TOSHKENT IRRIGATSIYA VA QISHLOQ XO'JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI

MAVZU

03

Chiziqli algebraik tenglamalar sistemasini Kramer va Gauss usullari hamda teskari matritsa yordamida yechish. ChTSning amaliy masalalarga tatbiqi

FAN: Iqtisodchilar uchun matematika

Reja:

1) *ChTSni* ***Kramer*** *usuli yordamida yechish;*

2) *ChTSni* ***Gauss*** *usuli yordamida yechish;*

3) *ChTSni* ***teskari matritsa*** *usuli yordamida yechish;*

4) *ChTSning amaliy masalalarga tatbiqi.*

Kramer usuli

Uch noma'lumli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases} \qquad (1)$$

Noma'lumlar oldidagi koeffitsiyentlardan 3-tartibli determinantni tuzamiz va uni determinant xossalariga ko'ra 1-ustun bo'yicha yoyamiz:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot A_{11} + a_{21} \cdot A_{21} + a_{31} \cdot A_{31}. \qquad (2)$$

Kramer usuli

Determinantning birinchi, ikkinchi va uchinchi ustunlarini mos ravishda ozod hadlar almashtirib uchda Δ_1, Δ_2, Δ_3 determinantlarni hosil qilamiz hamda ularni ham ozod hadlar turgan ustun bo'yicha yoyamiz:

$$\Delta_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix} = b_1 \cdot A_{11} + b_2 \cdot A_{21} + b_3 \cdot A_{31}, \qquad (3)$$

$$\Delta_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix} = b_1 \cdot A_{12} + b_2 \cdot A_{22} + b_3 \cdot A_{32}, \qquad (4)$$

$$\Delta_3 = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix} = b_1 \cdot A_{13} + b_2 \cdot A_{23} + b_3 \cdot A_{33}. \qquad (5)$$

Kramer usuli

(1) sistemaning 1-tenglamasini A_{11} algebraik to‘ldiruvchiga, 2-tenglamasini A_{21}ga, 3- tenglamasini A_{31}ga ko‘paytirib hadlab qo‘shamiz:

$$\begin{aligned} &(A_{11}a_{11} + A_{21}a_{21} + A_{31}a_{31})x_1 + \\ &+(A_{11}a_{12} + A_{21}a_{22} + A_{31}a_{32})x_2 + \\ &+(A_{11}a_{13} + A_{21}a_{23} + A_{31}a_{33})x_3 = \\ &= A_{11}b_1 + A_{21}b_2 + A_{31}b_3 \end{aligned} \qquad (6)$$

Yuqoridagi (2) va (3) munosabatlar hamda determinantlarning xossalariga ko‘ra:

$$a_{11} \cdot A_{11} + a_{21} \cdot A_{21} + a_{31} \cdot A_{31} = \Delta,$$

$$A_{11}a_{12} + A_{21}a_{22} + A_{31}a_{32} = 0,$$

$$A_{11}a_{13} + A_{21}a_{23} + A_{31}a_{33} = 0,$$

$$A_{11}b_1 + A_{21}b_2 + A_{31}b_3 = \Delta_1.$$

Kramer usuli

Natijada (6) tenglama quyidagi ko‘rinishga keladi:

$$\Delta \cdot x_1 = \Delta_1.$$

Xuddi yuqoridagidek, (1) sistemaning 1-tenglamasini A_{12} ga, 2-tenglamasini A_{22} ga va 3-tenglamasini A_{32} ga ko‘paytirib hadlab qo‘shamiz:

$$\Delta \cdot x_2 = \Delta_2.$$

(1) sistemaning 1-tenglamasini A_{13} ga, 2-tenglamasini A_{23} ga va 3-tenglamasini A_{33} ga ko‘paytirib hadlab qo‘shamiz:

$$\Delta \cdot x_3 = \Delta_3.$$

Natijada (1) sistemaga teng kuchli bo‘lgan

$$\begin{cases} \Delta \cdot x_1 = \Delta_1 \\ \Delta \cdot x_2 = \Delta_2 \\ \Delta \cdot x_3 = \Delta_3 \end{cases} \qquad (7)$$

sistemani hosil qilamiz.

(7) sistemaning yechimi unda qatnashgan determinantlarga boq‘liqdir.

1^0. $\Delta \neq 0$ bo‘lsin. U holda, (7) sistemadan noma‘lumlarning

$$x_1 = \frac{\Delta_1}{\Delta}, x_2 = \frac{\Delta_2}{\Delta}, x_3 = \frac{\Delta_3}{\Delta} \qquad (8)$$

qiymatini topamiz. (x_1, x_2, x_3) qiymatlar (1) sistemaning yagona yechimi bo’ladi. (8) formulaga ***Kramer formulasi***[1] deyiladi.

2^0. $\Delta = 0$ bo‘lib, Δ_1, Δ_2 va Δ_3 lardan hech bo‘lmaganda bittasi noldan farqli bo‘lsin. Bu holda (2) sistema yechimga ega bo‘lmaydi.

3^0. $\Delta = 0$ bo‘lib, $\Delta_1 = \Delta_2 = \Delta_3 = 0$ bo’lsin. Bu holda (2) sistema yoki cheksiz ko‘p yechimga ega bo‘ladi yoki bitta ham yechimga ega bo‘lmaydi.

1-misol. Tenglamalar sistemasini yeching.

$$\begin{cases} x_1 - x_3 = 2 \\ x_1 - x_2 + 3x_3 = -1 \\ 3x_1 + 2x_2 - 2x_3 = 5 \end{cases}$$

Yechish:

$$\Delta = \begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & 2 & -2 \end{vmatrix} = 2 + 0 - 4 - 3 - 6 - 0 = -11 \neq 0.$$

$\Delta \neq 0$ bo'lgani uchun berilgan tenglamalar sistemasi yagona yechimga ega bo'ladi. Kramer formulasiga ko'ra sistemaning yechimini topamiz.

Δ_1, Δ_2 va Δ_3 larning qiymatini aniqlaymiz:

$$\Delta_1 = \begin{vmatrix} 2 & 0 & -1 \\ -1 & -1 & 3 \\ 5 & 2 & -2 \end{vmatrix} = 4 + 0 + 2 - 5 - 12 - 0 = -11,$$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 5 & -2 \end{vmatrix} = 2 + 18 - 10 - 3 - 15 + 8 = 0,$$

$$\Delta_3 = \begin{vmatrix} 1 & 0 & 2 \\ 2 & -1 & -1 \\ 3 & 2 & 5 \end{vmatrix} = -5 + 0 + 8 + 6 + 2 - 0 = 11.$$

Kramer formulasiga ko'ra,

$$x_1 = \frac{\Delta_1}{\Delta} = \frac{-11}{-11} = 1, \quad x_2 = \frac{\Delta_2}{\Delta} = \frac{0}{-11} = 0,$$

$$x_3 = \frac{\Delta_3}{\Delta} = \frac{11}{-11} = -1.$$

*ChTSni **Gauss** usuli yordamida yechish*

Uch noma'lumli (учта чизикли) tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

Noma'lumlar oldidagi koeffisiyentlardan A matritsani tuzamiz:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

A matritsaning uchinchi ustunidan so‘ng ozod hadlardan iborat to‘rtinchi ustunni vertikal chiziq bilan ajratgan holda yozamiz va hosil bo‘lgan kengaytirilgan matritsani $\overline{A}$ bilan belgilaymiz:

$$\overline{A} = \left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right)$$

Kengaytirilgan $\overline{A}$ matritsada diagonal elementlaridan pastda joylashgan a_{21}, a_{31} va a_{32} elementlar o‘rnida nol hosil qilishimiz kerak. Birinchi yo‘l elementlarini a_{21} ga va ikkinch yo‘l elementlarini a_{11} ga ko‘paytirib, mos ravishda ayiramiz hamda hosil bo‘lgan natijani ikkinchi yo‘lga yozamiz. Xuddi shuningdek, birinchi yo‘l elementlarini a_{31} ga va uchinchi yo‘l elementlarini a_{11} ga ko‘paytirib, mos ravishda ayiramiz hamda hosil bo‘lgan natijani uchinchi yo‘lga yozamiz:

$$\begin{Bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{Bmatrix} \sim \begin{Bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a'_{22} & a'_{23} & b'_2 \\ 0 & a'_{32} & a'_{33} & b'_3 \end{Bmatrix}$$

bu yerda $a'_{22} = a_{12} \cdot a_{21} - a_{22} \cdot a_{11}$ $a'_{32} = a_{12} \cdot a_{31} - a_{32} \cdot$

a_{11} , $b'_2 = b_1 \cdot a_{21} - b_2 \cdot a_{11}$ va h.k.

Hosil bo‘lgan matritsaning ikkinchi yo‘l elementlarini $\acute{a}_{32}$ ga va uchinchi yo‘l elementlarini $\acute{a}_{22}$ ga ko‘paytirib, mos ravishda ayiramiz hamda hosil bo‘lgan natijani uchinchi yo‘lga yozamiz:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & | & b_1 \\ 0 & a'_{22} & a'_{23} & | & b'_2 \\ 0 & a'_{32} & a'_{33} & | & b'_3 \end{pmatrix} \sim \begin{pmatrix} a_{11} & a_{12} & a_{13} & | & b_1 \\ 0 & a'_{22} & a'_{23} & | & b'_2 \\ 0 & 0 & a''_{33} & | & b''_3 \end{pmatrix}$$

Bu yerda, $a''_{33} = a'_{23} \cdot a'_{32} - a'_{33} \cdot a'_{22}$, $b''_3 = b'_2 \cdot a'_{32} - b'_3 \cdot a'_{22}$

Hosil bulgan matritsani noma'lumlar orqali ifodalaymiz:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & | & b_1 \\ 0 & a'_{22} & a'_{23} & | & b'_2 \\ 0 & 0 & a''_{33} & | & b''_3 \end{pmatrix} \sim \begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a'_{22}x_1 + a'_{23}x_3 = b'_2 \\ a''_{33}x_3 = b''_3 \end{cases}$$

Sistemaning uchinchi tenglamasidan noma'lun koeffitsent $x_3$ning qiymatini topamiz:

$$x_3 = \frac{b''_3}{a''_{33}}$$

$x_3$ning qiymatini sistemaning ikkinchi tenglamasiga qo'yamiz va noma'lun koeffitsent $x_2$ning qiymatini topamiz. Shuningdek, 1-tenglamadan x_1 ning qiymati aniqlanadi.

*ChTSni **Gauss** usuli yordamida yechish*

3- misol. Tenglamalar sistemasini yeching.

$$\begin{cases} x_1 - x_3 = 2, \\ 2x_1 - x_2 + 3x_3 = -1, \\ 3x_1 + 2x_2 - 2x_3 = 5. \end{cases}$$

Yechish. Tenglamalar sistemasini Gauss usulida yechamiz.

Ushbu kengaytirilgan matritsani tuzamiz:

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 2 & -1 & 3 & -1 \\ 3 & 2 & -2 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & -5 & 5 \\ 0 & -2 & -1 & 1 \end{array}\right) \sim$$

$$\sim \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -5 & 5 \\ 0 & 0 & 11 & -11 \end{pmatrix} \sim \begin{cases} x_1 - x_3 = 2, \\ x_2 - 5x_3 = 5, \\ 11x_3 = -11. \end{cases}$$

Sistemaning uchinchi tenglamasidan noma'lum koeffitsiyent x_3 ning qiymatini topamiz:

$$11x_3 = -11 \Rightarrow x_3 = \frac{-11}{11} \Rightarrow x_3 = -1.$$

x_3 ning qiymatini sistemaning ikkinchi tenglamasiga qoyamiz va noma'lum koeffitsiyent x_2 ning qiymatini aniqlaymiz:

$$x_2 - 5 \cdot (-1) = 5 \Rightarrow x_2 + 5 = 5 \Rightarrow x_2 = 0.$$

Birinchi tenglamadan x_1 ning qiymatini aniqlaymiz:

$$x_1 - x_3 = 2 \Rightarrow x_1 - (-1) = 2 \Rightarrow x_1 = 1.$$

Demak, sistemaning yechimi $\{1; 0; -1\}$.

ChTSni ***teskari matritsa*** *usuli yordamida yechish*

Uch noma'lumli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases} \qquad (10)$$

Noma'lumlar oldidagi koeffisientlardan A matritsani, noma'lumlardan tashkil topgan X – ustun matritsani va ozod hadlardan B-ustun matritsani tuzamiz:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

u holda (10) tenglamalar sistemasini matritsali tenglama, ya'ni

$$AX=B \tag{11}$$

ko'rinishda ifodalash mumkin.

Agar A matritsa xosmas matritsa bo'lsa, u holda (11) tenglama quyidagicha yechiladi. (11) tenglamaning o'ng va chap qismini A matritsaga teskarisi matritsa A^{-1} ni ko`paytiramiz:

$$A^{-1}(AX) = A^{-1}B \quad \text{yoki } (A^{-1}A)X = A^{-1}B,$$

$A^{-1}A = E$ va $EX = X$ bo'lgani uchun tenglamaning

$$X = A^{-1}B \tag{12}$$

ko'rinishidagi yechimiga ega bo'lamiz.

Misol:

Ushbu tenglamalar sistemasini matritsalar yordamida yeching.

$$\begin{cases} x_1 - x_3 = 2, \\ 2x_1 - x_2 + 3x_3 = -1, \\ 3x_1 + 2x_2 - 2x_3 = 5. \end{cases}$$

Yechish.

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & 2 & -2 \end{pmatrix}; \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$$

A matritsa determinantini hisoblaymiz:

$$detA = \begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & 2 & -2 \end{vmatrix} = 2 + 0 - 4 - 3 - 6 - 0 = -11 \neq 0.$$

Demak, $detA \neq 0 \implies A^{-1}$ – mavjud. A^{-1} ni topamiz:

$$A^{-1} = \frac{1}{15} \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix},$$

bu yerda

$$A_{11}=(-1)^{1+1}\begin{vmatrix}-1 & 3\\ 2 & -2\end{vmatrix}=-4,$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix}2 & 3\\ 3 & -2\end{vmatrix}=13,$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix}2 & -1\\ 3 & 2\end{vmatrix}=7,$$

$$A_{21}=(-1)^{2+1}\begin{vmatrix}0 & -1\\ 2 & -2\end{vmatrix}=-2,$$

$$A_{22}=(-1)^{2+2}\begin{vmatrix}1 & -1\\ 3 & -2\end{vmatrix}=1,$$

$$A_{23}=(-1)^{2+3}\begin{vmatrix}1 & 0\\ 3 & 2\end{vmatrix}=-2,$$

$$A_{31}=(-1)^{3+1}\begin{vmatrix}0 & -1\\ -1 & 3\end{vmatrix}=-1,$$

$$A_{32}=(-1)^{3+2}\begin{vmatrix}1 & -1\\ 2 & 3\end{vmatrix}=-5,$$

$$A_{33}=(-1)^{3+3}\begin{vmatrix}1 & 0\\ 2 & -1\end{vmatrix}=-1.$$

Demak, berilgan matritsaga teskari matritsa quyidagi ko‘rinishga bo‘ladi:

$$A^{-1} = \frac{1}{-11}\begin{pmatrix} -4 & -2 & -1 \\ 13 & 1 & -5 \\ 7 & -2 & -1 \end{pmatrix}.$$

$$A^{-1} = \begin{pmatrix} \frac{4}{11} & \frac{2}{11} & \frac{1}{11} \\ -\frac{13}{11} & -\frac{1}{11} & \frac{5}{11} \\ -\frac{7}{11} & \frac{2}{11} & \frac{1}{11} \end{pmatrix}$$

(12) tenglikdan sistemaning yechimini topamiz:

$$X=\begin{pmatrix} \frac{4}{11} & \frac{2}{11} & \frac{1}{11} \\ -\frac{13}{11} & -\frac{1}{11} & \frac{5}{11} \\ -\frac{7}{11} & \frac{2}{11} & \frac{1}{14} \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}=\begin{pmatrix} \frac{8}{11}-\frac{2}{11}+\frac{5}{11} \\ -\frac{26}{11}+\frac{1}{11}+\frac{25}{11} \\ -\frac{14}{11}-\frac{2}{11}+\frac{5}{11} \end{pmatrix}=$$

$$=\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

Demak, tenglamalar sistemasining yechimi:

$$x_1=1;\ x_2=0;\ x_3=-1.$$

ChTSning amaliy masalalarga tadbiqi:

1-masala. *Zavodda 3 xil turdagi temir-buyum mahsulotlari ishlab chiqariladi. Mahsulotlar uchun 3 turdagi* $S_1, S_2\ va\ S_3$ *xom-ashyo ishlatiladi. Bitta mahsulot uchun har bir xom- ashyodan ishlatish me'yori va bir oylik xom-ashyo ishlatish hajmi 1-jadvalda berilgan. Zavodning har bir mahsulot bo'yicha bir oylik ishlab chiqarish hajmini toping.*

1-jadval

Xom-ashyo turlari	Bitta mahsulot ishlab chiqarish uchun xom-ashyo islatilishi me'yori (shartli birlikda)			Bir oylik xom-ashyo islatilishi (shartli birlikda)
	darvoza	deraza panjarasi	zinapoya to'siqlari	
S_1	2	0	3	69
S_2	1	2	1	60
S_3	5	0	4	120

Yechish. *Masalani chiziqli algebraik tenglamalar sistemasi yordamida yechamiz.*

Faraz qilaylik, zavod bir oyda x dona darvoza, y dona deraza panjarasi, z dona zinapoya to‘siqlari ishlab chiqarsin. U holda, har bir turdagi mahsulot uchun xom-ashyo sarflanishiga mos holda, quyidagi sistemani hosil qilamiz:

$$\begin{cases} 2x + 3z = 69, \\ x + 2y + z = 60, \\ 5x + 4z = 120. \end{cases}$$

Bu sistemani turli usullar bilan yechish mumkin. Biz Kramer usulidan foydalanamiz. Buning uchun asosiy determinantni tuzamiz va hisoblaymiz:

$$\Delta = \begin{vmatrix} 2 & 0 & 3 \\ 1 & 2 & 1 \\ 5 & 0 & 4 \end{vmatrix} = -14$$

Asosiy determinant noldan farqli, demak, sistema birgalikda va yagona yechimga ega. Yordamchi determinantlarni tuzamiz va hisoblaymiz:

$$\Delta x = \begin{vmatrix} 69 & 0 & 3 \\ 60 & 2 & 1 \\ 120 & 0 & 4 \end{vmatrix} = -168,$$

$$\Delta y = -231, \quad \Delta z = -210.$$

Kramer formulasiga asosan, masalaning yechimi quyidagicha bo'ladi:

$$x = \frac{-168}{-14} = 12, \quad z = \frac{-210}{-14} = 15,$$

$$y = \frac{-231}{-14} = 16.5,$$

Javob:

*Masala yechimi butun bo‘lishini hisobga olsak, sistemaning noma’lumlari qiymatidan quyidagi xulosaga kelamiz, ya’ni zavod bir oyda **12 ta darvoza, 16 ta deraza** va **15 ta zinapoya to‘siqlari**ni ishlab chiqaradi.*

Mustaqil yechish uchun misollar:

1.Tenglamalar sistemasini yeching:

1) $\begin{cases} 2x - 3y + z - 2 = 0, \\ x + 5y - 4z + 5 = 0, \\ 4x + y - 3z + 4 = 0. \end{cases}$

2) $\begin{cases} 7x + 2y + 3z = 15, \\ 5x - 3y + 2z = 15, \\ 10x - 11y + 5z = 36. \end{cases}$

3) $\begin{cases} x + 2y + 3z = 4, \\ 2x + y - z = 3, \\ 3x + 3y + 2z = 10. \end{cases}$

4) $\begin{cases} 2x - y + z = 2, \\ 3x + 2y + 2z = -2, \\ x - 2y + z = 1. \end{cases}$

Mustaqil yechish uchun misollar:

yeching

$$1)\ \begin{cases} x - y + 3z = -4, \\ 2x + 3y - 2z = 5, \\ 3x + 5y + z = 4. \end{cases}$$

$$2)\ \begin{cases} x - 2y - 3z = 8, \\ 3x + y + z = 3, \\ 4x + 3y - 2z = -1. \end{cases}$$

$$3)\ \begin{cases} x - 2y + 3z = 6, \\ 2x + 3y - 4z = 16, \\ 3x - 2y - 5z = 12. \end{cases}$$

$$4)\ \begin{cases} 2x + y - 3z = 3, \\ 3x + 4y - 5z = 9, \\ 2y + 7z = 11. \end{cases}$$

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