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THE NUMBER OF EIGENVALUE OF THE TWO PARTICLE SHRODINGER OPERATOR

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Let $\mathbb{T}^3 = (-\pi, \pi]^3$ be the three dimensional torus (Brillion zone) and let $L^2(\mathbb{T}^3)$ be the Hilbert space of square-integrable functions defined on the torus $\mathbb{T}^3$. We define the two particle discrete Shrödinger operator as wollows: \$$H^\gamma_\mu(K) = H^0_\gamma(K) + \mu V, (H^0_\gamma(K)f)(p) = E^\gamma_K(p)f(p), \quad f \in L^2(\mathbb{T}^3), \quad$ where\$

$$E^\gamma_K(p) = \varepsilon(p) + \gamma\varepsilon(K-p), \gamma > 0 \quad \varepsilon(p) = \sum_{i=1}^{3}(1 - \cos p^{(i)}), \quad p = (p^{(1)}, p^{(2)}, p^{(3)}) \in \mathbb{T}^3, \text{ and}$$

$$(Vf)(p) = \frac{1}{(2\pi)^3}\int_{\mathbb{T}^3} f(q)dq, \qquad f \in L^2(\mathbb{T}^3).$$

The perturbation V is positive operator of finite rank. Thus, by the well-known Weyl theorem [1] the essential spectrum fills the following segment on the real axis:

$$\sigma_{ess}(H^\gamma_\mu(K)) = \sigma_{ess}(H^0_\gamma(K)) = [\min_{p\in\mathbb{T}^3} E^\gamma_K(p), \max_{p\in\mathbb{T}^3} E^\gamma_K(p)],$$

We define the following number (using convergent integral)

$$\frac{1}{\mu_0} = \nu(K) = \frac{1}{(2\pi)^3}\int_{\mathbb{T}^3} \frac{dq}{\max_{p\in\mathbb{T}^3} E^\gamma_K(p) - E^\gamma_K(q)}$$

For any $\mu > 0$ we set:

$$M_<(\mu) = \{K \in \mathbb{T}^3 : 1 - \mu\nu(K) < 0\}$$
$$M_=(\mu) = \{K \in \mathbb{T}^3 : 1 - \mu\nu(K) = 0\}$$
$$M_>(\mu) = \{K \in \mathbb{T}^3 : 1 - \mu\nu(K) > 0\}.$$

The main result is the following theorem

Theorem. For any $K \in M_<(\mu_0)$ and $\gamma \neq 1$ the operator $H^\gamma_{\mu_0}(K)$ has a unique eigenvalue $E^\gamma_{\mu_0}(K)$ lying outside the essential spectrum $\sigma_{ess}(H_{\mu_0}(K))$ and this eigenvalue is even real analytic in $M_<(\mu_0)$

REFERENCES

1.M. Reed and B. Simon: Methods of modern mathematical physics. IV: Analysis of operators.// Academic Press, N.Y., 1978.

13. GEOMETRY AND TOPOLOGY

14. COMPUTATIONAL AND DISCRETE MATHEMATICS