bo'ladi.►

6.2.2. Kichik parametr usuli. Agar ushbu

$$\frac{dx_k}{dt}=f_k(t,\ x_1,\ x_2,\ ...,\ x_n,\ \mu),\ \ x_k(0)=a_k(\mu),\ k=\overline{1,\ n}, \qquad (1)$$

masalada f_k, a_k funksiyalar $x_1,\ x_2,\ ...,\ x_n,\ \mu$ o'zgaruvchilar bo'yicha analitik bo'lsa, u holda uning $x(t,\ \mu)$ vektor-yechimi

$$x(t,\mu)=y_0(t)+\mu y_1(t)+\mu^2 y_2(t)+... \qquad (2)$$

μ bo'yicha darajali qatorga yoyiladi va bu qator μ parametrning kichik qiymatlarida ($|\mu|\ll 1$) yaqinlashadi. $y_0, y_1, \ldots.$ funksiyalarni topish uchun (1) masalaning o'ng tomonidagi funksiyalarni μ ning darajalari bo'yicha qatorga yoyish kerak va u yerga (2) yoyilmani qo'yib, μ ning bir xil darajalari oldidagi koeffitsientlarni bir-birlariga tenglash kerak. Natijada tegishli boshlang'ich shartlar bilan differensial tenglamalar sistemasini hosil qilamiz. Bu sistemani yechib, birin-ketin $y_0, y_1, \ldots.$ funksiyalarni topamiz. Ixtiyoriy o'zgarmaslarni topish uchun

$$y_i(t_0)=\left(\alpha_{1i},\alpha_{2i},...,\alpha_{ni}\right),\ \ \alpha_{ki}=const$$

boshlang'ich shartlardan foydalanamiz.

Kichik parametr usuli yordamida

$$\ddot{x}+a^2x=\mu F(t,x,\dot{x},\mu) \qquad (3)$$

ko'rinishdagi tenglamalarning davriy yechimlarini taqribiy topish mumkin, bu yerda F - t bo'yicha ma`lum davriy funksiya. Bu holda y_0, $y_1, \ldots$ funksiyalarga nisbatan differensial tenglamalarni yechishda hosil bo'ladigan integrallash o'zgarmaslarining qiymatlari ko'rsatilgan differensial tenglamalar o'ng tomonlari nol bo'lganda topilgan funksiyalarning davriylik shartlaridan aniqlanadi.

Agar (3) tenglamaning o'ng tomoni t ga oshkor bog'liq bo'lmasa, u holda $x(t,\mu)$ yechimning davri oldindan ma`lum bo'lmaydi. Bunday holda (3) tenglamada

$$\tau = t\left(1 + b_1\mu + b_2\mu^2 + \ldots\right)$$

o'rniga qo'yishni bajarish (τ - yangi erkli o'zgaruvchi) va $\frac{2\pi}{a}$ davrli $x(t,\mu)$ yechimni izlash kerak. Bunda $b_1, b_2, \ldots$koeffitsientlar $y_0(\tau)$, $y_1(\tau), \ldots$ yechimlarning davriylik shartlaridan topiladi.

Quyidagi masalalarda yechimning μ kichik parametr darajalari bo'yicha yoyilmasining ikkita yoki uchta hadlarini toping (**218-219**).

218. $y' = \frac{2}{y} - 5\mu x,\ y(1) = 2.$

◄Tenglamaning o'ng tomonidagi funksiya $y > 0$ da y, μ o'zgaruvchilarning analitik funksiyasi ekanligini e`tiborga olib berilgan tenglamani $yy' = 2 - 5\mu xy$ ko'rinishda tasvirlab olamiz va kichik parametr usulidan foydalanib, yechimni

$$y(t,\mu) = y_0(x) + \mu y_1(x) + \mu^2 y_2(x) + \ldots \tag{4}$$

ko'rinishda izlaymiz. So'ngra,

$$y(x,0) = y_0(x),\quad \left.\frac{\partial y(x,\mu)}{\partial \mu}\right|_{\mu=0} = y_1(x),\ \left.\frac{\partial^2 y(x,\mu)}{\partial \mu^2}\right|_{\mu=0} = 2y_2(x),\ \ldots,$$

$$y'_x(x,0)=y'_0(x),\ \frac{\partial}{\partial\mu}y'_x(x,\mu)\bigg|_{\mu=0}=y'_1(x),\ \frac{\partial^2}{\partial\mu^2}y'_x(x,\mu)\bigg|_{\mu=0}=2y'_2(x),...$$

munosabatlarni hisobga olib, berilgan tenglamani μ parametr bo'yicha differensiallab, topamiz:

$$y_0y'_0=2,\quad y_0y'_1+y_1y'_0=-5xy_0,\ y_0y'_2+y_1y'_1+y_2y'_0=-5xy_1,.... \tag{5}$$

$y(1)=2$ boshlang'ich shartdan kelib chiqib, (4) dan $y_k,\ k=0,1,2,...$ funksiyalar uchun boshlang'ich shartlarga ega bo'lamiz:

$$y_0(1)=2,\ y_1(1)=y_2(1)=...=0. \tag{6}$$

Endi (5) tenglamalarni ketma-ket integrallab va (6) shartlardan foydalanib, topamiz:

$$y_0(x)=2\sqrt{x},\ y_1(x)=\frac{2}{\sqrt{x}}-2x^2, \tag{7}$$

$$y_2(x)=\frac{1}{4}x^{7/2}-\frac{4}{3}x+\frac{25}{12\sqrt{x}}-\frac{1}{x\sqrt{x}},.... \tag{7'}$$

Aniqlangan (7) va (7') funksiyalarni (4) ga qo'yib, qo'yilgan masalaning yechimini hosil qilamiz:

$$y(x,\mu)=2\sqrt{x}+2\mu\left(\frac{1}{\sqrt{x}}-x^2\right)+$$
$$+\mu^2\left(\frac{1}{4}x^{7/2}-\frac{4}{3}x+\frac{25}{12\sqrt{x}}-\frac{1}{x\sqrt{x}}\right)+....\blacktriangleright$$

219. $y'=\frac{6\mu}{x}-y^2,\ y(1)=1+3\mu.$

◀Oldingi misoldagi kabi

$$y(t,\mu)=y_0(x)+\mu y_1(x)+\mu^2y_2(x)+...$$

deb olamiz, bu yerda

$$y_0(x)=y(x,0),\quad y_1(x)=\left.\frac{\partial y(x,\mu)}{\partial\mu}\right|_{\mu=0},\ y_2(x)=\left.\frac{1}{2}\frac{\partial^2 y(x,\mu)}{\partial\mu^2}\right|_{\mu=0},\ \ldots,$$

$$y_0'(x)=y_x'(x,0),\ y_1'(x)=\left.\frac{\partial}{\partial\mu}y_x'(x,\mu)\right|_{\mu=0},\ \ y_2'(x)=\left.\frac{1}{2}\frac{\partial^2}{\partial\mu^2}y_x'(x,\mu)\right|_{\mu=0},$$

....

Shu munosabatlardan foydalanib, berilgan tenglamadan topamiz:

$$y_0'=-y_0^2,\ y_1'=-2y_0y_1+\frac{6}{x},\quad y_2'=-y_1^2-2y_0y_2,\ \ldots. \tag{8}$$

Bunda boshlang'ich shartlar

$$y_0(1)=1,\ y_1(1)=3,\ y_2(1)=y_3(1)=\ldots=0 \tag{9}$$

ko'rinishni oladi. (8) ning birinchi tenglamasidan $y_0(x)=C_1/x$ kelib chiqadi. (9) dagi birinchi boshlang'ich shartga binoan $y_0(x)=1/x$ ga teng. So'ngra, (8) ning ikkinchi tenglamasidan $y_1(x)=(3x^2+C_2)/x^2$ yechimni topish qiyin emas. C_2 o'zgarmasni $y_1(1)=3$ shartdan topamiz: $C_2=0$. Shunga ko'ra, $y_1(x)=3$. Xuddi shunga o'xshash,

$$y_2'+\frac{2}{x}y_2=-9,\ y_2(1)=0$$

masalani yechib, $y_2(x)=\dfrac{3}{x^2}-3x$ funksiyaga ega bo'lamiz. Shunday qilib, qo'yilgan masalaning yechimi $y(x,\mu)=\dfrac{1}{x}+3\mu+3\mu^2\left(\dfrac{1}{x^2}-x\right)+\ldots$ ko'rinishda topiladi.►

Kichik parametr usuli yordamida quyidagi tenglamalarning taqribiy davriy yechimini toping. Izlanayotgan yechimning davri o'ng tomondagi funksiyaning davriga teng bo'lsin; μ - kichik parametr bo'lsin (**220-221**).

220. $\ddot{x}+5x=\cos 2t+\mu x^2$.

◄Kichik parametr usuliga binoan davriy yechimni

$$x(t,\mu)=x_0(t)+\mu x_1(t)+\mu^2 x_2(t)+\dots, \qquad (10)$$

bu yerda x_k $(k=0,1,2,\dots)$ - $\pi-$davrli funksiyalar (chunki tenglamaning o'ng tomoni shunday funksiya). (10) yoyilmani tenglamaga qo'yib va μ ning bir xil darajalari oldidagi koeffitsientlarni bir-birlariga tenglashtirib, quyidagi tenglamalar sistemasiga ega bo'lamiz:

$$\ddot{x}_0+5x_0=\cos 2t,\ \ddot{x}_1+5x_1=x_0^2,\ \ddot{x}_2+5x_2=2x_0x_1,\dots. \qquad (11)$$

Birinchi tenglama

$$x_0(t)=C_{10}\sin\sqrt{5}t+C_{20}\cos\sqrt{5}t+\cos 2t$$

ko'rinishdagi umumiy yechimga ega. $\pi-$davrli yechimni topish talab etilayotganligi uchun oxirgi tenglikda $C_{10}=C_{20}=0$ deb olish kerak. Demak, $x_0(t)=\cos 2t$. Shu ifodani e`tiborga olib, (11) sistemaning ikkinchi tenglamasidan topamiz:

$$x_1(t)=C_{11}\sin\sqrt{5}t+C_{21}\cos\sqrt{5}t+\frac{1}{10}-\frac{1}{22}\cos 4t.$$

$x_1(t)$ funksiyaning $\pi-$davrlilik shartidan

$$x_1(t)=\frac{1}{10}-\frac{1}{22}\cos 4t$$

funksiyaga ega bo'lamiz. Xuddi shunga o'xshash, (11) sistemaning uchinchi tenglamasidan topamiz:

$$x_2(t)=\frac{17}{110}\cos 2t+\frac{1}{682}\cos 6t.$$

Shunday qilib, qo'yilgan masalaning yechimi

$$x(t,\mu)=\cos 2t+\mu\left(\frac{1}{10}-\frac{1}{22}\cos 4t\right)+$$

$$+\mu^2\left(\frac{17}{110}\cos 2t+\frac{1}{682}\cos 6t\right)+O(\mu^3)$$

ko'rinishda bo'ladi.►

221. $\ddot{x}+x^2=1+\mu\sin t.$

◀Ushbu qatorni

$$x(t,\mu)=x_0(t)+\mu x_1(t)+\mu^2x_2(t)+...$$

berilgan tenglamaga qo'yib, ma`lum usullar yordamida quyidagi tenglamalar sistemasini olamiz:

$$\ddot{x}_0+x_0^2=1,\ \ddot{x}_1+2x_0x_1=\sin t,\ \ \ddot{x}_2+x_1^2+2x_0x_2=0,\ ....$$

Bu sistema birinchi tenglamasining yechimi $x_0=\pm1$ ekanligini ko'rish qiyin emas. Aniqlik uchun, $x_0=1$ bo'lsin. Bu holda sistemaning keyingi tenglamalarini ketma-ket yechib, topamiz:

$$x_1(t)=\sin t,\quad x_2(t)=-\frac{1}{4}-\frac{1}{4}\cos 2t\,.$$

Xuddi shunga o'xshash, $x_0=-1$ bo'lgan holda

$$x_1(t)=-\frac{1}{3}\sin t,\quad x_2(t)=\frac{1}{36}-\frac{1}{108}\cos 2t$$

funksiyalarni olamiz.

Endi olingan natijalarni jamlab, qo'yilgan masalaning yechimlarini yozamiz:

$$x^+(t,\mu)=1+\mu\sin t-\frac{1}{4}\mu^2(1+\cos 2t)+O(\mu^3);$$

$$x^-(t,\mu)=-1-\frac{1}{3}\mu\sin t+\frac{1}{36}\mu^2\left(1-\frac{1}{3}\cos 2t\right)+O(\mu^3).$$ ▶

222. Kichik parametr usuli yordamida $x''+x+x^3=0$ tenglamaning taqribiy davriy yechimini toping.

◀ x ni yetarlicha kichik deb hisoblab, kichik parametr sifatida $x''+x=0$ tenglamaning yechimi bo'lgan tebranishlar amplitudasini olamiz. Aniqlik uchun $x\big|_{t=0}=\mu$ (μ −kichik parametr) deb olib, berilgan tenglamaning davriy yechimini

$$x = \mu x_0(\tau) + \mu^2 x_1(\tau) + \mu^3 x_2(\tau) + \ldots$$

ko'rinishda izlaymiz, bu yerda

$$\tau = t\left(1 + b_1\mu + b_2\mu^2 + \ldots\right).$$

Bu yoyilmalarni tenglamaga qo'yib va μ ning bir xil darajalari oldidagi koeffitsientlarni bir-birlariga tenglashtirib, quyidagi tenglamalar sistemasiga ega bo'lamiz:

$$\ddot{x}_0 + x_0 = 0,\ \ddot{x}_1 + x_1 = -2b_1\ddot{x}_0,$$

$$\ddot{x}_2 + x_2 + 2b_1\ddot{x}_1 + \left(b_1^2 + 2b_2\right)\ddot{x}_0 + x_0^3 = 0,$$

$$\ddot{x}_3 + 2b_1\ddot{x}_2 + (b_1^2 + 2b_2)\ddot{x}_1 + 2\left(b_1 b_2 + b_3\right)\ddot{x}_0 + x_3 + 3x_0^2 x_1 = 0, \ldots.$$

Bu sistemadagi birinchi tenglamaning $x_0(0) = 1$ boshlang'ich shartni qanoatlantiradigan yechimi $x_0 = \cos\tau$ ekanligi ravshan. Sistema ikkinchi tenglamasining yechimi $x_1(0) = 0$ shartni qanoatlantirishi kerak. Bunday yechimga $b_1 = 0$ bo'lgandagina erishiladi: $x_2 = 0$. Olingan natijalarga binoan sistemaning uchinchi tenglamasi

$$\ddot{x}_2 + x_2 = 2b_2\cos\tau - \cos^3\tau$$

ko'rinishni oladi va bu tenglamaning $x_2(0) = 0$ boshlang'ich shartni qanoatlantiradigan yechimi $b_2 = 3/8$ da

$$x_2 = \frac{1}{32}(\cos 3\tau - \cos\tau)$$

ekanligiga ishonch hosil qilish qiyin emas. Hisoblashlarni shu tarzda davom ettirib, $x_3 = 0,\ b_3 = 0,\ \ldots$ ekanligini ko'ramiz. Shunday qilib,

$$x = \mu\cos\tau + \frac{1}{32}\mu^3(\cos 3\tau - \cos\tau) + O(\mu^5),$$

$$\tau = t\left(1 + \frac{3}{8}\mu^2 + O(\mu^4)\right). \blacktriangleright$$

6.3. DIFFERENSIAL TENGLAMALARNI YECHISHNING SONLI USULLARI

6.3.1. Eylerning siniq chiziqlar usuli. Quyidagi masalani qaraymiz:

$$y' = f(x,y),\ \ y(x_0) = y_0,\ \ x_0 \le x \le b, \tag{1}$$

bu yerda

$$y = y(x) = \left(y_1(x), y_2(x), ..., y_m(x)\right),$$
$$f(x,y) = \left(f_1(x, y_1, y_2, ..., y_m),\ f_2(x, y_1, y_2, ..., y_m), ...\right)$$

- yetarlicha marta uzluksiz differensiallanuvchi funksiyalar.

(1) differensial masalani sonli yechish uchun $[x_0,\ b]$ integrallash oralig'i har birining uzunligi $h = \dfrac{b - x_0}{n}$ bo'lgan teng qismlarga bo'linadi va $y(x_0 + ih) = y_i$ qiymatidan foydalanib

$$y_{s+1} = y_k + hy'_k + \frac{h^2}{2!} y''_k + ... + \frac{h^s}{s!} y_k^{(s)}, \tag{2}$$

formula yordamida $y\left(x_0 + (k+1)h\right) = y_{k+1}$ qiymatlar taqribiy hisoblanadi, bu yerda

$$y'_k = f(x_k, y_k),$$

$$y''_k = \frac{d}{dx} f(x,y)\Bigg|_{\substack{x=x_k \\ y=y_k}} = \frac{\partial f(x_k, y_k)}{\partial x} + \frac{\partial f(x_k, y_k)}{\partial y} \cdot f(x_k, y_k),$$

$$\frac{\partial f}{\partial y} = \begin{pmatrix} \dfrac{\partial f_1}{\partial y_1} & \dfrac{\partial f_1}{\partial y_2} & ... & \dfrac{\partial f_1}{\partial y_m} \\ ... & ... & ... & ... \\ \dfrac{\partial f_m}{\partial y_1} & \dfrac{\partial f_m}{\partial y_2} & ... & \dfrac{\partial f_m}{\partial y_m} \end{pmatrix},\ \ x_k = x_0 + k\,h,\ \ k = 0, 1, 2, ..., n-1.$$

$[x_k,\ x_{k+1}]$ integrallash qadamidagi xatolik $O\left(h^{s+1}\right)$ tartibli bo'ladi.

Bayon etilgan usul *Eylerning siniq chiziqlar usuli* yoki $s-$*tartibli Eyler usuli* deyiladi.

Eylerning siniq chiziqlar usuli yordamida quyidagi differensial masalalarning yechimlarini ko’rsatilgan oraliqda taqribiy toping. Hisoblashlarni $h=0,2$ qadam bilan verguldan keyin uchta xona aniqligida bajaring (**223-225**).

223. $y'=\dfrac{1}{y}+x,\ 0\le x\le 1;\ y(0)=1.$

◄ $k=2$ bo’lsin. Har bir qadamdagi xatolik $O\left(h^3\right)\approx 0,008$ miqdorga teng bo’lganligi uchun hisoblashlarni verguldan keyin uchta xona aniqligida bajarish mumkin. Biz qarayotgan holda o’ng tomon $f(x,y)=x+y^{-1}$ ga teng, shuning uchun

$$y_{k+1}=y_k+hy'_k+\frac{h^2}{2!}y''_k=y_k+h\left(\frac{1}{y_k}+x_k\right)+\frac{h^2}{2!}\left(1-\frac{1}{y_k^2}\left(\frac{1}{y_k}+x_k\right)\right),$$
$$k=0,1,2,3,4,$$

yoki

$$y_{k+1}=y_k+0,2\left(\frac{1}{y_k}+0,2k\right)+0,02\left(1-\frac{1}{y_k^3}-\frac{0,2k}{y_k^2}\right).$$

Bu yerda ketma-ket $k=0,1,2,3,4$ deb olib va boshlang’ich shartni e`tiborga olib, topamiz:

$$y_1=1+0,2\cdot 1+0,02\cdot 0=1,2,$$

$$y_2=1,2+0,2\left(\frac{1}{1,2}+0,2\cdot 1\right)+0,02\left(1-\frac{1}{1,2^3}-\frac{0,2\cdot 1}{1,2^2}\right)=1,413,$$

$$y_3=1,413+0,2\left(\frac{1}{1,413}+0,2\cdot 2\right)+0,02\left(1-\frac{1}{1,413^3}-\frac{0,2\cdot 2}{1,413^2}\right)=1,644,$$

$$y_4=1,644+0,2\left(\frac{1}{1,644}+0,2\cdot 3\right)+0,02\left(1-\frac{1}{1,644^3}-\frac{0,2\cdot 3}{1,644^2}\right)=1,897,$$

$$y_5 = 1{,}897 + 0{,}2\left(\frac{1}{1{,}897} + 0{,}2\cdot 4\right) + 0{,}02\left(1 - \frac{1}{1{,}897^3} - \frac{0{,}2\cdot 4}{1{,}897^2}\right) = 2{,}172.$$

▶

224. $y' = \dfrac{2y}{x} + 2x^3, 1 \le x \le 2;\ y(1) = 2.$

◀ $k = 3$ bo'lsin, ya`ni uchinchi tartibli Eyler usulini qo'llaymiz. Bu holda y_{k+1} qiymatlar quyidagicha hisoblanadi:

$$y_{k+1} = y_k + hy_k' + \frac{h^2}{2!}y_k'' + \frac{h^3}{3!}y_k''',\ k = 0,1,2,3,4,$$

bu yerda

$$y_k' = f(x_k, y_k),\quad y_k'' = \frac{d}{dx}f(x,y)\Big|_{\substack{x=x_k\\ y=y_k}} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\cdot f,$$

$$y_k''' = \frac{d^2}{dx^2}f(x,y)\Big|_{\substack{x=x_k\\ y=y_k}} = \frac{\partial^2 f}{\partial x^2} + 2\frac{\partial^2 f}{\partial x\partial y}\cdot f + \frac{\partial^2 f}{\partial y^2}\cdot f^2 + \frac{\partial f}{\partial x}\cdot\frac{\partial f}{\partial y} + \left(\frac{\partial f}{\partial y}\right)^2\cdot f,$$

$$f = f(x_k, y_k).$$

Shunga binoan, tenglamaning o'ng tomoni $f(x,y) = \dfrac{2y}{x} + 2x^3$ ko'rinishda bo'lganda

$$y_k' = \frac{2y_k}{x_k} + 2x_k^3,\quad y_k'' = \frac{2y_k}{x_k^2} + 10x_k^2,\quad y_k''' = 4x_k + \frac{4y_k}{x_k^3} + \frac{4}{x_k^2} - \frac{12x_k^2}{y_k}$$

formulalar o'rinli.

Endi ketma-ket $k = 0,1,2,3,4$ deb olib va boshlang'ich shartni e`tiborga olib, topamiz ($h = 0{,}2$):

$$y_0 = 2,$$

$$y_1 = 2 + 0{,}2(4+2) + 0{,}02(4+10) + \frac{0{,}008}{6}(4\cdot 1 + 4\cdot 2 + 4 - 6) = 3{,}4933,$$

$$y_2 = 3,4933 + 0,2\left(\frac{2\cdot 3,4933}{1,2} + 2\cdot 1,2^3\right) + 0,02\left(\frac{2\cdot 3,4933}{1,2^2} + 10\cdot 1,2^2\right) +$$

$$+\frac{0,008}{6}\left(4\cdot 1,2 + \frac{4\cdot 3,4933}{1,2^3} + \frac{4}{1,2^2} - \frac{12\cdot 1,2^2}{3,4933}\right) = 5,7302,$$

$$y_3 = 5,7302 + 0,2\left(\frac{2\cdot 5,7302}{1,4} + 2\cdot 1,4^3\right) + 0,02\left(\frac{2\cdot 5,7302}{1,4^2} + 10\cdot 1,4^2\right) +$$

$$+\frac{0,008}{6}\left(4\cdot 1,4 + \frac{4\cdot 5,7302}{1,4^3} + \frac{4}{1,4^2} - \frac{12\cdot 1,4^2}{5,7302}\right) = 8,9898,$$

$$y_4 = 13,546, \ \ y_5 = 19,7339.$$

Boshqa tomondan, qo'yilgan masalani kvadraturalarda, masalan, o'zgarmasni variatsiyalash usulida yechib, $y = x^2 + x^4$ aniq yechimni topishimiz ham mumkin. Aniq yechimning qiymatlarini $x_k = 1 + 0,2k$, $k = 1,2,3,4,5$ da hisoblaymiz:

$$y_1^* = 1,2^2 + 1,2^4 = 3,5136, \quad y_2^* = 1,4^2 + 1,4^4 = 5,8016,$$

$$y_3^* = 1,6^2 + 1,6^4 = 9,1136, \quad y_4^* = 1,8^2 + 1,8^4 = 13,7376,$$

$$y_5^* = 2^2 + 2^4 = 20.$$

Aniq yechimning bunday qiymatlarini keltirishdan maqsad masalaning taqribiy yechimi integrallash oralig'ining nuqtalarida aniq yechimdan ma`lum miqdorga farq qilishini ta`kidlashdir. k-tartibli Eyler usulida k soni kattalashgani sari aniq va taqribiy yechimlar orasidagi farqlar ham kamayib borishini tushunish qiyin emas.►

225. $\dot{x} = 64t + x - y, \ \dot{y} = 8t^2 + 5x - y,$

$$0 \le x \le 0,5; \quad x(0) = 17, \ y(0) = 1.$$

◄ $h = 0,1; \ k = 2$ bo'lsin (2-tartibli Eyler usuli). x_{k+1}, y_{k+1} miqdorlarni quyidagi munosabatlardan aniqlaymiz:

$$x_{k+1} = x_k + h\dot{x}_k + \frac{h^2}{2!}\ddot{x}_k = x_k + h(64t_k + x_k - y_k) + \frac{h^2}{2}(64 + \dot{x}_k - \dot{y}_k) =$$

$$= x_k + h\ (64t_k + x_k - y_k) + \frac{h^2}{2}\left[64 + (64t_k + x_k - y_k) - (8t_k^2 + 5x_k - y_k)\right] =$$

$$= 64ht_k + (32 + 32t_k - 4t_k^2)h^2 + \left(1 + h - 2h^2\right)x_k - hy_k =$$

$$= 0,32 + 6,72t_k - 0,04t_k^2 + 1,08x_k - 0,1y_k,$$

$$y_{k+1} = y_k + h\dot{y}_k + \frac{h^2}{2!}\ddot{y}_k = y_k + h(8t_k^2 + 5x_k - y_k) + \frac{h^2}{2}\left(16t + 5\dot{x}_k - \dot{y}_k\right) =$$

$$= y_k + h(8t_k^2 + 5x_k - y_k) + \frac{h^2}{2}\left[16t_k + 5(64t_k + x_k - y_k) - (8t_k^2 + 5x_k - y_k)\right] =$$

$$= 8ht_k^2 + 4t_k(12 - t_k)h^2 + 5hx_k + (1 - h - 2h^2)y_k =$$

$$= 0,48t_k + 0,76t_k^2 + 0,5x_k + 0,88y_k,\ k = 0,1,2,3,4.$$

Bu formulalarda $x_0 = 17,\ y_0 = 1$ boshlang'ich shartlardan foydalanib, ketma-ket topamiz:

$$x_1 = 18,58, \qquad y_1 = 9,38,$$

$$x_2 = 20,12, \qquad y_2 = 17,6,$$

$$x_3 = 21,632, \qquad y_3 = 25,6744,$$

$$x_4 = 23,1276, \qquad y_4 = 33,6219,$$

$$x_5 = 24,6172, \qquad y_5 = 41,4647.$$

Boshqa tomondan, qo'yilgan masalani, masalan, aniqmas koeffitsientlar usuli yordamida yechib,

$$x = -2t^2 + 16t + 17,\ \ y = -2t^2 + 84t + 1$$

ko'rinishdagi aniq yechimni topamiz. Taqqoslash uchun aniq yechimning qiymatlarini keltiramiz:

$$x_1^* = 18,58, \qquad y_1^* = 9,38,$$

$$x_2^* = 20,12, \qquad y_2^* = 17,72,$$

$$x_3^* = 21,62, \qquad y_3^* = 26,02,$$

$$x_4^* = 23,08, \qquad y_4^* = 33,28,$$

$$x_5^* = 24,5, \qquad y_5^* = 42,5. \blacktriangleright$$

6.3.2. Runge-Kutta usuli. 6.3.1-banddagi masalani qaraymiz va u yerdagi belgilashlarni saqlab qolamiz. Bunga qo'shimcha, $y_{ik} = y_i(x_k)$ belgilash kiritamiz.

*Runge-Kutta usuli*da avvalo quyidagi sonlar aniqlab olinadi:

$$r_{i1} = f_i(x_k, y_{1k}, y_{2k}, ..., y_{mk}),$$

$$r_{i2} = f_i\left(x_k + \frac{h}{2},\ y_{1k} + \frac{hr_{11}}{2},\ y_{2k} + \frac{hr_{21}}{2},\ ...,\ y_{mk} + \frac{hr_{m1}}{2}\right),$$

$$r_{i3} = f_i\left(x_k + \frac{h}{2},\ y_{1k} + \frac{hr_{12}}{2},\ y_{2k} + \frac{hr_{22}}{2},\ ...,\ y_{mk} + \frac{hr_{m2}}{2}\right),$$

$$r_{i4} = f_i\left(x_k + h,\ y_{1k} + hr_{13},\ y_{2k} + hr_{23},\ ...,\ y_{mk} + hr_{m3}\right)$$

va bular yordamida $y_{i,k+1}$ miqdorlar hisoblab topiladi:

$$y_{i,k+1} = y_{ik} + \frac{h}{6}\left(r_{i1} + 2r_{i2} + 2r_{i3} + r_{i4}\right),\ i = 1,2,...,n.$$

$[x_i,\ x_{i+1}]$ integrallash qadamidagi xatolik $O\left(h^5\right)$ tartibli bo'ladi.

Runge-Kutta usuli yordamida quyidagi differensial masalalarning yechimlarini taqribiy hisoblang (hisoblashlarni verguldan keyin to'rtta xona aniqligida bajaring) (**226-227**).

226. $y' = 2x(x^2 + y),\quad 0 \le x \le 0,5;\ y(0) = -1.$

$\blacktriangleleft\ h = 0,1$ bo'lsin. Bu holda, yuqoridagi formulalarga binoan, yozamiz:

$$r_{k1} = 2x_k(x_k^2 + y_k),$$

$$r_{k2} = 2(x_k + 0,05)\left[(x_k + 0,05)^2 + y_k + 0,05r_{k1}\right],$$

$$r_{k3} = 2(x_k + 0,05)\left[(x_k + 0,05)^2 + y_k + 0,05r_{k2}\right],$$

$$r_{k4} = 2(x_k + 0,1)\left[(x_k + 0,1)^2 + y_k + 0,1r_{k3}\right],$$

$$y_{k+1} = y_k + \frac{0,1}{6}\left(r_{k1} + 2r_{k2} + 2r_{k3} + r_{k4}\right),\ x_k = 0,1k,\ y_0 = -1,$$

$$k = 0,1,2,3,4.$$

Endi ketma-ket $k = 0,1,2,3,4$ deb olib, topamiz:

$$x_0 = 0,\ y_0 = -1;\ r_{01} = 2x_0(x_0^2 + y_0) = 0,$$

$$r_{02} = 2(x_0 + 0,05)\left[(x_0 + 0,05)^2 + y_0 + 0,05r_{01}\right] = -0,09975,$$

$$r_{03} = 2(x_0 + 0,05)\left[(x_0 + 0,05)^2 + y_0 + 0,05r_{02}\right] = -0,1047,$$

$$r_{04} = 2(x_0 + 0,1)\left[(x_0 + 0,1)^2 + y_0 + 0,1r_{03}\right] = -0,2001,$$

$$y_1 = y_0 + \frac{0,1}{6}\left(r_{01} + 2r_{02} + 2r_{03} + r_{04}\right) = -1,0102;$$

$$x_1 = 0,1,\ \ r_{11} = 2x_1(x_1^2 + y_1) = -0,2000,$$

$$r_{12} = 2(x_1 + 0,05)\left[(x_1 + 0,05)^2 + y_1 + 0,05r_{11}\right] = -0,2993,$$

$$r_{13} = 2(x_1 + 0,05)\left[(x_1 + 0,05)^2 + y_1 + 0,05r_{12}\right] = -0,3008,$$

$$r_{14} = 2(x_1 + 0,1)\left[(x_1 + 0,1)^2 + y_1 + 0,1r_{13}\right] = -0,4001,$$

$$y_2 = y_1 + \frac{0,1}{6}\left(r_{11} + 2r_{12} + 2r_{13} + r_{14}\right) = -1,0402;$$

$$x_2 = 0,2,\ \ r_{21} = 2x_2(x_2^2 + y_2) = -0,4001,$$

$$r_{22} = 2(x_2 + 0,05)\left[(x_2 + 0,05)^2 + y_2 + 0,05r_{21}\right] = -0,4988,$$

$$r_{23} = 2(x_2 + 0,05)\left[(x_2 + 0,05)^2 + y_2 + 0,05r_{22}\right] = -0,5013,$$

$$r_{24}=2(x_2+0,1)\left[(x_2+0,1)^2+y_2+0,1r_{23}\right]=-0,6002,$$

$$y_3=y_2+\frac{0,1}{6}\left(r_{21}+2r_{22}+2r_{23}+r_{24}\right)=-1,0902;$$

$$x_3=0,3,\ \ r_{31}=2x_3(x_3^2+y_3)=-0,6001,$$

$$r_{32}=2(x_3+0,05)\left[(x_3+0,05)^2+y_3+0,05r_{31}\right]=-0,6984,$$

$$r_{33}=2(x_3+0,05)\left[(x_3+0,05)^2+y_3+0,05r_{32}\right]=-0,7018,$$

$$r_{34}=2(x_3+0,1)\left[(x_3+0,1)^2+y_3+0,1r_{33}\right]=-0,8003,$$

$$y_4=y_3+\frac{0,1}{6}\left(r_{31}+2r_{32}+2r_{33}+r_{34}\right)=-1,1602;$$

$$x_4=0,4,\ \ r_{41}=2x_4(x_4^2+y_4)=-0,8002,$$

$$r_{42}=2(x_4+0,05)\left[(x_4+0,05)^2+y_4+0,05r_{41}\right]=-0,8979,$$

$$r_{43}=2(x_4+0,05)\left[(x_4+0,05)^2+y_4+0,05r_{42}\right]=-0,9023,$$

$$r_{44}=2(x_4+0,1)\left[(x_4+0,1)^2+y_4+0,1r_{43}\right]=-1,0004,$$

$$y_5=y_4+\frac{0,1}{6}\left(r_{41}+2r_{42}+2r_{43}+r_{44}\right)=-1,2502.\ \blacktriangleright$$

227. $xyy''+xy'^2=2yy',\ \ 1\le x\le 1,6;\ y(1)=2,\ \ y'(1)=3.$

◀ $z(x)=y'(x)$ yangi o’zgaruvchini kiritib, differensial tenglamalar sistemasiga o’tamiz:

$$y'=z,\ \ z'=\frac{2z}{x}-\frac{z^2}{y};\ 1\le x\le 1,6;\ y(1)=2,\ z(1)=3.$$

$h=0,2$ bo’lsin. U holda, Runge-Kutta usuliga binoan, yozamiz:

$y_0=2,\ z_0=3,\ \ x_k=1+0,2k,$

$$r_{k1} = z_k, \qquad p_{k1} = \frac{2z_k}{x_k} - \frac{z_k^2}{y_k},$$

$$r_{k2} = z_k + 0{,}1p_{k1}, \qquad p_{k2} = \frac{2(z_k + 0{,}1p_{k1})}{x_k + 0{,}1} - \frac{(z_k + 0{,}1p_{k1})^2}{y_k + 0{,}1r_{k1}},$$

$$r_{k3} = z_k + 0{,}1p_{k2}, \qquad p_{k3} = \frac{2(z_k + 0{,}1p_{k2})}{x_k + 0{,}1} - \frac{(z_k + 0{,}1p_{k2})^2}{y_k + 0{,}1r_{k2}},$$

$$r_{k4} = z_k + 0{,}2p_{k3}, \qquad p_{k4} = \frac{2(z_k + 0{,}2p_{k3})}{x_k + 0{,}2} - \frac{(z_k + 0{,}2p_{k3})^2}{y_k + 0{,}2r_{k3}},$$

$$y_{k+1} = y_k + \frac{0{,}1}{3}\left(r_{k1} + 2r_{k2} + 2r_{k3} + r_{k4}\right),$$

$$z_{k+1} = z_k + \frac{0{,}1}{3}\left(p_{k1} + 2p_{k2} + 2p_{k3} + p_{k4}\right), \; k = 0,1,2.$$

Endi ketma-ket $k = 0,1,2$ deb olib, topilgan natijalarni keltiramiz:

$$r_{01} = 3, \qquad p_{01} = 1{,}5,$$

$$r_{02} = 3{,}15, \qquad p_{02} = 1{,}4132,$$

$$r_{03} = 3{,}1413, \qquad p_{03} = 1{,}449,$$

$$r_{04} = 3{,}2898, \qquad p_{04} = 1{,}3652,$$

$$y_1 = y_0 + \frac{0{,}1}{3}\left(r_{01} + 2r_{02} + 2r_{03} + r_{04}\right) = 2{,}6291$$

$$z_1 = z_0 + \frac{0{,}1}{3}\left(p_{01} + 2p_{02} + 2p_{03} + p_{04}\right) = 3{,}2863,$$

$$r_{11} = 3{,}2863, \qquad p_{11} = 1{,}3694,$$

$$r_{12} = 3{,}4232, \qquad p_{12} = 1{,}3045,$$

$$r_{13} = 3{,}4168, \qquad p_{13} = 1{,}3276,$$

$$r_{14} = 3{,}5518, \qquad p_{14} = 1{,}2656,$$

$y_2 = 3,3130,$ $z_2 = 3,5496,$

$r_{21} = 3,5496,$ $p_{21} = 1,2678,$

$r_{22} = 3,6764,$ $p_{22} = 1,2998,$

$r_{23} = 3,6796,$ $p_{23} = 1,3095,$

$r_{24} = 3,8115,$ $p_{24} = 1,1764,$

$y_3 = 4,0488,$ $z_3 = 3,8050.$ ►

6.3.3. Shtermer usuli. 6.3.1-banddagi masalani qaraymiz va u yerdagi belgilashlarni saqlab qolamiz. Bu yerda ham $[x_0,\ b]$ integrallash oralig'i har birining uzunligi h bo'lgan teng qismlarga bo'linadi va qo'yilgan masalaning yechimi quyidagi *Shtermer formulalari*dan bittasi yordamida hisoblanadi:

$$y_{i,k+1} = y_{ik} + q_{ik} + \frac{1}{2}\Delta q_{i,k-1}\ ,$$

$$y_{i,k+1} = y_{ik} + q_{ik} + \frac{1}{2}\Delta q_{i,k-1} + \frac{5}{12}\Delta^2 q_{i,k-2}\ ,$$

$$y_{i,k+1} = y_{ik} + q_{ik} + \frac{1}{2}\Delta q_{i,k-1} + \frac{5}{12}\Delta^2 q_{i,k-2} + \frac{3}{8}\Delta^3 q_{i,k-3}\ ,$$

…………………………………………..

bu yerda

$$i = 1,2,...,n,\ y_{ik} = y_i(x_k),\ x_k = x_0 + kh,\ q_{ik} = y_i'(x_k)h,$$

$$\Delta q_{i,k-1} = q_{ik} - q_{i,k-1},\ \ \Delta^2 q_{i,k-2} = \Delta q_{i,k-1} - \Delta q_{i,k-2},$$

$$\Delta^3 q_{i,k-3} = \Delta^2 q_{i,k-2} - \Delta^2 q_{i,k-3}.$$

Har bir integrallash oralig'idagi xatolik bu formulalarda mos ravishda $O(h^3),\ O(h^4),\ O(h^5)$ ni tashkil etadi.

Shtermer formulasi yordamida hisoblashni boshlash uchun $y_i(x_k)$ ning dastlabki bir necha qiymatlari ma`lum bo'lishi kerak. Bunday qiymatlar Eyler, Runge-Kutta yoki darajali qatorlar usullaridan biri yordamida avvaldan topib olinishi mumkin.

Quyidagi masalalarda yozilgan tenglamalarning ko'rsatilgan oraliqdagi taqribiy yechimlarini Shtermer usuli yordamida toping. Hisoblashlarni verguldan keyingi to'rtta xona aniqligida bajaring. Izlanayotgan yechimning yetishmayotgan qiymatlarini topish uchun Runge-Kutta yoki darajali qatorlar usullaridan birini qo'llang (**228-229**).

228. $y' = \frac{y}{x} + x,\ 1 \le x \le 1,5;\ y(1) = 2.$

◀ y_{k+1} ni hisoblash formulasida $h = 0,1$ deb olamiz. U holda

$$y_{k+1} = y_k + q_k + \frac{1}{2}\Delta q_{k-1} = y_k + hy'_k + \frac{1}{2}(q_k - q_{k-1}) =$$

$$= y_k + hy'_k + \frac{h}{2}(y'_k - y'_{k-1}) = y_k + \frac{h}{2}(3y'_k - y'_{k-1}) =$$

$$= y_k + \frac{0,1}{2}\left(\frac{3y_k}{x_k} + 3x_k - \frac{y_{k-1}}{x_{k-1}} - x_{k-1}\right),\ x_k = 1 + 0,1k,\ k = 1,\ 2,\ 3,\ 4.$$

Bu formuladan ko'rinadiki, $k = 1$ bo'lganda y_2 ni hisoblash uchun $x_0 = 1,\ y_0 = 2$ boshlang'ich qiymatlar berilgani holda y_1 ning qiymati ma`lum emas. y_1 ni Runge-Kutta usuli bilan topamiz:

$$r_{01} = \frac{y_0}{x_0} + x_0 = 3,\quad r_{02} = \frac{y_0 + 0,05r_{01}}{x_0 + 0,05} + x_0 + 0,05 = 3,0976,$$

$$r_{03} = \frac{y_0 + 0,05r_{02}}{x_0 + 0,05} + x_0 + 0,05 = 3,1023,$$

$$r_{04} = \frac{y_0 + 0,1r_{03}}{x_0 + 0,1} + x_0 + 0,1 = 3,2002,$$

$$y_1 = y_0 + \frac{0,1}{6}\left(r_{01} + 2r_{02} + 2r_{03} + r_{04}\right) = 2,31.$$

Endi ma`lum qiymatlar asosida y_2 ni hisoblaymiz:

$$y_2 = y_1 + \frac{0,1}{2}\left(\frac{3y_1}{x_1} + 3x_1 - \frac{y_0}{x_0} - x_0\right) = 2,64.$$

y_{k+1} ning navbatdagi qiymatlari ham shu kabi topiladi:

$$y_3 = 2,99,\ \ y_4 = 3,36,\ \ y_5 = 3,75. \blacktriangleright$$

229. $xy'' + 2y' + xy = 0,\ \ 0 \le x \le 0,5;\ y(0) = 1,\ \ y'(0) = 0.$

◀ $z(x) = y'(x)$ yangi o'zgaruvchini kiritib, differensial tenglamalar sistemasiga o'tamiz:

$$y' = z,\ \ z' = -y - \frac{2z}{x};\ \ 0 \le x \le 0,5;\ y(0) = 1,\ z(0) = 0.$$

Qadamni $h = 0,1$ deb olamiz. Shtermer formulalaridan birinchisini mazkur sistemaga qo'llaymiz:

$$y_{k+1} = y_k + 0,1p_k + 0,05\Delta p_{k-1}\ ,\ \ z_{k+1} = z_k + 0,1q_k + 0,05\Delta q_{k-1}, \quad (1)$$

bu yerda

$$p_k = z_k,\ \ q_k = -y_k - \frac{2z_k}{x_k},\ \Delta p_{k-1} = p_k - p_{k-1},\ \ \Delta q_{k-1} = q_k - q_{k-1},$$

$$x_k = 0,1k,\quad y_0 = 1,\ \ z_0 = 0.$$

Hisoblashlarni boshlash uchun bizga $y(0,1) = y_1,\ z(0,1) = z_1 = y'(0,1)$ hamda q_0 qiymatlar ($0/0$ aniqmaslik tufayli) kerak bo'ladi. Bu miqdorlarni darajali qatorlar usuli bilan topamiz. Berilgan masalaning yechimini

$$y = 1 + a_2 x^2 + a_3 x^3 + \ldots$$

ko'rinishda izlaymiz. Bu qatorni qaralayotgan tenglamaga qo'yib va x ning bir xil darajalari oldidagi koeffitsientlarni bir-biriga tenglashtirib, topamiz:

$$a_2=-\frac{1}{6},\, a_3=0,\;\; a_4=\frac{1}{120},\; ....$$

Shunga ko'ra, $$y=1-\frac{x^2}{3!}+\frac{x^4}{5!}+....$$

Bu yerdan y_1 va z_1 miqdorlarni topish qiyin emas:

$$y_1=y(0,1)=0,9983,\quad z_1=y'(0,1)=-0,0333.$$

So'ngra, $$z'=-y-\frac{2z}{x},\;\; q_k=-y_k-\frac{2z_k}{x_k}$$

munosabatlardan foydalanib, $q_0=z'(0)=y''(0)=-0,3333$ ekanligiga ishonch hosil qilamiz.

Shundan so'ng hisoblashlarni (1) formulalar yordamida bajaramiz.

Buning uchun ularni qulayroq ko'rinishda yozib olamiz:

$$y_{k+1}=y_k+0,15z_k-0,05z_{k-1}\;,$$

$$z_{k+1}=z_k-0,15\left(y_k+\frac{2z_k}{x_k}\right)+0,05\left(y_{k-1}+\frac{2z_{k-1}}{x_{k-1}}\right),\;\; k=1,2,....$$

Bu formulalarda ketma-ket $k=1,2,...$ deb olib, quyidagi miqdorlarni olamiz:

$$y_2=0,9933,\quad y_3=0,98,\quad y_4=0,9701,\quad y_5=0,953,$$

$$z_2=-0,0664,\; z_3=-0,0992,\; z_4=-0,1472,\quad z_5=-0,1664.\blacktriangleright$$

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M60. Quyidagi masalalarda taqribiy yechimning ko'rsatilgan oraliqdagi xatoligini baholang (taqribiy yechim to'lqin bilan belgilangan).

1. $y'=2xy^2+1,\; y(0)=1;\; \tilde{y}=\frac{1}{1-x},\;\; |x|\le\frac{1}{4}.$

2. $\dot{x} = x - y,\ \dot{y} = tx,\ x(0) = 1,\ \ y(0) = 0;\ \tilde{x} = 1 + t + \dfrac{t^2}{2},\ \ \tilde{y} = \dfrac{t^2}{2},\ \ |t| \le 0{,}1.$

3. $y'' - x^2 y = 0,\ y(0) = 1,\ y'(0) = 0;\ \tilde{y} = \exp\left(x^4/12\right),\ |x| \le 0{,}5.$

Quyidagi masalalarda parametr yoki boshlang'ich shartlar bo'yicha hosilalarni toping.

4. $y' = y + \mu(x + y^2),\ \ y(0) = 1;\ \left.\dfrac{\partial y}{\partial \mu}\right|_{\mu=0} = ?.$

5. $y' = y + y^2 + x\,y^3,\ \ y(2) = y_0;\ \left.\dfrac{\partial y}{\partial y_0}\right|_{y_0=0} = ?.$

6. $\begin{cases} \dot{x} = x + y, & x(0) = 1 + \mu, \\ \dot{y} = 2x + \mu\, y^2, & y(0) = -2; \end{cases}\ \ \left.\dfrac{\partial y}{\partial \mu}\right|_{\mu=0} = ?.$

7. $\ddot{x} - \dot{x} = (x+1)^2 - \mu\, x^2;\ \ x(0) = \dfrac{1}{2},\ \dot{x}(0) = -1;\ \left.\dfrac{\partial x}{\partial \mu}\right|_{\mu=1} = ?.$

Quyidagi masalalarning har birida berilgan boshlang'ich shartlarni qanoatlantiradigan yechimni darajali qator ko'rinishida izlang. Qatorning dastlabki bir necha (x^0, x^1, x^2, x^3, x^4 darajalar oldidagi) koeffitsientlarini hisoblang.

8. $y' = y^2 - x;\ y(0) = 1.$ 9. $y' = y + xe^y;\ \ y(0) = 0.$

10. $y'' = xy' - y^2;\ y(0) = 1,\ \ y'(0) = 2.$

Quyidagi masalalarda yechimning μ kichik parametr darajalari bo'yicha yoyilmasining ikkita yoki uchta hadlarini toping.

11. $y' = 4\mu x - y^2,\ y(1) = 1.$ 12. $xy' = \mu\, x^2 + \ln y,\ y(1) = 1.$

13. $y' = e^{y-x} + \mu\, y,\ y(0) = -\mu.$

Kichik parametr usuli yordamida quyidagi tenglamalarning taqribiy davriy yechimini toping. Izlanayotgan yechimning davri o'ng tomondagi funksiyaning davriga teng bo'lsin; μ- kichik parametr bo'lsin.

14. $\ddot{x}+3x=2\sin t+\mu\dot{x}^2$. 15. $\ddot{x}+3x+\dot{x}^3=2\mu\cos t$.

16. $\ddot{x}+\sin x=\mu\sin 2t$.

Kichik parametr usuli yordamida quyidagi tenglamalarning taqribiy davriy yechimini toping.

17. $\ddot{x}+x-x^2=0$. 18. $\ddot{x}+x=\mu\left(1-x^2\right)\dot{x}$. 19. $\ddot{x}+x=\mu\left(\dot{x}-\dot{x}^3\right)$.

Eylerning siniq chiziqlar usuli yordamida quyidagi differensial masalalarning yechimlarini ko'rsatilgan oraliqda taqribiy toping. Hisoblashlarni $h=0,2$ qadam bilan verguldan keyin uchta xona aniqligida bajaring.

20. $y'=y^2+x,\ 0\le x\le 1;\ y(0)=0,3$.

21. $y'=\dfrac{x}{y}-y,\ 0\le x\le 1;\ y(0)=1$.

Runge-Kutta yoki Shtermer usuli yordamida quyidagi differensial masalalarning yechimlarini taqribiy hisoblang (hisoblashlarni verguldan keyin to'rtta xona aniqligida bajaring).

22. $y'=y,\ 0\le x\le 1;\ y(0)=1$. 23. $y'=\dfrac{1}{y}-x,\ 0\le x\le 1;\ y(0)=1$.

24. $y'=y^2-x,\ 0\le x\le 1;\ y(0)=0,5$.

25. $y''=xy,\ 0\le x\le 1;\ y(0)=1,\ y'(0)=0$.

Quyidagi differensial tenglamalarning berilgan boshlang'ich shartni qanoatlantiruvchi xususiy yechimini Teylor qatorining dastlabki uchta hadi ko'rinishida toping

26. $y'-e^{x-1}+2x-y^2=0,\ y(1)=-1$. 27. $y'=2e^y+xy,\ y(0)=0$.

28. $y'-\sin x-y^3+1=0,\ y(0)=1$.

29. $y''-xy'+y-1=0,\ y(0)=y'(0)=0$.

30. $y''=2xy,\ y(0)=1,\ y'(0)=0$.

6-BOBNI TAKRORLASHGA DOIR ARALASH MASALALAR

Darajali qatorlarga yoyish yo'li bilan quyidagi Koshi masalalarining yechimlarini quring:

1. $y' = x + y,\ y(0) = 1.$ **2.** $y' = xy,\ y(0) = 1.$

3. $y' = x - 2xy,\ y(0) = 3.$ **4.** $y'' = xy' - y,\ y(0) = 1,\ y'(0) = 0.$

5. $y''' = -x^2 y'' + y' + 2y,\ y(0) = 1,\ y'(0) = 0,\ y''(0) = 0.$

Yechimni to'rtinchi tartibli ko'phad ko'rinishida quring:

6. $y' = y^2 - x,\ y(0) = 1.$ **7.** $y' = xe^y + y,\ y(0) = 0.$

8. $y' = x^2 + y^2,\ y(1) = 1.$ **9**. $y'' = x - y'^2,\ y(0) = 2,\ y'(0) = 0.$

10. $y''' = y''^2 + y' + y - x,\ y(0) = 1,\ y'(0) = y''(0) = 0.$

Quyidagi chegaraviy masalalarning taqribiy yechimlarini quring:

11. $y' = x^2 - y^2,\ y(1) + y(2) = 1, 1 < x < 2.$

12. $y' = x + y^{-1},\ y(0) - 4y(1) = 5,\ 0 < x < 1.$

13. $y'' = xy' + y^2,\ y(0) = 0,\ y'(1) = 2,\ 0 < x < 1.$

14. $y'' = y'^2 + y,\ y(1) = 2,\ y(2) = 4/3,\ 1 < x < 2.$

Quyidagi Koshi masalalarining taqribiy yechimlarini μ kichik parametrga nisbatan uchinchi darajali ko'phad ko'rinishida quring:

15. $y' = \dfrac{2}{y} - 5\mu x,\ y(1) = 2.$ **16.** $y' = \dfrac{6\mu}{x} + y^2,\ y(1) = 1 + 3\mu.$

17. $y' = \mu x^3 + y^2,\ y(0) = e^{-\mu}.$ **18.** $y' = 1 + x + \mu y^3,\ y(0) = \sin\mu.$

19. $y' = \cos x + \mu \ln(1 + y),\ y(0) = \mu.$

20. $y' = \sin x + \mu e^y,\ y(0) = 1 - \mu.$

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M2.1.$y = xy'$. 2.$y = xy' + y'^2$. 3.$y'' = 0$. 4.$y = 2xy'$. 5.$\left(1+y'^2\right)y^2 = 1$. 6.$xy' = \sqrt{1-y^2}\arcsin y$. 7.$xyy'' + xy'^2 - yy' = 0$. 8.$xy' = y + 2x^3e^{x^2}$. 9.$y\ln y = xy'$. 10.$y'^3 = 27y^2$. 11.$xy' = 3y$. 12.$y^2 + y'^2 = 1$. 13.$x^2y' - xy = yy'$. 14.$2xyy' - y^2 = 2x^3$. 15.$y'^3 = 4y(xy' - 2y)$. 16.$y\arccos y' = x\sqrt{1-y'^2}$. 17.$x(x-2)y'' - \left(x^2-2\right)y' + 2(x-1)y = 0$. 18.$\left(yy'' + y'^2\right)^2 = -y^3y''$. 19.$(1 - x\,ctg\,x)y'' - xy' + y = 0$. 20.$y'''y' = 3y''^2$. 21.$xy' + y = xy^2\ln x$. 22.$xy' + y'\ln y' = y$. 23.$2xy^2 - y + xy' = 0$. 24.$y' = y\,tgx + \cos x$. 25.$xy' = 4y + x^2\sqrt{y}$. 26.$2xyy' + x^2 - y^2 = 0$. 27.$y'\cos x + y\sin x = 1$. 28.$\left(1+x^2\right)y' + 1 + y^2 = 0$. 29.$yy' - 2y + x = 0$. 30.$xy' + 2y = x^4$.

M3.1.$e^{3y} = 3\left(C - xe^{-x} - e^{-x}\right)$. 2.$\ln y = C\,tg\,x$. 3.$\cos y = Ce^{x-x^2}$. 4.$tgx\cdot tgy = C$. 5.$(x+1)e^{-x} + y - \ln\left(e^y+1\right) = C$. 6.$\ln\left(x^2+3\right) + 2(y+1)e^{-y} = C$. 7.$\cos y = C\cos x$. 8.$(2y+1)\cos^2 x = C$. 9.$tg\,y = C + 2\cos x$. 10.$\sin y = C\left(e^x-1\right)^3$. 11.$4\ln|\sin y| = C + 2x - \sin 2x;\ y = \pi n,\ n = 0, \pm1, \pm2, \ldots$. 12.$y^2 = 2\ln C\left(1+e^x\right)$. 13.$2y(\ln y - 1) = e^{2x} + C$. 14.$2\cdot 3^y = 3^{-x^2} + C$. 15.$\sin 2y = tg\,x + C$. 16.$2arctg\,y = C + e^{x^2}$. 17.$tg^2 y = ctg^2 x + C$. 18.$y = C\sin x - 2$. 19.$e^y + 1 = Ce^{-x}$. 20.$y = C\cos x + 2$. 21.$2y + \sin 2y = C - 2e^{x^2}$. 22.$\ln\left|tg\left(\frac{\pi}{4} + \frac{y}{2}\right)\right| = C - e^x;\ y = \pi n,\ n = 0, \pm1, \pm2, \ldots$

$23.\ 2y+\sin 2y=4\sin 2x+C.$ $24.\ ctg\, y=C-\sin 2x.$ $25.\ 2\ln\left|tg\left(\frac{\pi}{4}+\frac{y}{2}\right)\right|+$
$+y=\ln\left|x+\sqrt{1+x^2}\right|+C;\ y=(2n-1)\pi/2,\ n=0,\pm1,\pm2,\ldots.$ $26.\ tg\, y=C+$
$+\arcsin x;$ $26.\ tg\, y=\arcsin x+C;\ y=(2n-1)\pi/2,\ n=0,\pm1,\pm2,\ldots.$
$27.\ tg\, y=C/\left(e^x-1\right);\ x=1;$ $28.\ \ln\left|tg(y/2)\right|=C-2\sin x.$
$29.\ 3^{-x^2}-3^{-y^2}=C.$ $30.\ 3x^2=2\ln\left(1+e^{3y}\right)+C.$

M4. $1.\ \left(1+x^2\right)\left(1+y^2\right)=Cx^2.$ $2.\ 5^{-y}=3\cdot5^{-x}+C.$ $3.\ (y-2)(2x+1)=Cx.$
$4.\ (x+1)(y-1)=Cx.$ $5.\ (x+4)^4y=Ce^x;\ x=-4.$ $6.\ y+1=Ce^xy,\ y=0.$
$7.\ 2y^{-1}+\ln y^2=C+\ln^2x,\ y=0.$ $8.\ y+2\ln|y-1|-\ln|xy|=C;\ x=0;\ y=0;$
$y=1.$ $9.\ y-2=Cye^{2x};\ y=0.$ $10.\ 2x^3+3\left(x^2+y^2\right)+6\ln|y|=C;\ y=0.$
$11.\ \left(x^2-1\right)^3\left(y^3+1\right)^2=C.$ $12.\ 2arctg\, x-\ln\left(y^2+1\right)=C.$ $13.\ x^2-\ln|2y+1|$
$=C;\ y=-1/2.$ $14.\ y-3=Cx/(3x+1).$ $15.\ x^2+2y^2-2\ln|x|=C.$ $16.\ y=$
$=C\sqrt{x^2-1}.$ $17.\ y^3=3\left(C-x+\ln|x+1|\right);\ x=-1.$ $18.\ \frac{1}{x^2}+\frac{1}{y^2}+2x+\ln y^2=C.$
$19.\ y/(y+1)=Cx;\ y=-1.$ $20.\ y^2+1=\ln^2Cx;\ x=0.$ $21.\ y/(y+2)=$
$=Ce^{x^2};\ y=-2.$ $22.\ x^{-1}-\ln\left|y^2-2\right|=C;\ y=\pm\sqrt{2}.$ $23.\ y=(x+C)/(1-Cx).$
$24.\ \left(y^2+1\right)^3=\left(C+x^3\right)^2.$ $25.\ 2y+2\ln|y|=2\arcsin x+x^2+C;\ y=0.$
$26.\ \left(x+\sqrt{1+x^2}\right)y=C\sqrt{1+x^2}.$ $27.\ 4\ln|x|+2\left(x^2-y^2\right)+y^4=C.$ $28.\ \frac{y^2}{2}-$
$-2y+\ln\left|\frac{y}{x}\right|-\frac{1}{x}=C;\ x=0;\ y=0.$ $29.\ (y-1)^2-\ln\left|(x-1)/(x+1)\right|$
$+2\ln|y+1|=C;\ y=-1;$ $30.\ \sqrt{1-y^2}=\arcsin x+C;\ y=\pm1.$

M6. $1.\ y=-x\arcsin(\ln Cx).$ $2.\ x^2-y^2=Cy^3;\ y=0.$ $3.\ y=Cx^2-x.$
$4.\ 2arctg(y/x)+\ln\left(x^2+y^2\right)=C.$ $5.\ x(x-y)=Cy;\ y=0.$ $6.\ y=x\ln Cy;$
$y=0.$ $7.\ y=x\arcsin Cx.$ $8.\ y=-x\ln\ln Cx.$ $9.\ x+y=xe^{Cx}.$ $10.\ \ln(y/x)=$

$= 2arcctg\left(\ln Cx\right);\ y = x.$ 11. $4y = x\ln^2 Cx;\ \ y = 0.$ 12. $y = x\sin\ln Cx;\ y = x.$

13. $y = -x\ln\ln\left(C/x\right).$ 14. $y = -x\ln\left(Cx\right).$ 15. $2xy = C - x^2.$

16. $x = y\ln^2 Cy; x = 0.$ 17. $y + \sqrt{x^2 + y^2} = Cx^2.$ 18. $\left(x + y\right)^2\left(x^2 + y^2\right)^3$ $= C.$ 19. $y = x/\ln Cx;\ y = 0.$ 20. $t + 2\ln|t| = \ln Cx,\ t = y/x;\ y = 0.$

21. $z - 2\ln|z| = -\ln Cx, z = x/y;\ y = 0.$ 22. $y = x\ln^2 Cx; y = 0.$ 23. $y = xe^{C/x}.$

24. $y^2 = x^2\ln Cx^2.$ 25. $y - 3x = Cx^3 y;\ y = 0.$ 26. $x^2\left(x + 3y\right) = C.$

27. $arctg\left(y/(2x)\right) + \ln\left(4x^2 + y^2\right) = C.$ 28. $y^2 = x^2/\ln Cx;\ y = 0.$

29. $y = -x/\ln Cx;\ y = 0.$ 30. $3xy^2 + x^3 = C.$

M9. 1. $y = \left(x^3 + 3x\right)/\left(x^2 + 1\right)^2.$ 2. $y = \sin x.$ 3. $y = -e^{-x}\ln|1 - x|.$

4. $y = x^4 - x^2.$ 5. $y = x^2 + 1 - e^{x^2}.$ 6. $y = (x+1)e^x.$ 7. $y = e^{-x^2}/(2x).$

8. $x = \left(\sin^2 y - \dfrac{1}{2}\right)\dfrac{1}{\cos y}.$ 9. $y = -\ln|x|/x.$ 10. $x = y^3 + y^2.$ 11. $x = 2\ln y +$ $+ 1 - y.$ 12. $x = y^2 - y^3.$ 13. $x\left(y - 1\right)^2 = y - \ln|y| - 1.$ 14. $y = e^x\ln|x|.$

15. $y = x\left(\sin x - 1\right).$ 16. $y = \ln^2 x - \ln x.$ 17. $x = 2sh\,y.$ 18. $xy = \left(x^3 - 1\right)e^{-x}.$

19. $x = y^2 - y.$ 20. $2x = -\sin 2y.$ 21. $y = x^3.$ 22. $y = -x^2\ln|x|.$ 23. $xy =$ $= 1 - \cos x.$ 24. $y = x^2 - 1.$ 25. $y = x + \sqrt{1 - x^2}.$ 26. $3y\cos x = 6\sin x - 2\sin^3 x.$

27. $y = -1/x.$ 28. $2y = x^2 e^{-x^2}.$ 29. $3y = x^3 e^{x^3}.$ 30. $y = \ln x.$

M11. 1. $y = \left(x - 2 + Ce^{-x/2}\right)^2;\ y = 0.$ 2. $xy^2 = \left(y\,tg\,y + \ln|\cos y| + \right.$ $\left. + C\right)^2; x = 0.$ 3. $\left(Ce^{2x} + e^x\right)y = 1;\ \ y = 0.$ 4. $y\cos x \cdot \sqrt[3]{C - 3\sin x} = 1.$

5. $y^2 = 2\left(Cx^2 - x\right).$ 6. $2x^4 y^2\left(C + e^x\right) = 1;\ y = 0.$ 7. $x = \sqrt{y/\left(C - \cos y\right)}.$

8. $xy\left(C - \ln^2 y\right) = 1.$ 9. $y^2 = \left(C - \sqrt{x^2 - 1}\right)\sqrt{x^2 - 1}.$ 10. $4y = x^4\left(C + \ln|x|\right)^2;$ $y = 0.$ 11. $y = \sqrt[3]{3x^2\left(Cx - 1\right)}.$ 12. $(x+1)y\ln\dfrac{C}{x+1} = 1;\ y = 0.$ 13. $xy\ln Cx = 1;$ $y = 0.$ 14. $2y^2 = e^{x^2}/(x + C);\ y = 0.$ 15. $y = x\left(x + C\right)^2;\ y = 0.$

$16.\ y^2=1/\left(x^2+1+Ce^{x^2}\right);\ y=0.\ 17.\ y=e^x\sqrt{x^2+C}.\ 18.\ xy\left(C+\ln|y|\right)=1;$
$x=0.\ 19.\ 2y^2=(x-1)^2/\left(x-\ln|x|+C\right);\ y=0.\ 20.\ y^2=x^{-3}\left(C-x\right).$

$21.\ y-xy^2=Cx.\quad 22.\ 216y^2=\left(2x+1+Ce^{2x}\right)^3;\ y=0.\quad 23.\ (\ln x+1+$
$+Cx)\,y=1;\ y=0.\ 24.\ x^2=y^2\left(C-y^2\right).\ 25.\ y^2=4/\left(2x^2+1+Ce^{2x^2}\right);$
$y=0.\quad 26.\ 3y^3=3x-1+Ce^{-3x}.\quad 27.\ (x+C)y\cos x=1;\ y=0.\quad 28.\ x^2y=$
$=\left(xtgx+\ln|\cos x|+C\right)^2;\ y=0.\quad 29.\ y=2/\left(\cos x+\sin x+Ce^{-x}\right);\ y=0.$
$30.\ 9y=\left(x^2-1+C\sqrt[4]{x^2-1}\right)^2;\ y=0.$

M16. $1.\ y=Cx.\ 2.\ x=Cy.\quad 3.\ x^2+y^2-xy+x-y=C.\ 4.\ 2arctg(x/y)$
$+x^2+y^2=C.\ 5.\ x^2-y^2=(C+x)^2.\ 6.\ e^y-1=C\left(x^2+1\right);\ 7.\ x^2-y^2=Cy^3.$
$8.\ x-ye^{x/y}=C.\ 9.\ x^4+x^2y^2+y^4=C.\ 10.\ x^3+3x^2y^2+y^4=C.$
$11.\ x+y\sqrt{x^2+y^2}+y\ln|xy|=Cy.\ 12.\ x^5tg\,y+x^2y^4+y^3=Cx^2.$
$13.\ x^3y+x^2-y^2=Cxy.\ 14.\ x^2y+y^3+2\sin^2x=Cy.\ 15.\ x^3+y^3-x^2-$
$-xy+y^2=C.\ 16.\ y+x\sqrt{x^2+y^2}=Cx.\ 17.\ x^3y+xy^3=C.\ 18.\ \left(x^2+y^2\right)^2+$
$+2a^2\left(y^2-x^2\right)=C.\ 19.\ x\sin y-y\cos x+\ln|xy|=C.\ 20.\ tg\,xy-\cos x+$
$+\cos y=C.\ 21.\ x^3+xy-\sin xy=C.\ 22.\ 3x^4+8y^2-e^{x/y}=C.\ 23.\ \sqrt{xy}-$
$-\cos x^2y+4x=C.\ 24.\ 3^{xy}-3y=C.\qquad 25.\ \ln|x-y|+x^3y^7=C.$
$26.\ x^2\sin xy-y=Cx^2.\ \ 27.\ x^2-\arcsin xy=C.\ 28.\ x^5y^4-y^3+4x^7=C.$
$29.\ e^{x^2+y^2}+2x-3y=C.\ 30.\ 7x-y^2+y^3\sin 3x=C.$

M20. $1.\ x^2+y^2=\left(x+C\right)^2.\quad 2.\ 4y=\left(\pm\sqrt{5}-1\right)x^2+C.\ 3.\ 15y+C=$
$=6\sqrt{(1-x)^5}-10\sqrt{(1-x)^3}.\ 4.\ y=Ce^{\pm x}.\ 5.\ y^2=\left(x+C\right)^3;\ y=0.\ 6.\ \left(x+C\right)^2$
$+y^2=1;\ y=\pm1.\ 7.\ y\left(x+C\right)^2=1;\ y=0.\ 8.\ y\left[1+\left(x+C\right)^2\right]=1;\ y=0;$
$y=\pm1.\ 9.\ \left(x-y\right)^2=2C(x+y)-C^2;\ y=0.\ 10.\ \sqrt[3]{(x-1)^4}+\sqrt[3]{y^4}=C.$

$11. y^2(1-y)=(x+C)^2;\ y=1. 12. x^2y=C;\ y=Cx. 13. x^2+C^2=2Cy;$
$y=\pm x. 14. (x+C)^2=4Cy;\ y=0;\ y=x. 15. \ln\left|1\pm 2\sqrt{2y-x}\right|=2(x+$
$+C\pm\sqrt{2y-x}). 16. 4e^{-y/3}=(x+2)^{4/3}+C. 17. y=2x^2+C;\ y=-x^2+C.$

$18. y=Cx^{-3}\pm 2\sqrt{x}. 19. \ln Cy=x\pm 2e^{x/2},\ y=0.\ 20. \ln Cy=x\pm\sin x;$
$y=0.\ 21. arctgu+\frac{1}{2}\ln\left|\frac{u-1}{u+1}\right|=\pm x+C,\ u=\sqrt[4]{1-y^{-2}};\ y=0;\ y=\pm 1.$
$22. x^2+(Cy+1)^2=1;\ y=0.\ 23. (x-2C)^2+y^2=2C^2. 24. y=Ce^{\pm x}-x^2.$
$25. y^2=C^2x-C; 4xy^2=-1.\ 26. y^2=2Cx-C^2,\ \ y=\pm x.$
$27. y=1/(x+C);\ y=Ce^{x^2/2}. 28. xy=C^2x+C;\ 4x^2y=-1.$
$29. y=\cos x+C;\ \ y=Ce^{x^2}. 30. y=x^2+C.$

M21. $1. x=\frac{1}{\sqrt{1+p^2}}+\frac{1}{2}\ln\frac{\sqrt{1+p^2}-1}{\sqrt{1+p^2}+1}+C,\ y=\frac{p}{\sqrt{1+p^2}}.\ \ 2. x=\frac{1}{1+p^2},$

$y=\frac{p}{1+p^2}-arctg\ p+C.$ $3. x=2p+3p^2+C, y=p^2+2p^3.$

$4. x=6p+4\sqrt{1+p^2},\ y=3p^2+6p\sqrt{1+p^2}+2\ln\left(p+\sqrt{1+p^2}\right)+C.$

$5. x=2p+3p^2,\ y=p^2+2p^3+C.\ \ 6. x=p+\ln\left(p+\sqrt{1+p^2}\right)+C,\ y=p+$
$+\sqrt{1+p^2}. 7. x=p\sin p,\ y=(p^2-1)\sin p+p\cos p+C.\ \ 8. x=(p+1)e^p+$
$+C,\ y=p^2e^p.$ $9. x=p\sqrt{p^2+1}, 3y=(2p^2-1)\sqrt{p^2+1}+C.$
$10. x=\frac{2p}{p^2-1},\ y=\frac{2}{p^2-1}-\ln\left|p^2-1\right|+C. 11. x=\ln p+\frac{1}{p},\ y=p-$
$-\ln p+C.\ \ 12. x=2arctg\ p,\ y=\ln(1+p^2);\ y=0.$

$13. x=\ln|p|\pm\frac{3}{2}\ln\left|\frac{\sqrt{p+1}-1}{\sqrt{p+1}+1}\right|\pm 3\sqrt{p+1}+C,\ y=p\pm(p+1)^{3/2},\ \ y=\pm 1.$
$14. x=e^p+C,\ y=(p-1)e^p;\ y=-1.$

$15. x=\pm 2\sqrt{p^2-1}\pm \arcsin\left(1/|p|\right)+C,\ y=\pm p\sqrt{p^2-1};\ y=0.$

$16. x=\pm\ln\left|\frac{1-\sqrt{1-p}}{1+\sqrt{1-p}}\right|\pm 3\sqrt{1-p}+C,\ y=\pm p\sqrt{1-p};\ y=0.$

$17. x=\pm 2\sqrt{p^2+1}-\ln\left(\sqrt{p^2+1}\pm 1\right)+C,\ y=-p\pm p\sqrt{p^2+1};\ y=0.$

$18. 4y=C^2-2\left(x-C\right)^2;\ 2y=x^2.$ $19. 2x=-p+C,\ 20y=C^2-5p^2;$
$x^2=4y.$ $20. \pm xp\sqrt{2\ln Cp}=1,\ y\sqrt{2\ln Cp}=\mp 2\ln Cp\pm 1.$
$21. pxy=y^2+p^3,\ y^2\left(2p+C\right)=p^4;\ y=0.$ $22. 2x=1+2\ln|y|;$
$y^2=C\left(2x-\ln C\right).$ $23. Cx=\ln Cy;\ y=ex.$ $24. xp^2=C\sqrt{|p|}-1,$
$y=xp-x^2p^3;\ y=0.$ $25. y^2=2C^3x+C^2;\ 27x^2y^2=1.$
$26. 2p^2x=C-C^2p^2,\ py=C;\ 32x^3=-27y^4.$
$27. x=8p+10\sqrt{1+p^2};\ y=4p^2+15p\sqrt{1+p^2}+5\ln\left(p+\sqrt{1+p^2}\right)+C.$
$28. x=4p+9p^2;\ y=2p^2+6p^3+C.$
$29. x=2p+b\sqrt{1+p^2};\ y=p^2+21p\sqrt{1+p^2}+7\ln\left(p+\sqrt{1+p^2}\right)+C.$
$30. x=8p+9p^2;\ y=4p^2+6p^3+C.$

M22. $1. x=\frac{2\ln p-2p+C}{\left(p-1\right)^2},\quad y=\frac{p^2\left(2\ln p-2p+C\right)}{\left(p-1\right)^2}+2p;\quad y=0.$

$2. x=\frac{C}{p^2}-\frac{2}{3}p,\ y=2px+p^2;\ y=0.$ $3. x=\frac{3p^2+C-2p^3}{2\left(p-1\right)^2},\ y=p^2x+$
$+p^3.$ $4. y=Cx-C^2;\ 4y=x^2.$ $5. y=Cx+\arcsin C;\ x=-\frac{1}{\sqrt{1-p^2}},\ y=$
$=\arcsin p-\frac{p}{\sqrt{1-p^2}}.$ $6. y=Cx+C-C^2;\ 4y=\left(x+1\right)^2.$ $7. y=Cx+\frac{2}{C};$
$y^2=8x.$ $8. y=Cx+\sqrt{1+C^2};\ x=-\frac{p}{\sqrt{1+p^2}},\ y=\frac{1}{\sqrt{1+p^2}}.$

$9. y=Cx-\frac{1}{C};\ y^2=-4x.$ $10. y=Cx+2C^2-C;\ 8y=-(x-1)^2.$

$11. x=3p^2+C|p|^{-3/2},\ y=2p^3+3Cp|p|^{-3/2};\ y=0.$ $12. x\sqrt{p}=\ln p+$ $+C,\ y=\sqrt{p}\left(4-\ln p-C\right);\ y=0.$ $13. y=Cx-C-2.$ $14. x=Cp,\ 2y=$ $=C\left(p^2+1\right);\ y=\pm x.$ $15. x=C\left(p-1\right)^{-2}+2p+1,\ y=Cp^2(p-1)^{-2}+$ $+p^2;\ y=0;\ y=x-2.$ $16. y=Cx-\ln C;\ y=\ln x+1.$ $17. y=2\sqrt{Cx}+$ $+C;\ y=-x.$ $18. 2C^2\left(y-Cx\right)=1;\ 8y^3=27x^2.$

$19. y=Cx+\sin C;\ x=-\cos p,\ y=px+\sin p.$

$20. 2y=2Cx+\ln C;\ 2y+1+\ln\left(-2x\right)=0.$

$21. x=\frac{C}{p^2}+\frac{8}{3}p, y=\frac{4}{3}p^2+\frac{2C}{p};\ y=0.$ $22. y=\left(\sqrt{x+1}+C\right)^2;\ y=0.$

$23. x=Ce^{-p}-2p+2, y=C(p+1)e^{-p}-p^2+2.$

$24. x=\frac{C}{3p^2}-\frac{2}{3}p,\ y=\frac{2C-p^3}{3p}.$ $25. y=Cx-\frac{1}{C^2}; y^3=-\frac{27}{4}x^2.$

$26. y=Cx+\frac{1}{C}; y^2=4x.$ $27. y=C(x+1).$ $28. y=Cx+\sqrt{1-C^2}; y^2=x+1.$

$29. y=Cx+\frac{3}{C^2}; 4y^3=81x^2.$ $30. x=\frac{2}{p^3}\left(C-\left(p^2-2p+2\right)e^p\right),$

$y=\frac{3}{2}px+e^p.$

М23. $1. y=-2e^{3x}.$ $2. y=5e^{7x}.$ $3. y=3e^{2x+2}.$ $4. y=4e^{6x+12}.$ $5. y=-e^{5x+10}.$ $6. y=-2e^{4x-12}.$ $7. y=5x^8/256.$ $8. y=-x\sqrt{x}/\left(3\sqrt{3}\right).$ $9. y=-x^9/11664.$ $10. y=x^3/256.$ $11. y=4-x^2/16.$ $12. y=-8+x^2/32.$ $13. y=1-x^2/4.$ $14. y=-3+x^2/12.$ $15. \left(x-13/4\right)^2+y^2=169/16.$ $16. \left(x+17/8\right)^2+y^2==289/64.$ $17. \left(x-2,5\right)^2+y^2=6,25.$ $18. \left(x+2\right)^2+y^2=4.$ $19. \left(x-25/8\right)^2+y^2=625/64.$ $20. \left(x-2,5\right)^2+y^2=6,25.$ $21. y=17x/4-x^2.$ $22. y=-9x/2-x^2.$ $23. y=7x/3-x^2.$ $24. y=4x-x^2.$ $25. y=3x-x^2.$ $26. y=6x-x^2.$ $27. y=5\sqrt{x}/3-x.$ $28. y=7\sqrt{x}-x.$ $29. y=4\sqrt{x}-x.$ $30. y=-6\sqrt{x}-x.$

M24. 1. $y = Ce^{x/a}$. 2. $y^2 = 2px$. 3. $\pm y = \sqrt{4-x^2} + \ln\dfrac{2-\sqrt{4-x^2}}{2+\sqrt{4-x^2}}$.
4. $(x+1)^2 + y^2 = Ce^x - 1$. 5. $\left(x^2y^2 - xy - x^2\right)^{\sqrt{5}}\left(2y - x - x\sqrt{5}\right) = C\left(2y - x + \right.$
$\left.+x\sqrt{5}\right)$. 6. $y = Cx$. 7. $y = Ce^{x/y}$. 8. $x^2 + y^2 = x^2(Cx+1)^2$. 9. $x^2 + y^2 = C^2$.
10. $(x-C)^2 + y^2 = C^2$. 11. $y = C(x-a)+b$. 12. $y = (x+C)/x$.
13. $\left(2x - 1 + C(x-1)^2\right)y - x^2 = 0$. 14. $\ln\left(y + \sqrt{y^2-1}\right) = C \pm x$; $y = \pm 1$.
15. $xy = 2$. 16. $4xy = 1$. 17. $x = -\dfrac{a}{1+p^2} - \dfrac{C}{\sqrt{1+p^2}} + \dfrac{a}{\sqrt{1+p^2}}$,
$y = \dfrac{ap}{1+p^2} + \dfrac{Cp}{\sqrt{1+p^2}}$. 18. $x^{2/3} + y^{2/3} = a^{2/3}$. 19. $xy = y$.
20. $k(x-1)y - y + 1 = 0$ 21. $y = Cx^n$. 22. $\rho^2 = 2(\varphi + C)$. 23. $\rho = C\varphi^{\pm 1}$
$(\varphi \ne 0)$. 24. $b\ln y - y = \pm x + C,\ 0 < y < b$. 25. $a\ln\left(a \pm \sqrt{a^2-y^2}\right) \mp$
$\mp\sqrt{a^2-y^2} = x + C$. 26. $y = Cx^2$. 27. $r(1 \pm \cos\varphi) = C$. 28. $x^2 + y^2 = Cx$.
29. $x = p\left(p^2+2\right)/u,\ y = p^2/u;\ x = p/u,\ y = \left(2p^2+1\right)/u$, bu yerda
$u = \sqrt{\left(p^2+1\right)^3}$. 30. $xy = \pm a^2$.

M25. 1. 0,5 kG. 2. $y^2 = C^2 + 2Cx$. 3. $y^2\left(2x^2 + y^2\right) = C$. 4. $b - \dfrac{b-a}{60k}\left(1 - e^{-60k}\right)$. 5. $a\cos\left(\sqrt{\dfrac{2k}{m}}\,t\right)$. 6. $975 \cdot 10^6$ yil. 7. $\cong 2,4$ kg. 8. 1,75 sekund, 16,3 m; 2 sekund, 20 m. 9. 1,87 sekund, 16,4 m/sek. 10. 17,3 minut. 11. $ab \cdot \dfrac{1 - e^{-k(b-a)t}}{b - ae^{-k(b-a)t}}$. 12. 27 sekund. 13. 260 sekund; 200 sekund. 14. $0,5kPl$. 15. 5350 kG. 16. $m_0 - v\left(q_1 - q_0\right)\left(1 - e^{-kt}\right)$, bu yerda k - proporsionallik koeffisienti. 17. $c(\ln M - \ln m)$. 18. Aylanma jismning kesimi $f(h) = C\sqrt[4]{h}$ ko’rinishdagi egri chiziq bo’lishi kerak. 19.

$T = \ln 2 / (\ln 10 - \ln 9) \approx 6$ sutka 14 soat. 20. $T = arctg\left(31{,}62\sqrt{k}\right) / \left(3{,}162\sqrt{k}\right)$. 21. $y = \omega^2 x^2 / (2g) + \quad + C$. 22. 1,28 km/soat. 23. $v \approx 0{,}93$ m/sek. 24. 2,5 kg. 25. $v(t) = ma\left(1 - -e^{-\gamma t/m}\right) / \gamma$. 26. $s(t) = \sqrt{25 + 200t}$, $v(t) = 100 / s(t)$; $s(10) = 45$ m., $v(10) = 20/9$ m/sek. 27. $8/\ln 2 \approx 11{,}5$ m. 28. $F(t) = (b - kv_0)e^{-kt/m}$.
29. $3s = 3s_0 + 2\sqrt{2kt^3 / m}$. 30. $v(2) \approx 4{,}43$ m/sek.

M26. 1. 1,23. 2. 0,38. 3. 1,23. 4. 6,07. 5. 4,37. 6. 0,44. 7. 0,77. 8. 1,22. 9. 3,58. 10. 5,57. 11. 3,93. 12. 5,31. 13. 0,15. 14. –0,39. 15. 25,08. 16. 0,34. 17. –0,01. 18. 0,14. 19. 7,85. 20. 1,00. 21. –0,78. 22. 0,08. 23. 12,56. 24. –1,00. 25. 4,14. 26. 1,90. 27. 3,47. 28. 1,62. 29. 4,31. 30. 5,14.

M27. 1. $y = \arcsin^2 x + C_1 \arcsin x + C_2$. 2. $9C_1^2 (y - C_2)^2 = 4(C_1 x + 1)^3$; $y = \pm x + C$. 3. $xy = C_1 x \ln|x| + C_2 x + 1$.
4. $2y = -2x - \sin 2x + C_1 \sin x + C_2$. 5. $y = C_1 x(\ln x - 1) + C_2$.
6. $y = (x-1)e^x + C_1 x^2 + C_2$. 7. $y = C_1\left(x\ln^2 x - 2x\ln x + 2x\right) + C_2$.
8. $2y = \ln^2|x| + C_1 \ln|x| + C_2$. 9. $2y = C_1^2 \arcsin(x / C_1) + x\sqrt{C_1^2 - x^2} + C_2$. 10. $y = C_1 x^2 + C_2$. 11. $2y = -x^2 - 2x + C_1 e^x + C_2$.
12. $3y = x^3 + C_1 x^2 + C_2$. 13. $C_1^2 y = (C_1 x - 1)e^{C_1 x + 1} + C_2$.
14. $y = -2x + (x + C_1)\ln x + C_2$. 15. $y = -x + C_1 \cos x + C_2$.
16. $2y = C_1 \ln \dfrac{C_1 x - 1}{C_1 x + 1} + C_2$; $y = C$. 17. $3C_1 y = 2(C_1 x - 1)^{3/2} + C_2$.
18. $24y = 3x^4 - 4x^3 + C_1 x^2 - 2C_1 x + C_2$.
19. $3y = -\sin^3 x + 2C_1 x - C_1 \sin 2x + C_2$.
20. $48y = 8x^3 - 6x^2 + 3x + C_1 e^{-4x} + C_2$.
21. $y = 2(x-1)e^x + C_1 x^2 + C_2$. 22. $4y = -x^2 + C_1 \ln|x| + C_2$.
23. $20y = 2\sin 2x - \cos 2x + C_1 e^{-4x} + C_2$.
24. $2y = -\sin x - \cos x + C_1 e^{-x} + C_2$.

25. $y=C_1x-C_1^2\ln\left(x+C_1\right)+C_2$. 26. $3C_1y=2\left(C_1x+4\right)^{3/2}+C_2$.
27. $y=C_1x^2\left(2\ln x-3\right)+C_2x+C_3$. 28. $y=2x+C_1\sin x+C_2$.
29. $y=C_1x^3+3C_1x+C_2$. 30. $y=-\sin x+C_1\cos x+C_2(x+1)$.

M31. 1. $y=-\ln(1-x)$. 2. $4y^3=(2+3x)^2$. 3. $y=\sqrt{2x+1}$. 4. y^3-
$-3x-3y-2=0$. 5. $y=arcctg(2-2x)$. 6. $4y=(x+2)^2$.
7. $y=1/(1-x)$. 8. $4y^2=1\pm4\sqrt{2}x$. 9. $x=\pm\ln\left|e^y+\sqrt{e^y-1}\right|$.
10. $12y=(x+3)^3$. 11. $4y=x^2+4x+8$. 12. $y=2(1+\sin x)$.
13. $y=\sqrt{x^2+1}$. 14. $\left(1-2x\right)y=1$. 15. $y=-\ln\left|2-e^x\right|$.
16. $(x+1)(y-1)+1=0$. 17. $1-4x=(y+1)^{-4}$. 18. $2y+3=3e^x$.
19. $x=2\ln\left|y+1+\sqrt{(y+1)^2-4}\right|-2\ln 2$. 20. $(1-2x)(y-1)=1$.
21. $x=\ln\left|y+\sqrt{y^2-1}\right|$. 22. $y^3=6x+1$. 23. $y=e^{2x}$.
24. $x=\ln\left|\ln y+\sqrt{\ln^2 y+1}\right|$. 25. $(x+1)(1-\ln y)=1$. 26. $y=2e^x$.
27. $y=(x+1)^2$. 28. $x=2arctg\sqrt{e^y-1}$. 29. $y=e^{tgx}$. 30. $3x=2y^{3/4}$.

M36. 1. $y=C_1+C_2e^{-x}+x^2-3x$.

2. $y=e^x\left(C_1\cos 2x+C_2\sin 2x\right)+e^{-x}\left(\cos 2x-2\sin 2x\right)$.
3. $y=C_1e^{-2x}+C_2e^{4x}+3\cos 2x$. 4. $y=\left(C_1+C_2x+7x^2\right)e^{6x}$.
5. $y=C_1e^x+C_2e^{2x}+\left(4-2x\right)e^{-x}$.
6. $y=e^{3x}\left(C_1\cos x+C_2\sin x\right)+3e^{-x}$.
7. $y=C_1\cos x+C_2\sin x+\left(x^2+2x\right)\cos x$.
8. $y=e^{-3x}\left(C_1\cos x+C_2\sin x\right)+2e^{3x}$.
9. $y=C_1e^x+C_2e^{2x}+6\cos x+\sin x$.
10. $y=\left(C_1+C_2x\right)e^{-3x}+\left(3x-1\right)e^x$.
11. $y=C_1+C_2e^{-5x}+3e^{2x}$. 12. $y=C_1e^{-x}+C_2e^{6x}+\cos x-2\sin x$.
13. $y=C_1e^{2x}+C_2e^{6x}+3x^4-x^2$.
14. $y=e^{-4x}\left(C_1\cos 3x+C_2\sin 3x\right)+e^{5x}$.

15. $y = C_1e^{4x} + C_2e^{5x} + 3e^{-2x}$.
16. $y = C_1\cos 6x + C_2\sin 6x - x^3 + 2x + 1$.
17. $y = C_1\cos x + C_2\sin x + x(\cos x - 2\sin x)$.
18. $y = C_1e^{-6x} + C_2e^{4x} + \sin 3x$.
19. $y = e^{-3x}(C_1\cos 2x + C_2\sin 2x) + 4\cos 2x - 3\sin 2x$.
20. $y = C_1 + C_2e^{-5x} + 4\cos 3x + 5\sin 3x$.
21. $y = e^{2x}(C_1\cos 5x + C_2\sin 5x) + 5\cos 5x + \sin 5x$.
22. $y = e^{2x}(C_1\cos x + C_2\sin x) + e^{-2x}(\cos x + \sin x)$.
23. $y = C_1\cos 4x + C_2\sin 4x + x\sin 4x$..
24. $y = C_1\cos 3x + C_2\sin 3x + x^4 - 3$.
25. $y = e^{6x}(C_1\cos 2x + C_2\sin 2x) + e^{6x}$.
26. $y = C_1 + C_2e^{-4x} + 2e^x\sin 2x$.
27. $y = (C_1 + C_2x)e^{-x} + 3x^2e^{-x}$.
28. $y = e^{-x}(C_1\cos 6x + C_2\sin 6x) + x^2 - x + 2$.
29. $y = C_1e^{x/2} + C_2e^{-x/3} + e^{2x}$.
30. $y = C_1e^{-3x} + C_2e^{-x/2} + 6\cos x + 7\cos 3x + 5\sin 3x$.

M37. 1. $y = e^{4x}(C_1\cos x + C_2\sin x) + 2e^{2x}$. 2. $y = C_1e^{-3x} + C_2e^{2x} + (x-1)e^{3x}$. 3. $y = C_1e^{3x} + C_2e^{4x} + 3xe^{4x}$. 4. $y = C_1 + C_2e^{2x} + 4x^3 + 3x^2$. 5. $y = e^{3x}(C_1\cos 5x + C_2\sin 5x) + 2\cos 5x$. 6. $y = (C_1 + x^2 + x)e^{2x} + C_2$. 7. $y = (C_1 + C_2x)e^{-x} + 4x^3 - 2x$. 8. $y = C_1 + C_2e^{4x} + 2x^2 - x$. 9. $y = (C_1 + C_2x + 2x^2)e^x$. 10. $y = e^{4x}(C_1\cos 2x + C_2\sin 2x) + \sin 2x$. 11. $y = e^{3x}(C_1\cos 2x + C_2\sin 2x) + e^{-3x}(2\cos 2x + 3\sin 2x)$. 12. $y = C_1e^{-3x} + C_2e^x + (x^3 - x)e^x$. 13. $y = (C_1 + C_2x + 3x^2)e^{-2x}$. 14. $y = C_1 + C_2e^{-3x} - x^2 + 4x$. 15. $y = (C_1 + C_2x)e^{-5x} - 8x^3 + 4x$. 16. $y = e^{-2x}(C_1\cos 4x + C_2\sin 4x) - 3\cos 4x + \sin 4x$. 17. $y = e^{-2x} \times (C_1\cos x + C_2\sin x) + x^2 - 8x + 7$. 18. $y = (C_1 + C_2x + 2x^3 - 5x^2)e^{-x}$. 19. $y = C_1e^{-2x} + (C_2 - 3x^2 - x)e^{2x}$.+ 20. $y = (C_1 + C_2x)e^{-3x} + 2e^{3x}$.

$21.\ y=C_1\cos 4x+C_2\sin 4x+4e^{2x}$. $22.\ y=C_1+C_2e^{-4x}+3e^{x}$.
$23.\ y=C_1e^{-2x}+C_2e^{x}-2\cos x+3\sin x$. $24.\ y=\left(C_1+C_2x+3x^3+\right.$
$\left.+4x^2\right)e^{-x}$. $25.\ y=\left(C_1+C_2x\right)e^{7x}+\cos 7x$. $26.\ y=C_1\cos 3x+$
$+C_2\sin 3x+e^{3x}$. $27.\ y=\left(C_1+C_2x\right)e^{x/2}+3\cos x+4\sin x$.
$28.\ y=C_1e^{-x/3}+C_2e^{2x}+\cos 2x-2\sin 2x$. $29.\ y=e^{-2x}\left(C_1\cos 5x+\right.$
$\left.+C_2\sin 5x\right)+e^{-x}$. $30.\ y=C_1e^{x/4}+C_2e^{-x}-\cos x+2\sin x$.

M38. $1.\ y=-2\left(2x+1\right)e^{x}+3\sin 2x$. $2.\ y=2\left(11x-3\right)e^{3x}+$
$+x^2-3x+5$. $3.\ y=e^{-x}\left(\cos x+3\sin x\right)+x^2+2x$. $4.\ y=e^{3x}\left(2\cos 4x-\right.$
$\left.-3\sin 4x\right)+\sin 4x$. $5.\ y=3e^{7x}\sin 2x+x^3+x$. $6.\ y=\left(e^{x}-1\right)\cos 4x+$
$+\sin 4x$. $7.\ y=e^{2x}\left(x+\cos 4x+\sin 4x\right)$. $8.\ y=(1-2x)e^{6x}+\cos 2x$.
$9.\ y=4\cos x+2\sin x+x^3-4x^2+x-2$. $10.\ y=2shx+\left(4x^2-3x\right)e^{-x}$.
$11.\ y=-2\left(3x+1\right)e^{-4x}+x^2-2x+5$. $12.\ y=e^{-5x}\left(-1+\cos 3x+\right.$
$\left.+2\sin 3x\right)$. $13.\ y=e^{3x}\left(4\cos 4x-3\sin 4x\right)+2x\sin 3x$. $14.\ y=$
$=2\cos 5x-\sin 5x+e^{x}\cos 5x$. $15.\ y=e^{-x}\left(2(1+x)\cos 2x+3\sin 2x\right)$.
$16.\ y=\left(x^2-2x+3\right)e^{5x}$. $17.\ y=e^{3x}+e^{-4x}+\left(2x+1\right)e^{4x}$. $18.\ y=e^{x}\times$
$\times\left(2\cos 2x-\sin 2x\right)+x^2+2x-2$. $19.\ y=4xe^{-4x}+x^3-x+1$. $20.\ y=$
$=\left(3x+1\right)e^{x}\sin 6x$. $21.\ y=2e^{8x}+4x^4-2x-3$. $22.\ y=\left(11x+2\right)e^{-6x}+$
$+(x-1)\left(2x^2+1\right)$. $23.\ y=4e^{-3x}+\left(4x+3\right)e^{2x}-7$. $24.\ y=2e^{6x}-3e^{3x}+$
$+\cos x-\sin x$. $25.\ y=2e^{-8x}-x^4+3x^3+3$. $26.\ y=e^{x}+2e^{2x}-\cos x+$
$+2\sin x$. $27.\ y=-e^{-2x}+x^3-x^2$. $28.\ y=e^{4x}+\cos 4x-\sin 4x$.
$29.\ y=2e^{-2x}+e^{-3x}-5\cos 2x+\sin 2x$. $30.\ y=3e^{-2x}+2(x-1)e^{2x}$.

M41. $1.a)\ y_1=x\left(Ax+B\right)e^{3x},\ b)\ y_1=A\cos 3x+B\sin 3x$.
$2.a)\ y_1=x\left(Ax+B\right)e^{2x},\ b)\ y_1=A\cos 2x+B\sin 2x$.
$3.a)\ y_1=x\left(Ax^2+Bx+C\right)e^{-x}$, $b)\ y_1=\left(Ax+B\right)\cos x+\left(Cx+D\right)\sin x$.

$4.a)\ y_1 = Axe^{4x},\ b)\ y_1 = e^x\left(A\cos 4x + B\sin 4x\right).$

$5.a)\ y_1 = Ax^3 + Bx^2 + Cx + D,\ b)\ y_1 = x\left(A\cos 7x + B\sin 7x\right).$

$6.a)\ y_1 = Axe^{-3x},\ b)\ y_1 = A\cos 3x + B\sin 3x.$

$7.a)\ y_1 = Ax + B,\ y_2 = Axe^x,\ b)\ f\left(x\right) = A\cos 4x + B\sin 4x.$

$8.a)\ y_1 = A\cos 2x + B\sin 2x,\ y_2 = Ae^x,\ b)\ y_1 = Ax^2 + Bx + C.$

$9.a)\ y_1 = xe^x\left(A\cos x + B\sin x\right),\ b)\ y_1 = Ax + B.$

$10.a)\ y_1 = x\left(Ax^2 + Bx + C\right),\ b)\ y_1 = e^{-x}\left(A\cos 2x + B\sin 2x\right).$

$11.a)\ y_1 = x\left(Ax + B\right)e^{-4x},\qquad b)\ y_1 = \left(Ax + B\right)\cos x + \left(Cx + D\right)\sin x.$

$12.a)\ y_1 = \left(Ax + B\right)e^{-x},\ b)\ y_1 = x\left(A\cos 6x + B\sin 6x\right).$

$13.a)\ y_1 = x^2\left(Ax + B\right)e^{3x},\ b)\ y_1 = A\cos x + B\sin x.$

$14.a)\ y_1 = x\left(Ax + B\right)e^x,\ b)\ f\left(x\right) = e^x\left(A\cos x + B\sin 3x\right).$

$15.a)\ y_1 = Axe^{-2x},\ b)\ y_1 = (Ax + B)\cos 2x + \left(Cx + D\right)\sin 2x.$

$16.a)\ y_1 = x\left(Ax + B\right)e^{3x},\ b)\ y_1 = A\cos x + B\sin x.$

$17.a)\ y_1 = Axe^{4x},\ b)\ y_1 = A\cos x + B\sin x.$

$18.a)\ y_1 = x\left(Ax + B\right)e^{4x},\ b)\ y_1 = A\cos 4x + B\sin 4x.$

$19.a)\ y_1 = \left(Ax + B\right)e^{4x},\ b)\ y_1 = xe^x\left(A\cos x + B\sin x\right).$

$20.a)\ y_1 = x\left(Ax^2 + Bx + C\right)e^x,\ b)\ y_1 = A\cos x + B\sin x.$

$21.a)\ y_1 = Ax^3 + Bx^2 + Cx + D,\ b)\ y_1 = A\cos 2x + B\sin 2x.$

$22.a)\ y_1 = \left(Ax + B\right)e^{3x},\ b)\ y_1 = \left(Ax + B\right)\cos 5x + \left(Cx + D\right)\sin 5x.$

$23.a)\ y_1 = Ax^3 + Bx^2 + Cx + D,\ b)\ y_1 = e^{3x}\left(A\cos x + B\sin x\right).$

$24.a)\ y_1 = x\left(Ax + B\right)e^{3x},\ y_2 = Ae^x,\ b)\ y_1 = \left(Ax + B\right)\cos 2x + \left(Cx + D\right)\sin 2x.$

$25.a)\ y_1 = x\left(Ax^2 + Bx + C\right),\ b)\ y_1 = e^{2x}\left[\left(Ax + B\right)\cos x + \left(Cx + D\right)\sin x\right].$

$26.a)\ y_1 = \left(Ax + B\right)e^x,\qquad b)\ y_1 = \left(Ax + B\right)\cos 2x + \left(Cx + D\right)\sin 2x.$

$27.a)\ y_1 = x\left(Ax + B\right)e^{-x},\ b)\ y_1 = A\cos x + B\sin x.$

$28.a)\ y_1 = x^2\left(Ax + B\right)e^{4x},\ b)\ y_1 = A\cos 4x + B\sin 4x.$

$29.a)\ y_1 = x\left(Ax + B\right)e^x,\qquad b)\ y_1 = \left(Ax + B\right)\cos 2x + \left(Cx + D\right)\sin 2x.$

$30.a)\ y_1 = \left(Ax + B\right)e^{-x},\ b)\ y_1 = \left(Ax^2 + \ldots\right)\cos 2x + \left(Dx^2 + \ldots\right)\sin 2x.$

M43. 1. $2y=\left(-e^{x}+\ln\left(e^{x}+1\right)+C_1\right)e^{-x}+\left(x-\ln\left(e^{x}+1\right)+C_2\right)e^{x}.$

2. $4y=\left(\ln\left|\cos 2x\right|+C_1\right)\cos 2x+\left(2x+C_2\right)\sin 2x.$

3. $y=e^{2x}\left(\ln\left|\cos x\right|+C_1\right)\cos x+e^{2x}\left(x+C_2\right)\sin x.$

4. $y=\sec x+C_1+\left(\ln\left|\cos x\right|+C_2\right)\cos x+\left(x-tgx+C_3\right)\sin x.$

5. $9y=\left(-3x+C_1\right)\cos 3x+\left(\ln\left|\sin 3x\right|+C_2\right)\sin 3x.$

6. $4y=\left(C_1+C_2x\right)e^{-x}+(x-1)e^{x}+4xe^{-x}\ln\left|x\right|.$

7. $y=\left(\ln\left|\cos x\right|+C_1\right)e^{-x}\cos x+\left(x+C_2\right)e^{-x}\sin x.$

8. $y=\left(\ln\left|ctg\left(x/2\right)\right|+C_1\right)e^{x}\cos x-e^{x}+C_2e^{x}\sin x.$

9. $y=C_1e^{-x}\cos x+C_2e^{-x}+e^{-x}\sin x\cdot\ln\left|tg\left(x/2\right)\right|.$

10. $y=\left(-x+C_1\right)e^{x}\cos x+(\ln\left|\sin x\right|+C_2)e^{x}\sin x.$

11. $y=\left(-\ln\left|x\right|+C_1\right)e^{x}-e^{x}+C_2xe^{x}.$

12. $y=C_1\cos x+C_2\sin x-\cos x\cdot\ln\left|tg\left(x/2+\pi/4\right)\right|.$

13. $4y=C_1\cos 2x+C_2\sin 2x+\sin 2x\cdot\ln\left|tg\, x\right|.$

14. $y=C_1\cos x+C_2\sin x+\sin x\cdot\ln\left|tg\left(x/2\right)\right|.$

15. $y=\left(-x+C_1\right)e^{x}+\left(\ln\left|x\right|+C_2\right)xe^{x}.$

16. $y=\left(-x+C_1\right)e^{-x}+\left(\ln\left|x\right|+C_2\right)xe^{-x}.$

17. $y=\left(\ln\left|\cos x\right|+C_1\right)\cos x+\left(x+C_2\right)\sin x.$

18. $y=\left(-x+C_1\right)\cos x+\left(\ln\left|\sin x\right|+C_2\right)\sin x.$

19. $4y=\left(-2x+C_1\right)\cos 2x+\left(\ln\left|\sin 2x\right|+C_2\right)\sin 2x.$

20. $4y=C_1\cos 2x+C_2\sin 2x-\cos 2x\cdot\ln\left|tg\left(x+\pi/4\right)\right|.$

21. $2y=\left(C_1+C_2x+1/x\right)e^{-2x}$. 22. $2y=\left(C_1+C_2x+1/x\right)e^{2x}.$

23. $y=\left(-\left(6/5\right)\sqrt{(x+1)^5}+2\sqrt{(x+1)^3}+C_1\right)e^{-x}+\left(2\sqrt{(x+1)^3}+C_2\right)xe^{-x}.$

24. $y=C_1\cos x+C_2\sin x+2+\cos x\cdot\ln\left|tg\left(x/2\right)\right|.$

25. $y=C_1+C_2e^{x}-\cos\left(e^{x}\right)$. 26. $y=C_1+C_2e^{x}-\sin\left(e^{x}\right).$

27. $y = C_1\cos x + C_2\sin x - 2 + \sin x\cdot\ln\left|tg\left(x/2+\pi/4\right)\right|$.

28. $y = C_1\cos x + C_2\sin x - 2 + 2\cos x\cdot\ln\left|ctg\left(x/2\right)\right|$.

29. $4y = \left(-2x + C_1\right)e^{-x}\cos 2x + \left(\ln\left|\sin 2x\right| + C_2\right)e^{-x}\sin 2x$..

30. $9y = \left(\ln\left|\cos 3x\right| + C_1\right)\cos 3x + \left(3x + C_2\right)\sin 3x$.

M45. 1. $y = C_1x^2 + C_2x^3$. 2. $y = C_1x^2 + C_2x^4$. 3. $y = C_1x^{-2} + C_2x^4$. 4. $y = C_1x^{-5} + C_2x^7$. 5. $y = C_1x^6 + C_2x^7$. 6. $y = C_1x^4 + C_2x^5$. 7. $y = C_1x^{-3} + C_2x^{-2}$. 8. $y = C_1x^{-2} + C_2x^3$. 9. $y = C_1x^{-2} + C_2x^5$. 10. $y = C_1x^{-5} + C_2x^{-4}$. 11. $y = C_1x^{-3} + C_2x^5$. 12. $y = C_1x^7 + C_2x^9$. 13. $y = C_1x^{-7} + C_2x^9$. 14. $y = C_1x^{-7} + C_2x^{-5}$. 15. $y = C_1x^{-7} + C_2x^{-6}$. 16. $y = C_1x^{-4} + C_2x^5$. 17. $y = C_1x^{-5} + C_2x^4$. 18. $y = C_1x^{-3} + C_2x^{-2}$. 19. $y = C_1x^{-4} + C_2x^2$. 20. $y = C_1x^{-6} + C_2x^7$. 21. $y = C_1x^{-7} + C_2x^6$. 22. $y = C_1x^3 + C_2x^5$. 23. $y = C_1x^7 + C_2x^8$. 24. $y = C_1x^5 + C_2x^8$. 25. $y = C_1x^5 + C_2x^6$. 26. $y = C_1x^{-5} + C_2x^6$. 27. $y = C_1x^{-6} + C_2x^5$. 28. $y = C_1x^{-4} + C_2x^7$. 29. $y = C_1x^{-7} + C_2x^{-4}$. 30. $y = C_1x^{-3} + C_2x^7$.

M47. 1. $y = 2b^2/\left(C \pm x\right)$. 2. $y = C\left(x^2 + y^2\right)$. 3. $xy = 2a^2 + Cx^3$. 4. $xy = a^2 + Cy^2$. 5. $Cx = x^2 + y^2$. 6. $y = Cx^2$. 7. $y \pm x = a\ln y + C$ $(0 < y < a)$. 8. $y = Cx^3$. 9. $x = l\ln\left(l \pm \sqrt{l^2 - y^2}\right) \mp \sqrt{l^2 - y^2} + C$. 10. $y = 2\sqrt{3}x\left(x^2 - 1\right)^{-1/2}$. 11. $y = x\left(3\ln x + 5\right)$. 12. $y^3 = 8x$. 13. $y = e^{6x-12}$. 14. $y = 2(y - x)^2$. 15. $y = -2e^{3x}$. 16. $y^2 = Cx - x^2$. 17. $y = Cx^n$. 18. $xy = C$. 19. $2Cy = C^2x^2 - 1$. 20. $x^2 + y^2 = Cx^4$. 21. $x^2 + y^2 = Cy, y^2 = C^2 - 2Cx,$ $xy = C$. 22. $\pm y = \sqrt{4 - x^2} + \ln\left(2 - \sqrt{4 - x^2}\right) - \ln\left(2 + \sqrt{4 - x^2}\right)$. 23. $y = C\,x$. 24. $x(y - 1) = C$. 25. $y = Cx^2$. 26. $y = Cx^{(n-1)/n} - x$. 27. $y^2 = 2x^2 \times$ $\times\ln\left(C/x\right)$. 28. $y = Cx - x^2$. 29. $x^2 + (C + 2)y^2\ln y = 0$. 30. $x = (5 - y)y$.

M48. 1. $y = C_1 y_1 + C_2\left(1+\frac{1}{x}\right)e^{-x}$. 2. $y = C_1 y_1 + C_2\left(2+\frac{1}{x}\right)e^{-2x}$.
3. $y = C_1 y_1 + C_2\left(1+\frac{2}{x}\right)e^{-x}$. 4. $y = C_1 y_1 + C_2\left(\frac{2}{x}+\frac{3}{x^2}\right)e^{-2x}$.
5. $xy = C_1 + C_2 e^{x^2/2}$. 6. $y = \left(C_1 x^3 + C_2 x^{-2}\right)e^{-x^2/4}$. 7. $xy = C_1 + C_2\left(ax+b\right)^3$.
8. $y = C_1 y_1 + C_2\left(\sin x + \frac{x}{\cos x}\right)$. 9. $xy = C_1 + C_2\left(7x+4\right)^3$.
10. $xy = C_1\left(3x-1\right) + C_2\left(3x+1\right)e^{-6x}$. 11. $y = C_1 y_1 + C_2 \sin\left(\sin x\right)$.
12. $xy = C_1\left(4x-1\right) + C_2\left(4x+1\right)e^{-8x}$. 13. $xy = C_1 + C_2\left(x+1\right)^3$.
14. $xy = C_1\left(3x-2\right) + C_2\left(3x+2\right)e^{-3x}$. 15. $y = C_1 y_1 + C_2 e^{-\sin x}$.
16. $xy = C_1 + C_2\left(2x+1\right)^3$. 17. $xy = C_1\left(5x-2\right) + C_2\left(5x+2\right)e^{-5x}$.
18. $y = C_1 y_1 + C_2 \sin\left(\cos x\right)$. 19. $xy = C_1 + C_2\left(x+2\right)^3$.
20. $xy = C_1\left(7x-2\right) + C_2\left(7x+2\right)e^{-7x}$. 21. $y = C_1 y_1 + C_2\left(1 - x\,ctgx\right)$.
22. $xy = C_1 + C_2\left(2-x\right)^3$. 23. $xy = C_1\left(9x-2\right) + C_2\left(9x+2\right)e^{-9x}$.
24. $y = C_1 y_1 + C_2 \sin x$. 25. $xy = C_1 + C_2\left(2x-1\right)^3$.
26. $xy = C_1\left(x-1\right) + C_2\left(x+1\right)e^{-2x}$.
27. $y = C_1 y_1 + C_2\left(1 + x\ln|x|\right)x^2$. 28. $xy = C_1 + C_2\left(2x+3\right)^3$.
29. $xy = C_1\left(2x-1\right) + C_2\left(2x+1\right)e^{-4x}$. 30. $y = C_1 y_1 + C_2\left(x + \ln|x|\right)x$.

M49. 1. $y = \left(C_1 + C_2 x\right)e^{-x^2}$. 2. $y = \left(C_1 + C_2 x\right)e^{x^2}$.
3. $y = \left(C_1 e^{x} + C_2 e^{-x} - 1\right)e^{x^2}$. 4. $y = \left(C_1 chx + C_2 shx\right)e^{-x^2/2}$.
5. $y = \left(C_1 \cos x + C_2 \sin x\right)e^{x^2/2}$. 6. $y = \left(C_1 ch2x + C_2 sh2x\right)e^{-2x^2}$.
7. $y = \left(C_1 \cos 2x + C_2 \sin 2x\right)e^{2x^2}$. 8. $y = C_1\left(x+1\right) + C_2 e^{x}$.
9. $y = C_1\left(3x+1\right)e^{-x} + C_2 e^{2x}$. 10. $y = \left(C_1 + C_2 x^3\right)e^{-x}$.
11. $y = C_1\left(x - tgx\right) + C_2\left(1 + x\,tgx\right)$. 12. $y = x^2 + C_1 x + C_2 x^{-1}$.

$13.\ y=\left(C_1+C_2x^3\right)e^x.$ $14.\ y=\left(C_1\cos x+C_2\sin x\right)x.$ $15.\ y=C_1x+C_2xe^{2x}.$
$16.\ y=\left(C_1+C_2x^5\right)e^x.$ $17.\ y=\left(C_1\cos 2x+C_2\sin 2x\right)x.$
$18.\ y=C_1x+C_2\left(x^2-1\right).$ $19.\ y=\left(C_1+C_2x^3\right)e^{2x}.$
$20.\ y=\left(C_1\cos 3x+C_2\sin 3x\right)x.$ $21.\ y=C_1x+C_2\left(x^2+1\right).$
$22.\ y=\left(C_1+C_2x^5\right)e^{3x}.$ $23.\ y=\left(C_1\cos 4x+C_2\sin 4x\right)x.$
$24.\ y=C_1\left(x^2+x+3\right)+C_2e^{-x}.$ $25.\ y=\left(C_1+C_2x^5\right)e^{2x}.$
$26.\ y=\left(C_1\cos 5x+C_2\sin 5x\right)x.$ $27.\ 4y=\left(C_1+C_2\ln|x|\right)x+3x^3.$
$28.\ y=\left(C_1+C_2x^3\right)e^{-2x}.$ $29.\ y=\left(C_1+C_2x^7\right)e^x.$ $30.\ y=\left(C_1+C_2x^3\right)e^{3x}.$

M50. $1.\ y=e^x-2.$ 2. Yechim yo\`q. $3.\ y=1+2e^x.$ $4.\ y=\dfrac{\sin x}{\sin a.}y_0.$ $5.\ xy=1.$ $6.\ y=x^2/2.$ $7.\ y=-2e^{-x}.$ $8.\ y=-e^{(-1-i)x}.$ $9.\ y=-x^{-3}.$ $10.\ y=1-\sin x-\cos x.$ $11.\ y=e^{-3x}.$ 12. Chegaraviy masala yechimga ega emas. $13.\ a=(2n-1)^2\pi^2,\ n=1,2,...$ $14.\ G(x,s)=(s-1)x\ (0\le x\le s),$ $G(x,s)=s(x-1)\ (s\le x\le 1).$ $15.\ G(x,s)=-e^{-s}ch\,x\ (0\le x\le s),$ $G(x,s)=-e^{-x}ch\,s\quad (s\le x\le 2).$ $16.\ G(x,s)=\dfrac{1}{x}-1\quad (1\le x\le s),$ $G(x,s)=\dfrac{1}{s}-1$ $(s\le x\le 3).$ $17.\ G(x,s)=-1\quad (0\le x\le s),$ $G(x,s)=-e^{s-x}\quad (s\le x<\infty).$ $18.\ G(x,s)=-\ln x\quad (1\le x\le s),$ $G(x,s)=-\ln s\quad (s\le x<\infty).$ $19.\ G(x,s)=\dfrac{x\left(s^3-1\right)}{3s^2}\ (0\le x\le s),$ $G(x,s)=\dfrac{s\left(x^3-1\right)}{3x^2}\ (s\le x\le 1).$ $20.\ G(x,s)=-\dfrac{1}{2}e^{-|x-s|}.$
$21.\ G(x,s)=-\dfrac{x^2}{3s^3}\ (0\le x\le s),$ $G(x,s)=-\dfrac{1}{3x}\ (s\le x<\infty).$ $22.\ G(x,s)=-\dfrac{(1-s)(1+x)}{2}\ (-1\le x\le s),$ $G(x,s)=-\dfrac{(1+s)(1-x)}{2}\ (s\le x\le 1).$
$$23.\ y=\frac{1}{a-b}\left[(b-x)\int_a^x(s-a)f(s)ds+(x-a)\int_x^b(b-s)f(s)ds\right].$$
$24.\ G(x,s)=1+\ln s\quad (0<x\le s),$ $G(x,s)=1+\ln x\quad (s\le x<1);$ $2y=x^2+1.$ $25.\ a\ne k^2\pi^2,\ k=1,2,...$ $26.\ \lambda_k=-k^2,\ y_k=\cos kx,\ k=0,1,2,...$

27. $\lambda_k = -\left(k-\frac{1}{2}\right)^2$, $y_k = \sin\left(k-\frac{1}{2}\right)x$, $k = 1, 2, 3, \ldots$ 28. $\lambda_k = -k^2\pi^2 - \frac{1}{4}$, $y_k = \sqrt{x}\sin k\pi \ln x$, $k = 1, 2, \ldots$. 29. $\lambda_k = -\left(\frac{k\pi}{l}\right)^2$, $y_k = \cos\frac{k\pi x}{l}$, $k = 0, 1, 2, \ldots$ 30. $\lambda_k = -\left(\frac{k\pi}{\ln a}\right)^2 - \frac{1}{4}$, $y_k = \sqrt{x}\sin\frac{k\pi \ln x}{\ln a}$, $k = 1, 2, 3, \ldots$

M51. 1. $x = C_1e^t + C_2e^{5t}, y = -C_1e^t + 3C_2e^{5t}$.
2. $x = C_1e^{-t} + C_2e^{3t}, y = 2C_1e^{-t} - 2C_2e^{3t}$.
3. $x = 4C_1e^{-3t} + 2C_2e^{3t}, y = -C_1e^{-3t} + C_2e^{3t}$.
4. $x = 3C_1e^{6t} - C_2e^{10t}, y = C_1e^{6t} + C_2e^{10t}$.
5. $x = C_1 + C_2e^{5t}, y = C_1 - 4C_2e^{5t}$.
6. $x = C_1e^{-t} + C_2e^{t}, y = C_1e^{-t} + 3C_2e^{t}$.
7. $x = C_1e^{3t} + C_2e^{5t}, y = 3C_1e^{3t} + C_2e^{5t}$.
8. $x = C_1 + C_2e^{-t}, y = -2C_1 - 3C_2e^{-t}$.
9. $x = C_1e^{-8t} - 5C_2e^{3t}, y = 2C_1e^{-8t} + C_2e^{3t}$.
10. $x = -2C_1e^{2t} + C_2e^{5t}, y = C_1e^{2t} + C_2e^{5t}$.
11. $x = -C_1e^{-5t} + 5C_2e^{6t}, y = 2C_1e^{-5t} + C_2e^{6t}$.
12. $x = C_1e^{2t} + C_2e^{8t}, y = -C_1e^{2t} + 2C_2e^{8t}$.
13. $x = C_1e^{2t} + 3C_2e^{7t}, y = 2C_1e^{2t} + C_2e^{7t}$.
14. $x = C_1e^{2t} + C_2e^{4t}, y = -C_1e^{2t} + C_2e^{4t}$.
15. $x = C_1e^{-t} + 3C_2e^{7t}, y = -C_1e^{-t} + 5C_2e^{7t}$.
16. $x = 2C_1 + C_2e^{7t}, y = -C_1 + 3C_2e^{7t}$.
17. $x = C_1e^{t} + C_2e^{9t}, y = -C_1e^{t} + C_2e^{9t}$.
18. $x = C_1e^{-t} + C_2e^{5t}, y = -C_1e^{-t} + 2C_2e^{5t}$.
19. $x = 2C_1e^{-t} + 2C_2e^{3t}, y = -C_1e^{-t} + C_2e^{3t}$.
20. $x = 2C_1e^{4t} + C_2e^{7t}, y = -C_1e^{4t} - 2C_2e^{7t}$.
21. $x = C_1e^{4t} + 2C_2e^{7t}, y = -2C_1e^{4t} - C_2e^{7t}$.

22. $x = C_1e^{4t} + 3C_2e^{8t}, y = -C_1e^{4t} + C_2e^{8t}$.

23. $x = C_1e^{3t} + C_2e^{5t}, y = C_1e^{3t} - C_2e^{5t}$.

24. $x = 4C_1 + 2C_2e^{6t}, y = -C_1 + C_2e^{6t}$.

25. $x = 4C_1e^{-t} + 2C_2e^{9t}, y = -3C_1e^{-t} + C_2e^{9t}$.

26. $x = C_1e^{-t} + C_2e^{5t}, y = -4C_1e^{-t} + 2C_2e^{5t}$.

27. $x = C_1e^{-4t} + 5C_2e^{2t}, y = C_1e^{-4t} - C_2e^{2t}$.

28. $x = 2C_1e^{-4t} + C_2e^{-7t}, y = C_1e^{-4t} - C_2e^{-7t}$.

29. $x = 3C_1e^{-2t} + C_2e^{3t}, y = -8C_1e^{-2t} - C_2e^{3t}$.

30. $x = C_1e^{-4t} + C_2e^{12t}, y = C_1e^{-4t} - C_2e^{12t}$.

M52.1. $x = 2C_1e^{7t} + C_2e^{-4t}, y = C_1e^{7t} - 5C_2e^{-4t}$.

2. $x = C_1e^{-7t} + 5C_2e^{4t}, y = -2C_1e^{-7t} + C_2e^{4t}$.

3. $x = C_1e^{-6t} + C_2e^{4t}, y = -9C_1e^{-6t} + C_2e^{4t}$.

4. $x = 3C_1e^{6t} + C_2e^{-4t}, y = C_1e^{6t} - 3C_2e^{-4t}$.

5. $x = 9C_1e^{11t} + C_2e^{-5t}, y = 2C_1e^{11t} + 2C_2e^{-5t}$.

6. $x = C_1e^{-9t} + 5C_2e^{2t}, y = -2C_1e^{-9t} + C_2e^{2t}$.

7. $x = -5C_1e^{9t} + C_2e^{-2t}, y = C_1e^{9t} + 2C_2e^{-2t}$.

8. $x = C_1e^{-8t} + 2C_2e^{5t}, y = -C_1e^{-8t} + 11C_2e^{5t}$.

9. $x = C_1e^{-8t} + 11C_2e^{5t}, y = C_1e^{-8t} - 2C_2e^{5t}$.

10. $x = 2C_1e^{7t} + C_2e^{-7t}, y = C_1e^{7t} - 3C_2e^{-7t}$.

11. $x = 3C_1e^{8t} + C_2e^{-6t}, y = 2C_1e^{8t} - 4C_2e^{-6t}$.

12. $x = C_1e^{9t} + C_2e^{-5t}, y = -C_1e^{9t} + 6C_2e^{-5t}$.

13. $x = C_1e^{-8t} + 13C_2e^{7t}, y = -C_1e^{-8t} + 2C_2e^{7t}$.

14. $x = C_1e^{8t} + 2C_2e^{-7t}, y = C_1e^{8t} - 13C_2e^{-7t}$.

15. $x = 7C_1e^{10t} + C_2e^{-8t}, y = 4C_1e^{10t} - 2C_2e^{-8t}$.

16. $x = 4C_1e^{-10t} + 2C_2e^{8t}, y = -7C_1e^{-10t} + C_2e^{8t}$.

17. $x = 13C_1e^{-10t} + C_2e^{8t}, y = -5C_1e^{-10t} + C_2e^{8t}$.

18. $x = 3C_1e^{5t} + C_2e^{-5t}, y = C_1e^{5t} - 3C_2e^{-5t}$.

19. $x = 5C_1e^{10t} + C_2e^{-8t}$, $y = 3C_1e^{10t} - 3C_2e^{-8t}$.
20. $x = C_1e^{11t} + C_2e^{-7t}$, $y = C_1e^{11t} - 5C_2e^{-7t}$.
21. $x = 3C_1e^{-12t} + 3C_2e^{6t}$, $y = -C_1e^{-12t} + 5C_2e^{6t}$.
22. $x = 4C_1e^{12t} + C_2e^{-6t}$, $y = C_1e^{12t} - 2C_2e^{-6t}$.
23. $x = 2C_1e^{11t} + C_2e^{-7t}$, $y = C_1e^{11t} - 4C_2e^{-7t}$.
24. $x = 2C_1e^{5t} + C_2e^{-2t}$, $y = C_1e^{5t} + 4C_2e^{-2t}$.
25. $x = C_1e^{7t} + 3C_2e^{-3t}$, $y = 2C_1e^{7t} + C_2e^{-3t}$.
26. $x = C_1e^{-7t} + 2C_2e^{6t}$, $y = 7C_1e^{-7t} + C_2e^{6t}$.
27. $x = 13C_1e^{8t} + C_2e^{-3t}$, $y = -2C_1e^{8t} - C_2e^{-3t}$.
28. $x = 5C_1e^{8t} + C_2e^{-4t}$, $y = 3C_1e^{8t} + 3C_2e^{-4t}$.
29. $x = 5C_1e^{-9t} + C_2e^{3t}$, $y = C_1e^{-9t} + C_2e^{3t}$.
30. $x = C_1e^{-6t} + 4C_2e^{5t}$, $y = 3C_1e^{-6t} + C_2e^{5t}$.

M53 1. $x = \left(C_1 + 5C_2t\right)e^{-4t}$, $y = (-C_1 + C_2 - 5C_2t)e^{-4t}$.
2. $x = \left(C_1 + 3C_2t\right)e^{-2t}$, $y = (2C_1 + C_2 + 6C_2t)e^{-2t}$.
3. $x = \left(C_1 + 9C_2t\right)e^{4t}$, $y = (C_1 - C_2 + 9C_2t)e^{4t}$.
4. $x = (C_1 + C_2 + 6C_2t)e^{-t}$, $y = \left(c_1 + 6c_2t\right)e^{-t}$.
5. $x = 8\left(C_1 + C_2t\right)e^{-t}$, $y = (C_1 + C_2 + 8C_2t)e^{-t}$.
6. $x = (C_1 + C_2 + 10C_2t)e^{3t}$, $y = 2\left(C_1 + 10C_2t\right)e^{3t}$.
7. $x = (C_1 + C_2 + 5C_2t)e^{3t}$, $y = \left(C_1 + 5C_2t\right)e^{3t}$.
8. $x = \left(C_1 + 7C_2t\right)e^{-t}$, $y = (-C_1 + C_2 - 7C_2t)e^{-t}$.
9. $x = 3\left(C_1 + 3C_2t\right)e^{t}$, $y = (-2C_1 + C_2 - 6C_2t)e^{t}$.
10. $x = \left(C_1 + 7C_2t\right)e^{3t}$, $y = (-C_1 + C_2 - 7C_2t)e^{3t}$.
11. $x = \left(C_1 + 9C_2t\right)e^{2t}$, $y = (C_1 + C_2 + 9C_2t)e^{2t}$.
12. $x = (C_1 + C_2 + 7C_2t)e^{2t}$, $y = \left(C_1 + 7C_2t\right)e^{2t}$.
13. $x = (2C_1 + C_2 + 8C_2t)e^{-3t}$, $y = \left(C_1 + 4C_2t\right)e^{-3t}$.
14. $x = \left(C_1 + 4C_2t\right)e^{-t}$, $y = (C_1 + C_2 + 4C_2t)e^{-t}$.

15. $x=2\left(C_1+6C_2t\right)e^{-t}$, $y=(C_1+C_2+6C_2t)e^{-t}$.
16. $x=\left(C_1+C_2t\right)e^{3t}$, $y=(-C_1+C_2-C_2t)e^{3t}$.
17. $x=2\left(C_1+2C_2t\right)e^{5t}$, $y=(-C_1+C_2-2C_2t)e^{5t}$
18. $x=(C_1+C_2-3C_2t)e^{-2t}$, $y=\left(-C_1+3C_2t\right)e^{-2t}$
19. $x=(C_1+C_2+C_2t)e^{-3t}$, $y=\left(C_1+C_2t\right)e^{-3t}$.
20. $x=(3C_1+C_2+3C_2t)e^{4t}$, $y=-\left(C_1+C_2t\right)e^{4t}$.
21. $x=(C_1+C_2+C_2t)e^{6t}$, $y=\left(C_1e+C_2t\right)e^{6t}$.
22. $x=(2c_1+c_2+2c_2t)e^{6t}$, $y=-\left(c_1+c_2t\right)e^{6t}$.23.
23. $x=3\left(C_1+3C_2t\right)e^{-6t}$, $y=(5C_1+C_2+3C_2t)e^{-6t}$.
24. $x=(2C_1+3C_2t)e^{t}$, $y=\left(C_1+3C_2t\right)e^{t}$.
25. $x=C_2(1-2t)e^{-t}$, $y=\left(c_1+2c_2t\right)e^{-t}$.
26. $x=(C_1+C_2-C_2t)e^{2t}$, $y=\left(-C_1+C_2t\right)e^{2t}$.
27. $x=(C_1+C_2+2C_2t)e^{t}$, $y=\left(2C_1+4C_2t\right)e^{t}$.
28. $x=\left(C_1+C_2+C_2t\right)e^{3t}$, $y=\left(C_1+C_2t\right)e^{3t}$.
29. $x=\left(C_1+C_2t\right)e^{2t}$, $y=\left(-C_1+C_2-C_2t\right)e^{2t}$.
30. $x=(C_1+C_2+C_2t)e^{t}$, $y=-\left(C_1+C_2t\right)e^{t}$.

M54. 1. $x=e^{-t}\left(5C_1\cos 6t+5C_2\sin 6t\right)$,
$y=(4C_1+3C_2)e^{-t}\cos 6t-(3C_1-4C_2)e^{-t}\sin 6t$.
2. $x=C_1e^{t}\cos 5t+C_2e^{t}\sin 5t$, $y=(C_1-C_2)e^{t}\cos 5t+(C_1+C_2)e^{t}\sin 5t$.
3. $x=2(2C_1+C_2)e^{t}\cos 2t+2(-C_1+2C_2)e^{t}\sin 2t$,
$y=5C_1e^{t}\cos 2t+5C_2e^{t}\sin 2t$.
4. $x=(3C_1+C_2)e^{t}\cos t+(-C_1+3C_2)e^{t}\sin t$, $y=2C_1e^{t}\cos t+2C_2e^{t}\sin t$.
5. $x=3(C_1+C_2)\cos 3t+3(-C_1+C_2)\sin 3t$, $y=2C_1\cos 3t+2C_2\sin 3t$.
6. $x=5e^{t}\left(C_1\cos 3t+C_2\sin 3t\right)$,
$y=(4C_1+3C_2)e^{t}\cos 3t+(-3C_1+4C_2)e^{t}\sin 3t$.
7. $x=(C_1+C_2)e^{t}\cos 2t-(C_1-C_2)e^{t}\sin 2t$, $y=2C_1e^{t}\cos 2t+2C_2e^{t}\sin 2t$.

8. $x=(3C_1+2C_2)e^{2t}\cos 2t-(2C_1-3C_2)e^{2t}\sin 2t,$

$y=13e^{2t}\left(C_1\cos 2t+13C_2\sin 2t\right).$

9. $x=(3C_1+4C_2)\cos 4t-(4C_1-3C_2)\sin 4t,\ y=5C_1\cos 4t+5C_2\sin 4t.$

10. $x=(6C_1+3C_2)e^{t}\cos 3t-(3C_1-6C_2)e^{t}\sin 3t,$

$y=5C_1e^{t}\cos 3t+5C_2e^{t}\sin 3t.$

11. $x=(2C_1+3C_2)e^{2t}\cos 6t-(3C_1-2C_2)e^{2t}\sin 6t,$

$y=2C_1e^{2t}\cos 6t+2C_2e^{2t}\sin 6t.$

12. $x=(5C_1+6C_2)e^{t}\cos 6t-(6C_1-5C_2)e^{t}\sin 6t,$

$y=61C_1e^{t}\cos 6t+61C_2e^{t}\sin 6t.$

13. $x=(3C_1+5C_2)e^{8t}\cos 5t+(-5C_1+3C_2)e^{8t}\sin 5t,$

$y=17C_1e^{8t}\cos 5t+17C_2e^{8t}\sin 5t.$

14. $x=C_1e^{10t}\cos 3t+3C_2e^{10t}\sin 3t,$

$y=(C_1+C_2)e^{10t}\cos 3t-(10C_1+3C_2)e^{10t}\sin 3t.$

15. $x=(C_1+C_2)e^{7t}\cos 2t+(C_2-C_1)e^{7t}\sin 2t,y=2e^{7t}\left(C_1\cos 2t+C_2\sin 2t\right).$

16. $x=(C_1+C_2)e^{2t}\cos t+(C_1-C_2)e^{2t}\sin t,y=-2C_1e^{2t}\cos t+2C_2e^{2t}\sin t.$

17. $x=C_1e^{3t}\cos 2t+C_2e^{3t}\sin 2t,$

$y=(C_1+2C_2)e^{3t}\cos 2t+(-2C_1+C_2)e^{3t}\sin 2t.$

18. $x=2C_1e^{2t}\sin 2t+2C_2e^{2t}\cos 2t,y=-C_1e^{2t}\cos 2t+C_2e^{2t}\sin 2t.$

19. $x=C_1e^{-4t}\cos 4t+C_2e^{-4t}\sin 4t,$

$y=-(C_1+4C_2)e^{-4t}\cos 4t+(4C_1-C_2)e^{-4t}\sin 4t.$

20. $x=-(2C_1+3C_2)e^{t}\cos 3t+(3C_1-2C_2)e^{t}\sin 3t,y=C_1e^{t}\cos 3t+C_2e^{t}\sin 3t.$

21. $x=(C_1+C_2)e^{-t}\cos 2t+(C_1-C_2)e^{-t}\sin 2t,y=2e^{-t}\left(-C_1\cos 2t+C_2\sin 2t\right).$

22. $x=(-3C_1+4C_2)e^{-2t}\cos 4t-(4C_1+3C_2)e^{-2t}\sin 4t,$

$y=5C_1e^{-2t}\cos 4t+5C_2e^{-2t}\sin 4t.$

23. $x=-2C_1e^{-3t}\cos 3t+2C_2e^{-3t}\sin 3t,$

$y=(C_1-3C_2)e^{-3t}\cos 3t+(3C_1+C_2)e^{-3t}\sin 3t.$

24. $x=(C_1+C_2)e^{4t}\cos t-(C_1-C_2)e^{4t}\sin t,y=2C_1e^{4t}\cos t+2C_2e^{4t}\sin t.$

25. $x=(C_1+C_2)e^{-4t}\cos 3t+(C_1-C_2)e^{-4t}\sin 3t,$

$y=-2C_1e^{-4t}\cos 3t+2C_2e^{-4t}\sin 3t.$

26. $x = C_1e^{2t}\sin t + C_2e^{2t}\cos t, y = -C_1e^{2t}\cos t + C_2e^{2t}\sin t.$

27. $x = (C_1 + C_2)\cos t + (-C_1 + C_2)\sin t, y = C_1\cos t + C_2\sin t.$

28. $x = (2C_1 + C_2)e^{2t}\cos t + (2C_2 - C_1)e^{2t}\sin t, y = C_1e^{2t}\cos t + C_2e^{2t}\sin t.$

29. $x = C_1e^{2t}\cos t + C_2e^{2t}\sin t, y = (C_1 + C_2)e^{2t}\cos t + (-C_1 + C_2)e^{2t}\sin t.$

30. $x = 2C_1e^t\cos 2\sqrt{3}t + 2C_2e^t\sin 2\sqrt{3}t, y = \sqrt{3}C_2e^t\cos 2\sqrt{3}t - \sqrt{3}C_1e^t\sin 2\sqrt{3}t.$

M55.1. $x = C_2e^{2t} + C_3e^{3t}, y = C_1e^t + C_2e^{2t}, z = C_1e^t + C_2e^{2t} + C_3e^{3t}.$

2. $x = C_1e^{3t} + 2C_2e^t(\cos 3t + \sin 3t) + 2C_3e^t(\cos 3t - \sin 3t),$
$y = C_2e^t(3\cos 3t - 2\sin 3t) - C_3e^t(2\cos 3t + 3\sin 3t),$
$z = C_2e^t(2\cos 3t + 3\sin 3t) + C_3e^t(3\cos 3t - 2\sin 3t).$

3. $x = (-C_1 + C_2 + 2C_3t)e^{3t}, y = (C_1 + C_3(t+1))e^{3t}, z = (C_2 + 3C_3t)e^{3t}.$

4. $x = \left(C_1 - 2C_2t - (C_1 + 2C_2 + C_3)t^2\right)e^t, y = \left(C_2 + (C_1 + 2C_2 + C_3)t\right)e^t,$
$z = \left(C_3 - 2(C_1 + C_2 + C_3)t + (C_1 + 2C_2 + C_3)t^2\right)e^t.$

5. $x = C_1e^{-t} + C_2e^t, y = C_2e^t + C_3e^{2t}, z = -C_1e^{-t} + 2C_3e^{2t}.$

6. $x = C_1e^t + C_2e^{2t}, y = C_2e^{2t} + C_3e^{3t}, z = C_1e^t + C_2e^{2t} + C_3e^{3t}.$

7. $x = C_1e^t + (C_2 + C_3)e^{3t}, y = C_1e^t + C_2e^{3t}, z = C_1e^t + C_3e^{3t}.$

8. $x = C_1 + 2C_3t, y = C_2 + C_3t, z = -2C_1 + 2C_2 + C_3(1 - 2t).$

9. $x = C_2(\cos t - \sin t) + C_3(\cos t + \sin t),$
$y = C_1e^t + C_2(\cos t - \sin t) + C_3(\cos t + \sin t),$
$z = C_1e^t + C_2(\cos t - 2\sin t) + C_3(2\cos t + \sin t).$

10. $x = C_1e^{-t} + C_3e^{2t}, y = (C_2 - C_1)e^{-t} + C_3e^{2t}, z = -C_2e^{-t} + C_3e^{2t}.$

11. $x = 2C_1e^t + 7C_2e^{2t} + 3C_3e^{3t}, y = C_1e^t + 3C_2e^{2t} + C_3e^{3t},$
$z = -2C_1e^t - 8C_2e^{2t} - 3C_3e^{3t}.$

12. $x = C_1e^t + 3C_2e^{2t}\cos 3t + 3C_3e^{2t}\sin 3t,$
$y = 2C_1e^t + C_2e^{2t}(4\cos 3t - \sin 3t) + C_3e^{2t}(\cos 3t + 4\sin 3t),$
$z = C_1e^t + 2C_2e^{2t}(\cos 3t - \sin 3t) + 2C_3e^{2t}(\cos 3t + \sin 3t).$

13. $x = C_2e^t + 3C_3e^{2t}, y = C_1e^{-t} + 3C_2e^t + 7C_3e^{2t},$
$z = C_1e^{-t} + 2C_2e^t + 5C_3e^{2t}.$

14. $x=-C_1e^{t}-3C_2e^{2t}+3C_3e^{5t},\ y=C_1e^{t}-2C_2e^{2t}+C_3e^{5t},$

$z=C_1e^{t}+C_2e^{2t}+C_3e^{5t}.$

15. $x=C_2e^{-t}\cos 2t+C_3e^{-t}\sin 2t,\ y=C_1e^{-2t}-C_2e^{-t}\sin 2t+C_3e^{-t}\cos 2t,$

$z=-C_1e^{-2t}+C_2e^{-t}\cos 2t+C_3e^{-t}\sin 2t.$

16. $x=C_1e^{-t}-C_2e^{t}-C_3e^{2t},\ y=2C_2e^{t}+5C_3e^{2t},$

$z=2C_1e^{-t}-C_2e^{t}+C_3e^{2t}.$

17. $x=2C_1e^{-t}+3C_2e^{2t}+C_3e^{3t},\ y=-C_1e^{-t}-C_2e^{2t},$

$z=C_1e^{-t}+2C_2e^{2t}+2C_3e^{3t}.$

18. $x=-2C_1e^{t}+3C_2e^{2t}+C_3e^{3t},\ y=2C_1e^{t}-2C_2e^{2t}-C_3e^{3t},$

$z=-C_1e^{t}+3C_2e^{2t}+C_3e^{3t}.$

19. $x=\left[3C_1+3C_2(1+t)+C_3\left(8+6t+3t^2\right)\right]e^{t},$

$y=\left[C_1+C_2(t-1)+C_3\left(-2-2t+t^2\right)\right]e^{t},\ z=\left[C_1+C_2t+C_3t^2\right]e^{t}.$

20. $x=-C_1e^{-t}+2C_2e^{t}+2C_3e^{2t},\ y=-C_1e^{-t}-C_2e^{t}-2C_3e^{2t},$

$z=C_1e^{-t}+2C_2e^{t}+3C_3e^{2t}.$

21. $x=2C_1e^{t}+C_2e^{2t}+2C_3e^{3t},\ y=C_1e^{t}+C_3e^{3t},$

$z=-2C_1e^{t}-C_2e^{2t}-3C_3e^{3t}.$

22. $x=\left(-2C_2\sin 2t+2C_3\cos 2t\right)e^{t},\ y=\left(-C_1+C_2\cos 2t+C_3\sin 2t\right)e^{t},$

$z=\left(C_1+3C_2\cos 2t+3C_3\sin 2t\right)e^{t}.$

23. $x=-3C_2e^{3t}+2C_3e^{6t},\ y=C_1e^{2t}-C_2e^{3t}+2C_3e^{6t},\ z=C_1e^{2t}+C_3e^{6t}.$

24. $x=C_1e^{-3t}+7C_2e^{t}-4C_3e^{5t},\ y=-C_2e^{t}+C_3e^{5t},\ z=C_1e^{-3t}+2C_2e^{t}.$

25. $x=C_1+C_2e^{t},\ y=3C_1+C_3e^{t},\ z=-C_1+\left(C_2-C_3\right)e^{t}.$

26. $x=\left(C_1+C_2\right)e^{t}+C_3e^{4t},\ y=-C_1e^{t}+C_3e^{4t},\ z=-C_2e^{t}+C_3e^{4t}.$

27. $x=4C_1e^{-3t}+2C_2e^{-2t}-C_3e^{-t},\ y=C_1e^{-3t}+3C_2e^{-2t}-C_3e^{-t},$

$z=-4C_1e^{-3t}-5C_2e^{-2t}+2C_3e^{-t}.$

28. $x=C_1-C_2e^{-6t}+2C_3e^{3t},\ y=4C_2e^{-6t}+C_3e^{3t},\ z=-C_1-C_2e^{-6t}+2C_3e^{3t}.$

29. $x=C_1e^{-t}+C_2e^{t}+C_3e^{2t},\ y=-3C_1e^{-t}+C_2e^{t},\ z=-5C_1e^{-t}+C_2e^{t}+C_3e^{2t}.$

30. $x=-\left(5C_1+3C_2\right)e^{3t}+C_3e^{6t},\ y=C_1e^{3t}+C_3e^{6t},\ z=C_2e^{3t}-3C_3e^{6t}.$

М60. 1. $|\tilde{y}-y|<0{,}034$. 2. $|\tilde{x}-x|+|\tilde{y}-y|<0{,}0012$. 3. $|\tilde{y}-y|<$ $<0{,}002$. 4. $e^{2x}-x-1$. 5. e^{x-2}. 6. $e^{2t}+2e^{-t}-3e^{-2t}$. 7. $\frac{1}{8}+\frac{e^{2t}}{36}-\frac{e^{-2t}}{4}+$ $+\left(\frac{2}{9}-\frac{t}{3}\right)e^{-t}$. 8. $y=1+x+\frac{x^2}{2}+\frac{2x^3}{3}+\frac{7x^4}{12}+\ldots$. 9. $y=\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{6}+\ldots$. 10. $y=1+2x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{3}-\ldots$. 11. $y=\frac{1}{x}+\mu\left(x^2-\frac{1}{x^2}\right)+\mu^2\left(-\frac{x^5}{7}+\right.$ $\left.+\frac{2x}{3}-\frac{32}{21x^2}+\frac{1}{x^3}\right)+O\left(\mu^3\right)$. 12. $y=1+\mu\left(x^2-x\right)+\frac{1}{6}\mu^2x(1-x)^3+$ $+O\left(\mu^3\right)$. 13. $y=x-\mu(x+1)+(\mu^2/2)\left(e^x-x^2-2x-1\right)+O\left(\mu^3\right)$. 14. $y=\sin t+\mu\left(\frac{1}{6}-\frac{1}{2}\cos 2t\right)+\mu^2\left(\frac{1}{2}\sin t-\frac{1}{6}\sin 3t\right)+O\left(\mu^3\right)$.

15. $x=\mu\cos t+\mu^3\left(-\frac{3}{8}\cos t+\frac{1}{24}\cos 3t\right)+O\left(\mu^5\right)$. 16. $x_1=-\frac{\mu}{3}\sin 2t+$ $+\frac{\mu^3}{648}\left(\sin 2t-\frac{1}{35}\sin 6t\right)+O\left(\mu^5\right)$, $x_2=\pi-\frac{\mu}{5}\sin 2t-\frac{\mu^3}{1000}\left(\frac{1}{5}\sin 2t-\right.$ $\left.-\frac{1}{111}\sin 6t\right)+O\left(\mu^5\right)$. 17. $x=C\cos\tau+C^2\left(\frac{1}{2}-\frac{1}{3}\cos\tau-\frac{1}{6}\cos 2\tau\right)+$ $+O\left(C^3\right)$, $\tau=t\left(1-\frac{5}{12}C^2+O\left(C^3\right)\right)+C_2$. 18. $x=2\cos\tau-\frac{\mu}{4}\sin 3\tau+O\left(\mu^2\right)$, $\tau=t\left(1-\frac{\mu^2}{16}+O\left(\mu^4\right)\right)+C$. 19. $x=\frac{2}{\sqrt{3}}\cos\tau+\frac{\mu}{12\sqrt{3}}\sin 3\tau+O\left(\mu^2\right)$, $\tau=\left(1-\frac{\mu^2}{16}+O\left(\mu^3\right)\right)t+C$. 25. $y\approx-1-\frac{1}{2}(x-1)^2+\frac{1}{2}(x-1)^3$. 26. $y\approx 2-4x+4x^2-\frac{8}{3}x^3$. 27. $y\approx 2x+2x^2+\frac{10}{3}x^3$. 28. $y\approx 1+\frac{1}{2}x^2$ $+\frac{1}{2}x^3$. 29. $y\approx\frac{1}{2}x^2+\frac{1}{24}x^4+\frac{1}{240}x^6$. 30. $y\approx 1+\frac{1}{3}x^3+\frac{1}{45}x^6$.

JAVOBLAR II

TAKRORLASHGA DOIR ARALASH MASALALARNING JAVOBLARI

1-*bob*

5. $y = e^{xy'/y}$. **6.** $x^2y' - xy = yy'$. **7.** $x(x-2)y'' - (x^2-2)y' + 2(x-1)y = 0$. **8.** $y'''y' = 3y''^2$. **9.** $(y-2x)^2(y'^2+1) = (2y'^2+1)^2$. **10.** $x^2 + y^2 = z^2 - 2yz + 2xzy'$; $x + yy' = zz' - z'(y - xy')$. **11.** $(x^2+y^2)y' = 2xy$. **12.** $\left(x \mp y\sqrt{3}\right)y' = y \pm x\sqrt{3}$. **13.** $\left(3x \mp y\sqrt{3}\right)y' = y \pm 3x\sqrt{3}$. **14.** $\rho' = \frac{1}{2}\rho\, ctg\varphi$. **15.** $\rho' = \rho\, ctg(\varphi \pm 45^\circ)$. **16.** $(x+2y)y' = -3x - y$; $(3x+2y)y' = y - x$. **17.** $y\left(\ln\left|x^2-1\right| + C\right) = 1$, $y = 0$; $y\left(\ln\left(1-x^2\right)+1\right) = 1$. **18.** $y^2 - 2 = Ce^{1/x}$. **19.** $\left(C\exp(-x^2) - 1\right)y = 2$; $y = 0$. **20.** $x^2 + t^2 - 2t = C$. **21.** $ctg\frac{y-x}{2} = x + C$; $y - x = 2\pi k$, $k = 0, \pm 1, \pm 2, \ldots$ **22.** $x + 2y + 2 = Ce^y$; $x + 2y + 2 = 0$. **24.** $a\ln\left(a \pm \sqrt{a^2-y^2}\right) \mp \sqrt{a^2-y^2} = x + C$. **25.** CO_2 ning hajmi $x(t) = 0,08 + 0,22e^{-t/10}$ kub metr; $t = 10\ln 11 \approx 24$ minutda $x(t) = 0,1$. **26.** Jismning harorati $x(t) = 20 + 80\cdot 2^{-t/10}$; $t = 40$ minutda $x(t) = 25$. **27.** Moddaning qolgan miqdori $x(t) = x(0)\cdot 2^{-t/30}$; $t = 60/\lg 2 \approx 200$ kunda $x(t) = 0,01x(0)$ bo\`ladi. **28.** Tezlik $v(t) = 50\, th\frac{t}{5}$, yo\`l (metrlarda) $s(t) = 250\ln ch\frac{t}{5}$; $ch\frac{t}{5} = e^4$, $t \approx 5(4 + \ln 2) \approx 23$ sekundda $s(t) = 1000$. **29.** $\sqrt{H} - \sqrt{h(t)} = kt$, $k = \frac{\sqrt{H}}{5}\left(1 - \frac{1}{\sqrt{2}}\right)$; $t = 5(2+\sqrt{2}) \approx 17$ minutda $h(t) = 0$. **30.** $x + y = Cx^2$; $x = 0$. **31.** $y = Ce^{y/x}$. **32.** $x\ln Cx = 2\sqrt{xy}$; $y = 0$. **33.** $(y - x + 2)^2 + 2x = C$.

34. $\ln\frac{y+x}{x+3}=1+\frac{C}{x+y}$. **35.** $\left(2\sqrt{y}-x\right)\ln C\left(2\sqrt{y}-x\right)=x;\ 2\sqrt{y}=x.$ **36.** x^2+ $+y^2=Cx.$ **37.** $y=(2x+1)\left(C+\ln\left|2x+1\right|\right)+1$. **38.** $y=x(C+\sin x)$. **39.** $x=$ $=y^2+Cy;\ y=0.$ **40.** $x=Cy^3+y^2;\ y=0.$ **41.** $y^{-3}=C\cos^3 x-3\sin x\cos^2 x;$ $y=0.$ **42.** $y^{-2}=x^4(2e^x+C);\ y=0.$ **43.** $y=\frac{2}{x}+\frac{4}{Cx^5-x};\ y=\frac{2}{x}.$ **44.** $y=\frac{2Cx^3+1}{Cx^4-x};\ y=\frac{2}{x}$. **45.** $y=e^x-\frac{x}{x+C};\quad y=e^x.$ **46.** $xe^{-y}-y^2=C.$ **47.** $4y\ln x+y^4=C.$ **48.** $x^2+\frac{2}{3}(x^2-y)^{3/2}=C.$ **49.** $x-y^2\cos^2 x=C.$ **50.** $2x+\ln(x^2+y^2)=C.$ **51.** $y^2=x^2(C-2y);\ x=0.$ **52.** $\frac{1}{2}x^2+xy+\ln\left|y\right|=C;$ $y=0.$ **53.** $x^2y\ln Cxy=-1;\ x=0;\ y=0.$ **54.** $y=C\ln x^2y.$ **55.** $x\sqrt{1+z^2}+$ $\ln\left(z+\sqrt{1+z^2}\right)=C,$ bu yerda $z=y/x;\quad x=0.$ **56.** a) $y_0=1,\ \ y_1=x^3,$ $y_2=1+x^3-x+(x^7-1)/7;$ b) $y_0=1,\quad y_1=1+2x,\ 2y_2=e^{2x}+1+2x+$ $+2x^2;$ **c**) $y_0=2\pi,,\ y_1=\pi+x,\ \ y_2=2\pi+x+x\cos x-\sin x.$ **57.** a) $0,87\le$ $\le x\le 1,13.$ b) $-0,1\le t\le 0,1.$ **58.** $0<a<1$ bo\`lganda $Ox$ o\`qning nuqta-larida. **60.** a) x_0 va y_0 ixtiyoriy, $y_0=\frac{\pi}{2}+\pi k,\ k=0,\pm1,\pm2,\dots$ b) $x_0\ne-1,$ $y_0>0,\ \ y_0'$ ixtiyoriy. **c**) t_0 va y_0 ixtiyoriy, $x_0\ne 0.$ **61.** $n=1$ holda yechim yo\`q, $n=2$ holda bitta yechim bor, $n=3$ holda cheksiz ko\`p yechimlar bor. **62.** a)**3.** b)**4.** **c**)**3.** **63.** a) $0\le a\le 1.$ b) $1\le a\le\frac{3}{2}.$ **65.** $y=Ce^{\pm x}.$ **66.** $y(x+C)^2=1;\ y=0.$ **67.** $(y-x)^2=2C(x+y)-C^2;\ y=0.$ **68.** $(x-1)^{4/3}+$ $+y^{4/3}=C.$ **69.** $4y=\left(\pm\sqrt{5}-1\right)x^2+C.$ **70.** $x^2+C^2=2Cy;\ \ y=\pm x.$ **71.** $y=$ $=2x^2+C;\ \ y=-x^2+C.$ **72.** $\ln Cy=x\pm 2e^{x/2},\ y=0.$ **73.** $\ln Cy=x\pm\sin x;$ $y=0.$ **74.** $2(x-C)^2+2y^2=C^2;\ y=\pm x.$ **75.** $x=p\sqrt{p^2+1},\quad 3y=C+$ $=(2p^2-1)\sqrt{p^2+1}.$ **76.** $x=\ln p+\frac{1}{p},\ \ y=p-\ln p+C.$ **77.** $x=e^p+C,\ y=$ $=(p-1)e^p;\ y=-1.$ **78.** $\pm x=2\sqrt{p^2-1}+\arcsin\left(1/\left|p\right|\right)+C,\quad y=\pm p\sqrt{p^2-1};$

$y=0.$ **79.** $4y=C^2-2(x-C)^2;\quad 2y=x^2.$ **80.** $5y=-5x^2+10Cx-4C^2;$ $x^2=4y.$ **81.** $pxy=y^2+p^3,\ y^2(2p+C)=p^4;\ y=0.$ **82.** $Cx=\ln Cy;\ y=ex.$ **83.** $xp^2=C\sqrt{|p|}-1,\ y=xp-x^2p^3;\ y=0.$ **84.** $y^2=2C^3x+C^2;\ 27x^2y^2=1.$ **85.** $x\sqrt{p}=\ln p+C,\ y=\sqrt{p}(4-\ln p-C);\ y=0.$ **86.** $x=3p^2+Cp^{-2},\ y=2p^3+$ $+2Cp^{-1};\ y=0.$ **87.** $y=Cx-C-2.$ **88.** $x=C(p-1)^{-2}+2p+1,\quad y=p^2+$ $+Cp^2(p-1)^{-2};\quad y=0;\quad y=x-2.$ **89.** $y=Cx-\ln C;\ y=1+\ln x.$ **90.** $y=\pm2\sqrt{Cx}+C;\ y=-x.$ **91.a)** $4y=x^4;$ **b)** $y=0,\ y=-4x;$ **c)** $y=0,$ $27y=4x^3.$ **92.** $xy=\pm a^2.$ **93.** $x=p(p^2+2)/u,\ y=p^2/u$ va $x=p/u,$ $y=(2p^2+1)/u,$ bu yerda $u=\left(\sqrt{p^2+1}\right)^3.$

2-*bob*

1. $C_1y^2-1=(C_1x+C_2)^2.$ **2.** $y=C_1\,tg(C_1x+C_2);\ \ln\left|\frac{y-C_1}{y+C_1}\right|=2C_1x+C_2;$ $y(C-x)=1;\ y=C.$ **3.** $y=C_1(x-e^{-x})+C_2.$ **4.** $y=C_1[1\pm ch(x+C_2)];$ $y=Ce^{\pm x}.$ **5.** $y=\frac{1}{3}C_1x^2-C_1^2x+C_2;\quad y=(x^3/12)+C.$ **6.** $e^y\sin^2(C_1x+C_2)=2C_1^2;\quad e^y sh^2(C_1x+C_2)=2C_1^2;\quad e^y(x+C)^2=2.$ **7.** $3C_1y=(x-C_1)^3+C_2;\quad y=C;\quad y=C-2x^2.$ **8.** $x=3C_1p^2+\ln C_2p;$ $y=2C_1p^3+p;\ y=C.$ **9.** $\ln y=C_1\,tg(C_1x+C_2);\ \ln\left|(\ln y-C_1)/\ln y+C_1\right|=$ $2C_1x+C_2;\quad (C-x)\ln y=1;\quad y=C.$ **10.** $C_1^2y=(C_1^2x^2+1)arctgC_1x-$ $-C_1x+C_2;\quad 2y=k\pi x^2+C,\ k=0,\pm1,\pm2,...$ **11.** $C_1^2y+1=\pm ch(C_1x+C_2);$ $C_1^2y-1=\sin(C_1x+C_2);\ 2y=(x+C)^2;\quad y=0.$ **12.** $C_2^2(x+C_3)^2=C_2y^2-$ $-C_1;\ \ y=C.$ **13.** $C_1y-1=C_2e^{C_1x};\ y=C-x;\ y=0.$ **14.** $y^2=x^2+C_1x++C_2.$ **15.** $y=C_2\left(x+\sqrt{x^2+1}\right)^{C_1}.$ **16.** $y^2=C_1x^3+C_2.$ **17.** $y=C_2\,xe^{-C_1/x}.$ **18.** $y=C_2\left|\frac{x}{x+C_1}\right|^{1/C_1};\quad y=C;\quad y=Ce^{-1/x}.$ **19.** $\ln|y|=\ln\left|x^2-2x+C_1\right|+$ $+\int\frac{2dx}{(x-1)^2+C_1-1}+C_2;\quad y=C.$ **20.** $y=-x\ln(C_2\ln C_1x);\quad y=Cx.$ **21.**

$(3-x)y^5=8(x+2)$. **22.** $(1-\ln x)^2 y=x^2$. **23.** $ay=ch(ax+C_1)+C_2$; $a=q/T$, q- arqon uzunlik birligining vazni, T- arqon taranglik kuchining gorizontal tashkil etuvchisi. **24.** $y=C_1e^x+C_2e^{-2x}$.
25. $y=C_1\cos 2x+C_2\sin 2x$. **26.** $y=C_1e^x+C_2e^{-x}+C_3\cos x+C_4\sin x$.
27. $y=C_1\cos x+C_2\sin x+e^{x\sqrt{3}}(C_3\cos x+C_4\sin x)+e^{-x\sqrt{3}}(C_5\cos x+C_6\sin x)$.
28. $y=C_1+C_2e^x+C_3e^{-x}+C_4e^{3x}+C_5e^{-3x}$. **29.** $y=e^x(C_1+C_2x+C_3x^2)$.
30. $y=e^x(C_1+C_2x)+C_3e^{-2x}$. **31.** $y=C_1e^{-x}+C_2e^{3x}+0,2e^{4x}$. **32.** $y=$ $=C_1e^x+C_2e^{-4x}-\dfrac{x}{5}e^{-4x}-\left(\dfrac{x}{6}+\dfrac{1}{36}\right)e^{-x}$. **33.** $y=e^{2x}(C_1\cos 2x+C_2\sin 2x)+$ $+0,25e^{2x}+0,1\cos 2x+0,05\sin 2x$. **38.** $y=e^x(x\ln|x|+C_1x+C_2)$. **39.** $y=$ $=\left(C_1+\ln|\sin x|\right)\sin x+(C_2-x)\cos x$. **40.** $y=e^{-x}\left(\dfrac{5}{4}(x+1)^{5/2}+C_1+C_2x\right)$.
41. $y=2\cos x-5\sin x+2e^x$. **42.** $y=e^{-x}(x-\sin x)$. **43.** $y=(x-1)\times$ $\times(e^{2x}-e^{-x})$. **44.** $y=C_1x^2+C_2x^3$. **45.** $y=x\left(C_1+C_2\ln|x|+C_3\ln^2|x|\right)$.
46. $y=x\left(C_1+C_2\ln|x|\right)+2x^3$. **47.** $y=C_1x^2+\dfrac{1}{x}\left(C_2-\dfrac{2}{3}\ln x-\ln^2 x\right)$.
48. $y=C_1x^3+C_2x^{-2}+x^3\ln|x|-2x^2$. **49.** $y=(x-2)^2\left(C_1+C_2\ln|x-2|\right)+$ $+x-1,5$. **50.** $y=\dfrac{1-x}{16}e^{3x}-\dfrac{1+x}{16}e^{-x}+\left(\dfrac{x^3}{12}+C_1x+C_2\right)e^x$. **51.** $y=C_1e^{\left(\sqrt{3}+i\right)x}+$ $+C_2e^{\left(i-\sqrt{3}\right)x}+\left(C_3-\dfrac{x}{24}\right)e^{-2ix}+\dfrac{i}{32}e^{2ix}$. **52.** $y=x[C_1+(C_2+\ln|\ln x|)\ln x]+$ $+\dfrac{1+\ln x}{4x}$. **53.** $y=\left(C_1+C_2x+x\ln|x|\right)e^{-x}+\dfrac{x-1}{4}e^x$. **54.** $y'''-3y''+3y'-$ $-y=0$. **55.** $y'''-y''-y'+y=0$. **56.** $y^{IV}+y''=0.$. **57.** $b<0$ yoki $b\ge 0$, $a>0$. **58.** $x=4-2\cos t$. **59.** $I=\dfrac{V}{R}\left[1-\exp\left(-\dfrac{R}{L}t\right)\right]$. **60.** Yo\`q. **61.** Yo\`q.
62. Ha. **63.** Ha. **64.** Yo\`q. **65.** Yo\`q. **66.** $n\ge 2$. **67.** $(x-1)y''-xy'+y=0$. **68.** $(2x^2+6x-9)y''-(4x+6)y'+4y=0$. **69.** $(x^2-2x+2)y'''-x^2y''+$ $+2xy'-2y=0$. **70.** $6y=x^4+C_1x^{-2}+C_2x$. **71.** $y=e^x+C_1x^3+C_2x+C_3$.

3-*bob*

1. $y = e^x(C_1x^2 + C_2)$. **2.** $y = C_1 tg\, x + C_2(1 + x tg\, x)$. **3.** $y = C_1(3x+1)e^{-x} + +C_2e^{2x}$. **4.** $y = (C_1 + C_2x)\exp(-x^2)$. **5.** $y = C_1[x + (x^2+1)arctg\, x] + +C_2(x^2+1)$. **6.** $y = C_1x + C_2e^x + C_3(x^2-1)$. **7.** $y = \frac{C_1}{x+1} + \frac{C_2}{x-1} + x$. **9.** $z'' = 0$. **10.** $y''_{tt} - y = 0$. **11.** $y''_{tt} + t^2y = 0$. **12.** $y''_{tt} - y = 0$. **13-15** javoblarda y_2 yechim ko`rsatilmagan joylarda bu yechimni $y_1$ yechimdan cos ning o`rniga sin yozib hosil qilish mumkin. **13.** $y_{1,2} = = x^{-1/2}\exp(\pm x^2/2)\left(1 + O(x^{-2})\right)$. **14.** $y_1 = e^{-x/2}\cos e^x + O(e^{-3/2x})$. **15.** $y_1 = \left[(2x)^{-1/4}\cos\frac{(2x)^{3/2}}{3} + O(x^{-7/4})\right]\exp\left((x-1)^2/2\right)$.

4-*bob*

1. $y = C_2e^{C_1x}$, $z = x + \frac{C_2}{C_1}e^{C_1x}$; $y = 0$, $z = x + C$. **2.** $y = \frac{x+C_1}{x+C_2}$, $z = \frac{(C_2-C_1)x}{(x+C_2)^2}$. **3.** $x = \frac{1}{C_1} + +\frac{C_1}{4}(t+C_2)^2$, $y = -\frac{1}{C_1} + \frac{C_1}{2}(t+C_2) - \frac{C_1}{4}(t+C_2)^2$. **4.** $\frac{x}{y} = C_1$, $xy + z^2 = C_2$. **5.** $x^2 - z^2 = C_1$, $y^2 - u^2 = C_2$, $x + z = C_3(u+y)$. **6.** $y^2 + z^2 = = C_1$, $x(y-z) = C_2$. **7.** a) Ha. b) Yo`q. **8.** Bog'liq. **9.** $x = C_1e^{2t} + C_2e^{-2t}$, $y = C_1e^{2t} - C_2e^{-2t}$. **10.** $x = e^{3t}[(2C_1+C_2)\sin 2t + (C_1 - 2C_2)\cos 2t]$, $y = e^{3t}(C_1\cos 2t + C_2\sin 2t)$. **11.** $x = C_1e^{-t} + C_2e^{3t}$, $y = 2C_1e^{-t} - 2C_2e^{3t}$. **12.** $x = e^{2t}(C_1\cos t + +C_2\sin t)$, $y = e^{2t}[(C_1+C_2)\cos t + (C_2 - C_1)\sin t]$. **13.** $x = 2C_1e^{3t} - 4C_2e^{-3t}$, $y = C_1e^{3t} - C_2e^{-3t}$. **14.** $x = e^t(C_1\cos 3t + C_2\sin 3t)$, $y = e^t(C_1\sin 3t - C_2\cos 3t)$. **15.** $x = (C_1 + C_2t)e^{3t}$, $y = (C_1 + C_2 + +C_2t)e^{3t}$. **16.** $x = (C_1 + C_2t)e^t$, $y = (2C_1 - C_2 + 2C_2t)e^t$. **17.** $x = (C_1 + 2C_2t)e^{-t}$, $y = (C_1 + C_2 + +2C_2t)e^{-t}$. **18.** $x = (C_1 + 3C_2t)e^{2t}$, $y = (C_2 - C_1 - 3C_2t)e^{2t}$. **19.** $x = C_1e^t + C_2e^{2t} + C_3e^{5t}$, $y = C_1e^t - 2C_2e^{2t} + C_3e^{5t}$, $z = -C_1e^t - 3C_2e^{2t} + +3C_3e^{5t}$. **20.** $x = C_1e^{2t} + e^{3t}(C_2\cos t + C_3\sin t)$, $y = e^{3t}\left[(C_2+C_3)\cos t + +(C_3 - C_2)\sin t\right]$, $z = C_1e^{2t} + e^{3t}[(2C_2 - C_3)\cos t + (2C_3 + C_2)\sin t]$.

21. $x=C_2\cos t+(C_2+2C_3)\sin t,\ y=2C_1e^t+C_2\cos t+(C_2+2C_3)\sin t,$
$z=C_1e^t+C_3\cos t-(C_2+C_3)\sin t.$**22.** $x=C_1e^{2t}+C_3e^{-5t},\ y=C_2e^{2t}+$
$+3C_3e^{-5t},\quad z=(C_1-2C_2)e^{2t}+2C_3e^{-5t}.$ **23.** $x=C_1+C_2t+4C_3e^{3t},\ y=C_2-$
$-2C_1-2C_2t+4C_3e^{3t},\ z=C_1-C_2+C_2t+C_3e^{3t}.$**24.** $x=(C_1+C_3t)e^t,$
$y=(C_2+2C_3t)e^t,\ z=(C_1-C_2-C_3-C_3t)e^t.$**25.** $x=e^t(C_1\cos t+C_2\sin t)+$
$+e^{-t}(C_3\cos t+C_4\sin t),\ y=e^t(C_1\sin t-C_2\cos t)+e^{-t}(C_4\cos t-C_3\sin t).$
26. $x=3Ce^{-t},\quad y=Ce^{-t}.$**27.** $x=2C_1e^{2t}+2C_2e^{-2t}+2C_3\cos 2t+2C_4\sin 2t,$
$y=3C_1e^{2t}-3C_2e^{-2t}-C_3\sin 2t+C_4\cos 2t.$ **28.** $x=C_1+C_2e^t+C_3\cos t+$
$+C_4\sin t,\quad y=-C_1-C_2e^t+(0,6C_4-0,8C_3)\cos t-(0,6C_3+0,8C_4)\sin t.$
29. $x=C_1e^{3t}\begin{pmatrix}1\\0\end{pmatrix}+C_2e^{3t}\begin{pmatrix}0\\1\end{pmatrix}.$**30.** $x=C_1e^{2t}\begin{pmatrix}1\\1\end{pmatrix}+C_2e^{-t}\begin{pmatrix}1\\-2\end{pmatrix}.$**31.** $x=C_1e^{-t}\begin{pmatrix}1\\1\end{pmatrix}+$
$+C_2e^{-t}\begin{pmatrix}2t\\2t-1\end{pmatrix}.$ **32.** $x=C_1e^t\begin{pmatrix}\cos 2t\\ \cos 2t+\sin 2t\end{pmatrix}+C_2e^t\begin{pmatrix}\sin 2t\\ \sin 2t-\cos 2t\end{pmatrix}.$

33. $x=C_1e^t\begin{pmatrix}1\\0\\1\end{pmatrix}+C_2\begin{pmatrix}1\\1\\1\end{pmatrix}+C_3e^{-t}\begin{pmatrix}1\\1\\2\end{pmatrix}.$ **34.** $x=C_1e^t\begin{pmatrix}\cos t-\sin t\\ \cos t\\ \sin t\end{pmatrix}+$
$+C_2e^t\begin{pmatrix}\cos t+\sin t\\ \sin t\\ -\cos t\end{pmatrix}+C_3e^{-t}\begin{pmatrix}1\\1\\-1\end{pmatrix}.$ **35.** $x=C_1e^{3t}\begin{pmatrix}1\\1\\2\end{pmatrix}+C_2e^{-t}\begin{pmatrix}1\\-1\\0\end{pmatrix}+$
$+C_3e^{-t}\begin{pmatrix}1\\0\\-1\end{pmatrix}.$ **36.** $x=C_1e^{2t}\begin{pmatrix}1\\-1\\0\end{pmatrix}+C_2e^{2t}\begin{pmatrix}1\\0\\1\end{pmatrix}+C_3e^{2t}\begin{pmatrix}2t+1\\t\\3t\end{pmatrix}.$ **37.** $\begin{pmatrix}e^3&0\\0&e^{-2}\end{pmatrix}.$
38. $\begin{pmatrix}-1&-4\\1&3\end{pmatrix}.$**39.** $\begin{pmatrix}e^2&e^2&0,5e^2\\0&e^2&e^2\\0&0&e^2\end{pmatrix}.$**40.** $e^2.$**41.** $\dot x=x\cos^2 t-(1-\cos t\sin t)y,$
$\dot y=(1+\sin t\cos t)x+y\cdot\sin^2 t.$**42.** $x=C_1(\cos 2t-\sin 2t)+C_2(\cos 2t+$
$+\sin 2t),\ y=C_1\cos 2t+C_2\sin 2t+e^{-2t}.$**43.** $x=C_1e^t\cos t+C_2e^t\sin t+$
$+e^t+t-1,\ y=C_1e^t(-\sin t-\cos t)+C_2e^t(\cos t-\sin t)-2e^t-2t-1.$

44. $x = C_1 \cos t + C_2 \sin t + t \sin t - t \cos t,\ y = C_1(\sin t + \cos t) + C_2(\sin t -$
$- \cos t) - 2t\cos t + \sin t + \cos t.$ **45.** $x = C_1 e^t + C_2 e^{3t} + e^t(2\cos t - \sin t),$
$y = C_1 e^t - C_2 e^{3t} + e^t(3\cos t + \sin t).$ **46.** $x = C_1 e^t + 2C_2 e^{2t} - e^t \ln(e^{2t} + 1) +$
$+2e^{2t} arctg\, e^t,\ y = C_1 e^t + 3C_2 e^{2t} - e^t \ln(e^{2t} + 1) + 3e^{2t} arctg\, e^t.$**47.**
$x = C_1 \cos t + C_2 \sin t + t(\cos t + \sin t) + (\cos t - \sin t)\ln|\cos t|,$
$y = (C_1 - C_2)\cos t + (C_1 + C_2)\sin t + 2\cos t \ln|\cos t| + 2t \sin t.$

48. $x = d\cos at,\ ay = v\sin at;\ (x/d)^2 + (ay/v)^2 = 1$ **e**llips.

5-*bob*

1. $z = f(xy + y^2).$ **2.** $z = f\left(ye^x - e^{2x}\right).$ **3.** $u = f\left(\frac{x-y}{z}, \frac{(x+y+2z)^2}{z}\right).$
4. $F\left(x^2 - y^2, x - y + z\right) = 0.$ **5.** $F\left(x^2/y,\ (x^2y - 3z)/x\right) = 0.$ **6.** $F\left(x^2 + y^4,\right.$
$\left.y\left(z + \sqrt{z^2+1}\right)\right) = 0.$ **7.** $F\left(tg\, z + ctg\, x,\ 2y + 2tg\, z \cdot ctg\, x + ctg^2 x\right) = 0.$ **8.**
$F\left((x-y)/z,\ (2u + x + y)z,\ (u - x - y)/z^2\right) = 0.$ **9.** $z = y^2 \exp\left(2\sqrt{x} - 2\right).$
10. $x - 2y = x^2 + y^2 + z.$ **11.** $\sqrt{z/y^3}\sin x = \sin\sqrt{z/y}.$ **12.** $3(x+y+z)^2 =$
$= x^2 + y^2 + z^2.$ **13.** $(1+yz)^3 = 3yz(1+yz-x) + y^3.$ **14.** $2(x^3 - 4z^3 - 3yz)^2 =$
$= 9(y+z^2)^3.$**15.** $2x^2 + z^2 = z(x^2 + y^2 + z^2).$ **16.** Markazi Oz o`qida bo`lib, $z = 0$ tekislikka parallel tekisliklarda joylashgan barcha aylanalar. **17.** Yechimi yo`q. **18.** $z = 0.$. **19.** $x^3y^2z = C.$ **20.** $2xy + y^2 + 6xz^2 = C.$
21. $z = \frac{a}{3}x^3 - \frac{y^3}{3a} + b.$**22.** $xyz = ax + by.$**23.** $z = ax + by + a^3.$**24.** $z = x - y +$
$+a\ln|xy| + b.$ **25.** $4(1+a^2)z - (ax + y + b)^2 = 0.$ **26.** $a\ln|z| = 3a^2x + 3y + b,$
$a \neq 0.$**27.** $\left(2\sqrt{a(a+z)} + a\ln\left|\frac{\sqrt{a(a+z)} - a}{\sqrt{a(a+z)} + a}\right|\right)^2 - (ax + y + b)^2 = 0.$ **28.** $z^2 =$
$= 2ax + a^2y^2 + b.$**29.** $z = xy + 1.$**30.** $z = 3xy.$**31.** $z = x^2 + y^2.$ **32.** $z = xy +$
$+y\sqrt{x^2+1}.$ **33.** $2x = s(e^{-t} + 1),\ y = s^2(2e^{-t} - 1),\ z = s^3e^{-2t}.$ **34.** $x = 2t\cos a +$
$+\cos s,\ y = 2t\sin a + \sin s,\ 2z = 4t + s.$**35.** $x_1 = (e^t - 1)D + 1,\ x_2 = (5e^t - 4)s_1,$

$x_3=(s_1+s_2)e^t,\quad 2z=(5-D)e^t+2s_1^2+D-3,\qquad D=\pm\sqrt{5-20s_1^2}$. **36.** $x_1=\pm 2\sqrt{1-4(s_1-s_2)^2-4(s_2-s_3)^2-4s_3^2}\,t+1,\ x_2=4(s_1-s_2)t+s_1,$ $x_3=4(s_2-s_3)t+s_1+s_2,\quad x_4=4s_3t+s_1+s_2+s_3,\quad z=2t+s_1^2+s_2^2+s_3^2.$

6-*bob*

1. $y=1+x+2\sum_{k=2}^{\infty}\frac{x^k}{k!}$. **2.** $y=\sum_{k=0}^{\infty}\left(\frac{x^2}{2}\right)^k\frac{1}{k!}$. **3.** $y=3+\frac{5}{2}\sum_{k=1}^{\infty}(-1)^k\frac{x^{2k}}{k!}$. **4.** $y=$ $=1-\frac{x^2}{2}-\sum_{k=2}^{\infty}\frac{(2k-3)!!}{(2k)!}x^{2k}$. **5.** $y=\sum_{k=0}^{\infty}a_kx^k,\ a_0=1,\ a_1=a_2=a_4=0,\ a_3=\frac{2}{3!},$ $a_k=\left[\left(2-(k-3)(k-4)\right)a_{k-3}+(k-2)a_{k-2}\right]\left[k(k-1)(k-2)\right]^{-1},\ k=5,6,\ldots.$ **6.** $y=1+x+\frac{x^2}{2}+\frac{2}{3}x^3+\frac{5}{12}x^4+O\left(x^5\right)$. **7.** $y=\frac{x^2}{2!}+\frac{x^3(1+x)}{3!}+O\left(x^5\right)$. **8.** $y=1+2(x-1)+3(x-1)^2+\frac{11}{3}(x-1)^3+\frac{29}{6}(x-1)^4+O\left((x-1)^5\right)$. **9.** $y=2$ $+\frac{1}{6}x^3+O\left(x^5\right)$. **10.** $y=1+\frac{1}{3!}x^3-\frac{1}{4!}x^4+O\left(x^5\right)$. **11.** $y=a_0+a_1(x-1)+$ $+a_2(x-1)^2+O\left((x-1)^3\right),\ a_1=1-a_0^2,\ a_2=1-a_0+a_0^3,\quad a_0^3-a_0^2+a_0+3=0$ $\Rightarrow \tilde a_0=-1,\ \tilde a_1=0,\ \tilde a_2=2.$ **12.** $y=a_0+a_1x+a_2x^2+O\left(x^3\right),\ \tilde a_0=-1,\ \tilde a_1=1,$ $\tilde a_2=-\frac{1}{2}$. **13.** $y=a_0+a_1x+a_2x^2+a_3x^3+O\left(x^4\right),\ \tilde a_0=0,\ \tilde a_1=\frac{4}{3},\ \tilde a_2=0,$ $\tilde a_3=\frac{2}{3}$. **14.** $y=a_0+a_1(x-1)+a_2(x-1)^2+a_3(x-1)^3+O\left((x-1)^4\right),$ $\tilde a_0=2,\quad \tilde a_1=-1,\ \tilde a_2=\frac{3}{2},\ \tilde a_3=-\frac{7}{6}$. **15.** $y=a_0+a_1\mu+O\left(\mu^2\right),\quad a_0=x^{-1/2},$ $a_1=2\left(x^{-1/2}-x^2\right)$. **16.** $y=a_0+a_1\mu+O\left(\mu^2\right),\ a_0=x^{-1},\ a_1=3.$ **17.** $a_0=$ $=(1-x)^{-2},\quad a_1=(1-x)^{-2}\left(\frac{x^6}{6}-\frac{2}{5}x^5+\frac{x^4}{4}-1\right)$. **18.** $a_0=x+\frac{x^2}{2},\ a_1=\frac{x^7}{56}+$ $+\frac{x^6}{8}+\frac{3}{10}x^5+\frac{x^4}{4}+1,\ y=a_0+\mu a_1+O\left(\mu^2\right).$ **19.** $y=a_0+\mu a_1+O\left(\mu^2\right),$ $a_0=\sin x,\ a_1=1+\int_0^x\ln(1+\sin s)ds.$

MASHHUR OLIMLAR

Abel Niels Henric (1802-1829)

Bernoulli Jacob (1654-1705)

Bernoulli Johann (1667-1748) – Jacob Bernoullining ukasi

Bernoulli Nicolas (1687-1759) – Jacob va Johann Bernoullilarning jiyani

Bernoulli Nicolas (1695-1726) – Johann Bernoullining o'g'li

Bernoulli Daniel (1700-1782) – Johann Bernoullining o'g'li

Bernoulli Johann (1744-1807) – Johann Bernoullining nabirasi

Bernoulli Jacob (1759-1789) – Johann Bernoullining nabirasi

Bessel Friedrich Wilhelm (1784-1846)

Cauchy Augustin Louis (1789-1857)

Clairaut Alexis Claude (1713-1765)

Gauss Carl Friedrich (1777-1855)

Green George (1793-1841)

D'Alambert Jean Le Rond (1717-1783)

Descartes René (1596-1650)

Euler Leonhard (1707-1783)

Lagrange Joseph Louis (1736-1813)

Liouville Joseph (1809-1882)

Остроградский Михаил Васильевич (1801-1861)

Pfaff Johann Friedrich (1765-1825)

Riccati Jacopo Francesco (1676-1754)

Foydalanilgan adabiyotlar

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MUNDARIJA

3-BOB. IKKINCHI TARTIBLI DIFFERENSIAL TENGLAMALAR

4-BOB. DIFFERENSIAL TENGLAMALAR SISTEMASI

5-BOB. BIRINCHI TARTIBLI XUSUSIY HOSILALI DIFFERENSIAL TENGLAMALAR

6-BOB. DIFFERENSIAL TENGLAMALARNI TAQRIBIY YECHISH USULLARI

ОГЛАВЛЕНИЕ

ГЛАВА 3. ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ ВТОРОГО ПОРЯДКА

ГЛАВА 4. СИСТЕМЫ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ

ГЛАВА 5. ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ С ЧАСТНЫМИ ПРОИЗВОДНЫМИ ПЕРВОГО ПОРЯДКА

ГЛАВА 6. ПРИБЛИЖЕННЫЕ МЕТОДЫ РЕШЕНИЯ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ

CONTENT

T.G. ERGASHEV

DIFFERENSIAL TENGLAMALAR

/O'quv qo'llanma/

Muharrir: M.Mustafoyeva

Kompyuterda sahifalovchi: A.Abdullayev

Bosishga ruxsat etildi: 03.07.2023. Qog'oz o'lchami: 60x84 – 1/16

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"TIQXMMI" MTU bosmaxonasida chop etildi.

Toshkent – 100000, Qori-Niyoziy ko'chasi, 39 uy.