Combinatorics

Combinatorics *is a section mathematics, which studies questions choice or location elements sets according to with given rules.*

"Combinatorics" comes from the Latin the words **" combina ",** what is translated into english means **"combine".**

HISTORICAL REFERENCE

G.W. Leibniz

The term "combinatorics" was introduced into mathematical use by the world-famous German scientist G.W. Leibniz , who in 1666 published "Discourses on Combinatorial Art" .

In the 18th century, other outstanding mathematicians turned to solving combinatorial problems. Yes, Leonhard Euler considered problems on partitioning numbers, on matchings, on cyclic arrangements, on the construction of magic and Latin squares.

Combinatorics deals with various kinds of compounds (**permutations, placements, combinations**) that can be formed from elements of some finite set.

Combinatorial compounds

- **Permutations**

1. Permutations without repetition
2. Permutations with repetitions

- **Placements**

1. Placements without repetition
2. Placements with repetitions

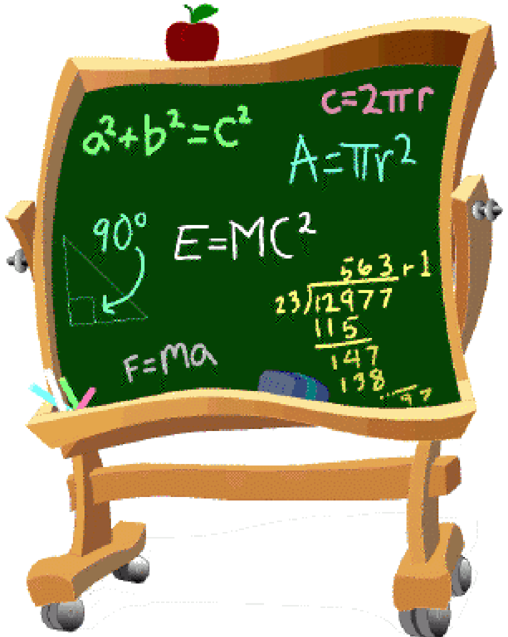
- **Combinations**

1. Combinations without repetition
2. Combinations with repetitions

Permutations are compounds that can be made up of n elements , changing their order in all possible ways.

Formula:

$$P_n = n!$$



Example

In how many ways can 8 people line up at the box office?

The solution of the problem:

There are 8 seats to be filled by 8 people.

Any of 8 people can take the first place, i.e. ways to win first place - 8.

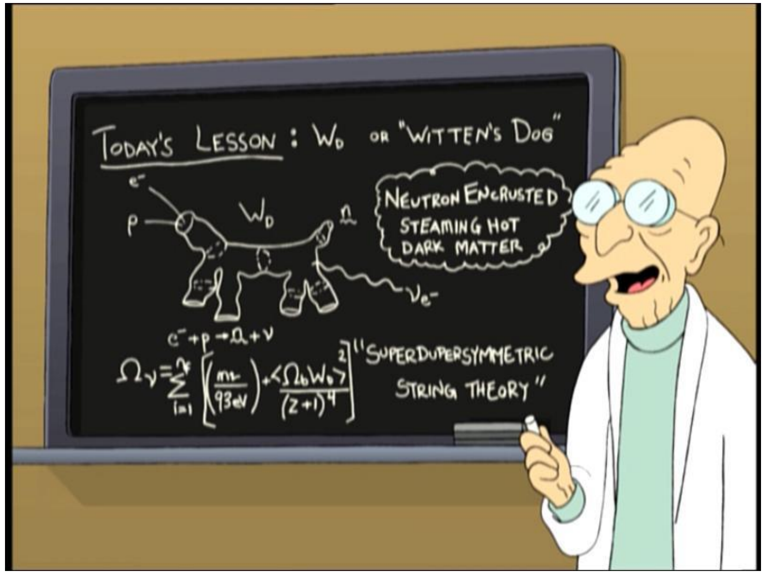
After one person took the first place, there are 7 places left and 7 people who can be placed on them, i.e. ways to take second place - 7. Similarly for the third, fourth, etc. places.

Using the principle of multiplication, we get the product. Such a product is designated as 8! (read 8 factorial) and is called a permutation of P 8.

Answer: P 8 = 8!

check yourself

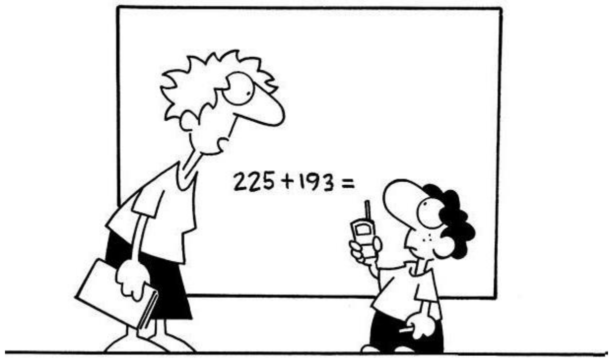
1) In how many ways can **four different** books be placed side by side on a shelf?



SOLUTION

check yourself

2) In how many ways can 10 different postcards be put into 10 existing envelopes (one postcard per envelope)?



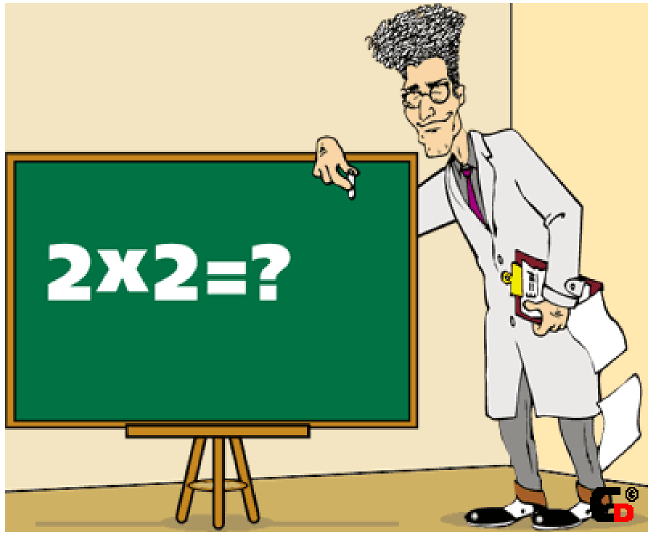
"You have to solve this problem by yourself. You can't call tech support."

SOLUTION

check yourself

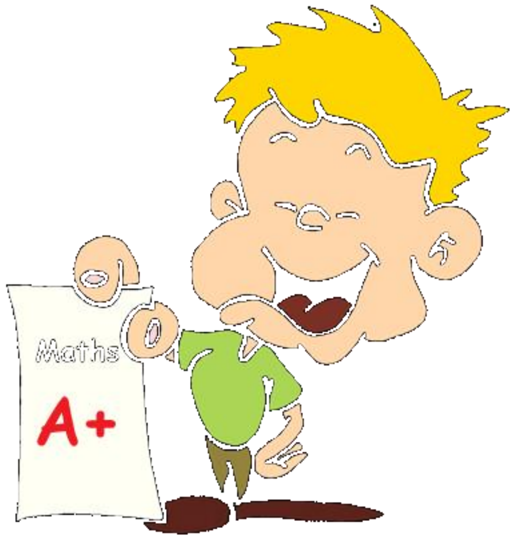
3) In how many ways can **eight** children be seated **on eight** chairs in the kindergarten's dining room?

SOLUTION



check yourself

4) How many different words can be formed by rearranging the letters in the word "**triangle**" (counting this word itself)?



SOLUTION

check yourself

5) In how many ways can one person be on duty per day among **seven** students in a group for 7 days (each must be on duty once)?

SOLUTION

Permutations with repetitions

Any arrangement with repetitions in which element a_1 is repeated k_1 times, element a_2 is repeated k_2 times, and so on. element a_n is repeated k_n times, where k_1, k_2, ..., k_n are given numbers, is called a permutation with repetitions of the order

$k_1 + k_2 + ... + kn=n$, in which the given elements a_1, a_2, ..., a_n are repeated respectively k_1, k_2, .., k_n times.

Permutations with repetitions

Theorem. The number of different permutations with repetitions from the elements {a 1 , ..., an} , in which the elements a 1 , ..., an are repeated k 1 , ..., kn times, respectively, is equal to

$$P_{k_1 k_2 k_3 \ldots k_n} = \frac{m!}{k_1! \cdot k_2! \cdot \ldots \cdot k_n!}$$

Example

Words and phrases with rearranged letters are called anagrams.
How many anagrams can you make from the word macaque?
Solution.

There are 6 letters in the word "MACAKA" ($m=6$) .
Determine how many times each letter is used in a word:
"M" - 1 time (k_1)
"A" - 3 times (k_2)
"K" - 1 times (k_3)
“C”-1 time

check yourself

2) In how many ways can a set of white pieces (a king, a queen, two rooks, two bishops and two knights) be placed on the first horizontal of the chessboard?

SOLUTION

check yourself

3) Mom has 2 apples, 3 pears and 4 oranges. Every day for nine consecutive days, she gives her son one of the remaining fruits. In how many ways can this be done?

SOLUTION

Placements

Accommodation out of n elements by k (k<=n)

is any set consisting of any k elements taken in a certain order from n elements.

Two arrangements of n elements are considered different if they differ in the elements themselves or in the order and x of the arrangement.

$$A_n^k = n(n-1)(n-2)\cdot\ldots\cdot(n-(k-1))$$

Example

In how many ways can an asset be identified out of 40 students in the following composition: the headman, the fizorg and the editor of the wall newspaper?

Solution:

Required select ordered three-element subsets of a set containing 40 elements, i.e. find the number of placements without repetitions from 40 elements of 3.

$$A_{40}^{3} = \frac{40!}{(40-3)!} = \frac{40!}{37!} = 38 \cdot 39 \cdot 40 = 59280$$

check yourself

1. Choose **four out of seven** different books . In how many ways can this be done?

SOLUTION

check yourself

2) Ten teams participate in the football championship . How many different possibilities are there for teams to take **the first three** places?

SOLUTION

check yourself

3. **7 subjects are studied in the class** . On Wednesday **there are 4 lessons** , and all are different. In how many ways can you schedule Wednesday?

SOLUTION

Placements with repetitions

- **Placements with repetitions** - compounds containing **n** elements selected from elements **of m** different types and differing from one another either in the composition or the order of the elements.
- **Their number**, assuming an unlimited number elements of each type is

$$\bar{A}_m^n = m^n$$

Usage example

5 schoolchildren came to the library, which has many identical textbooks **in ten subjects** , each of whom wants to take a textbook. The librarian writes in the journal in order the names (without number) of the borrowed textbooks without the names of the students who took them. How many different lists could appear in the magazine?

The solution of the problem

Since the textbooks for each subject are the same, and the librarian writes down only the name (without a number), the list is a repeating layout . the number of elements of the original set is 10, and the number of positions is 5.

Then the number of different lists is

$$\overline{A}_{10}^{5} = 10^5 = 100\ 000$$

Answer : 100 000

Check yourself!

1. Phone number consists of 7 digits. What is the maximum number of calls the loser Petya can make before he guesses the correct number.

SOLUTION

Check yourself!

2. In how many ways can you write a word made up of <u>four letters</u> of the English alphabet?

SOLUTION

Check yourself!

3. In a store where there are <u>4 types of balls</u>, they decided to put <u>8 balls in a row</u>. In how many ways can this be done if their location matters?

SOLUTION

Check yourself!

4) six buttons of one of the four colors be sewn on a clown costume in a line to get a pattern?

SOLUTION

Combinations

Combinations *- compounds containing m items out of n , differing from each other in at least one item .*

Combinations are finite sets in which the order does not matter.

Combinations

The formula for finding the number of combinations without repetitions:

$$C_n^m = \frac{n!}{m!(n-m)!}$$

Usage example:

In how many ways can two attendants be chosen from a class of 25 students?

Solution:

$m = 2$ (required number of attendants)

$n = 25$ (total students in the class)

$$C_{25}^{2} = \frac{25!}{2!\,(25-2)!} = \frac{24*25}{2} = 300$$

Check yourself!

1) In how many ways can three students be delegated to an interuniversity conference of 9 members of a scientific society?

SOLUTION

Check yourself!

2) Ten participants of the conference shook hands, shaking hands with each. How many handshakes were made in total?

SOLUTION

Check yourself!

3) There are 6 girls and 4 boys in the school choir. In how many ways can 2 girls and 1 boy be selected from the school choir to participate in the performance of the district choir?

SOLUTION

Check yourself!

4) In how many ways can 3 athletes be selected from a group of 20 to compete?

SOLUTION

Check yourself!

5) There are 10 subjects in the class and 5 different lessons per day. In how many ways can lessons be distributed on one day?

SOLUTION

Combinations with repetitions

Definition

- **Combinations with repetitions** from m to n call compounds consisting of n elements selected from elements of m different types, and differing from one another by at least one element.

Number of combinations from m to n

designate

$$\overline{C}_m^n$$

Combinations with repetitions

If from a set containing *n elements, m elements* are selected in turn , and the selected element is returned each time, then the number of ways to make an unordered sample is the number of *combinations with repetitions* - is

$$C_m^n = P_{m-1,\, n} = \frac{(m+n-1)!}{(m-1)!\,n!}$$

Usage example

Task #1

How many sets of 7 cakes can you make if you have 4 types of cakes?

Solution:

$$C_4^7 = \frac{(4+7-1)!}{(4-1)!4!} = 120$$

Usage example

Task #2

How many bones are there in a typical domino game?

Solution **: Domino bones can be considered as combinations with repetitions of two of the seven digits of the set (0,1,2,3,4,5,6). The number of all such combinations is**

$$\boxed{C_7^2 = \frac{(7+2-1)!}{(7-1)!2!} = 28}$$

check yourself

Task 1.

In the cafeteria of the Gymnasium, **5 varieties of pies are sold** : with apples, with cabbage, potatoes, meat and mushrooms. In how many ways can you make a purchase **out of 10 cakes?**

SOLUTION

check yourself

Task 2.

The box contains balls **of three** colors - red, blue and green. In how many ways can you make a set **of two** balls?

SOLUTION

check yourself

Task 3.

In how many ways can 4 coins be chosen from 4 5-kopeck coins and from 4 2-kopeck coins?

SOLUTION

check yourself

Task 4.

How many dominoes will there be if all numbers are used in their formation?

SOLUTION

check yourself

Task 5.

The palette of the young impressionist consists **of 8** different colors. The artist takes any of the paints at random with a brush and puts a colored spot on the paper. Then he takes the next brush, dips it in any of the paints and makes a second spot in the neighborhood. How many different combinations are there **for the six** spots?

SOLUTION