

МАТЕМАТИКА

1-MA'RUZA

MAVZU:

MATRITSA VA ULAR USTIDA

AMALLAR

REJA

1. MATRITSA TUSHUNCHASI
2. KVADRATIK MATRITSA
3. MATRITSALAR USTIDA AMALLAR

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \text{ yoki}$$

$$\left\| \begin{array}{cccc} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{array} \right\| \quad (1)$$

$$\text{Misol. } A = \begin{pmatrix} 2 & 1 & \pi \\ 1 & \sqrt{2} & -5 \end{pmatrix}$$

2×3 -o'lchamli **matritsa**.

$$B = \begin{pmatrix} e^t & 1 & -1 & \cos(t) \\ 0 & 4t & -7 & 1-t \end{pmatrix}$$

2×4 -o'lchamli **matritsa**.

- Matritsalarini A, B, C, \dots bosh harflar bilan belgilaymiz:

- $A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$ yoki

$$A = \||a_{ij}\|| = (a_{ij})$$

- ($i=1,2,\dots,m, \quad j=1,2,\dots,n$)
- a_{ij} -lar **matritsa elementlari** deyiladi, bu yerda birinchi indeks i – element turgan

“ \dots ” – element turadigan joyni bildiradi.

- Agar ixtiyoriy i va j larda $a_{ij} = b_{ij}$ bo'lsa, $A = (a_{ij})$ va $B = (b_{ij})$ **matritsalar teng** deyiladi va $A=B$ kabi yosiladi.

Resurslar	Iqtisodiyotning tarmoqlari	
	Qishloq xo'jaligi	Suv xo'jaligi
Suv	7,2	8,1
Mehnat	4,1	3,2
Elektroenergiya	5,2	6,3

Matritsalar yordamida ba'zi iqtisodiy bog'liqlarni ifodalash mumkin. Masalan, iqtisodiyotning ba'zi tarmoqlari bo'yicha resurslarning taqsimotini jadvalda ifodalaymiz.

Ushbu jadvalni resurslar taqsimotining ixcham matritsasi ko'rinishida ifodalash mumkin:

$$A = \begin{pmatrix} 7,2 & 8,1 \\ 4,1 & 3,2 \\ 5,2 & 6,3 \end{pmatrix}$$

jadvalga ko'ra, $a_{11}=7,2$ – matritsa elementi – qishloq xo'jaligiga qancha suv resursi, $a_{22}=3,2$ – element esa – suv xo'jaligiga qancha mehnat resursi sarflanishini ko'rsatadi.

Nol matritsa

- Agar (1) matritsaning barcha elementlari nolga teng bo'lsa,

- $0 = \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 \end{pmatrix}$

- u ***nol matritsa*** deyiladi.

Kvadratik matritsa

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \quad (2)$$

$$A_1 = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix} \quad \text{yoki}$$

$$B = \begin{pmatrix} b_{11} & 0 & \cdots & 0 \\ b_{21} & b_{22} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{pmatrix}.$$

A_1 – *yuqori uchburchakli*, B – *quyi uchburchakli* matritsadir.

$$\begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix} \quad (3)$$

diagonal matritsa, $a_{11} = a_{22} = \dots = a_{nn} = 1$

bo'lsa,

$$E = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

ko'rinishga keladi, uni **birlik matritsa** deb ataladi.

Matritsani songa ko'paytirish. $A = (a_{ij})$ ($i=1,2,\dots,m, j = 1,2,\dots,n$) matritsaning har bir elementini λ haqiqiy songa ko'paytirganda hosil bo'lgan matritsaga λ son bilan *A matritsa ko'paytmasi* deyiladi va λA kabi belgilanadi. Demak,

$$\lambda A = \begin{pmatrix} \lambda a_{11} & \lambda a_{12} & \cdots & \lambda a_{1n} \\ \lambda a_{21} & \lambda a_{22} & \cdots & \lambda a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \lambda a_{n1} & \lambda a_{n2} & \cdots & \lambda a_{nn} \end{pmatrix}.$$

Matritsalar ni qo'shish xossalari:

$$1^0. A + \mathbf{0} = \mathbf{0} + A = A$$

$$2^0. A + B = B + A.$$

$$3^0. (A + B) + C = A + (B + C) \text{ (assosiativ).}$$

Misol. 2×3 – o‘lchovli $A = \begin{pmatrix} 1 & 2 & -3 \\ 4 & 0 & 2 \end{pmatrix}$ va

$B = \begin{pmatrix} -1 & 6 & 3 \\ 8 & 12 & 14 \end{pmatrix}$ matritsalar berilgan bo‘lsin.

$A + B$ yig‘indini toping.

$$\begin{pmatrix} 1 & 2 & -3 \\ 4 & 0 & 2 \end{pmatrix} + \begin{pmatrix} -1 & 6 & 3 \\ 8 & 12 & 14 \end{pmatrix} = \begin{pmatrix} 0 & 8 & 0 \\ 12 & 12 & 16 \end{pmatrix}$$

Matritsalar ni songa ko‘paytirish xossalari:

$$4^0. (\lambda\mu)A = \lambda(\mu A), \quad (\lambda, \mu = \text{const})$$

$$5^0. \lambda(A+B) = \lambda A + \lambda B$$

$$6^0. (\lambda+\mu)A = \lambda A + \mu A.$$

$$7^0. (A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$8^0. (A+B) \cdot C = A \cdot C + B \cdot C, \quad A \cdot (B+C) = A \cdot B + A \cdot C$$

Misol. Matritsalar uchun berilgan amallarni bajaring.

$$1. A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & -4 & 6 \\ -1 & 1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} -4 & 0 & 0 \\ -2 & -1 & 6 \\ 8 & 15 & 4 \end{pmatrix}; \quad 2A - 3B$$

$$2. A = \begin{pmatrix} -2 & 2 \\ 0 & 1 \\ 14 & 2 \\ 6 & 8 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 4 \\ 2 & 1 \\ 14 & 16 \\ 1 & 25 \end{pmatrix}, \quad -5A + 3B$$

Matritsani songa ko 'paytirish.

$A = (a_{ij})$ ($i=1,2,\dots,m, j = 1,2,\dots,n$) matritsaning har bir elementini λ haqiqiy songa ko'paytirganda hosil bo'lgan matritsaga λ son bilan A *matritsa ko'paytmasi* deyiladi va λA kabi belgilanadi. Demak,

$$\lambda A = \begin{pmatrix} \lambda a_{11} & \lambda a_{12} & \cdots & \lambda a_{1n} \\ \lambda a_{21} & \lambda a_{22} & \cdots & \lambda a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \lambda a_{n1} & \lambda a_{n2} & \cdots & \lambda a_{nn} \end{pmatrix}.$$

Misol. $\mathbf{A} = \begin{pmatrix} -3 \\ 4 \\ 2t \\ \sin(2t) \end{pmatrix}$ matritsani $\sqrt{2}$ ga ko'paytiring.

$$\sqrt{2} \begin{pmatrix} -3 \\ 4 \\ 2t \\ \sin(2t) \end{pmatrix} = \begin{pmatrix} -3\sqrt{2} \\ 4\sqrt{2} \\ 2t\sqrt{2} \\ \sqrt{2}\sin(2t) \end{pmatrix}.$$

Misol. $\mathbf{A} = \begin{pmatrix} 2 & e^t \\ \sin(t) & 4 \end{pmatrix}$ matritsani $\cos(t)$ ga ko‘paytiring.

$$\cos(t) \begin{pmatrix} 2 & e^t \\ \sin(t) & 4 \end{pmatrix} = \begin{pmatrix} 2 \cos(t) & e^t \cos(t) \\ \cos(t) \sin(t) & 4 \cos(t) \end{pmatrix}$$

4-misol.

$$AB \neq BA$$

$$\begin{pmatrix} 1 & 0 \\ 2 & -4 \end{pmatrix} \begin{pmatrix} -2 & 6 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 8 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 \\ 8 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & -4 \end{pmatrix} = \begin{pmatrix} -14 & 24 \\ -5 & 12 \end{pmatrix}.$$

Demak, $A \cdot B \neq B \cdot A$.

Foydalanilgan adabiyotlar

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**ETIBORLARINGIZ UCHUN
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