

## MATHEMATICAL MODELING OF TRACTOR MOVEMENT ON THE SLOPE

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### ABSTRACT

*Researching issues of the controllability and stability of wheeled tractors on slopes with road irregularities using mathematical modeling and optimal control are considered in the paper. The static transverse stability at the limiting angle of the slope is pre-determined. In accordance with the calculation scheme, generalized mathematical models in the form of Lagrange equations of the second kind for horizontal and vertical oscillations of a four-wheeled tractor in the process of moving along the road-irregularities on the turning strip of the cotton field are developed. The theory of optimal systems is used to control the movement of the tractor, control purpose and functional in the form of performance evaluation are formulated. The Pontryagin maximum principle was used to study the necessary conditions for optimal control. The conjugate system with variation of design parameters and generalized models under different conditions of tractor movement are investigated by numerical method. The functioning conditions of the tractor and the main parameters arising from vibrations of the tractor in obstacle negotiation on the slopes with ensuring of the stabilization.*

**KEYWORDS:** Wheeled Tractor, Modeling, Optimal Control, Numerical Method, Oscillations

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### INTRODUCTION

In recent years the analysis of the methods of diagnosing and mathematical simulation of technological machines and systems operation has shown that the relevant studies have been conducted within the framework of a single-level structure of research. Such a structure does not provide accurate values of adjusting, damping, stiffness parameters of control and test objects for maintaining transient processes within specified limits due to the absence of interacting subsystems of state and control assessment. The need and relevance of research in this area is proved by the lack of knowledge on the application of multi-level simulation structure and optimal control of dynamic modes in the process of diagnosing the parameters of a four-wheel universal plough tractor [1-6].

### MATHEMATICAL MODELING OF TRACTOR MOVEMENT ON THE SLOPE

When the tractor is moving along the slope and its left and right wheels hit on an obstacle of the height higher than the limit one, it will be affected by an overturning moment leading to unstable equilibrium position (Figure 1). Later, the tractor may overturn on its side around the outer edge of the rear wheel tire as a result of displacement of its center of gravity. As is known, the condition for maintaining the stability of the tractor is achieved by the ratio [7, 8]:

$$M_{stb} \geq M_{ot} ,$$

Where  $M_{stb}$  is the moment of the forces tending to return the tractor to its original position, N·m;  $M_{ot}$  is the



Table 1: Calculated Reaction Value of the Left and Right Wheels of the Tractor

No.	Angle of slope	Weight on the left wheel of machine, kg	Left side reaction $R_A=R_L$ , N	Weight on the right wheel of machine, kg	Right side reaction $R_B=R_R$ , N
1.	$\beta = 0$	$\sin \beta = 0$ $\cos \beta = 1$	2375	23298.75	2375
2.	$\beta = 10^\circ$	$\sin \beta = 0.1736$ $\cos \beta = 0.9848$	2797.156	27440.1	1880.683
3.	$\beta = 20^\circ$	$\sin \beta = 0.3420$ $\cos \beta = 0.9397$	3134.32	30747.7	1329.215
4.	$\beta = 30^\circ$	$\sin \beta = 0.5$ $\cos \beta = 0.866$	3376.25	33121.05	737.365

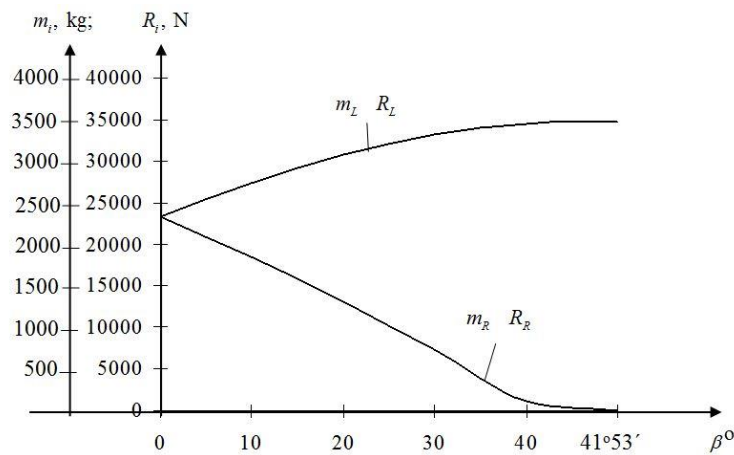


Figure 2: Graph of the Reactions of the Left and Right Wheels of the Tractor.

In accordance with the design scheme given in Figure 1, let's build a generalized mathematical model of horizontal and vertical oscillations of a 4-wheeled tractor in the process of moving over the roughness on the headland of the cotton field in the form of the Lagrange equations of the second kind [1, 2, 9, 10]:

- For Horizontal Oscillations

$$\left. \begin{aligned}
 m_t \ddot{x}_t &= F_x - b_1(\dot{x}_t - \dot{x}_{Lfw}) - c_1(x_t - x_{Lrw}) - b_2(\dot{x}_t - \dot{x}_{Rrw}) - c_2(x_t - x_{Rrw}) - \\
 &\quad - b_3(\dot{x}_t - \dot{x}_{Lfw}) - c_3(x_t - x_{Lfw}) - b_4(\dot{x}_t - \dot{x}_{Rfw}) - c_4(x_t - x_{Rfw}) \\
 m_{Lrw} \ddot{x}_{Lrw} &= b_1(\dot{x}_t - \dot{x}_{Lrw}) + c_1(x_t - x_{Lrw}) - m_{Lrw} \frac{2\pi^2 V_{Lrw}^2}{l_{gr}^2} h_s \sin \frac{2\pi V_{Lrw}}{l_{gr}} t \\
 m_{Rrw} \ddot{x}_{Rrw} &= b_2(\dot{x}_t - \dot{x}_{Rrw}) + c_2(x_t - x_{Rrw}) - m_{Rrw} \frac{2\pi^2 V_{Rrw}^2}{l_{gr}^2} h_s \sin \frac{2\pi V_{Rrw}}{l_{gr}} t \\
 m_{Lfw} \ddot{x}_{Lfw} &= b_3(\dot{x}_t - \dot{x}_{Lfw}) + c_3(x_t - x_{Lfw}) - m_{Lfw} \frac{2\pi^2 V_{Lfw}^2}{l_{gr}^2} h_s \sin \frac{2\pi V_{Lfw}}{l_{gr}} t \\
 m_{Rfw} \ddot{x}_{Rfw} &= b_4(\dot{x}_t - \dot{x}_{Rfw}) + c_4(x_t - x_{Rfw}) - m_{Rfw} \frac{2\pi^2 V_{Rfw}^2}{l_{gr}^2} h_s \sin \frac{2\pi V_{Rfw}}{l_{gr}} t
 \end{aligned} \right\} \tag{1a}$$

- For Vertical Oscillations

$$\left. \begin{aligned}
 m_t \ddot{y}_t &= F_y - b_1 \frac{B}{2} (\dot{\varphi}_t - \dot{\varphi}_{Lrw}) - c_1 \frac{B}{2} (\varphi_t - \varphi_{Lrw}) - b_2 \frac{B}{2} (\dot{\varphi}_t - \dot{\varphi}_{Rrw}) - c_2 \frac{B}{2} (\varphi_t - \varphi_{Rrw}) - \\
 &\quad - b_3 \frac{B}{2} (\dot{\varphi}_t - \dot{\varphi}_{Lfw}) - c_3 \frac{B}{2} (\varphi_t - \varphi_{Lfw}) - b_4 \frac{B}{2} (\dot{\varphi}_t - \dot{\varphi}_{Rfw}) - c_4 \frac{B}{2} (\varphi_t - \varphi_{Rfw}) \\
 m_{Lrw} \ddot{y}_{Lrw} &= b_1 \frac{B}{2} (\dot{\varphi}_t - \dot{\varphi}) + c_1 \frac{B}{2} (\varphi_t - \varphi_{Lrw}) - m_{Lrw} \frac{2\pi^2 V_{Lrw}^2}{l_{gr}^2} h_s (1 - \cos \frac{2\pi V_{Lrw}}{l_{gr}} t) \\
 m_{Rrw} \ddot{y}_{Rrw} &= b_2 \frac{B}{2} (\dot{\varphi}_t - \dot{\varphi}_{Rrw}) + c_2 \frac{B}{2} (\varphi_t - \varphi_{Rrw}) - m_{Rrw} \frac{2\pi^2 V_{Rrw}^2}{l_{gr}^2} h_s (1 - \cos \frac{2\pi V_{Rrw}}{l_{gr}} t) \\
 m_{Lfw} \ddot{y}_{Lfw} &= b_3 \frac{B}{2} (\dot{\varphi}_t - \dot{\varphi}_{Lfw}) + c_3 \frac{B}{2} (\varphi_t - \varphi_{Lfw}) - m_{Lfw} \frac{2\pi^2 V_{Lfw}^2}{l_{gr}^2} h_s (1 - \cos \frac{2\pi V_{Lfw}}{l_{gr}} t) \\
 m_{Rfw} \ddot{y}_{Rfw} &= b_4 \frac{B}{2} (\dot{\varphi}_t - \dot{\varphi}_{Rfw}) + c_4 \frac{B}{2} (\varphi_t - \varphi_{Rfw}) - m_{Rfw} \frac{2\pi^2 V_{Rfw}^2}{l_{gr}^2} h_n (1 - \cos \frac{2\pi V_{Rfw}}{l_{gr}} t) \\
 j_t \ddot{\varphi}_t &= e m_{Lws} \ddot{y}_t - h_c k_c m_{Rws} \ddot{y}_t \sin \frac{2\pi V_t}{l_{gr}} t
 \end{aligned} \right\} \tag{1b}$$

Where  $F_x, F_y$  - tractor pulling forces;  $\dot{x}_i, \ddot{x}_i, \dot{y}_i, \ddot{y}_i$  - linear speeds and acceleration of tractor for front and rear wheels of the tractor under horizontal oscillation and vertical oscillations;  $\dot{\varphi}_i, \ddot{\varphi}_i$  - angular speeds and acceleration of the front and rear wheels of the tractor;  $V_i$  - tractor speed and its wheels under horizontal and vertical oscillation;  $b_i, c_i$  - coefficients of viscous resistance and rigidity of the wheel tire of a tractor;  $m_i$  - distributed mass on supports of tractor wheels;  $h_s$  - height of the road roughness;  $l_{gr}$  - distance between the support and road roughness;  $e$  - shoulder tire bend,  $e = b_i / 6$ ;  $b_i$  - tire width;  $h_c$  - height of the tractor center of mass;  $k_c$  - adhesion coefficient of the wheels.

### OPTIMAL CONTROL OF TRACTOR MOTION ON A SLOPE

To solve the problem, the theory of optimal systems is used. The statement of the optimal control problem is as follows.

At the initial time, the test object is in the following state:

$$q_i(0) = q_0(0), \quad \dot{q}_i(0) = \dot{q}_0(0), \quad V_i(0) = V_0(0). \tag{2}$$

It is required to choose such a control  $u(t)$ , which will transfer the test object to a predetermined final state

$$q_i(t) = q_0(t), \quad \dot{q}_i(t) = \dot{q}_0(t), \quad V_i(t) = V_0(t) \quad (i = \overline{1, n}), \quad 0 \leq t \leq T. \tag{3}$$

Time of the transition process should be the shortest. The goal of control is reduced to minimizing the functional [11, 12].

$$J(q_0, u(t), q(t)) = \int_{t_0}^T f^0(q(t), u(t), t) dt + g^0(q_0, g(T)) \tag{4}$$

At the conditions equations (2)-(5)

$$\dot{q}(t) = f(q(t), u(t), t). \tag{5}$$

Let the functions be set as:

$$g^i(q_0, q(T)) \leq 0, \quad i=1, \dots, m; \quad g^i(q_0, q(T)) = 0, \quad i=m+1, \dots, s, \tag{6}$$

$$u \in U, \quad t_0 \leq t \leq T, \tag{7}$$

Where  $f(q(t), u(t), t)$  is the continuously differentiable function with its derivatives;  $u(t)$  is a piecewise continuous function on the interval  $[t_0, T]$ .

In machine testing under specified operating conditions, the performance criterion can be an evaluation of the speed of operation.

To study necessary conditions for optimal control, the Pontryagin maximum principle [11, 12] is used.

To formulate the maximum principle [13, 14], we introduce the Hamilton – Pontryagin function

$$H = (q, u, t, \psi_i, \psi_0) = -f^0(q, u, t) + \langle \psi, u \rangle \tag{8}$$

And a conjugated system for horizontal oscillations:

$$\left. \begin{aligned} \frac{d\psi_1}{dt} &= -\frac{\partial H_t}{\partial x_1} = -m_i^{-1}(c_1 + c_2 + c_3 + c_4)\psi_2, \\ \frac{d\psi_2}{dt} &= -\frac{\partial H_t}{\partial x_2} = -\psi_1 + m_t^{-1}(b_1 + b_2 + b_3 + b_4)\psi_2 \\ \frac{d\psi_1}{dt} &= -\frac{\partial H_{Lrw}}{\partial x_3} = -m_{Lrw}^{-1}c_1\psi_2, & \frac{d\psi_2}{dt} &= -\frac{\partial H_{Lrw}}{\partial x_4} = -\psi_1 + m_{Lrw}^{-1}b_1\psi_2 \\ \frac{d\psi_1}{dt} &= -\frac{\partial H_{Rrw}}{\partial x_5} = -m_{Rrw}^{-1}c_2\psi_2, & \frac{d\psi_2}{dt} &= -\frac{\partial H_{Rrw}}{\partial x_6} = -\psi_1 + m_{Rrw}^{-1}b_2\psi_2 \\ \frac{d\psi_1}{dt} &= -\frac{\partial H_{Lfw}}{\partial x_7} = -m_{Lfw}^{-1}c_3\psi_2, & \frac{d\psi_2}{dt} &= -\frac{\partial H_{Lfw}}{\partial x_8} = -\psi_1 + m_{Lfw}^{-1}b_3\psi_2 \\ \frac{d\psi_1}{dt} &= -\frac{\partial H_{Rfw}}{\partial x_9} = -m_{Rfw}^{-1}c_4\psi_2, & \frac{d\psi_2}{dt} &= -\frac{\partial H_{Rfw}}{\partial x_{10}} = -\psi_1 + m_{Rfw}^{-1}b_4\psi_2 \end{aligned} \right\} \tag{9}$$

With restriction on control  $|u| \leq 1$ .

To solve the problem under consideration, the following necessary condition must be met:

$$H(q_i(t), u(t), t, \psi_i, \psi_0) = \max_{u \in U} H(q_i(t), u, t, \psi_i(t), \psi_0). \tag{10}$$

Proceeding to determining the optimal control of tractor based on (8), the following function for horizontal oscillations is formed as:

$$\left. \begin{aligned} x_t &= x_1, \dot{x}_t = x_2, \dot{x}_2 = u_x - m_t^{-1}[b_1(x_2 - x_4) - c_1(x_1 - x_3) - \\ &\quad - b_2(x_2 - x_6) - c_2(x_1 - x_5) - b_3(x_2 - x_8) - c_3(x_1 - x_7) - \\ &\quad - b_4(x_2 - x_{10}) - c_4(x_1 - x_9)] \\ x_{Lrw} &= x_3, \dot{x}_{Lrw} = x_4, \dot{x}_4 = m_{Lrw}^{-1}[b_1(x_2 - x_4) + c_1(x_1 - x_3)] - u_1 \\ x_{Rrw} &= x_5, \dot{x}_{Rrw} = x_6, \dot{x}_6 = m_{Rrw}^{-1}[b_2(x_2 - x_6) + c_2(x_1 - x_5)] - u_2 \\ x_{Lfw} &= x_7, \dot{x}_{Lfw} = x_8, \dot{x}_8 = m_{Lfw}^{-1}[b_3(x_2 - x_8) + c_3(x_1 - x_7)] - u_3 \\ x_{Rfw} &= x_9, \dot{x}_{Rfw} = x_{10}, \dot{x}_{10} = m_{Rfw}^{-1}[b_4(x_2 - x_{10}) + c_4(x_1 - x_9)] - u_4 \end{aligned} \right\} \tag{11}$$

If  $f^0 \equiv 1, g^0 \equiv 0$ , then  $J(q_0, u(t), q(t)) = T - t_0$ , in this case the problem presented by equations (4)-(7) is called the problem of operation speed.

The object under consideration is a stationary system and the problem (4) means that  $f$  and  $U$  do not explicitly depend on time, i.e.

$$f(t, q, u) = f(q, u), \quad U(t) = U \tag{12}$$

If the stationary problems (4), (12) have an optimal control  $u(t)$  and an optimal path  $q_0(t)$ , then there exists a non-zero vector of conjugate variables  $(\psi_1(t), \psi_2(t)), \psi(t) \in R^n$  satisfying conditions by equation (2), i.e. the maximum condition is satisfied equation (10)

$$\psi_0(t) = const \leq 0. \tag{13}$$

Since conjugate systems (9) are homogeneous relative to  $\psi_i$ , a constant in equation (13) can be arbitrarily chosen as:

$$\psi_0(t) = -1 \quad 0 \leq t \leq T \tag{14}$$

From the condition  $\max_{|u| \leq 1} H$  follows  $u = sign\psi_2$  at  $\psi_2 \neq 0$ , then the boundary value problem of the maximum principle for horizontal oscillations is written in the following form:

$$\left. \begin{aligned} \dot{x}_2 &= sign\psi_2 - m_t^{-1} [b_1(x_2 - x_4) - c_1(x_1 - x_3) - \\ &\quad - b_2(x_2 - x_6) - c_2(x_1 - x_5) - b_3(x_2 - x_8) - \\ &\quad - c_3(x_1 - x_7) - b_4(x_2 - x_{10}) - c_4(x_1 - x_9)] \\ \dot{x}_4 &= m_{Lrw}^{-1} [b_1(x_2 - x_4) + c_1(x_1 - x_3)] - sign\psi_2 \\ \dot{x}_6 &= m_{Rrw}^{-1} [b_2(x_2 - x_6) + c_2(x_1 - x_5)] - sign\psi_2 \\ \dot{x}_8 &= m_{Lfw}^{-1} [b_3(x_2 - x_8) + c_3(x_1 - x_7)] - sign\psi_2 \\ \dot{x}_{10} &= m_{Rfw}^{-1} [b_4(x_2 - x_{10}) + c_4(x_1 - x_9)] - sign\psi_2 \end{aligned} \right\}. \tag{15}$$

The boundary value problem of the maximum principle in this case consists of system equation (15), boundary conditions (2) and (3) resulting from equation (10), and condition (14).

Hamilton-Pontryagin function for horizontal oscillations was composed. Condition (10) will mark out the function  $u = sign\psi_2, \psi_2 \neq 0$ . The boundary value problem (15) in this case has the form

$$H_i = -f^0 u + \psi_2(t) u_t. \tag{16}$$

Proceed to investigate equation (9), (16) in the area of equation (17)

$$u_k = sign\psi_2(t) = \begin{cases} 1, & \psi_2(t) > 1 \\ -1, & \psi_2(t) < -1 \end{cases}, \quad k=2,4,\dots,2n, \tag{17}$$

That is, the control  $u_k(t)$  can have only one switch point [11-14].

Thus, from the Pontryagin maximum principle, we obtain the structure of optimal control of motion of the tractor guide wheels.

To determine the auxiliary functions (9), the conjugate system with variation in design parameters  $b_i, c_i, m_i$  has been investigated by a numerical method.

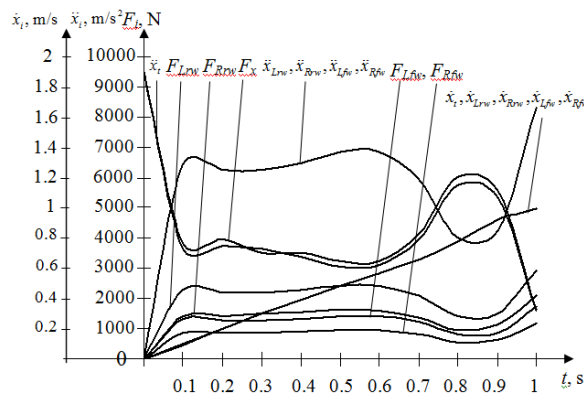
As a result, graphical dependences of the rates and accelerations of tractor oscillations, the maximum values of the H-function have been obtained and presented on Figures 3, 4.

**RESULTS AND DISCUSSION**

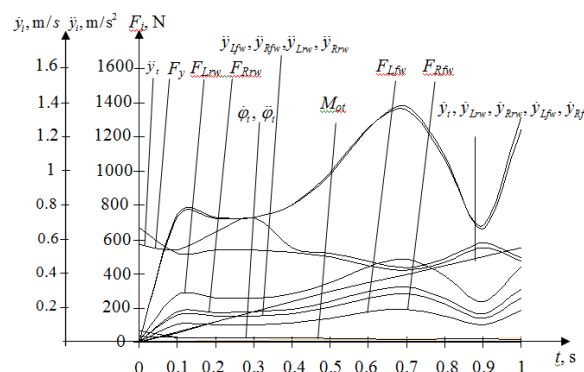
Systems (1a), (1b), (9), (15) are solved using the Runge–Kutta numerical methods. The control  $u_k(t)$ , bringing the maximum of function (10), is defined in area (18).

Computational experiment has been conducted at the following parameters:

$h_t=10 \text{ mm}=0.01\text{m}$  - tire deflection;  $F_x = 7500 \text{ N}; F_y = 5250 \text{ N}; c_1=c_{Lrw}=1476216.346 \text{ N/m}; b_1=b_{Lrw}=109016.25 \text{ N}\cdot\text{s/m}; c_2=c_{Rrw}=1474976.96 \text{ N/m}; b_2=b_{Rrw}=108924.727 \text{ N}\cdot\text{s/m}; c_3=c_{Lfw}=858921.76 \text{ N/m}; b_3=b_{Lfw}=63430.02 \text{ N}\cdot\text{s/m}; c_4=c_{Rfw}=860402.8 \text{ N/m}; b_4=b_{Rfw}=63539.39 \text{ N}\cdot\text{s/m}; m_t=4750 \text{ kg}; m_{Lfw}=1760.625 \text{ kg}; m_{Rrw}=1178.8 \text{ kg}; m_{Lfw}=1030.531 \text{ kg}; m_{Rfw}=692.883 \text{ kg}; m_{Lws}=2797.156 \text{ kg}; m_{Rws}=1880.683\text{kg}; r_{rw}=0.785 \text{ m}; r_{fw}=0.43 \text{ m}; h_s =0.07 \text{ m}; h_c=0.94 \text{ m}; e=0.0656 \text{ m}; k_c=0.04; b_t =0.394 \text{ m};  $\beta=10^0$ ;  $V=1 \text{ m/s}$ .$



**Figure 3: Change Pattern of the Parameters of Motion of a Four-Wheeled Tractor Under Horizontal Oscillations at  $h_t=10 \text{ mm}, \beta = 10^0$ .**



**Figure 4: Change Pattern of the Parameters of Motion of a Four-Wheeled Tractor Under Vertical Oscillations at  $h_t=10 \text{ mm}, \beta = 10^0$ .**

## CONCLUSIONS

- Thus, the smoothness of tractor motion depends on the mass and parameters of the controlled axes, the values of which are determined by numerical solution of the systems (1a), (1b) and the conjugate system (9) with variation in motion parameters  $F_i$  and design parameters  $b_i, c_i, m_i$  for given road roughness.
- The load on the side, located downward the slope, increases, and the load on the other side decreases. The redistribution of normal reactions along the sides depends not only on the machine gauge, but on the vertical coordinate of the center of mass as well. Machine is in a statically critical state when the reaction of the unloaded side is zero, i.e.  $R_B = 0$
- When determining the tractor stability against overturning, the limit angle of the slope is calculated, on which the braking tractor cannot be tipped, and the angle at which the tractor does not slip. The perfection of tractor design when moving down a slope under a load is characterized not only by lateral stability, but by the maximum slope angle of controllability as well.
- To obtain the operating condition of the tractor at different states of equilibrium, it is necessary to determine the principal parameters, such as: driving forces, tractor mass ratio, elastic-dissipative forces and resistance moments arising from tractor oscillations while overcoming the road roughness.
- When taking into account the stability of the tractor in an inclined state, it is necessary to meet the condition of maintaining stability  $M_{stb} \geq M_{or}$ . The limit position occurs at the moment when the vertical plane passing through the center of mass crosses the axis of tipping. The static lateral stability of the tractor is estimated by the limit angle of the slope when the tractor is at rest. The maximum angle of slope is estimated as equal to  $\beta = 41^\circ 53'$ .

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