TOSHKENT IRRIGATSIYA VA QISHLOQ XO'JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI

MAVZU

01

Matematika fanini o’qitishdan maqsad va vazifalari.Determinant tushunchasi.Determinantni hisoblash usullari.Determinantlarning asosiy xossalari.

Oliy matematika kafedrasi

Reja:

- 1.Matematika fanini o'qitishdan maqsad va vazifalari
- 2.Ikkinchi tartibli determinant
- 3.Uchinchi tartibli determinant
- 4.Determinantlarning asosiy xossalari
- 5.Minor va algebraik to'ldiruvchi
- 6.Determinantni satr yoki ustun elementlari bo'yicha yoyish

Matematik model tuzishga doir quyidagi masalalarni ko'ramiz:

1-masala:

Fermer xo'jaligi bosh binosidan xo'jalik tomonidan qurilgan chorvachilik kompleksiga suv yoki gaz tarmog'ini o'tkazish masalasini qaraymiz.

Aytaylik xo'jalik omborida 5 va 7 metrlik trubalar bor bo'lsin. Agar fermer xo'jaligini binosidan chorvachilik kompleksigacha masofa 191 metr bo'lsa, eng kam xarajat qilib, suv yoki gazni kompleksga tortib borish uchun ketadigan 5 metr va 7 metrli trubalar soni topilsin.

F 191 K $|FK| = 191$

5m x

7m y

$5x+7y=191 \Rightarrow 5x+7y-191=0$

$Ax+By+C=0$

$$\Rightarrow y = \frac{191 - 5x}{7}$$

$$(x_i\ y_i) \Rightarrow \begin{cases} (20, 13) & 20 + 18 = 33 - 1 = 32 \\ (18, 13) & 18 + 13 = 31 - 1 = 30 \\ (6, 23) & 6 + 23 = 29 - 1 = 28 \\ & \ldots \ldots \ldots \end{cases}$$

2-masala:

Fermer xo'jaligi (A) ga yerga paxta,bug'doy va sholi ekib, mos ravishda undan hosil olishni rejalashtirgan bo'lib, jami (B) million so'm sarf etib (D) million so'm daromad olishni rejalashtirgan bo'lsin.

Agar xar gektar paxta, bug'doy va sholi maydoni uchun mos ravishda (m), (n), (k) so'mdan xarajat qilib, yetishtirgan har bir tonna hosilni davlatga mos ravishda (a), (b) va (c) va so'mdan sotsa, fermerni paxta, bug'doy va sholidan oladigan hosil miqdori qanday bo'ladi.

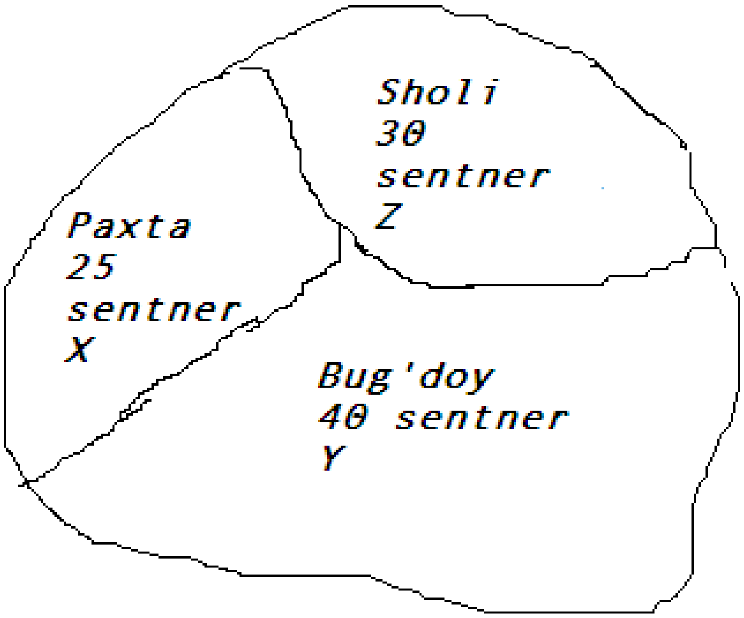
Yechish:

x, y va z lardan mos ravishda 25, 30 va 40sentner hosil olish rejalashtirilgan bo'lsin. U holda mavjud A gektar yer maydoni quyidsgicha taqsimlanadi →

$$Paxta\ maydoni \quad \frac{x}{2,5}$$

$$Bug'doy maydoni \quad \frac{y}{4}$$

$$Sholi\ maydoni \quad \frac{z}{3}$$



P	B	Sh
$\frac{x}{2,5}$	$\frac{y}{4}$	$\frac{z}{3}$
a	b	c

x=? y=? z=? m, n, k

Masala shartiga ko'ra quyidagi tenglamalarni tuzamiz:

1) Yerni taqsimlanish tenglamasi: $\frac{x}{2,5}+\frac{y}{4,0}+\frac{z}{3,0}=A$

2) Xarajatlar tenglamasi: $\frac{x}{2,5}m+\frac{y}{4,0}n+\frac{z}{3,0}k=B$

3) Daromad tenglamasi: $ax+by+cz=D$

$$\begin{cases} \frac{x}{2,5}+\frac{y}{4,0}+\frac{z}{3,0}=A \\ \frac{x}{2,5}m+\frac{y}{4,0}n+\frac{z}{3,0}k=B \\ ax+by+cz=D \end{cases}$$

1.Ikkinchi tartibli determinant

1-Tarif

*a_{11}, a_{12}, a_{21}, a_{22} sonlardan tuzilgan ushbu ko'rinishdagi ifodaga yoki jadvalga **ikkinchi tartibli determinant** deyiladi.*

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

a_{11}, a_{12}, a_{21}, a_{22} sonlarni determinantni elementlari deyiladi; bu sonlar ikkita satr va ikkita ustunga joylashtirilgan bo'ladi.

Ikkinchi tartibli determinant sondan iborat bo'lib, u bosh diagonal elementlari ko'paytmasidan ikkinchi diagonal elementlari ko'paytmasini ayirmasiga tengdir.

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Bu yerda a_{ij} – larni xaqiqiy sonlar deb qaraladi.a_{ij}- ni determinantni i-chi satr j-ustun elementi deyiladi.

Umumiy xolda a_{ij}-lar funksiyalardan iborat bo'lishi mumkin.

Misollar:

1-misol

$$\begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix} = 2x5-3x(-1)=13$$

2-misol

$$\begin{vmatrix} sinx & cosx \\ cosx & sinx \end{vmatrix} = sin^2x-cos^2x=-cos2x$$

2.Uchinchi tartibli determinant

2-tarif

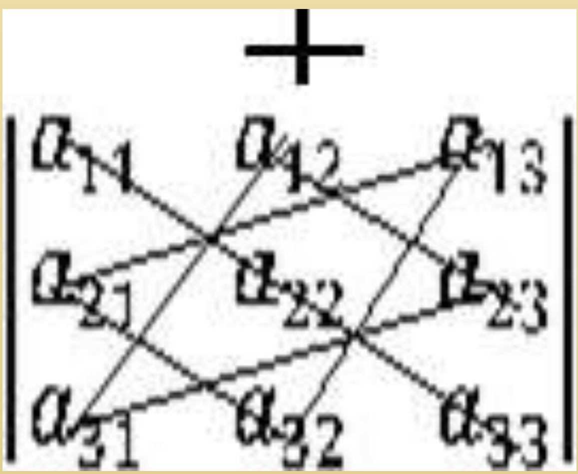
a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33} 9 ta sondan tuzilgan ushbu ko'rinishdagi ifodaga yoki sonlardan tuzilgan jadvalga uchinchi tartibli determinant deyiladi.

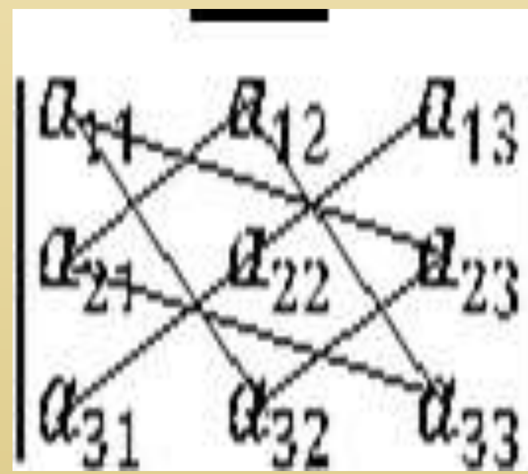
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Uchinchi tartibli determinantda 3 ta satr va 3 ta ustun bo'lib elementlari joylashtirilgan bo'ladi.

Uchinchi tartibli determinant ham sondan iborat bo'lib, uni quyidagi uchburchak yoki diagonal usulda hisoblanadi:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$





Uchburchak qoidasi

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{23}a_{12})$$

$$-(a_{13}a_{22}a_{31} + a_{23}a_{32}a_{11} + a_{33}a_{12}a_{21})$$

Uchburchak usuli orqali masalaning yechimi:

$$\begin{vmatrix} 2 & -2 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & 0 \end{vmatrix} = 2\cdot 2\cdot 0 + 1\cdot 1\cdot 3 + 3\cdot(-2)\cdot 1 - (3\cdot 2\cdot 3 + 2\cdot 1\cdot 1 + 1\cdot(-2)\cdot 0) =$$

$$= 0 + 3 - 6 - 18 - 2 - 0 = -23$$

3.Determinantning asosiy xossalari

1.Determinantda xamma satrlar mos ustunlar qilib yozilsa uning qiymati o'zgarmaydi

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

4-misol $\begin{vmatrix} 2 & 3 \\ 5 & 8 \end{vmatrix} = \begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix} = 16-15=1$

2.Determinantning istalgan ikki ustuni (yoki ikki satri) almashtirilsa uning faqat ishorasi o'zgaradi.

5-misol $\begin{vmatrix} 3 & 5 \\ 4 & 2 \end{vmatrix} = 6-20=-14$ $\begin{vmatrix} 4 & 2 \\ 3 & 5 \end{vmatrix} = 20-6=14$

3. Determinantda biror ustun yoki satrninng xamma elementlari boshqa ustunyoki satrning mos elementlariga teng yoki proporsianal bo'lsa, bunday determent nolga teng bo'ladi.

6-misol $\begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 12 - 12 = 0$

4.Agar determinantda ayrim ustun va satr elementlari umumiy ko'paytuvchilarga ega bo'lsa,ularni determinant belgisi oldiga chiqarish mumkin.

7-misol

$$\begin{vmatrix} 6 & 18 \\ 5 & 15 \end{vmatrix} = \begin{vmatrix} 6x1 & 6x3 \\ 5 & 5x3 \end{vmatrix} = 6x3x5 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 90x0 = 0$$

5.Determinantni m≠0 songa ko'paytirish uchun uning biror satri yoki ustunidagi xamma elementlarini shu (m) soniga ko'paytirish lozim.

8-misol

$2x\begin{vmatrix}3 & 4\\2 & 6\end{vmatrix} = \begin{vmatrix}2x3 & 2x4\\2 & 6\end{vmatrix} = \begin{vmatrix}6 & 8\\2 & 6\end{vmatrix}$ =36-16=20

6.Agar determinantda biror ustun m ta ko'shiluvchilar yig'indisidan iborat bo'lsa,u xolda D determinant m ta $D_1, D_2, \ldots, D_m$ determinantlar yig'indisiga yoyiladi.

$$\begin{vmatrix}2+3 & 4\\3+5 & 6\end{vmatrix}=\begin{vmatrix}2 & 4\\3 & 6\end{vmatrix}+\begin{vmatrix}3 & 4\\5 & 6\end{vmatrix}=-2$$

7.Determinantni biror ustun (satr)ga biror o'zgarmas $m \neq 0$ sonni ko'paytirib biror ustun (satr)ning mos elementriga qo'shilsa determinantni qiymati o'zgarmaydi.

9-misol

$$\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 5+2x2 \\ 3 & 6+3x2 \end{vmatrix} = \begin{vmatrix} 2 & 9 \\ 3 & 12 \end{vmatrix} =$$

=24-27=-3

4.Minor va algebraik to'ldiruvchi

3-tarif

n-tartibli D determinantning istalgan m ta satr va m ta ustunini ajrataylik. Bu satr va ustunlarning kesishgan joylaridagi elementlarini olib, ulardan m-tartibli M determinantni tuzamiz, bu M determinantni D determinantning m-tartibli minori deb ataladi. $(1 \leq m \leq n)$

1-misol

$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \\ 6 & 2 & 1 & 3 & 4 \\ 7 & 8 & 9 & 10 & 12 \\ 2 & 5 & 7 & 8 & 3 \end{vmatrix}$$

5-tartibli determinantda 1 va 5 chi satrlar,3va 4-ustunlarni ajrataylik.U xolda bu determinantni m=2-tartibli minor

$$\begin{vmatrix} 3 & 4 \\ 7 & 8 \end{vmatrix} = 24-28=-4$$

ga teng bo'ladi.Boshqa tartibli minorlar xam shu kabi aniqlanadi.

4-tarif (Qo'shimcha minor)

D determinantda m ta satr va m ta ustunni o'chiramiz. D qolgan elementlarini shu D dagidek olib, ulardan (n-m) tartibli $\overline{M}$ determinantni tuzamiz. $\overline{M}$ ni M ga qo'shimcha minor deyiladi.

Masalan: Yuqoridagi misol uchun (5-2=3-tartibli qo'shimcha minor hisoblanadi)

$$\begin{bmatrix} 5 & 4 & 1 \\ 6 & 2 & 4 \\ 7 & 8 & 12 \end{bmatrix}$$ *ga teng bo'ladi.*

<u>5-tarif</u> (Algebraik to'ldiruvchi)

$$(-1)^{\alpha_1+\alpha_2 \ldots+\alpha_n+\beta_1+\beta_2 \ldots+\beta_n}$$

darajaning $\overline{M}$ qo'shimcha minorga ko'paytmasi m-tartibli M minorning **algebraik to'ldiruvchisi** deyiladi.

$$A=\overline{M} * (-1)^{\alpha_1+\alpha_2 \ldots+\alpha_n+\beta_1+\beta_2 \ldots+\beta_n}$$

Bu yerda α1+ α2+...+ β1 + β2+... lar mos ravishda D ning M ga tegishli satr va ustunlaring nomerini bildiradi.

Yuqorida keltirilgan 1-misoldagi $\overline{M}$ *minorning algebraik to'ldiruvchisi*

$$A=\overline{M} * (-1)^{1+5+3+4} = (-1) * \begin{bmatrix} 5 & 4 & 1 \\ 6 & 2 & 4 \\ 7 & 8 & 12 \end{bmatrix} = 326.$$

5.Determinantni satr yoki ustun bo'yicha yoyish.Laplas teoremasi.

n-tartibli D determinantda istalgan i-satr (yoki j-ustun)ni ajratamiz.

Bu ajratilgan satr yoki ustun elementlaridan tuzilgan xamma birinchi tartibli minorlarni o'z algebraik to'ldiruvchilariga ko'paytirib natijani qo'shsak yig'indi D determinantga teng bo'ladi.

Bu teoremaga ko'ra i-satr ajratilgan bo'lsa, u holda

$$D=a_{ij}A_{j1}+a_{i2}A_{i2}+.........+a_{in}A_{in} \quad (4)$$

formulani D-ni i-satr elementlari bo'yicha yoyish deyiladi.

Agar j-ustun ajratilmagan bo'lsa

$$D=a_{1j}A_{1j}+a_{2j}A_{2j}+..........+a_{nj}A_{nj} \quad (5)$$

(5) formulani D-ni j-ustun elementlari bo'yicha yoyish deyiladi.

- Quyidagi determinant **yoyish** usuli bilan hisoblansin.

$$\boldsymbol{D}=\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{3} & \mathbf{1} & \mathbf{2} \\ \mathbf{2} & \mathbf{3} & \mathbf{1} \end{bmatrix}$$

Yechish:

Birinchi satr va uchinchi ustun elementlari bo'yicha yoyilsin.

$$D=(-1)^{1+1}\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}+(-1)^{1+2}x2\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}+$$

$$+(-1)^{1+3}x3\begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix}=(1-6)-2(3-4)+3(9-2)=$$

$$=-5+2+21=18$$

Endi uchinchi ustun elementlari bo'yicha yoyamiz:

$$D=(-1)^{1+}x3x\begin{vmatrix}3 & 1\\ 2 & 3\end{vmatrix}+(-1)^{3+2}x2x\begin{vmatrix}1 & 2\\ 2 & 3\end{vmatrix}+$$

$$+(-1)^{3+3}x1x\begin{vmatrix}1 & 2\\ 3 & 1\end{vmatrix}=3(9-2)-2(3-4)+$$

$$+(1-6)=21+2-5=23-5=18$$

Mustaqil yechish uchun misollar:

1. Determinantlarni hisoblang.

$$a)\begin{vmatrix} 3 & 1 & 2 & 3 \\ 4 & -1 & 2 & 4 \\ 1 & -1 & 1 & 1 \\ 4 & -1 & 2 & 5 \end{vmatrix}; \qquad b)\begin{vmatrix} 2 & -1 & 2 & 0 \\ 3 & 4 & 1 & 2 \\ 2 & -1 & 0 & 1 \\ 1 & 2 & 3 & -2 \end{vmatrix};$$

$$c)\begin{vmatrix} -1 & -2 & 4 & 1 \\ 2 & 3 & 0 & 6 \\ 2 & -2 & 1 & 4 \\ 3 & 1 & -2 & -1 \end{vmatrix}; \qquad d)\begin{vmatrix} 0 & 4 & 1 & 1 \\ -4 & 2 & 1 & 3 \\ 0 & 1 & 2 & -2 \\ 1 & 3 & 4 & -3 \end{vmatrix}.$$

Mustaqil yechish uchun misollar:

2. Determinant xossalaridan foydalanib hisoblang:

$$1)\ \begin{vmatrix} 8 & 7 & 2 & 10 \\ -8 & 2 & 7 & 10 \\ 4 & 4 & 4 & 5 \\ 0 & 4 & -3 & 2 \end{vmatrix};\qquad 2)\ \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & 2 \\ 1 & 1 & -1 & 3 \\ 1 & 1 & 1 & -1 \end{vmatrix};$$

$$3)\ \begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & 3 & 2 \\ a & b & c & d \\ 3 & -1 & 4 & 3 \end{vmatrix};\qquad 4)\ \begin{vmatrix} 5 & a & 2 & -1 \\ 4 & b & 4 & -3 \\ 2 & c & 3 & -2 \\ 4 & d & 5 & -4 \end{vmatrix}.$$

3. Berilgan tenglamalar dan x ni toping va ildizlarni determinantga qo‘yib tekshiring:

$$1)\ \begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0;\qquad 2)\ \begin{vmatrix} x^2 & 4 & 0 \\ x & 2 & 3 \\ 1 & 1 & 0 \end{vmatrix} = 0.$$

Mustaqil yechish uchun misollar:

1) $\begin{vmatrix} 8 & 7 & 2 & 10 \\ -8 & 2 & 7 & 10 \\ 4 & 4 & 4 & 5 \\ 0 & 4 & -3 & 2 \end{vmatrix}$

2) $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & 2 \\ 1 & 1 & -1 & 3 \\ 1 & 1 & 1 & -1 \end{vmatrix}$

3) $\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & 3 & 2 \\ a & b & c & d \\ 3 & -1 & 4 & 3 \end{vmatrix}$

4) $\begin{vmatrix} 5 & a & 2 & -1 \\ 4 & b & 4 & -3 \\ 2 & c & 3 & -2 \\ 4 & d & 5 & -4 \end{vmatrix}$

Berilgan tenglamalar dan x ni toping va ildizlarni determinantga qo‘yib tekshiring:

1) $$\begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0$$

2) $$\begin{vmatrix} x^2 & 4 & 0 \\ x & 2 & 3 \\ 1 & 1 & 0 \end{vmatrix} = 0$$

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