



TOSHKENT IRRIGATSIYA VA QISHLOQ
XO'JALIGINI MEXANIZATSIYALASH
MUHANDISLARI INSTITUTI



Fan: | Oliy matematika

Mavzu: | Kvadrat uchhad
qatnashgan
funktsiyalarni itegrallash



I.Ushbu integralni qaraymiz:

$$I_1 = \int \frac{dx}{ax^2 + bx + c}$$

Avval maxrajdagi kvadrat uchhadni yig'indi yoki ayirmaning kvadrati ko'rinishiga keltiramiz:

$$\begin{aligned} ax^2 + bx + c &= a \left[x^2 + \frac{b}{a}x + \frac{c}{a} \right] = a \left[x^2 + 2 \frac{b}{2a}x + \left(\frac{b}{2a} \right)^2 + \frac{c}{a} - \left(\frac{b}{2a} \right)^2 \right] = \\ &= a \left[\left(x + \frac{b}{2a} \right)^2 + \left(\frac{c}{a} - \frac{b^2}{4a^2} \right) \right] = a \left[\left(x + \frac{b}{2a} \right)^2 \pm k^2 \right] \end{aligned}$$

Bu yerda

$$\frac{c}{a} - \frac{b^2}{4a^2} = \pm k^2$$

Shunday qilib, I_1 integral ushbu ko'rinishni oladi.

$$I_1 = \int \frac{dx}{ax^2 + bx + c} = \frac{1}{a} \int \frac{dx}{\left[\left(x + \frac{b}{2a} \right)^2 \pm k^2 \right]}$$

so'nggi integralda o'zgaruvchini quyidagicha almashtiramiz:

u vaqtda

hosil bo'ladi.

$$x + \frac{b}{2a} = t, \quad dx = dt$$

$$I_1 = \frac{1}{a} \int \frac{dx}{t^2 \pm k^2}$$

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$$\int \frac{dx}{x^2 + 4x + 5} = \int \frac{dx}{(x + 2)^2 + 1} = \left| \begin{array}{l} x + 2 = t \\ dx = dt \end{array} \right|$$

$$= \int \frac{dt}{t^2 + 1} = \text{arctgt} + C = \text{arctg}(x + 2)$$

II. Umumiyroq ko'rinishdagi integralni qaraymiz:

$$I_2 = \int \frac{(Ax + B)dx}{ax^2 + bx + c}$$

Integral ostidagi funksiyani bunday almashtiramiz:

$$I_2 = \int \frac{(Ax + B)dx}{ax^2 + bx + c} = \int \frac{\frac{A}{2a}(2ax + b) + (B - \frac{Ab}{2a})}{ax^2 + bx + c} dx$$

So'nggi integralni ikki integralning yig'indisi ko'rinishida tasvirlaymiz:

$$I_2 = \frac{A}{2a} \int \frac{2ax + b}{ax^2 + bx + c} dx + (B - \frac{Ab}{2a}) \int \frac{dx}{ax^2 + bx + c}$$

Birinchi integralni o'zgaruvchini almashtiramiz:

$$ax^2+bx+c=t, \quad (2ax+b)Dx=dt,$$

Demak,

$$\int \frac{(2ax+b)}{ax^2+bx+c} dx = \int \frac{dt}{t} = \ln|t| + C = \ln|ax^2+bx+c| + C$$

Shunday qilib, javobni topamiz:

$$I_2 = \frac{A}{2a} \ln|ax^2+bx+c| + \left(B - \frac{Ab}{2a}\right) I_1$$

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$$\begin{aligned}\int \frac{x dx}{3x^2 + 2x - 4} &= \frac{1}{6} \int \frac{6x + 2 - 2}{3x^2 + 2x - 4} \\ &= \frac{1}{6} \int \frac{(6x + 2) dx}{3x^2 + 2x - 4} - \frac{2}{6} \int \frac{dx}{3 \left(x^2 + \frac{2}{3}x + \frac{1}{9} - \frac{1}{9} - \frac{4}{3} \right)} \\ &= \frac{1}{6} \int \frac{d(3x^2 + 2x - 4)}{3x^2 + 2x - 4} - \frac{1}{9} \int \frac{d \left(x + \frac{1}{3} \right)}{\left(x + \frac{1}{3} \right)^2 - \frac{13}{9}} \\ &= \frac{1}{6} \ln |3x^2 + 2x - 4| - \frac{1}{9} \cdot \frac{1}{2 \cdot \frac{\sqrt{13}}{3}} \ln \left| \frac{x + \frac{1}{3} - \frac{\sqrt{13}}{3}}{x + \frac{1}{3} + \frac{\sqrt{13}}{3}} \right| + C \\ &= \frac{1}{6} \ln |3x^2 + 2x - 4| - \frac{1}{6\sqrt{13}} \ln \left| \frac{3x + 1 - \sqrt{13}}{3x + 1 + \sqrt{13}} \right| + C\end{aligned}$$

III. Ushbu integralni qaraymiz:

$a > 0$ bo'lganda
$$\int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

yoki $a < 0$ bo'lganda
$$\int \frac{dx}{\sqrt{t^2 \pm k^2}}$$

bu integrallar jadvalida mavjud.
$$\int \frac{dx}{\sqrt{k^2 - t^2}}$$

IV. Ushbu integralni qaraymiz:

$$\int \frac{(Ax + B)dx}{\sqrt{ax^2 + bx + c}}$$

$$\int \frac{(Ax + B)dx}{\sqrt{ax^2 + bx + c}} = \int \frac{\frac{A}{2a}(2ax + b) + (B - \frac{Ab}{2a})}{\sqrt{ax^2 + bx + c}} dx =$$

$$\frac{A}{2a} \int \frac{2ax + b}{\sqrt{ax^2 + bx + c}} dx + (B - \frac{Ab}{2a}) \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

hosil bo'lgan integrallarning birinchisida
ushbu almashtirishni qo'llaymiz:

$$ax^2 + bx + c = t, \quad (2ax + b)dx = dt,$$

U holda quyidagini hosil qilamiz:

$$\int \frac{(2ax + b)dx}{\sqrt{ax^2 + bx + c}} = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} + C = 2\sqrt{ax^2 + bx + c} + C$$

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+ 998 71 237 0986