TOSHKENT IRRIGATSIYA VA QISHLOQ XO'JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI

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Мавзу

Funksiya hosilasi va uning tadbiqlari. Bir o‘zgaruvchili funksiyaning hosilasi, hosilaning geometrik va mexanik ma’nolari. Funksiyaning grafigiga berilgan nuqtada o‘tkazilgan urinma va normal tenglamalari. Murakkab, parametrik, teskari va oshkormas funksiyalarning hosilalari.

Hosilaning ta’rifi

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

yoki

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Demak, **berilgan $y = f(x)$ funksiyaning argument x bo‘yicha *hosilasi* deb, argument orttirmasi Δx ixtiyoriy ravishda nolga intilgan holda funksiya orttirmasi Δy ning argument orttirmasi Δx ga nisbatining limitiga aytiladi.**

Hosilaning $x=a$ dagi aniq qiymati $f'(a)$ yoki $y' \,/_{x=a}$ bilan belgilanadi.

Berilgan $f(x)$ funksiyadan hosila olish amali shu funksiyani ***differensiallash*** deyiladi.

Misol. $y = x^2$ funksiya berilgan, uning: 1) ixtiyoriy x nuqtadagi va 2) x=3 nuqtadagi y' hosilasi topilsin.

Yechish. 1)Argumentning x ga teng qiymatida $y = x^2$ ga egamiz. Argumentning $x + \Delta x$ ga teng qiymatida $y + \Delta y = (x + \Delta x)^2$ ga egamiz. Funksiya orttirmasini topamiz:

$\Delta y = (x + \Delta x)^2 - x^2 = 2x\Delta x + (\Delta x)^2.$

$\frac{\Delta y}{\Delta x}$ nisbatni tuzamiz:

$$\frac{\Delta y}{\Delta x} = \frac{2x\Delta x + (\Delta x)^2}{\Delta x} = 2x + \Delta x.$$

Limitga o‘tib, berilgan funksiyaning hosilasini topamiz:

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x) = 2x.$$

Demak, $y = x^2$ funksiyaning ixtiyoriy nuqtadagi hosilasi $y' = 2x$.

2) x=3 da quyidagini hosil qilamiz:

$y'|_{x=3} = 2 \cdot 3 = 6.$

Hosilaning geometric ma’nosi

$$tg\,\alpha = \lim_{\Delta x \to 0} tg\varphi = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x).$$

Demak,

$f'(x) = tg\,\alpha,$ (2)

Yani *argument x ning berilgan qiymatida* $f'(x)$ *hosilaning qiymati f(x) funksiyaning grafigiga uning* $M_0(x, y)$ *nuqtasidagi urinmasining Ox o‘qning musbat yo‘nalishi bilan hosil qilgan burchak tangensiga teng.*

2-misol. $y = x^2$ egri chiziqqa $M_1\left(\frac{1}{2}, \frac{1}{4}\right)$; $M_2(-1,1)$ nuqtalardagi urinmalar og‘malik burchaklarining tangenslari topilsin.

Yechish. Ma’lumki $y' = 2x$, demak,

$tg\alpha_1 = y'_{x=\frac{1}{2}} = 1;\ tg\alpha_2 = y'_{x=-1} = -2.$

Elementar funksiyalar hosilalari jadvali

1) $(C)' = 0, \quad C = const$;

2) $(x^n)' = nx^{n-1}$;

3) $(Cu)' = Cu'$;

4) $(u + v)' = u' + v'$;

5) $(u \cdot v)' = u'v + uv'$;

6) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$;

7) $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$;

8) $(sinx)' = cosx$;

9) $(cosx)' = -sinx$;

10) $(tgx)' = \frac{1}{cos^2 x}$;

11) $(ctgx)' = -\frac{1}{sin^2 x}$;

12) $(e^x)' = e^x$;

13) $(a^x)' = a^x lna$;

14) $(lnx)' = \frac{1}{x}$;

15) $(\log_a x)' = \frac{1}{xlna}$

Misollar. 1) $= 6\sqrt[3]{x} - 4\sqrt[4]{x}\,,\quad y' =?$

Yechish. $y' = 6\left(x^{\frac{1}{3}}\right)' - 4\left(x^{\frac{1}{4}}\right)' = 6\cdot\frac{1}{3}x^{-\frac{2}{3}} - 4\cdot\frac{1}{4}x^{-\frac{3}{4}} = \frac{2}{\sqrt[3]{x^2}} - \frac{1}{\sqrt[4]{x^3}}$

2) $y = \frac{tgx}{\sqrt{x}}\,,\quad y' =?$

Yechish. $y' = \frac{(tgx)'\sqrt{x} - tgx(\sqrt{x})'}{(\sqrt{x})^2} = \frac{\frac{1}{cos^2x}\sqrt{x} - tgx\frac{1}{2\sqrt{x}}}{x} = \frac{\frac{2x}{cos^2x} - tgx}{2x\sqrt{x}}$

3) $y = \frac{x^3}{3} - x^2 + x,\qquad y'(-1) =?$

Yechish. $y' = x^2 - 2x + 1,\ y'(-1) = (-1)^2 - 2\cdot(-1) + 1 = 4$

4) $y=lg(5x),\quad y' =?$

Yechish. $y' = \frac{1}{5x}\cdot 5 = \frac{1}{x}$

5) $= a^{sinx}\,,\quad y' =?$

Yechish. $y' = a^{sinx}\,lna(sinx)' = cosx\;a^{sinx}\,lna$

Murakkab funksiyaning hosilasi

Agar ***y=f(u)*** va ***u=φ(x)*** bo‘lsa, ***y*** *funksiyaning funksiysi* yoki *murakkab funksiya* deyiladi. U holda

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ $yoki$ $y' = f'(u) \cdot u'$

Misollar.

1) $y = \frac{1}{\sqrt[3]{(4+3x)^2}}$, $y = (4+3x)^{-\frac{2}{3}}$, $y' = ?$

Yechish. $y' = -\frac{2}{3}(4+3x)^{-\frac{2}{3}-1} \cdot (4+3x)' = -\frac{2}{3\sqrt[3]{(4+3x)^5}} \cdot 3 = -\frac{2}{\sqrt[3]{(4+3x)^5}}$

2) $= \frac{sin^2x}{cosx}$, $y' = ?$

Yechish. $y' = \frac{(sin^2x)'cosx - sin^2x(cosx)'}{cos^2x} = \frac{2sinxcosx \cdot cosx - sin^2x \cdot (-sinx)}{cos^2x} = \frac{sinx(2cos^2x + sin^2x)}{cos^2x}$

Urinma va normal tenglamalari

Y=f(x) funksiyaga $M(x_0, y_0)$ nuqtada o‘tkazilgan *urinma* tenglamasi

$$y - y_0 = k(x - x_0)$$

Normal tenglamasi

$$y - y_0 = -\frac{1}{k}(x - x_0)$$

Misol. $y = \frac{8}{4+x^2}$ funksiyaga *x=2* nuqtada o‘tkazilgan *urinma va normal* tenglamalarini yozing.

Yechish. $x_0 = 2 \Rightarrow y_0 = 1$, *M(2,1)*

$$y' = 8((4 + x^2)^{-1})' = -8(4 + x^2)^{-2} \cdot 2x = -\frac{16x}{(4+x^2)^2}$$

$$k = y'(2) = -\frac{1}{2}$$

$$y - 1 = -\frac{1}{2}(x - 2), \quad 2y - 2 = -x + 2, \quad x + 2y - 4 = 0;$$

$$y - 1 = 2(x - 2), \qquad 2x - y - 3 = 0.$$

Teskari trigonometric funksiyalarning hosilalari

$(arcsinu)' = \frac{u'}{\sqrt{1-u^2}}$; $(arccosu)' = -\frac{u'}{\sqrt{1-u^2}}$;

$(arctgu)' = \frac{u'}{1+u^2}$; $(arcctgu)' = -\frac{u'}{1+u^2}$.

Misollar. 1) $= \arccos\frac{4}{x}$, $y' = ?$

Yechish. $y' = -\frac{1}{\sqrt{1-\left(\frac{4}{x}\right)^2}} \cdot \left(\frac{4}{x}\right)' = -\frac{x}{\sqrt{x^2-16}} \cdot \left(-\frac{4}{x^2}\right) = \frac{4}{x\sqrt{x^2-16}}$

2) $y = arcctg(e^{3x})$, $y' = ?$

Yechish. $y' = -\frac{1}{1+(e^{3x})^2} \cdot (e^{3x})' = -\frac{3e^{3x}}{1+e^{6x}}$

Yuqori tartibli hosilalar

2-tartibli hosila: $(y')' = y'' = \frac{d^2y}{dx^2} = f''(x)$

3-tartibli hosila: $(y'')' = y''' = \frac{d^3y}{dx^3} = f'''(x)$ va h.k.

Misollar. 1) $y = tgx$, $y'' = ?$

Yechish. $y' = \frac{1}{cos^2x}$

$$y'' = \left(\frac{1}{cos^2x}\right)' = \left((cosx)^{-2}\right)' = -2(cosx)^{-3}(-sinx) = \frac{2sinx}{cos^3x}$$

Oshkormas funksiyaning hosilasi

Misollar. $x = y + arcctgy\ ,\ \ y' = ?$

Yechish. $x' = y' + (arcctgy)'$

$$1 = y' - \frac{1}{1+y^2} \cdot y'\ ,\ \ 1 = y'\left(1 - \frac{1}{1+y^2}\right),\ \ y' = \frac{1+y^2}{y^2}$$

Parametrik ko‘rinishda berilgan funksiyaning hosilasi

$$\begin{cases} x = f(t) \\ y = \varphi(t) \end{cases}, \quad \frac{dy}{dx} = \frac{y'_t}{x'_t}, \quad \frac{d^2y}{dx^2} = \frac{y''_{tt}x'_t - x''_{tt}y'_t}{(x'_t)^3}$$

Misollar. $\begin{cases} x = a(t - sint) \\ y = a(1 - cost) \end{cases}, \quad \frac{dy}{dx} = ?, \quad \frac{d^2y}{dx^2} = ?$

Yechish. $x'_t = a(1 - cost)$, $x''_{tt} = asint$

$y'_t = asint$, $y''_{tt} = acost$

$$\frac{dy}{dx} = \frac{asint}{a(1-cost)} = \frac{sint}{1-cost},$$

$$\frac{d^2y}{dx^2} = \frac{acosta(1-cost) - asintasint}{a^3(1-cost)^3} = \frac{cost-1}{a^2(1-cost)^3} = -\frac{1}{a^2(1-cost)^2}$$

Mustaqil yechish uchun misollar

1) $y = (1 + \sqrt[3]{x})^2$

2) $y = x^2 tgx$

3) $y = \frac{cosx}{1+2sinx}$

4) $y = tg^3 x - 3tgx + 3x$

5) $y = e^{\frac{x}{a}} cos\frac{x}{a}$

6) $y = \ln(1 + cosx)$

7) $y = arctg\sqrt{6x-1}$

8) $y = ctgx, \ \ y'' = ?$

9) $e^{y} - e^{-x} + xy = 0$

10) $\begin{cases} x = 2t - 1 \\ y = 1 - 4t^{2} \end{cases}$

11) $y^{2}sinx + x^{2}\sqrt{y} - 3xy = 0$

12) $ln(x^{2}y + xy^{2}) + tgxy = 0$

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