Experimental and analytical method for determining specific losses in the presence of a surface effect

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Abstract. In this article, based on the accumulated experience of determining the intensity of the internal loss function in the active nodes of electromechanical devices, an experimental and analytical method for determining this intensity in time by the coordinates of the area under study is proposed. The error in determining the intensity of internal losses is determined based on comparing the results of calculating the integration of two variants of the two-dimensional heat equation.

1 Introduction

The development and implementation in the national economy of power units based on electromechanical converters with distributed secondary parameters, as is known, is associated with the integration of Maxwell's equations for a specific case. The exact (analytical) or numerical integration of the system of Maxwell's equations, taking into account the real conditions associated with the energy conversion process, is complex and always succeeds. Therefore, the task of identifying the intensity of internal losses in electrical and magnetic conducting systems, as well as the distribution of these losses along the coordinates of the space of the studied area, is often solved in practice on physical models, sometimes on originals. The latter is complicated and expensive. If the problem is solved on models, then the transition to the originals is made, taking into account the features of physical modeling of devices in which eddy currents operate (are induced).

The most accessible and fairly accurate method (method) for determining the intensity and distribution over the coordinates of the space of the function of internal losses from the action of induced currents (which is especially important when eddy currents are operating) is the thermometric method [1-4]. This method also makes it possible to determine the magnetic field strength on the surface of the electro- and magnetically conducting mass under study. The latter can serve as structural components of various electromechanical energy converters: electric machines with a massive rotor, turbine generator rotors, jet tires of high-speed transport systems with magnetic suspension (with magnetic levitation), secondary systems of electrodynamic couplings and brakes, etc. The problems with similar

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formulations were considered in [5-20].

2 Materials and Methods

The advantages and disadvantages of this method are evaluated in [1-4]. Here, in [1-4], a brief description of the method is given, the essence of which is laid down in the inhomogeneous equation of thermal conductivity:

$$c\gamma\ \partial\vartheta/\partial\tau = q + div\ \lambda\ grad\ \vartheta;\ \vartheta = \vartheta(x, y, x, \tau); q = q(x, y, x, \tau)$$

or

$$\partial\vartheta/\partial\tau = q' + div\ a\ grad\ \vartheta,$$

where

$$q' = \frac{q}{c\gamma},\ {}^{o}C\ /\ c;\ q - W/m^3$$

The last entry is valid for the case of constant parameters.

The thermometric method assumes that at the first moment of occurrence of internal losses with a specific power q or q', the heating is adiabatic in nature, and the temperature at all points of the studied area is the same, i.e., $grad\ \vartheta = 0$, so the formulas can be written as follows:

$$c\gamma\ \partial\vartheta/\partial\tau = q$$

or

$$\partial\vartheta/\partial\tau = q'.$$

It follows from the relations for q and q' that at the first moment of occurrence of losses, their specific value is determined by the rate of increase in the temperature of the volume under study $\partial\vartheta/\partial\tau$. The value $\partial\vartheta/\partial\tau$ required to calculate q is defined as $\Delta\vartheta/\Delta\tau$ by drawing a tangent to the heating curve at the time of occurrence of loss sources.

The duration of recording the temperature change curve to determine the temperature increment $\Delta\vartheta$ during $\Delta\tau$ should not exceed the duration of the adiabatic process. With a uniform distribution of q in the volume under study, this duration, depending on the total heat capacity, can reach several tens of seconds. During this time, with the help of various recording devices, it is possible to obtain a reliable picture of the temperature increase under the condition $grad\ \vartheta = 0$.

A different picture is obtained with an uneven distribution of q or q', for example, in massive rotors of electromagnetic couplings, synchronous and special asynchronous machines, i.e., in those nodes of electric machine structures where the surface effect is strongly pronounced. While recording the heating curve using thermocouples, the colder areas of the body under study will divert heat from the more heated areas. The experimental heating curve will no longer satisfy the condition $grad\ \vartheta = 0$; respectively, it is illegal to use the relations for q or q'. Therefore, the obtained distribution pattern q will differ from the real one, and the analysis of thermal processes based on the revealed distribution pattern of local losses will give significant errors. The discrepancies between the actual and measured values of specific losses, depending on the depth of penetration of the electromagnetic wave into the mass of the body under study, can reach several hundred percent.

The errors of the thermometric method are greater; the weaker the field under study and its uneven distribution in the volume under study, the greater the heat transfer coefficient from the limiting surfaces, the inertia of the recording sensor (thermocouple) and recording equipment, the worse the contact of the thermocouple with the volume under study, etc. The measurement error, taking into account the necessary corrections, is more than 25%.

In [2], a method for calculating the correction factor ξ was developed to estimate the error of measuring local losses in the presence of a surface effect (with an uneven distribution of losses) in the case of changes in losses and temperature along one coordinate of space.

This article aims to estimate the error ξ in the case of two-dimensional thermal fields when the analytical solution of Maxwell's equations, which more reliably reflect the real picture of the processes occurring, encounters significant mathematical difficulties. In the case of a two-dimensional field, for example, in the rotor of an electromagnetic brake, heat during the recording of the heating curve passes from the more heated parts to the less heated ones both in thickness and in the radial direction; such a transition is explained by the natural process of thermal conductivity. The analysis of the effect of heat transfer in two directions on the initial intensity of losses is carried out based on solving the heat equation:

$$\frac{\partial\vartheta(x,y,\tau)}{\partial\tau} = a\frac{\partial^2\vartheta(x,y,\tau)}{\partial x^2} + a\frac{\partial^2\vartheta(x,y,\tau)}{\partial y^2} + q_0F(x,y,\tau),$$

where q_0 is the initial intensity of losses at $\tau = 0, x = 0, y = 0$; $F(x,y,\tau)$ is a function describing the distribution of sources along the coordinates of space x, y, and time τ.

The two-dimensional parabolic equation under consideration does not consider changes in thermophysical parameters, and they are not considered in boundary conditions. This is because during the registration of the temperature rise, the parameters change very slightly.

As in [2], the influence of the geometric dimensions of the two-dimensional region, the heat transfer coefficients α_i $(i = 1,2,3,4)$ from the bounding surfaces, the depth of penetration of the electromagnetic wave into the array of the studied region, and the attenuation intensity of the loss sources was investigated.

Thus, the error of the thermometric method is investigated in the space-time domain

$$D\{0 \le x \le \delta_1; 0 \le y \le \delta_2, 0 \le \tau \le T_{\text{конеч.}}\}$$

To determine the error ξ, the thermal conductivity equation was solved twice.

Case 1. It is assumed that the internal heat sources with intensity q_0 are distributed uniformly along the x and y axes and are independent of τ. The boundaries of the region D are thermally insulated – boundary value problem (1).

Case 2. It is assumed that the internal heat sources are distributed according to the law $q = q_0F(x,y,\tau)$, and the real conditions of heat exchange (Newton's law) are set at the boundaries of the region D – boundary value problem (2).

Boundary value problems (1) and (2) have the form:

$$\left.\begin{aligned}
\frac{\partial\vartheta(x,y,\tau)}{\partial\tau} = a\frac{\partial^2\vartheta(x,y,\tau)}{\partial x^2} + a\frac{\partial^2\vartheta(x,y,\tau)}{\partial y^2} + q_0;\\
\vartheta(x,y,0) = 0|_{\tau=0};\\
\lambda_1\frac{\partial\vartheta(x,y,\tau)}{\partial x} = 0|_{x=0};\\
\lambda_2\frac{\partial\vartheta(x,y,\tau)}{\partial x} = 0|_{x=\delta_1};\\
\lambda_3\frac{\partial\vartheta(x,y,\tau)}{\partial y} = 0|_{y=0};\\
\lambda_4\frac{\partial\vartheta(x,y,\tau)}{\partial y} = 0|_{y=\delta_2};
\end{aligned}\right\} \quad (1)$$

$$\left.\begin{array}{c}\frac{\partial\vartheta(x,y,\tau)}{\partial\tau}=a\frac{\partial^2\vartheta(x,y,\tau)}{\partial x^2}+a\frac{\partial^2\vartheta(x,y,\tau)}{\partial y^2}+q_0F(x,y,\tau);\\ \vartheta(x,y,\tau)=0|_{\tau=0};\\ -\alpha_1\vartheta(x,y,\tau)+\lambda_1\frac{\partial\vartheta(x,y,\tau)}{\partial x}=0|_{x=0};\\ \alpha_2\vartheta(x,y,\tau)+\lambda_2\frac{\partial\vartheta(x,y,\tau)}{\partial x}=0|_{x=\delta_1};\\ -\alpha_3\vartheta(x,y,\tau)+\lambda_3\frac{\partial\vartheta(x,y,\tau)}{\partial y}=0|_{y=0};\\ \alpha_4\vartheta(x,y,\tau)+\lambda_4\frac{\partial\vartheta(x,y,\tau)}{\partial y}=0|_{y=\delta_2};\end{array}\right\} \quad (2)$$

For two-dimensional thermal fields, in most practical cases, the following laws of change in the intensity of losses in electric machines are acceptable:

I.

$$q(x,y,\tau)=q_0F(x,y,\tau)=q_0e^{-kx}\left[R-(R-S)\frac{y}{\delta_2}\right]e^{-m\tau};$$

II.

$$q(x,y,\tau)=q_0F(x,y,\tau)=q_0e^{-kx}\left[R-(R-S)\frac{y}{\delta_2}\right]\left(A-B\frac{\tau}{L}\right);$$

III.

$$q(x,y,\tau)=q_0F(x,y,\tau)=q_0e^{-kx}\left[R-(R-S)\frac{y}{\delta_2}\right]\left(1-e^{-\tau/T}\right);$$

IV.

$$q(x,y,\tau)=q_0F(x,y,\tau)=q_0.$$

In option I, the intensity of losses decreases exponentially along the x coordinate, along the y axis – according to a linear law; in option II, the intensity of losses decreases exponentially along x, and along y and τ – according to a linear law; in option III, the intensity of losses decreases exponentially along x, along y – according to a linear law, according to τ – increases exponentially; in variant IV, the intensity of losses does not depend on the x, y coordinates and the time of τ.

In the given dependencies for $q(x, y, \tau)$, the values of various coefficients and constants are selected based on the known electrodynamic relations and the available information about the intensity of attenuation or increasing losses in space and time coordinates. Solutions to boundary value problems (1) and (2) are obtained using finite integral transformations, the method of application of which in solving boundary value problems is given in [2].

Solution of equation (1) for option IV:

$$\vartheta_1^{\mathrm{IV}}(x,y,\tau)=M\sum_{1n=1}^{\infty}\sum_{2n=1}^{\infty}\frac{sin\varepsilon_{1n}\delta_1 sin\varepsilon_{2n}\delta_2}{\varepsilon_{1n}\varepsilon_{2n}}\frac{1-e^{-a(\varepsilon_{1n}^2\delta_1^2+\varepsilon_{2n}^2\delta_2^2)\tau}}{a(\varepsilon_{1n}^2\delta_1^2+\varepsilon_{2n}^2\delta_2^2)}cos\varepsilon_{1n}x cos\varepsilon_{2n}y,$$

where $\varepsilon_{in}-$ the roots of $sin\varepsilon_i\delta_i=0;\ i=1,2.$

Solution of equation (2) for option I:

$$\vartheta_2^{\mathrm{I}}(x,y,\tau)=M\sum_{1n=1}^{\infty}\sum_{2n=1}^{\infty}N\cdot G\cdot P\frac{1}{a(\nu_{1n}^2\delta_1^2+\nu_{2n}^2\delta_2^2)-m}\left\{e^{\left[a(\nu_{1n}^2\delta_1+\nu_{2n}^2\delta_2)-m\right]\tau}-1\right\}.$$

Solution of equation (2) for option II:

$$\vartheta_2^{\mathrm{II}}(x,y,\tau) = M\sum_{1n=1}^{\infty}\sum_{2n=1}^{\infty} N\cdot G\cdot P\left\{A\frac{e^{a(\nu_{1n}^2\delta_1^2+\nu_{2n}^2\delta_2^2)\tau}-1}{a(\nu_{1n}^2\delta_1^2+\nu_{2n}^2\delta_2^2)}-\right.$$
$$\left.-\frac{B}{L}\frac{e^{a(\nu_{1n}^2\delta_1^2+\nu_{2n}^2\delta_2^2)\tau}-1}{a^2(\nu_{1n}^2\delta_1^2+\nu_{2n}^2\delta_2^2)^2}\left[a(\nu_{1n}^2\delta_1^2+\nu_{2n}^2\delta_2^2)\tau-1\right]\right\}.$$

Solution of equation (2) for option III:

$$\vartheta_2^{\mathrm{III}}(x,y,\tau) = M\sum_{1n=1}^{\infty}\sum_{2n=1}^{\infty} N\cdot G\cdot P\left\{\frac{e^{a(\nu_{1n}^2\delta_1^2+\nu_{2n}^2\delta_2^2)\tau}-1}{a(\nu_{1n}^2\delta_1^2+\nu_{2n}^2\delta_2^2)}-\right.$$
$$\left.-\frac{e^{\left[a(\nu_{1n}^2\delta_1^2+\nu_{2n}^2\delta_2^2)-\frac{1}{T}\right]\tau}-1}{a(\nu_{1n}^2\delta_1^2+\nu_{2n}^2\delta_2^2)-\frac{1}{T}}\right\}.$$

Solution of equation (2) for option IV:

$$\vartheta_2^{\mathrm{IV}}(x,y,\tau) = M\sum_{1n=1}^{\infty}\sum_{2n=1}^{\infty} N\cdot\frac{e^{a(\nu_{1n}^2\delta_1^2+\nu_{2n}^2\delta_2^2)\tau}-1}{a(\nu_{1n}^2\delta_1^2+\nu_{2n}^2\delta_2^2)}\times$$
$$\times\left(sin\nu_{1n}\delta_1-\frac{H}{\nu_{2n}}cos\nu_{1n}\delta_1+\frac{H_1}{\nu_{1n}}\right)\left(sin\nu_{2n}\delta_2-\frac{H_3}{\nu_{2n}}cos\nu_{2n}\delta_2+\frac{H_3}{\nu_{2n}}\right).$$

In solutions $\vartheta_2^{\mathrm{I}}(x,y,\tau)-\vartheta_2^{\mathrm{IV}}(x,y,\tau)$, the values ϑ_{1n} and ϑ_{2n} are respectively the roots of the transcendental equations.

The last equations for real α_i and λ_i are solved by the iterative method.

In the solutions $\vartheta_1^{\mathrm{IV}}(x,y,\tau)$; $\vartheta_2^{\mathrm{I}}(x,y,\tau)-\vartheta_2^{\mathrm{IV}}(x,y,\tau)$, the following short designations are adopted:

$$M=\frac{4q_0}{\delta_1\delta_2 c\gamma};$$
$$N=\frac{e^{-a(\nu_{1n}^2\delta_1^2+\nu_{2n}^2)\tau}(\nu_{1n}cos\nu_{1n}x+H_1sin\nu_{1n}x)(\nu_{2n}cos\nu_{2n}y+H_3sin\nu_{2n}y)}{(\nu_{1n}^2+H_1^2)(\nu_{2n}^2+H_3^2)\left(1+\frac{H}{\nu_{1n}^2\delta_1+H_2^2\delta_1}+H_1\delta_1\right)\left(1+\frac{H_4}{\nu_{2n}^2\delta_2+H_4^2\delta_2}+H_3\delta_2\right)};$$
$$G=\nu_{1n}\frac{e^{-k\delta_1}}{k^2+\nu_{1n}^2}(-k\;cos\nu_{1n}\delta_1+\nu_{1n}sin\nu_{1n}\delta_1+k)+$$
$$+H_2\frac{e^{-k\delta_1}}{k^2+\nu_{1n}^2}(-k\;sin\nu_{1n}\delta_1-\nu_{1n}cos\nu_{1n}+\nu_{1n});$$
$$P=Rsin\nu_{2n}\delta_2-\frac{H_3R}{\nu_{2n}}(cos\nu_{2n}\delta_2-1)-$$
$$-\frac{R-S}{\delta_2}\left(\frac{cos\nu_{2n}\delta_2}{\nu_{2n}}+\delta_2sin\nu_{2n}\delta_2-\frac{1}{\nu_{2n}}\right)-$$
$$-\frac{(R-S)H_3}{\delta_2}\left(\frac{sin\nu_{2n}\delta_2}{\nu_{2n}^2}-\frac{\delta_2cos\nu_{2n}\delta_2}{\nu_{2n}}\right).$$

The error ξ due to heat dissipation during the recording or recording of temperature at the point under study is defined as the ratio:

$$\xi=\frac{\vartheta_1^{\mathrm{IV}}(x,y,\tau)}{\vartheta_2^{\mathrm{I}}(x,y,\tau)},\ \text{where } i=\mathrm{I,II,III,IV}.$$

Here we consider only four options for changing q by coordinates x,y,τ. The same calculation formulas can be obtained for other practical (realizable) cases of changing the functions $q(x,y,\tau)$. For example, for the pressure plate of a turbo generator, for which $q = q_0(fy^2 + gy + h)e^{-kx}$, the following solution is obtained:

$$\vartheta_2^{plate}(x,y,\tau) = M \sum_{1n=1}^{\infty} \sum_{2n=1}^{\infty} N \cdot G \cdot Q \frac{e^{a(\nu_{1n}^2\delta_1^2+\nu_{2n}^2\delta_2^2)\tau} - 1}{a(\nu_{1n}^2\delta_1^2 + \nu_{2n}^2\delta_2^2)},$$

Where

$$Q = \frac{f2\delta_2 cos\nu_{2n}\delta_2}{\nu_{2n}} + \frac{f(\nu_{2n}^2\delta_2^2 - 2)sin\nu_{2n}\delta_2}{\nu_{2n}^2} + \frac{gcos\nu_{2n}\delta_2}{\nu_{2n}} + g\delta_2 sin\nu_{2n}\delta_2 + hsin\nu_{2n}\delta_2 +$$
$$+\frac{H_3 f2\delta_2 sin\nu_{2n}\delta_2}{\nu_{2n}^2} - \frac{H_3 f(\nu_{2n}^2\delta_2^2 - 2)cos\nu_{2n}\delta_2}{\nu_{2n}^2} + \frac{H_3 gsin\nu_{2n}\delta_2}{\nu_{2n}^2} - \frac{H_3 g\delta_2 cos\nu_{2n}\delta_2}{\nu_{2n}} -$$
$$-\frac{H_3 hcos\nu_{2n}\delta_2}{\nu_{2n}} - \frac{2H_3 f}{\nu_{2n}^3} - \frac{g}{\nu_{2n}}.$$

3 Results and Discussion

The results of numerical calculations are shown as graphs in Figures 1-5.

Calculations were performed at α_1=100; α_2=120; α_3=80;

α_4=130 $\frac{W}{m^2 \cdot {}^oC}$ - curves 1-12;

α_1=5000; α_2=6000; α_3=4000;

α_4=6500 $\frac{W}{m^2 \cdot {}^oC}$ - curves 12-21.

The errors ξ are calculated for a steel strip with dimensions of 10×30 mm.

Figure 1-4 shows graphs of the change in ξ when

$$q(x,y,\tau) = q_0 e^{-kx}\left[R - (R - S)\frac{y}{\delta_2}\right] e^{-m\tau},$$

for different values of k, R, S.

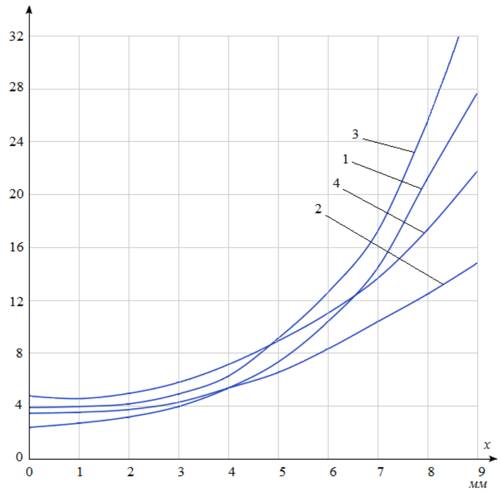

Fig. 1. Error change ξ by x
(y=0;1,3- τ=0.5 s.,2,4- τ=1.0 s.)

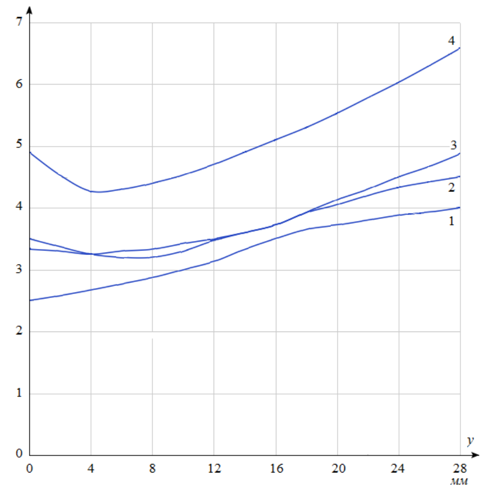

Fig. 2. Error change ξ by y
(x=0;1,3- τ=0.5 s.,2,4- τ=1.0 s.)

The value of ξ, as can be seen from the calculation formulas obtained, does not depend on q_0. Analysis of the obtained results shows (as in the case of one-dimensional temperature changes [2]) that ξ increases with increasing heat transfer coefficients; the initial error does not depend on the geometric dimensions of the area under study if the depth of penetration of the electromagnetic wave into the mass under consideration is two or more times less than the limiting dimensions of this area.

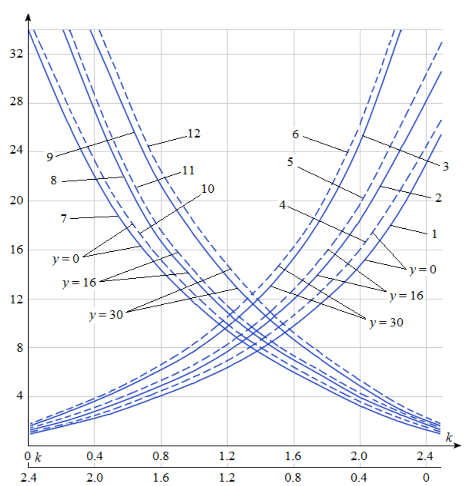

Fig. 3. Change of the error ξ at different values of k, α_i, x, y
1, 2, 3, 4, 5, 6 - $\tau = 0.5$ s. (on the upper axis);
7, 8, 9, 10, 11, 12 - $\tau = 1.0$ s. (on the lower axis)

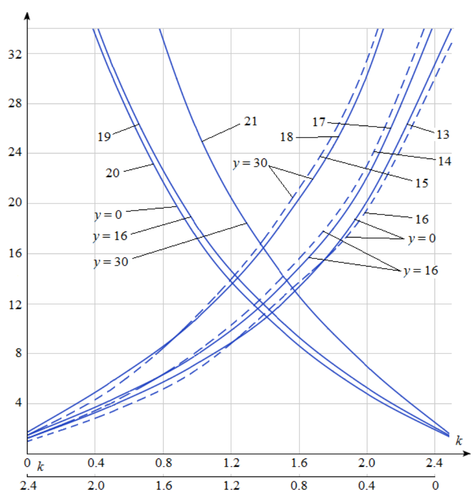

Fig. 4. Change of the error ξ at different values of k, α_i, x, y
13, 14, 15, 16, 17, 18 – $\tau = 0.5$ s. (on the upper axis); 19, 20, 21, 22, 23, 24 - $\tau = 1.0$ s. (on the lower axis)

When the size of the area under study in the direction of the electromagnetic wave hitting the area array differs slightly from the thickness of the area under consideration, the internal loss function q(x,y,τ) takes on a complex character. This option is not considered here.

Figure 5 shows a graph of the error variation ξ in the case of a two-way (symmetrical) wave hitting a ferromagnetic array. The obtained calculation formulas and graphs of changes will allow us to determine the real value of the intensity of losses with the two-dimensional nature of the change in the thermal field in space.

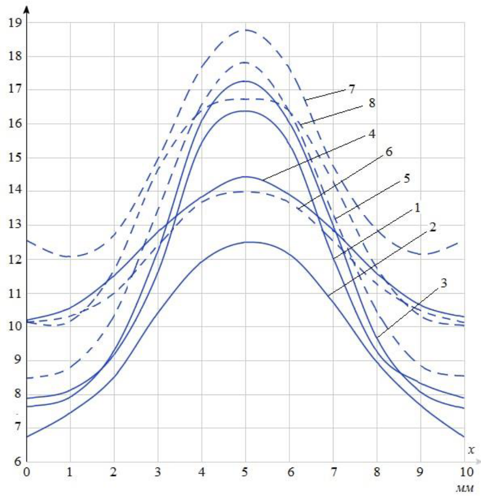

Fig. 5. Error change ξ with a two-way electromagnetic wave hit
1, 3, 5, 7 – $\tau = 0.5$ $s.$, $y = 15$ mm; 2, 4, 6, 7 - $\tau = 1.0$ $s.$, $y = 15$ mm

4 Conclusion

In its original form, the thermometric method assumes a uniform distribution of sources of internal losses and adiabatic heating of the studied area. In reality, these conditions are usually not met. The results obtained in the work allow:

1. When determining the initial rate of temperature rise, take into account the real intensity distribution of internal sources of losses at any point in the study area.
2. Calculating various characteristics of electromechanical energy converters to evaluate and consider the real conditions of coupling limiting the studied area with the environment (boundary conditions).
3. Reliably calculate the effect of temperature (i.e., changes in the material parameters of secondary systems) on the characteristics of an electromechanical converter.

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