Matematika

Mavzu: Kompleks sonlarga doir masalalar yechish

O'QITUVCHI:TOG'AYNAZAROV SIROJIDDIN

DARSNING MAQSADI

- **M1.** Kompleks sonlarni Dekart koordinatalar sistemasida tasvirlashni o‘rganish
- **M2.** Kompleks sonning turli ko‘rinishlari va ular orasidagi bog‘lanishlarni o‘rganish
- **M3.** Kompleks sonlar ustida arifmetik amallarni bajarish, ularni darajaga ko‘tarishni va ulardan ildiz chiqarishni o‘rganish.

OLDINGI DARSLARGA BIR NAZAR

Oldingi darsimizda, n-darajali ildiz va ratsional ko‘rsatkichli darajaga oid misollarni yechishni ko‘rgan edik.

Bugungi darsimizda kompleks sonlarga doir masalalar yechishni o‘rganamiz. Haqiqiy sonlar to‘plamida barcha turdagi masalalar yoki tenglamalarni yecha olmaymiz. Masalan, kvadrat tenglamalarni yechayotganda diskriminant manfiy son bo‘lib qolgan holat. Bunga o‘xshagan holatlar uchun yechim olishda biz kompleks sonlar to‘plamidan foydalanamiz

KOMPLEKS SON TUSHUNCHASI

$$\sqrt{-16}=?$$

ifodaning son qiymatini topish masalasini qaraylik.

Bu ko‘rinishdagi haqiqiy bo‘lmagan va bizga mavhum sonlarning hammasini $\sqrt{-1}$ ko‘paytuvchi orqali hosil qilish mumkin.

$$\sqrt{-16}=\sqrt{16}\cdot\sqrt{-1}$$

Bizga mavhum bo‘lgan bu birlik ifoda $\boldsymbol{i}$ orqali belgilanadi va *mavhum birlik* deyiladi: $\boldsymbol{i=\sqrt{-1}\ \ =>\ \ i^2=-1}$

Demak, $\sqrt{-16}=\sqrt{16}\cdot\sqrt{-1}=4\cdot i=4i$ munosabat o‘rinli.

KOMPLEKS SON TUSHUNCHASI

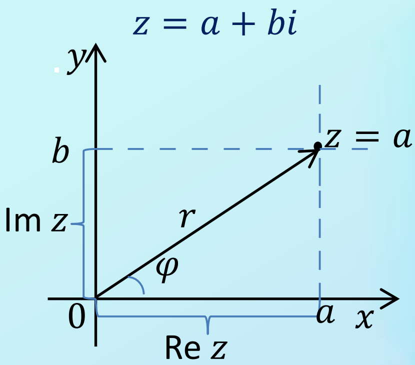
a va b haqiqiy sonlardan tuzilgan $a + bi$ ifoda kompleks son deb nomlanadi. Kompleks sonni z orqali belgilaylik.

U holda $z = a + bi$ kompleks son uchun

a soni z kompleks sonning *haqiqiy qismi* deyiladi va $\mathrm{Re}\, z$ orqali belgilanadi, $a = \mathrm{Re}\, z$

b soni esa z kompleks sonning *mavhum qismi* deyiladi va $\mathrm{Im}\, z$ orqali belgilanadi, $b = \mathrm{Im}\, z$

KOMPLEKS SONNING GEOMETRIK TASVIRI



$z = a + bi$ kompleks son *moduli* $r = |z|$ kabi belgilanadi, $r = \sqrt{a^2 + b^2}$

$z = a + bi$ kompleks sonning *argumenti* $\varphi = arg\ z$ belgilanadi,

$cos\varphi = \frac{a}{r}, \ sin\varphi = \frac{a}{r}, 0 \leq \varphi < 2\pi$

$z = a + bi$ $\quad \bar{z} = a - bi$ ko‘rinishidagi kompleks sonlar o‘zaro qo‘shma kompleks sonlar deyiladi.Masalan, $z = 2 + 3i$ $\quad \bar{z} = 2 - 3i$

KOMPLEKS SONNING KO‘RINISHLARI

1. Vektor ko‘rinishi: $z = \overrightarrow{(x; y)}$

2. Algebraik ko‘rinishi : $z = x + iy$

3. Trigonometrik ko‘rinishi : $z = r \cdot (\cos\varphi + i \cdot \sin\varphi)$

4. Ko‘rsatkichli ko‘rinishi : $z = r \cdot e^{i\varphi}$

KOMPLEKS SONLAR USTIDA ARIFMETIK AMALLAR

1. $(a+bi)+(c+di)=(a+c)+(b+d)i$

$z_1=-0{,}13+2i$ va $z_2=7+3{,}6i$ sonlarini qo'shing.

$$z_1+z_2=(-0{,}13+2i)+(7+3{,}6i)=(-0{,}13+7)+(2+3{,}6)i=$$
$$=6{,}87+5{,}6i$$

2. $(a+bi)-(c+di)=(a-c)+(b-d)i$

$z_1=13-7i$ va $z_2=-5+4i$ sonlarini ayiring.

$$z_1+z_2=(13-7i)-(-5+4i)=\big(13-(-5)\big)+(-7-4)i=$$
$$=18-11i$$

KOMPLEKS SONLAR USTIDA ARIFMETIK AMALLAR

3. $(a+bi)\cdot(c+di)=(ac-bd)+(ad+bc)i$

$z_1=3+2i$ va $z_2=7+6i$ sonlarini ko‘paytiring.

$$z_1\cdot z_2=(3+2i)\cdot(7+6i)=3\cdot7+3\cdot6i+2i\cdot7+2i\cdot6i=$$
$$=21+18i+14i+12(i)^2=9+32i$$

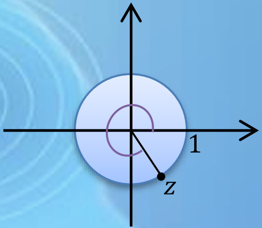
4. $\frac{a+bi}{c+di}=\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}i$

$z_1=2-i$ sonni $z_2=-3+2i$ songa bo‘ling.

$$\frac{2-i}{-3+2i}=\frac{(2-i)(-3+2i)}{(-3+2i)(-3-2i)}=\frac{-6+4i+3i-2}{(-3)^2-(2i)^2}=\frac{-8-i}{13}=$$
$$=\frac{-8}{13}-\frac{1}{13}i$$

Masalalar yechish

1-Masala Quyidagi sonni trigonometrik va ko‘rsatkichli ko‘rinishga keltiring: $z=\frac{1}{2}-\frac{\sqrt{3}}{2}i$



Yechish: Dastlab r va φ ni topib olamiz:

$$r=\sqrt{\left(\frac{1}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{3}{4}}=\sqrt{1}=1$$

$$\begin{cases} \cos\varphi=\frac{1}{2} \\ \sin\varphi=-\frac{\sqrt{3}}{2} \end{cases} \Rightarrow \varphi=$$

$$z=1\cdot\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$$

$$z=1\cdot e^{i\cdot\frac{5\pi}{3}}$$

$\frac{5\pi}{3}=300°$

KOMPLEKS SONLAR USTIDA ARIFMETIK AMALLAR

Trigonometrik ko‘rinishdagi kompleks sonlarni ko‘paytirishni qaraymiz.

$$z_1 = r_1(cos\varphi_1 + i\, sin\varphi_1), \quad z_2 = r_2(cos\varphi_2 + i\, sin\varphi_2)$$

$$z_1 \cdot z_2 = r_1(cos\varphi_1 + i\, sin\varphi_1) \cdot r_2(cos\varphi_2 + i\, sin\varphi_2) =$$
$$= r_1 \cdot r_2 \cdot \big((cos\varphi_1 \cdot cos\varphi_2 - sin\varphi_1 \cdot sin\varphi_2) +$$
$$+ i \cdot (cos\varphi_1 \cdot sin\varphi_2 + sin\varphi_1 \cdot cos\varphi_2)\big)$$

$$z_1 \cdot z_2 = r_1 \cdot r_2 \cdot \big(cos(\varphi_1 + \varphi_2) + i\, sin(\varphi_1 + \varphi_2)\big)$$

$$z_1 = 6(cos70° + i\, sin70°), \quad z_2 = 4(cos25° + i\, sin25°)$$

$$z_1 \cdot z_2 = 6 \cdot 4\big(cos(70° + 25°) + i\, sin(70° + 25°)\big) =$$
$$= 24(cos95° + i\, sin95°)$$

KOMPLEKS SONLAR USTIDA ARIFMETIK AMALLAR

Trigonometrik ko‘rinishdagi kompleks sonlarni bo‘lishni qaraymiz.

$$z_1 = r_1(cos\varphi_1 + i\, sin\varphi_1), \qquad z_2 = r_2(cos\varphi_2 + i\, sin\varphi_2),$$

$$\frac{z_1}{z_2} = \frac{r_1(cos\varphi_1 + i\, sin\varphi_1)}{r_2(cos\varphi_2 + i\, sin\varphi_2)} = \frac{r_1}{r_2} \cdot \frac{(cos\varphi_1 + i\, sin\varphi_1)(cos\varphi_2 - i\, sin\varphi_2)}{(cos\varphi_2 + i\, sin\varphi_2)(cos\varphi_2 - i\, sin\varphi_2)} =$$

$$= \frac{r_1}{r_2} \cdot \frac{(cos\varphi_1 \cdot cos\varphi_2 + sin\varphi_1 \cdot sin\varphi_2) + i(sin\varphi_1 \cdot cos\varphi_2 - cos\varphi_1 \cdot sin\varphi_2)}{cos^2\varphi_1 + \; sin^2\varphi_2}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\big(cos(\varphi_1 - \varphi_2) + i\, sin(\varphi_1 - \varphi_2)\big)$$

$$z_1 = 6(cos70° + i\, sin70°), \qquad z_2 = 4(cos25° + i\, sin25°),$$

$$\frac{z_1}{z_2} = \frac{6}{4}\big(cos(70° - 25°) + i\, sin(70° - 25°)\big) = 1{,}5\,(cos45° - i\, sin45°)$$

Kompleks sonni natural darajaga ko‘tarish

$z = r(\cos\varphi + i\sin\varphi)$ kompleks son uchun $\forall n \in \mathbb{N}$ soni uchun Muavr formulasi:

$$z^n = r^n(\cos\varphi + i\sin\varphi)^n = r^n\big(\cos(n\varphi) + i\sin(n\varphi)\big)$$

$z = 3(\cos 15° + i\sin 15°)$ kompleks son 4-darajasini toping:

$$z^4 = 3^4\big(\cos(4\cdot 15°) + i\sin(4\cdot 15°)\big) = 81\cdot(cos60° + i\,sin60°) =$$

$$= 81\cdot\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \frac{81}{2}\left(1+\sqrt{3}i\right)$$

Javob: $\frac{81}{2}\left(1+\sqrt{3}i\right)$

Masalalar yechish

2 masala $z=\frac{1}{2}-\frac{\sqrt{3}}{2}i$ kompleks sonning 10-darajasini toping.

Yechish: Dastlab trigonometrik ko‘rinishda yozib olamiz:

$$r=\sqrt{\left(\frac{1}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{3}{4}}=\sqrt{1}=1, \quad \varphi=\frac{5\pi}{3}=300°$$

$$z=1\cdot(cos300°+i\,sin300°)$$

$$z^{10}=1^{10}(cos3000°+i\,sin3000°)=1\cdot(cos120°+i\,sin120°)=$$

$$=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

Javob: $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

Masalalar yechish

3 masala

$(1+i)^{100}$ ifodaning qiymatini toping.

Yechish: $(1+i)^{100} = \left((1+i)^2\right)^{50} = (1+2i$

Javob: -2^{50}

Masalalar yechish

4 masala $a^2+a+1=0$ tenglama uchun $a^{200}+\frac{1}{a^{200}}$ ifodani qiymatini toping.

Yechish: $a^2+a+1=0$ tenglama uchun $D=1^2-4=-3=\left(\sqrt{3}i\right)^2$

$$a_1=\frac{-1-\sqrt{3}i}{2}=-\frac{1}{2}-\frac{\sqrt{3}}{2}i \quad r=\sqrt{\left(-\frac{1}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{3}{4}}=1$$

$$cos\varphi=-\frac{1}{2}, sin\varphi=-\frac{\sqrt{3}}{2} => \varphi=\frac{4\pi}{3}=240° \quad a_1=cos240°+$$

$$i\ sin240°$$

$$a_1^{200}=(cos240°+i\ sin240°)^{200}=cos120°+i\ sin120°$$

$$a_1^{-200}=(cos240°+i\ sin240°)^{-200}=cos120°-i\ sin120°$$

$$a_1^{200}+a_1^{-200}=cos120°+i\ sin120°+cos120°-i\ sin120°=$$

$$=2\cdot cos120°=2\cdot\frac{-1}{2}=-1.$$

a_2 uchun mustaqil bajarib ko‘rasiz

Kompleks sondan ildiz chiqarish

Kompleks sondan natural tartibli ildiz olishni ildiz tartibiga teskari qiymatli darajaga oshirish sifatida qabul qilish mumkin. Bunda ham Muavr formulasidan foydalanib quyidagicha formula xosil qilish mumkin

$$\sqrt[n]{z} = z^{\frac{1}{n}} = r^{\frac{1}{n}}(\cos\varphi + i\sin\varphi)^{\frac{1}{n}} =$$

$$= r^{\frac{1}{n}}\left(\cos\left(\frac{\varphi + 2\pi k}{n}\right) + i\sin\left(\frac{\varphi + 2\pi k}{n}\right)\right)$$

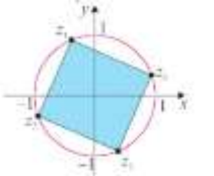
Bu formulada k o‘rniga $0,\ \pm1,\ \pm2, \ldots$. qiymatlarni qo‘yib turli ildizlarni topamiz.

Masalalar yechish

5 masala $z = \frac{1}{2} + \frac{\sqrt{3}}{2} i$ kompleks sondan 4-darajali ildiz chiqaring.

Yechish: Dastlab trigonometrik ko‘rinishda yozib olamiz:

$$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1, \quad \varphi = \frac{\pi}{3} = 60°$$



$$z = 1 \cdot (cos60° + i\ sin60°)$$

$$\sqrt[4]{z} = z^{\frac{1}{4}} = 1^{\frac{1}{4}}\left(cos\frac{60°+2\pi k}{4} + i\ sin\frac{60°+2\pi k}{4}\right)$$

$k = 0 \ \ z_0 = cos15° + i\ sin15°$

$k = 1 \ \ z_1 = cos105° + i\ sin105°$

$k = 2 \ \ z_2 = cos195° + i\ sin195°$

$k = 3 \ \ z_3 = cos285° + i\ sin285°$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

1(M3). $2 - i$ sonini $3 + 4i$ soniga qo‘shgandagi natijani toping

A) $5 + 3i$ B) $4 + i$ C) $2 + 7i$ D) $6 - 4i$

2(M3). $11 + 0{,}7i$ sonidan $0{,}3 - 2i$ soni ayrilgandagi natijani toping

A) $10{,}7 + 2{,}7i$ B) $3{,}3 - 1{,}4i$ C) $1{,}7 + 9{,}2i$ D) $11 - 2i$

3(M3). $5{,}7 - 3i$ sonini $6 + 2{,}1i$ soniga qo‘shgandagi natijani toping

A) $12{,}7 + 3i$ B) $4{,}6 + 2i$ C) $11{,}7 - 0{,}9i$ D) $7{,}5 - 2{,}5i$

4(M3). $4 - 2i$ sonini $1{,}5 + 2i$ soniga ko‘paytirgandagi natijani toping

A) $5 + 10i$ B) $6 - 2i$ C) $10 + 5i$ D) $4 - 1{,}5i$

5(M3). $3 + 0{,}1i$ sonini $1 + i$ soniga ko‘paytirgandagi natijani toping

A) $3 + i$ B) $2{,}9 + 3{,}1i$ C) $2{,}7 + 0{,}4i$ D) $5 - 1{,}7i$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

6(M3). 16 sonini $4+4i$ soniga bo‘lgandagi natijani toping

A) $2+8i$ B) $4-4i$ C) $2-2i$ D) $16-4i$

7(M3). $1-i$ sonini $1+i$ soniga bo‘lgandagi natijani toping

A) i B) $1-i$ C) $-i$ D) -1

8(M3). $\sqrt{3}+i$ sonini kubga oshirgandagi natijani toping

A) 27 B) $8i$ C) 9 D) $7{,}5i$

9(M3). $2-2i$ sonini $8-$darajaga oshirgandagi natijani toping

A) 1024 B) 2048 C) 4096 D) $8192i$

10(M3). $z^8+16=0$ tenglama nechta kompleks yechimga ega?

A) Yechimi yo’q B) 16 C) 8 D) 4

Darsni yakunlash

Bugungi darsimizda kompleks sonlarga doir masalalar yechishni o‘rganishda bosqichma-bosqich yondashdik.

Kompleks sonlarning Dekart koordinatalar sistemasida tasviri, kompleks sonning turli ko‘rinishlari, kompleks sonlar ustida arifmetik amallarning o‘rganish va ularning haqiqiy sonlardan farqlarini ko‘rish orqali bugungi mavzuni o‘zlashtirib oldik. Haqiqiy sonlar to‘plami kompleks sonlar to‘plamining qism to‘plami ekanligini ko‘rdik.

Foydalanilgan adabiyotlar

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