Mavzu: Chiziqli algebraik tenglamalar sistemasi. Kramer qoidasi va Gauss usuli.

Reja:

1. Chiziqli tenglamalar sistemasi va uning ishlash usullari
2. Kramer qoidasi
3. Gauss usuli

Ikkita x va y noma'lum chiziqli tenglamalardan iborat ushbu

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \qquad (1)$$

Sistema ikki noma'lumli chiziqli tenglamalar sistemasi deyiladi, bunda a_{11}, a_{12}, a_{21}, a_{22}, – (1) sistema koeffitsientlari, b_1 va b_2 – berilgan sonlardir.

(1) sistemani o’rganishda bu sistemaning koeffisent-laridan tuzilgan.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21} \qquad (2)$$

determinant (uni (1) sistemaning *bosh determinanti* deyiladi) hamda bu determinantning birinchi va ikkinchi ustunlarini mos ravishda ozod hadlar bilan ushbu

$$\Delta_x = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} = b_1 a_{22} - a_{12} b_2, \qquad (3)$$

$$\Delta_y = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix} = a_{11} b_2 - b_1 a_{21} \qquad (4)$$

determinantlar muhim ahamiyatga ega.

(1) tenglamalar sistemasini yechish uchun avvalo bu sistemaning birinchi tenglamasini a_{22} ga, ikkinchi tenglamasini esa -a_{12} ga ko'paytirib, keyin hadlab qo'shib

$$\begin{cases} a_{11}a_{22}x + a_{12}a_{22}y = a_{22}b_1 \\ -a_{21}a_{12}x - a_{22}a_{12}y = -a_{12}b_2 \end{cases} \Rightarrow (a_{11}a_{22} - a_{12}a_{21})x = a_{22}b_1 - a_{12}b_2$$

bo'lishini topamiz. So'ngra (1) sistemaning birinchi tenglamasini $-a_{21}$ ga, ikkinchi tenglamasini esa a_{11} ga ko'paytirib keyin hadlab qo'shib

$$\begin{cases} -a_{11}a_{21}x - a_{12}a_{21}y = -b_1a_{21} \\ a_{11}a_{21}x + a_{11}a_{22}y = b_2a_{11} \end{cases} \Rightarrow (a_{11}a_{22} - a_{12}a_{21})y = a_{11}b_2 - a_{21}b_1$$

bo'lishini topamiz. Natijada (1) sistemaga teng kuchli

bo'lgan ushbu

$$(a_{11}a_{22} - a_{12}a_{21})x = b_1a_{22} - a_{12}b_2$$
$$(a_{11}a_{22} - a_{12}a_{21})y = b_2a_{11} - a_{21}b_1$$

sistemaga kelamiz. Bu sistema yuqoridagi (2), (3) va (4) munosabatlarda hisobga olganda quyidagicha yoziladi:

$$\begin{cases} \Delta \cdot x = \Delta_x \\ \Delta \cdot y = \Delta_y \end{cases} \qquad (1')$$

(1\`) sistemaning yechimi Δ, Δ_x, Δ_y larga bog'liq. $\Delta \neq 0$ bo'lsin. Bu holda (1) sistemadan

$$x = \frac{\Delta_x}{\Delta}, \qquad y = \frac{\Delta_y}{\Delta} \qquad (5)$$

bo'lishini topamiz. Bu topilgan x va y lar (1\`) tenglamaning yechimi bo'ladi. (1) sistemaning topishning bu usuli *Kramer qoidasi* deyiladi. (5) formula esa *Kramer formulasi* deyiladi.

Gauss usuli

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

Tenglamalar sistemasi berilgan bo'lsin. Uni quyidagicha yozib olamiz

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & | & b_1 \\ a_{21} & a_{22} & a_{23} & | & b_2 \\ a_{31} & a_{32} & a_{33} & | & b_3 \end{pmatrix}$$

$$a'_{21} = a_{11}a_{21} - a_{21}a_{11} = 0$$

$$a'_{21} = a_{12}a_{21} - a_{22}a_{11}$$

$$a'_{23} = a_{13}a_{21} - a_{23}a_{11}$$

$$b'_2 = b_1a_{21} - b_2a_{11}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & | & b_1 \\ a_{21} & a_{22} & a_{23} & | & b_2 \\ a_{31} & a_{32} & a_{33} & | & b_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & | & b_1 \\ 0 & a'_{22} & a'_{23} & | & b'_2 \\ 0 & 0 & a''_{33} & | & b''_3 \end{pmatrix} = \begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a'_{22}y + a'_{23}z = b'_2 \\ a''_{33}z = b''_3 \end{cases}$$

$$z = \frac{b''_3}{a''_{33}}$$