ЎзР ФА В.И. Романовский номидаги Математика институти
Математика институти Бухоро бўлинмаси

ДИФФЕРЕНЦИАЛ ТЕНГЛАМАЛАР ВА АНАЛИЗНИНГ ТУРДОШ МАСАЛАЛАРИ
хорижий олимлар иштирокидаги илмий конференцияси

МАТЕРИАЛЛАРИ

Бухоро, Ўзбекистон, 04–05 ноябр, 2021 йил

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Институт Математики имени В.И. Романовского АН РУз
Бухарское отделение института Математики

ТЕЗИСЫ ДОКЛАДОВ

Республиканской научной конференции с участием зарубежных ученых

ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ И РОДСТВЕННЫЕ ПРОБЛЕМЫ АНАЛИЗА
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ABSTRACTS

of the Republican Scientific Conference with the participation of foreign scientists

DIFFERENTIAL EQUATIONS AND RELATED PROBLEMS OF ANALYSIS
Bukhara, Uzbekistan, November 04–05, 2021

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Theorem. *Let* $K \in \mathbf{T}^{\mathrm{d}}$ *be a fixed and* $\min\limits_{k\in\mathbf{T}^{\mathrm{d}}} \Delta_K(k; m_K) \geq 0$. *Then for the essential spectrum of* $\mathcal{A}(K)$ *the following equality*

$$\sigma_{\mathrm{ess}}(\mathcal{A}(K)) = \begin{cases} [m_K; M_K], \ \textit{if} \ \max\limits_{k\in\mathbf{T}^{\mathrm{d}}} \Delta_K(k\,; M_K) \leq 0; \\ \left[m_K; E_{\max}^{(r)}(K)\right], \ \textit{if} \ \min\limits_{k\in\mathbf{T}^{\mathrm{d}}} \Delta_K(k\,; M_K) \leq 0 \ \textit{and} \ \max\limits_{k\in\mathbf{T}^{\mathrm{d}}} \Delta_K(k\,; M_K) > 0; \\ [m_K; M_K] \cup [E_{\min}^{(r)}(K); E_{\max}^{(r)}(K)], \ \textit{if} \ \min\limits_{k\in\mathbf{T}^{\mathrm{d}}} \Delta_K(k\,; M_K) > 0; \end{cases}$$

For the cases $\min\limits_{p,q\in\mathbf{T}^{\mathrm{d}}} \triangle_K(k; m_k) \leq 0$, $\max\limits_{p,q\in\mathbf{T}^{\mathrm{d}}} \triangle_K(k; m_k) > 0$ and $\max\limits_{p,q\in\mathbf{T}^{\mathrm{d}}} \triangle_K(k; m_k) < 0$, one can formulate similar assertions.

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THE EXISTENCE OF EIGENVALUE OF THE TWO PARTICLE SHRÖDINGER OPERATOR

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Let $\mathbb{T}^3 = (-\pi, \pi]^3$ be the three dimensional torus (Brillion zone) and let $L^2(\mathbb{T}^3)$ be the Hilbert space of square-integrable functions defined on the torus $\mathbb{T}^3$. We define the two particle discrete Shrödinger operator as wollows:

$$H_\mu(K) = H^0(K) + \mu V,$$

here $H^0(K)$ is a multiplication operator in $L^2(\mathbb{T}^3)$ by the function $E_K(p)$, i.e.

$$(H^0(K)f)(p) = E_K(p)f(p), \quad f \in L^2(\mathbb{T}^3), \quad \text{where}$$

$$E_K(p) = \varepsilon(p) + \frac{1}{2}\varepsilon(K-p), \quad \varepsilon(p) = \sum_{i=1}^{3}(1 - \cos p^{(i)}), \quad p = (p^{(1)}, p^{(2)}, p^{(3)}) \in \mathbb{T}^3.$$

V is an integral operator:

$$(Vf)(p) = \frac{1}{(2\pi)^3}\int_{\mathbb{T}^3} f(q)dq, \qquad f \in L^2(\mathbb{T}^3).$$

The perturbation V is positive operator of finite rank. Thus, by the well-known Weyl theorem [1] the essential spectrum fills the following segment on the real axis:

$$\sigma_{ess}(H_\mu(K)) = \sigma_{ess}(H^0(K)) = [E_{\min}(K),\, E_{\max}(K)\,],$$

$$E_{\min}(K) = \min_{p\in\mathbb{T}^3} E_K(p), \qquad E_{\max}(K) = \max_{p\in\mathbb{T}^3} E_K(p)$$

We define the following number (using convergent integral)

$$\nu(K) = \frac{1}{(2\pi)^3} \int_{\mathbb{T}^3} \frac{dq}{E_{\max}(K) - E_K(q)}$$

For any $\mu > 0$ we set:

$$M_<(\mu) = \{K \in \mathbb{T}^3 : 1 - \mu\nu(K) < 0\}$$
$$M_=(\mu) = \{K \in \mathbb{T}^3 : 1 - \mu\nu(K) = 0\}$$
$$M_>(\mu) = \{K \in \mathbb{T}^3 : 1 - \mu\nu(K) > 0\}.$$

Let

$$\mu_0 = \frac{1}{\nu(K)}$$

for some $K \in \mathbb{T}^3$.

The main result is the following theorem

Theorem. For any $K \in M_<(\mu_0)$ the operator $H_{\mu_0}(K)$ has a unique eigenvalue $E_{\mu_0}(K)$ lying outside the essential spectrum $\sigma_{ess}(H_{\mu_0}(K))$ and this eigenvalue is even real analytic in $M_<(\mu_0)$, as well as satisfies the following condition $E_{max}(K) < E_{\mu_0}(K) < E_{max}(0), K \neq 0$. Furthermore

$$\psi_{\mu_0,K}(\cdot) = \frac{\mu_0 \cdot c}{E_{\mu_0}(K) - E_K(\cdot)}, \qquad c = constant$$

is real analytic as function $\psi_{\mu_0,K} : M_<(\mu_0) \longrightarrow L^2(\mathbb{T}^3)$.

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MAXIMAL OPERATORS ASSOCIATED WITH SINGULAR SURFACES

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We study maximal operators defined by

$$\mathcal{M}f(y) := \sup_{t>0} |\mathcal{A}_t f(y)|,$$

where

$$\mathcal{A}_t f(y) := \int_S f(y - tx)\psi(x)dS(x)$$

is the averaging operator, S is a hypersurface in $\mathbb{R}^{n+1}$, ψ is a fixed non-negative smooth function with compact support, that is, $\psi \in C_0^\infty(\mathbb{R}^{n+1})$ and $f \in C_0^\infty(\mathbb{R}^{n+1})$.

Let us given singular surface $S_1 \subset \mathbb{R}^3$ defined by the parametric equations

$$x_1(u_1, u_2) = u_1^{a_1} u_2^{a_2} g_1(u_1, u_2),\ x_2(u_1, u_2) = u_1^{b_1} u_2^{b_2} g_2(u_1, u_2),\ x_3(u_1, u_2) = 1 + u_1^{c_1} u_2^{c_2} g_3(u_1, u_2),$$