TOSHKENT IRRIGATSIYA VA QISHLOQ
XO'JALIGINI MEXANIZATSIYALASH
MUHANDISLARI INSTITUTI

MAVZU
09

Fazoda tekislik va uning tenglamari

Oliy mstematika kafedrasi

Reja:

1. Kirish.
2. Berilgan nuktadan o’tuvchiva berilgan vektorga perpendikular bo’gan tekislik tenglamasi.
3. Tekislikning umumiy tenglamasi va uning xususiy xollar.
4. Berilgan nuktadan o’tib, berilgan ikki nokolinear vektorlarga parallel bo’lgan tekislik tenglamasi.
5. Berilgan uchta nuqtadan o’tuvchi tekislik tenglamasi;
6. Tekislikning koordinata o’qlaridan ajratgan kesmalar bo’yicha tenglamasi;
7. Tekislikning normal tenglamasi;
8. Berilgan   nuqtadan tekislikkacha bo’lgan masofa;
9. Ikki tekislik orasidagi burchak;
10. Tekisliklarning o’zaro parallellik va perpendikulyarlik shartlari;

Kirish:

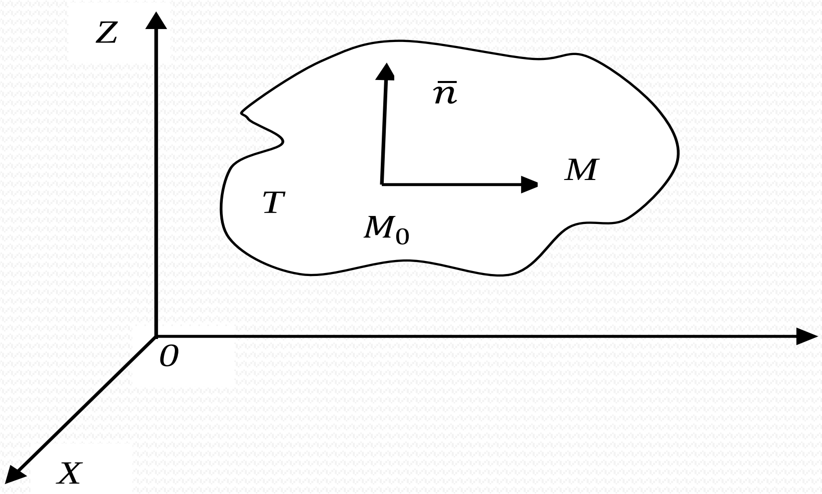
Ko’pincha amaliy masalalar yechish jarayonida fazoda berilgan tekislik yoki to’g’ri chiziq tenglamalari bila nish ko’rishga to’g’ri keladi.

Fazoda berilgan ikki nuqtadan bir xil masofada turgan nuqtalar to’plamiga tekislik deyiladi.

Tekislikni turli xil usullarda berish mumkin. Masalan, tekislikka tegishli nuqta va shu nuqtada tekislikka pependikulyar bo’lgan vektor orqali, tekislikka tegishli uchta nuqta orqali, o’qlardan ajratgan kesmalar bo’yicha va boshqalar.

2. Berilgan nuqtadan o'tuvchi va berilgan vektorga perpendikulyar bo'lgan tekislik tenglamasi

Aytaylik, fazoda T tekislik va unda yotuvchi $M_0(x_0, y_0, z_0)$ nuqta xamda shu nuqtada tekislikka pependikulyar bo'lgan $\bar{n}(A, B, C)$ vektor berilgan bo'lsin. Shu tekislikni tenglamasini topish talab etiladi.



Buning uchun T tekislikka tegishli M(x,y,z) nuqtani olamiz. $\overline{\mathrm{M}_0\mathrm{M}}$ vektorni tuzamiz. Ya'ni,

$$\overline{\boldsymbol{M_0}\mathbf{M}}\,(\boldsymbol{x}-\boldsymbol{x_0},\boldsymbol{y}-\boldsymbol{y_0},\boldsymbol{z}-\boldsymbol{z_0})$$

$\bar{n}$ vektor $\overline{M_0M}$ vektorga $\perp$ bo'ladi. U holda, bu vektorning skalyar ko'paytmasi nolga teng bo'ladi.

Ya'ni, $\boldsymbol{\bar{n}}\cdot\overline{\boldsymbol{M_0M}} \rightarrow \boldsymbol{A}(\boldsymbol{x}-\boldsymbol{x_0})+\boldsymbol{B}(\boldsymbol{y}-\boldsymbol{y_0})+\boldsymbol{C}(\boldsymbol{z}-\boldsymbol{z_0})=\mathbf{0}$ (1)

Tenglamani berilgan nuqtadan o'tib, berilgan vektorga perpendikulyar bo'lgan tekislik tenglamalari deyiladi.

Agar (1) da – $(Ax_0 - B_0 - Cz_0) = D$ deb belgilasak,

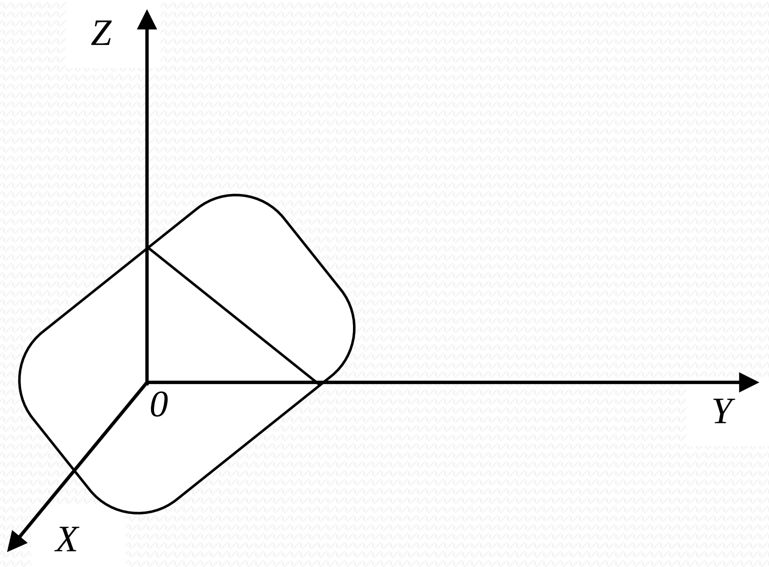
$$Ax+By+Cz+D=0 \qquad (2)$$

tenglama hosil bo'ladi. (2) tenglama tekislikning umumiy tenglamasi deyiladi.

3. A,B,C,D koeffitsientlarni turli qiymatlarida tekislikning turli tenglamalari hosil bo'ladi.

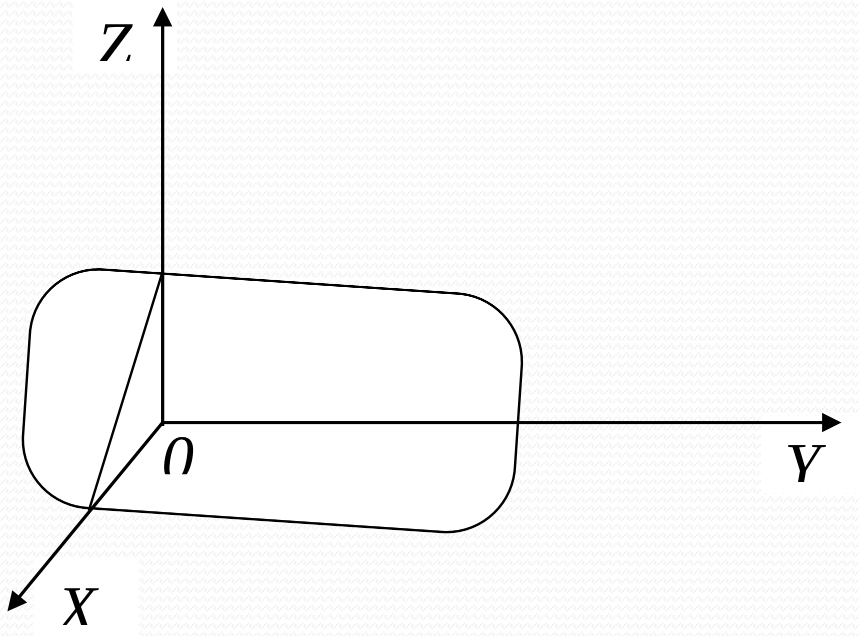
XUSUSIY XOLLAR

Agar A=0 bo'lsa, *By*+*Cz*+*D*=*0* tekislik *OX* o'qiga parallel bo'ladi (1-rasm);



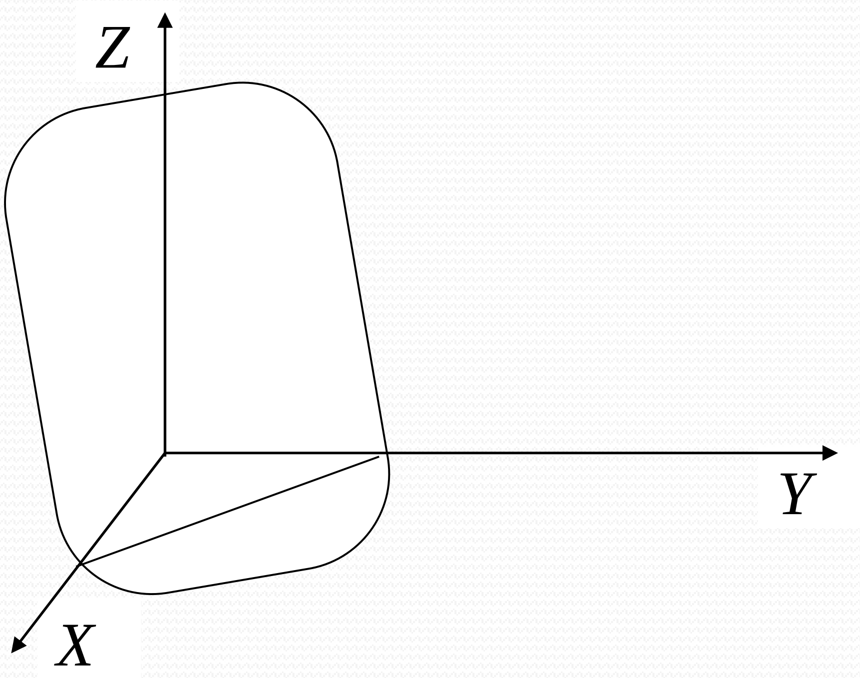
1-chizma.

Agar B=0 bo’lsa, $Ax+By+Cz+D=0$ tekislik OY o’qiga paralell bo’ladi (2-chizma);



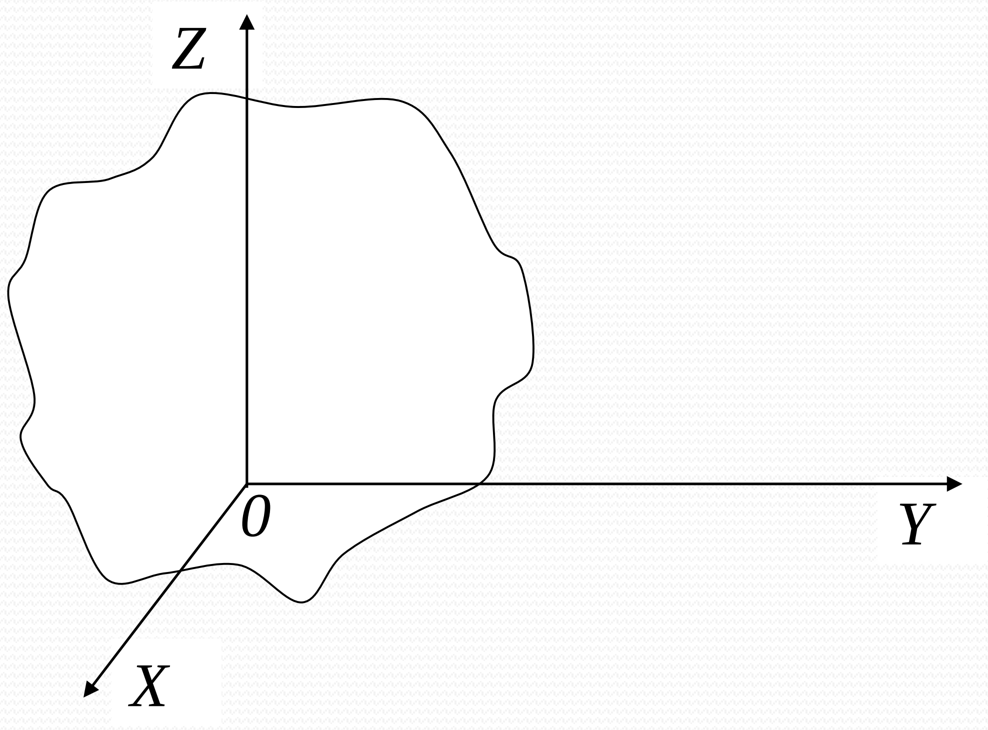
2-chizma.

Agar C=0 bo'lsa, $Ax+By+D=0$ tekislik OZ o'qiga paralell bo'ladi (3-chizma);



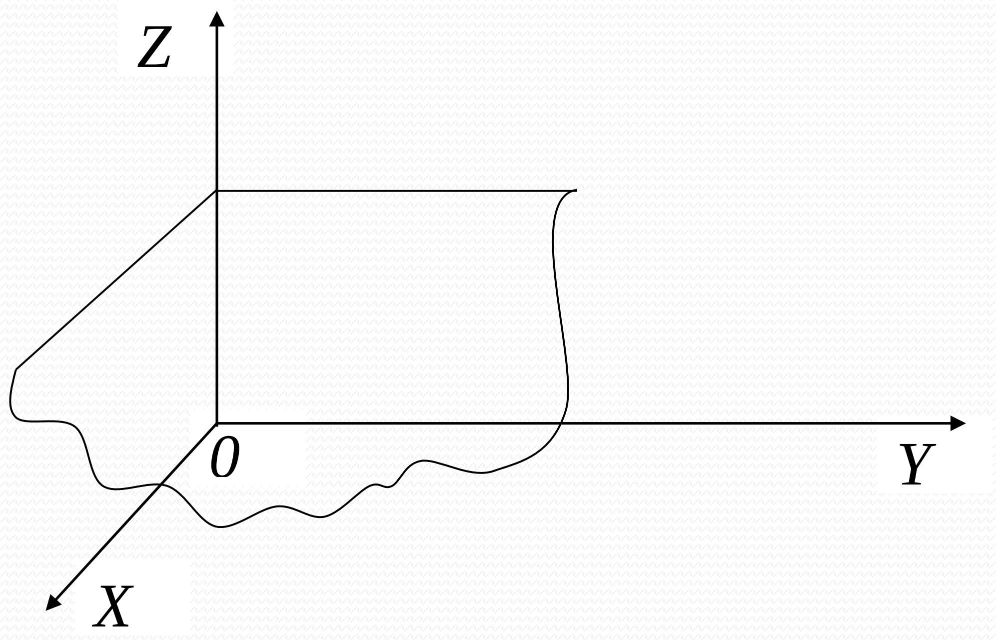
3-chizma.

Agar D=0 bo'lsa, $Ax+By+Cz=0$ tekislik koordinatalar boshidan o'tadi (4-chizma).



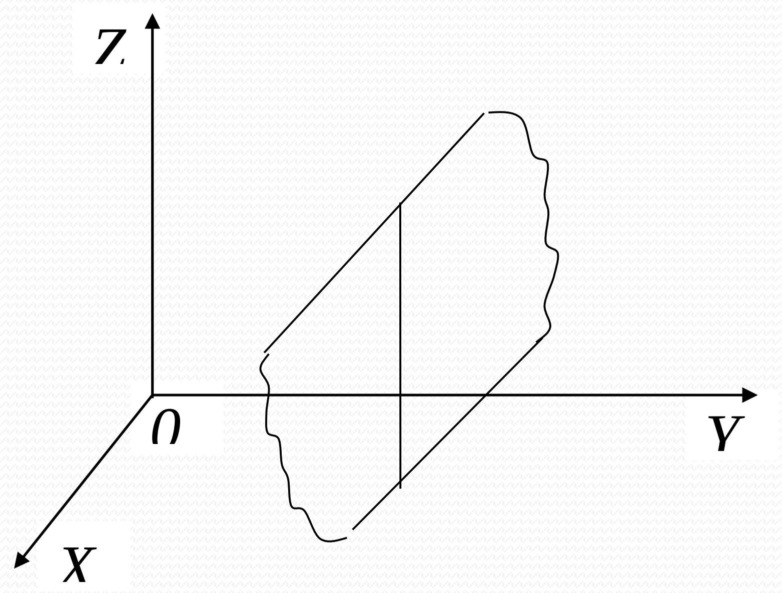
4-chizma.

Agar $A=B=0$ bo'lsa, $CZ+D=0$ tekislik OZ o'qiga perpendikulyar (OXY tekislikka paralell bo'ladi) 5-chizma;



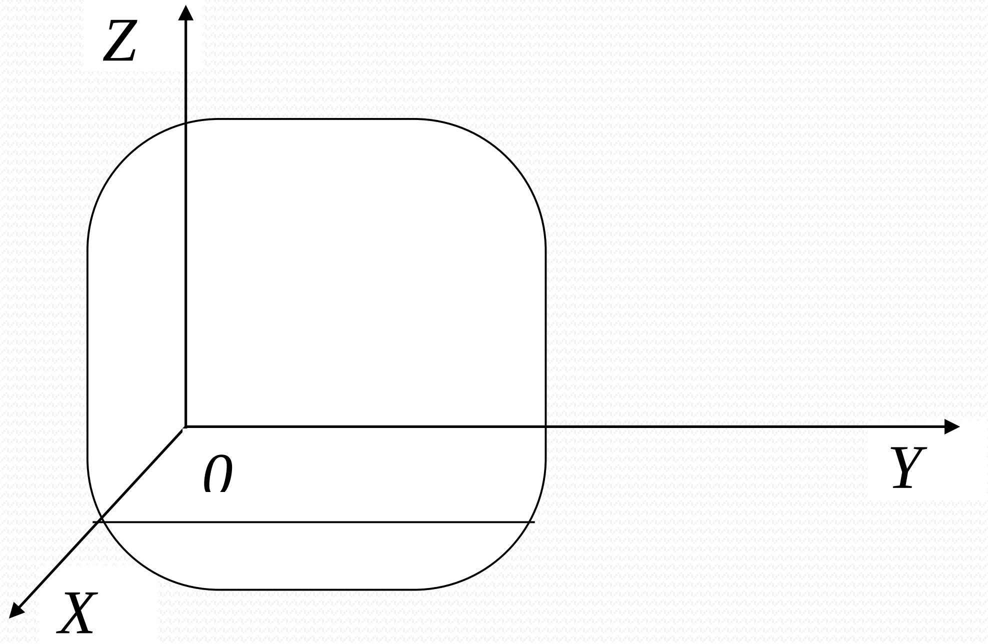
5-chizma

Agar $A=C=0$ bo'lsa, $By+D=0$ tekislik Oy o'qiga $\perp$ bo'lib, XOZ tekislikka paralell bo'ladi (6-chizma).



6-chizma

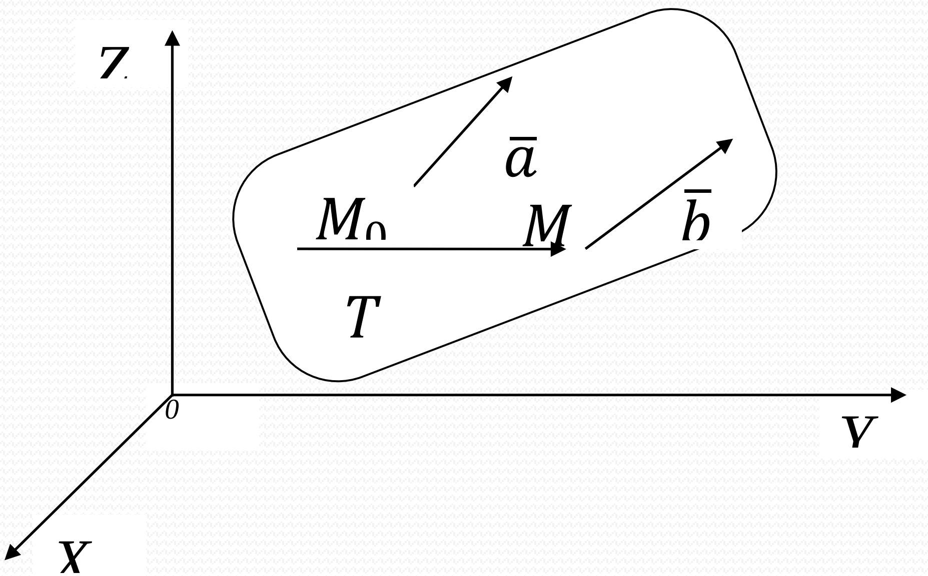
Agar $B=C=0$ bo'lsa, $AX+D=0$ tekislik OX o'qiga perpendikulyar bo'lib, YOZ tekisligiga parallel bo'ladi (7-chizma).



7-chizma

4. Berilgan nuqtadan o'tib, berilgan ikki nokollinear vektorga parallel bo'lgan tekislik tenglamasi.

Aytaylik, $M_0(x_0, y_0, z_0)$ nuqta T tekislikka tegishli bo'lib, $\bar{a}(a_1, a_2, a_3)$ va $\bar{b}(b_1, b_2, b_3)$ vektorlar T tekislikka parallel bo'lsin. Shu tekislikning tenglamasini topish talab etiladi.



T tekislikka tegishli ixtiyoriy M nuqtani olamiz. U holda $\overline{M_0M}$ *vektor* $\bar{a}$ ва $\bar{b}$ *vektorlar bilan komplanar bo'ladi, demak bu vektorlar chiziqli bog'liq bo'lib, ularning koordinatalaridan tuzilgan uchinchi tartibli determinant nolga teng bo'ladi.*

Ya'ni,

$$\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = 0 \quad (3)$$

(3) shart bajarilsa, M (x,y,z) nuqta T tekislikka tegishli bo'ladi.

Bundan (3) tenglama izlanayotgan tekislikni tenglamasi ekanligi kelib chiqadi.

6. Tekislikning koordinata o'qlaridan ajratgan kesmalar bo'yicha tenglamasi

Aytaylik, tekislik koordinatalar boshidan o'tmasin va u (O_x, O_y, O_z) o'qlarni mos ravishda $M_1(a;0;0)$, $M_2(0;b;0)$, $M_3(0;0;C)$ nuqtalarda kesib o'tsin. U holda (4) tenglama quyidagi ko'rinishni oladi:

$$\begin{vmatrix} x-a & y-0 & z-0 \\ 0-a & b-0 & 0 \\ 0-a & 0 & c-0 \end{vmatrix} = 0 \rightarrow$$

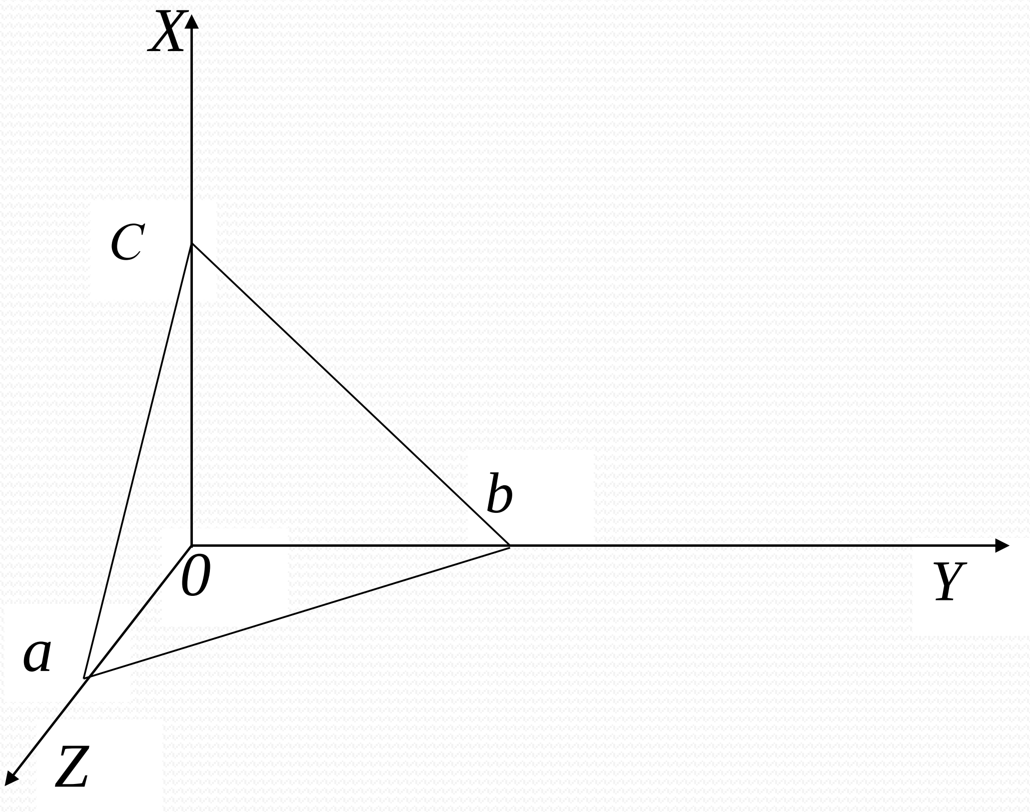
$$\begin{vmatrix} x-a & y & z \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = 0$$

$$(x-a)bc + abz + ayc = 0$$

$$\rightarrow xbc + ayc + abz = abc \ /\!:abc \neq 0$$

$$\frac{x}{a} + \frac{y}{b} + \frac{c}{z} = 1 \qquad (5)$$

(5) Tenglamani tekislikning kesmalar bo'yicha tenglamasi deyiladi.



7. Tekislikning normal tenglamasi

Fazoda berilgan tekislikka koordinatorlar boshidan tushurilgan perependikularni uzunligi P va bu ⊥ ni *Ox*, *Oy*, *Oz* o'qlarining musbat yo'nalishlari bilan hosil qilgan burchaklari mos ravishda α, β va γ bo'lsin.

Bu tekislikni tenglamasi

$$\boldsymbol{x\cos\alpha + y\cos\beta + z\cos\gamma - P = 0} \qquad (6)$$

Bu yerda ,

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad (6)$$

tenglamani tekisling normal tenglamasi deyiladi.

8. $M_0(x_0, y_0, z_0)$ nuqtadan $Ax+By+Cz=0$ tekislikkacha bo'lgan masofa:

$$d = \frac{|\mathrm{Ax_0+By_0+Cz_0+D}\,|}{\sqrt{A^2+B^2+C^2}} \qquad (7)$$

9. $\boldsymbol{A_1x + B_1y + C_1z + D_1 = 0}$ **va**

$$\boldsymbol{A_2x + B_2y + C_2z + D_2 = 0}$$

tekisliklar orasidagi burchak

Berilgan tekisliklar orasidagi burchak bu tekisliklarning yo'naltiruvchi vektorlari

$\bar{n}_1 = \{A_1, B_1, C_1\}$ *va* $\bar{n}_2 = \{A_2, B_2, C_2\}$ *vektorlar orasidagi burchakka teng bo'ladi.*

U holda bu tekisliklar orasidagi burchak:

$$\cos\varphi = \frac{\bar{n}_1 . \bar{n}_2}{|\bar{n}_1| \cdot |\bar{n}_2|} \rightarrow$$

$$\cos\varphi = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \cdot \sqrt{A_2^2 + B_2^2 + C_2^2}} \qquad (8)$$

10.Tekisliklarning o'zaro perpendikularning sharti

Agar ikki tekislik o'zaro perpendikulyar bo'lsa,ular orasidagi burchak 90° bo'lib , Cos 90°=0,bo'ladi. Bu holda (8) formuladan

$$\boldsymbol{A_1A_2+B_1B_2+C_1C_2=0} \qquad (9)$$

kelib chiqadi. (9) formula ikki tekislikni ⊥ lik sharti deyiladi.

$\bar{n}_1=(A_1,B_1,C_1)$ $\bar{n}_2=(A_2,B_2,C_2)$ *vektorlar tekisliklarga perpendikulyar bo'lsa,* $\bar{n}_1.\bar{n}_2=0\rightarrow$ $A_1A_2+B_1B_2+C_1C_2=0$ *bo'ladi.*

Tekisliklarning o'zaro paralellik sharti

Agar ikki tekislik o'zaro parallel bo'lsa, ularga koordinatalar boshidan tushirilgan perpendikulyarning Ox,Oy,Oz o'qlari bilan hosil qilgan burchaklari bir biriga teng bo'ladi. U holda bu perpendikulyarlarning yo'naltiruvchi cosinuslari ham bir-biriga teng bo'ladi, ya'ni:

$$\cos\alpha = \cos\alpha_{1,}\ \cos\beta = \cos\beta_1, \cos\gamma = \cos\gamma_1$$

Bulardan esa,

$$\frac{A_1}{\sqrt{\Delta_1}} = \pm\frac{A_2}{\sqrt{\Delta_2}},$$

$$\frac{B_1}{\sqrt{\Delta_1}} = \pm\frac{B_2}{\sqrt{\Delta_2}}, \qquad \frac{C_1}{\sqrt{\Delta_1}} = \pm\frac{C_2}{\sqrt{\Delta_2}}$$

tengliklar yoki proportsiyalar kelib chiqadi.

Bu yerda:

$\Delta_1 = {A_1}^2 + {B_1}^2 + {C_1}^2 \quad \Delta_2 = {A_2}^2 + {B_2}^2 + {C_2}^2$

Oxirgi tengliklarni quyidagicha yozish mumkin :

$$\frac{A_1}{A_2} = \pm\sqrt{\frac{\Delta_1}{\Delta_2}},$$

$$\frac{B_1}{B_2} = \pm\sqrt{\frac{\Delta_1}{\Delta_2}}, \qquad \frac{C_1}{C_2} = \pm\sqrt{\frac{\Delta_1}{\Delta_2}}$$

O'ng tomonlari tengligidan chap tomonlarning xam tengligi kelib chiqadi. Ya'ni,

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} \qquad (10)$$

(10)-chi formulaga ikki tekislikni parallellik sharti deyiladi.

1-Misol:

2x-3y+5z+7=0

6x-9y+15z+5=0

tekisliklar o'zaro paralleldir. Haqiqatan ham, koeffitsentlar nisbati

$$\frac{2}{6} = \frac{-3}{-9} = \frac{5}{15} = \frac{1}{3}$$ bir biriga tengdir.

2-misol :

$$2x+y-5z+4=0$$

$$3x+4y+2z-1=0$$

tekisliklar o'zaro perpendikulyardir. Haqiqatan ham, (9) shart bajariladi:

$$2\cdot3+1\cdot4-5\cdot2=10-10=0$$

3-misol: Quyidagi ikki tekislik orasidagi burchakni toping:

$$\begin{cases} 2x + y + 4z + 2 = 0 \\ x + 2y - z + 1 = 0 \end{cases}$$

$$A_1A_2 + B_1B_2 + C_1C_2 = 0$$

Demak,

$$\cos\alpha = 0 \rightarrow \varphi = \frac{\pi}{2} \text{ ёки } \varphi_2 = \frac{3\pi}{2}$$

Mavzuga doir misollar

1-misol.

M(2;3;-1) nuqtadan o'tuvchi va 5x-3y+2z-10=0 tekislikka parallel bo'lgan tekislik tenglamasi topilsin.

Yechish: *izlanayotgan tekislik M(2;3;-1) nuqtadan o'tganligi uchun uning tenglamasi A(x-2)+V(u-3)+S(z+1)=0 bo'ladi.*

Bu tekislikka parallel bo'lgani uchun

$$\frac{A}{5} = \frac{B}{-3} = \frac{C}{2} = m \rightarrow A = 5m, B = -3m, C = 2m$$

A=5m, B=-3m, C=2m

U holda, 5m(x-2)-3m(y-3)+2m(z+1)=0

5x-10-3y+9+2z+2=0

5x-3y+2z+1=0 hosil bo'ladi.

2-misol.

M(2;-1,5) nuqtadan o'tib,

$3x-2y+z+7=0$ va $5x-4y+3z+1=0$ tekisliklarga ⊥ bo'lgan tekislik tenglamasini toping.

Yechish: *Izlanayotgan tekislik M(2;-1,5) nuktadan o'tganliki uchun*

$A(x-2)+B(y+1)+C(Z-5)=0$ bo'ladi.

Bu tekislik misol shartida berilgan tekisliklarga ⊥ bulgani uchun

$$3A-2B+C=0$$

$$5A-4B+3C=0$$ bo'ladi.

Demak, uch noma'lumli uchta jinsli chiziqli tenglamalar sistemasiga ega bo'lamiz.

Ya'ni,

$$\begin{cases} \boldsymbol{A(x-2)+B(y+1)+C(x+5)=0} \\ \boldsymbol{3A-2B+C=0} \\ \boldsymbol{5A-4B+3C=0} \end{cases}$$

Bu yerda A, B, C koeffitsentlarini nomaьlumlar sifatida qaraladi.

(*) sistema noldan farqli yechimlarga ega bo'lishi uchun sistemani determinanti nolga teng bo'lishi kerak .

Ya'ni,

$$\begin{vmatrix} x-2 & y+1 & x-5 \\ 3 & -2 & 1 \\ 5 & -4 & 3 \end{vmatrix} = 0 \rightarrow$$

x+2y+z-5=0 izlanayotgan tekislik tenglamasi bo'ladi . Haqiqatan ham ko'rish mumkinki : 3·1+2·(-2)+1·1=0 va 5·1-4·2+3·1=0 bo'ladi.

Tekislik tenglamalriga doir masalalar

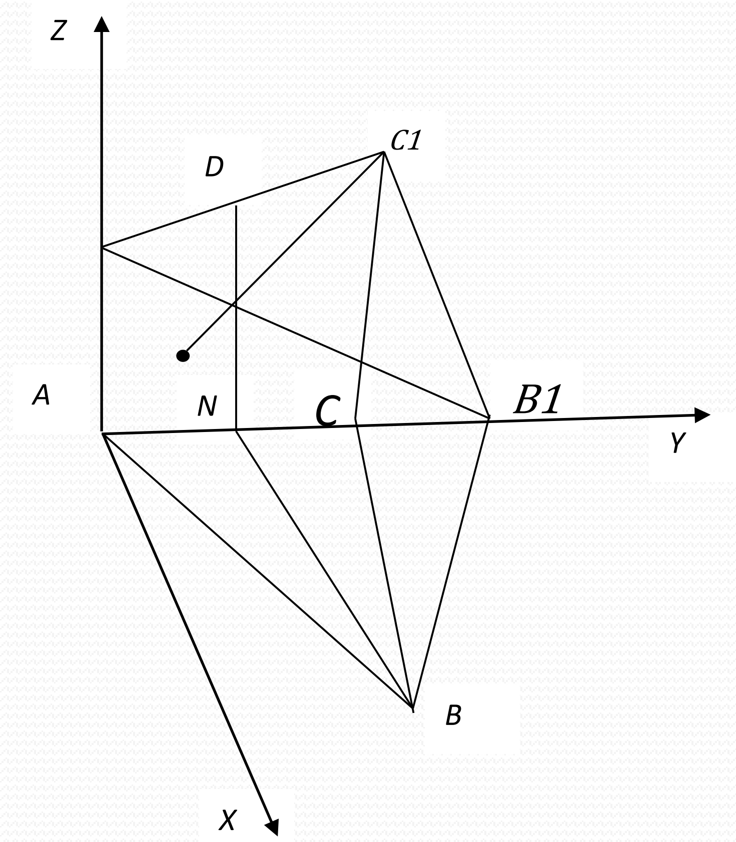
1-masala.

Muntazam uchburchakli prizma asosining tomoni

|AB|=|AC|=|BC|=4 sm, qirrasi $|AA_1| = |BB_1| = |CC_1| = 3\ cm$ ga teng.

D nuqta A_1C_1 tomonni o'rtasi bo'lsa,

C_1 uchidan ADB tekislikkacha bo'lgan masofa topilsin.



Yechish: $OXYZ$ koordinatalar sistemasini quyidagicha olamiz. Koordinatalar boshi A nuqtada, Oy uqini AC orqali, OZ o'qini AA_1 qirra orqali o'tkazamiz.

OX o'q ABC uchburchakning BN medianasiga parallel bo'ladi. $BN \perp AC$, A,B,C va C_1 nuqtalarni koordinatalarini topamiz:

$A(0;0;0), B\left(2\sqrt{3};2;0\right), D(0;2;3)$ ва

$C_1(0;4;3)$ *bo'ladi.*

Endi ABD tekislik tenglamasini topamiz. Bizga ma'lumki, $M_0(x_0, y_0, z_0)$ nuqtadan o'tuvchi tekislik tenglamasi $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

Bu tenglamaga A,B,D nuqtalarni koordinatalarini qo'yamiz:

$$A(0;0;0)$$

$$B(2\sqrt{3};2;0) \quad \begin{cases} ax+by+cz=0 \\ 2\sqrt{3}a+2b=0 \\ 2b+3c=0 \end{cases} \rightarrow$$

$$D(0;2;3)$$

$$\rightarrow \begin{cases} 2\sqrt{3}a = -2b \\ 2b = -3c \end{cases} \rightarrow \begin{cases} 2\sqrt{3}a = 3c \\ 2b = -3c \end{cases} \rightarrow \begin{cases} a = \dfrac{\sqrt{3}}{2}c \\ b = -\dfrac{3}{2}c \end{cases}$$

$$\rightarrow \begin{cases} 2\sqrt{3}a = -2b \\ 2b = -3c \end{cases} \rightarrow \begin{cases} 2\sqrt{3}a = 3c \\ 2b = -3c \end{cases} \rightarrow \begin{cases} a = \dfrac{\sqrt{3}}{2}c \\ b = -\dfrac{3}{2}c \end{cases}$$

U holda, $\frac{\sqrt{3}}{2}\mathrm{c}x - \frac{3}{2}cy + cz = 0/:$ $c \neq o$

$$\frac{\sqrt{3}}{2}x - \frac{3}{2}y + z = 0\ yoki$$

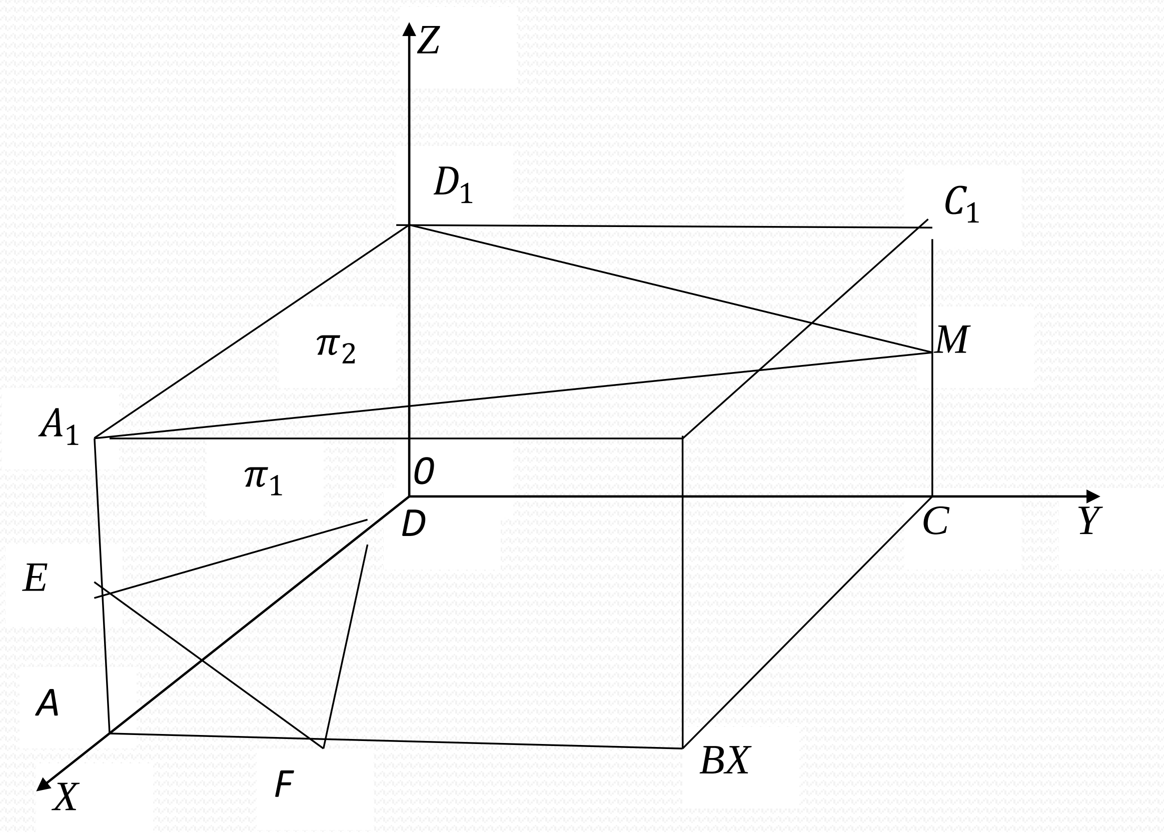
$$\sqrt{3}x - 3y + 2z = 0 \rightarrow$$

ABD ***tekislik tenglamasi*** *bo'ladi.*

$C_1(0;4;3)$ *Demak,* C_1 *nuqtadan ABD tekislikkacha bo'lgan masofada* $|C_1E|$

$$d = \frac{|\sqrt{3}\cdot 0 - 3\cdot 4 + 2\cdot 3|}{\sqrt{(\sqrt{3})^2+(-3)^2+2^2}} = \frac{6}{4} = \frac{3}{2} = 1.5$$

2-masala. Kubda E,F,M nuqtalar AA_1, AB, CC_1 qirralarini o’rtalari bo’lsin. EFD va A_1D_1M tekisliklar orasidagi burchak topilsin.



Yechish. *Kubning qirralari orqali Ox,Oy,Oz o'qlarini o'tkazamiz. Kubni qirrasini a bilan belgilaymiz. D, E, F,* A_1 *va M nuqtalarni koordinatalarini topamiz:*

D (o;o;o) , E (a;o; $\frac{a}{2}$ *) ,* $F(a, \frac{a}{2}, 0)$ *,*

A(a;o;a), D(o;o;a), M(o;a; $\frac{a}{2}$ *).*

Berilgan uchta nuqtadan o'tuvchi tekislik tenglamasini topish formulasiga ko'ra, ya'ni (4) formulaga ko'ra DEF va A_1D_1M tekisliklarni tenglamalarini topamiz:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

tenglamaga ko'ra:

$$EFD: \begin{vmatrix} x-0 & y-0 & z-0 \\ a & 0 & \frac{a}{2} \\ a & \frac{a}{2} & 0 \end{vmatrix} = 0 \rightarrow$$

$$y\frac{a^2}{2} + z\frac{a^2}{2} - x\frac{a^2}{4} = 0 \rightarrow \boldsymbol{x - 2y - 2z = 0}$$

$$, \overline{\boldsymbol{n}}_{\boldsymbol{1}}(\boldsymbol{1}; \boldsymbol{-2}; \boldsymbol{-2})$$

$$A_1D_1M : \begin{vmatrix} x-a & y-0 & z-a \\ -a & 0 & 0 \\ -a & a & -\frac{a}{2} \end{vmatrix} = 0 \rightarrow$$

$$-a^2(z-a)-\frac{a^2}{2}y=0\rightarrow \boldsymbol{y+2z-2a=0},$$

$$\boldsymbol{\bar{n}_2(0;1;2)}$$

Demak, ikki tekislik orasidagi burchakni topish formulasiga asosan, ya'ni

$$\cos\alpha=\frac{|\bar{n}_1\cdot\bar{n}_2|}{|\bar{n}_1|\cdot|\bar{n}_2|}=\frac{|1\cdot 0-2\cdot 1-2\cdot 2|}{\sqrt{9}\cdot\sqrt{5}}$$

$$=\frac{6}{3\sqrt{5}}$$

yoki $\cos\alpha=\frac{2}{\sqrt{5}}\rightarrow\alpha=arc\cos\frac{2}{\sqrt{5}}$

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