TOSHKENT IRRIGATSIYA VA QISHLOQ XO'JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI

Fan: Oliy matematika

Mavzu:

Irratsional va trigonometrik funksiyalarni itegrallash

Trigonometrik funksiyalarni integrallash

1

Quyidagi ko'rinishdagi integralda :

$$\int R(\sin x, \cos x)\,dx$$

O'zgaruvchini almashtirish :

$$t = tg\frac{x}{2}$$

Universal trigonometric almashtirish

U holda

$$x = 2arctgt$$

$$dx = (2arctgt)'dt = \frac{2}{1+t^2}dt$$

Demak

$$\sin x = \frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}$$

Misol.

Integralni hisoblang:

$$\int \frac{1}{\sin x} dx$$

Yechilishi:

$$\int \frac{1}{\sin x}\,dx = \left| \begin{array}{c} t = tg\,\dfrac{x}{2} \\ dx = \dfrac{2}{1+t^2}\,dt \\ \sin x = \dfrac{2t}{1+t^2} \end{array} \right| =$$

$$= \int \frac{1}{\dfrac{2t}{\cancel{1+t^2}}} \cdot \frac{2}{\cancel{1+t^2}}\,dt = \int \frac{1}{t}\,dt = \ln|t| + C = \ln\left|tg\,\frac{x}{2}\right| + C$$

2

R(sin x,cos x) ifodada sin x ni (-sin x) ga almashtirsak, u holda

$$\int R(\sin x, \cos x)\,dx$$

Quyidagi almashtirish:

$$t = \cos x$$

Misol.

Integralni hisoblang:

$$\int \frac{\sin^3 x}{\cos^4 x} dx$$

Yechilishi:

$$\frac{-(\sin x)^3}{\cos^4 x} = -\frac{\sin^3 x}{\cos^4 x}$$

Yuqorida keltirilgan almashtirishni bajaramiz:

$$\int \frac{\sin^3 x}{\cos^4 x} dx = \left| \begin{array}{c} t = \cos x \\ dt = -\sin x dx \\ \sin^2 x = 1 - t^2 \end{array} \right| =$$

$$= -\int \frac{1-t^2}{t^4}\,dt = -\int \frac{1}{t^4}\,dt - \int \frac{t^2}{t^4}\,dt =$$

$$= -\int \frac{1}{t^4}\,dt - \int \frac{1}{t^2}\,dt = \frac{t^{-3}}{3} - t^{-1} + C =$$

$$= \frac{\cos^{-3} x}{3} - \cos^{-1} x + C$$

3

Quyidagi ko’rinishdagi integralda:

$$\int R(\sin x, \cos x)dx$$

O’zgaruvchini almashtirish

$$t = \sin x$$

Misol.

Integralni hisoblang:

$$\int \sin^2 x \cos^3 x dx$$

Misol:

$$\sin^2 x \cdot (-\cos x)^3 = -\sin^2 x \cos^3 x$$

Yuqorida keltirilgan almashtirishni bajaramiz:

$$\int \sin^2 x \cos^3 x dx = \left| \begin{array}{c} t = \sin x \\ dt = \cos x dx \\ \cos^2 x = 1 - t^2 \end{array} \right| =$$

$$=\int t^2(1-t^2)dt=\int(t^2-t^4)dt=$$

$$=\int t^2dt-\int t^4dt=\frac{t^3}{3}-\frac{t^5}{5}+C=$$

$$=\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}+C$$

4

Quyidagi ko'rinishdagi integralda:

$$\int \sin \alpha x \cos \beta x dx$$

$$\int \sin \alpha x \sin \beta x dx$$

$$\int \cos \alpha x \cos \beta x dx$$

Bu yerda α va β – haqiqiy sonlar, ko'paytmani yig'indiga keltirish formulalaridan foydalaniladi

Almashtirish formulalari:

$$\sin\alpha x\cos\beta x=\frac{1}{2}(\sin(\alpha+\beta)x+\sin(\alpha-\beta)x)$$

$$\cos\alpha x\cos\beta x=\frac{1}{2}(\cos(\alpha+\beta)x+\cos(\alpha-\beta)x)$$

$$\sin\alpha x\sin\beta x=\frac{1}{2}(\cos(\alpha-\beta)x-\cos(\alpha+\beta)x)$$

Misol.

Integralni hisoblang:

$$\int \sin 3x \cos 5x dx$$

Yechilishi:

$$\int \sin 3x \cos 5x dx =$$

$$= \frac{1}{2}\int (\sin 8x - \sin 2x) dx =$$

$$= \frac{1}{2}\left(-\frac{1}{8}\cos 8x + \frac{1}{2}\cos 2x \right) + C$$

Irratsional funksiyalarni integrallash

Irratsional funksiyalarni integrallashda o’zgaruvchini almashtirish yordamida ratsional funksiyalarni integrallashga keltiriladi.

Irratsional funksiyalarning berilishiga qarab, turlicha almashtirishlar bajariladi. Natijada integral ostida integrallash mumkin bo’lgan ratsional funksiya hosil bo’ladi.

1

Quyidagi ko'rinishdagi integralda:

$$\int R\left(x, \sqrt[n]{x}\right)dx$$

O'zgaruvchini almashtirish:

$$t = \sqrt[n]{x}$$

Misol.

Integralni hisoblang:

$$\int \frac{1}{\sqrt{x} + \sqrt[3]{x}} dx$$

Yechilishi:

$$\int \frac{1}{\sqrt{x}+\sqrt[3]{x}}dx = \left| \begin{matrix} t = \sqrt[6]{x} \\ x = t^6 \\ dx = 6t^5dt \end{matrix} \right| = \int \frac{6t^5}{t^3+t^2}dt =$$

$$= 6\int \frac{t^3}{t+1}dt = \left| \begin{matrix} t+1 = u \\ dt = du \end{matrix} \right| = 6\int \frac{(u-1)^3}{u}du =$$

$$= 6\int \frac{u^3-3u^2+3u-1}{u}du =$$

$$= 6\int u^2 du - 18\int u du + 18\int du - 6\int \frac{1}{u} du =$$

$$= 2u^3 - 9u^2 + 18u - 6\ln|u| + C =$$

$$= 2(t+1)^3 - 9(t+1)^2 + 18(t+1) -$$
$$- 6\ln|t+1| + C =$$

$$= 2(\sqrt[6]{x}+1)^3 - 9(\sqrt[6]{x}+1)^2 + 18(\sqrt[6]{x}+1) -$$
$$- 6\ln|\sqrt[6]{x}+1| + C$$

2

Quyidagi ko'rinishdagi integralda :

$$\int R\left(x, \sqrt[n]{\frac{ax+b}{cx+d}}\right)dx$$

O'zgaruvchini almashtirish :

$$t = \sqrt[n]{\frac{ax+b}{cx+d}}$$

Misol.

Integralni hisoblang:

$$\int \sqrt{\frac{1-x}{1+x}} \cdot \frac{1}{1+x} dx$$

Yechilishi:

$$\int\sqrt{\frac{1-x}{1+x}}\cdot\frac{1}{1+x}dx=\left|\begin{array}{ll} t=\sqrt{\frac{1-x}{1+x}} & x=\frac{1-t^2}{1+t^2} \\ dx=-\frac{4t}{(1+t^2)^2} & 1+x=\frac{2}{1+t^2}\end{array}\right|=$$

$$=\int t\cdot\frac{1+t^2}{2}\cdot\left(-\frac{4t}{(1+t^2)^2}\right)dt=$$

$$=-2\int\frac{t^2}{1+t^2}dt=-2\int\frac{(t^2+1)-1}{1+t^2}dt=$$

$$= -2\int \frac{(t^2+1)}{1+t^2}\,dt + 2\int \frac{1}{1+t^2}\,dt =$$

$$= -2t + 2arctgt + C =$$

$$= -2\sqrt{\frac{1-x}{1+x}} + 2arctg\sqrt{\frac{1-x}{1+x}} + C$$

3

Quyidagi ko’rinishdagi integralda :

$$\int \frac{Ax+B}{\sqrt{ax^2+bx+c}}\,dx$$

Ildiz ostida o'zgaruvchining to'la kvadrati ajratiladi va elementar o'zgartirishlar yordamida quyidagi ko'rinishga keltiriladi:

$$\int \frac{Mx+N}{\sqrt{ex^2+f}}dx = M\underbrace{\int \frac{x}{\sqrt{ex^2+f}}dx}_{1} + N\underbrace{\int \frac{1}{\sqrt{ex^2+f}}dx}_{2}$$

Birinchi integralni hisoblash uchun quyidagi almashtirish bajariladi:

$$t = ex^2 + f$$

U holda

$$\int \frac{x}{\sqrt{ex^2+f}}dx = \left| \begin{matrix} ex^2+f=t \\ dt=2exdx \end{matrix} \right| = \frac{1}{2e}\int \frac{1}{\sqrt{t}}dt =$$

$$= \frac{1}{e}\sqrt{t} + C = \frac{1}{e}\sqrt{ex^2+f} + C$$

Ikkinchi integral $e \cdot f > 0$ **bo'ganda:**

$$\int \frac{1}{\sqrt{x^2+a}}dx = \ln\left|x+\sqrt{x^2+a}\right| + C$$

va $e \cdot f < 0$ **bo'lganda:**

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\frac{x}{a} + C$$

Misol.

1

Integralni hisoblang:

$$\int \frac{x}{\sqrt{x^2+4x+5}}dx$$

Yechilishi:

$$\int \frac{x}{\sqrt{x^2+4x+5}}dx = \int \frac{x}{\sqrt{(x+2)^2+1}}dx =$$

$$= \begin{vmatrix} x+2=t \\ dt=dx \end{vmatrix} = \int \frac{t-2}{\sqrt{t^2+1}}dt =$$

$$= \int \frac{t}{\sqrt{t^2+1}}dt - 2\int \frac{1}{\sqrt{t^2+1}}dt =$$

$$= \begin{vmatrix} t^2+1=u \\ du=2tdt \end{vmatrix} =$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du - 2\ln\left|t + \sqrt{t^2+1}\right| =$$

$$= \sqrt{u} - 2\ln\left|t + \sqrt{t^2+1}\right| + C =$$

$$= \sqrt{t+1} - 2\ln\left|t + \sqrt{t^2+1}\right| + C =$$

$$= \sqrt{x+2+1} - 2\ln\left|x+2 + \sqrt{(x+2)^2+1}\right| + C$$

2

Integralni hisoblang:

$$\int \frac{x}{\sqrt{8+4x-4x^2}}\,dx$$

Yechilishi:

$$\int \frac{x}{\sqrt{8+4x-4x^2}}dx = \int \frac{x}{\sqrt{9-(1-2x)^2}}dx =$$

$$= \left| \begin{matrix} 1-2x=t \\ dt=-2dx \end{matrix} \right| = \int \frac{\frac{1-t}{2}}{\sqrt{9-t^2}} \cdot \left(-\frac{1}{2}\right) dt =$$

$$= -\frac{1}{4}\int \frac{1-t}{\sqrt{9-t^2}}dt =$$

$$= -\frac{1}{4}\int \frac{1}{\sqrt{9-t^2}}dt + \frac{1}{4}\int \frac{t}{\sqrt{9-t^2}}dt =$$

$$=\left|\begin{matrix} 9-t^2=u \\ du=-2tdt \end{matrix}\right|= -\frac{1}{4}\arcsin\frac{t}{3}-\frac{1}{8}\int\frac{1}{\sqrt{u}}du=$$

$$=-\frac{1}{4}\arcsin\frac{t}{3}-\frac{1}{4}\sqrt{u}+C=$$

$$=-\frac{1}{4}\arcsin\frac{t}{3}-\frac{1}{4}\sqrt{9-t^2}+C=$$

$$=-\frac{1}{4}\arcsin\frac{1-2x}{3}-\frac{1}{4}\sqrt{9-(1-2x)^2}+C$$

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