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Optimal Control of Grinding Processes of Non-Rigid Shafts in Elastically Deformed State

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Optimal Control of Grinding Processes of Non-Rigid Shafts in Elastically Deformed State

B Azimov, L Sulyukova

Scientific and Innovation Centre of Information and Communication Technologies under Tashkent IT University, Kichik halka yuli st., 2, Tashkent, Uzbekistan
E-mail: slf72@yandex.com

Abstract. Application issues of modern methods and algorithms for optimal control of the process of technological system functioning in dynamic modes ensuring the efficiency of design of technological process of grinding for elastically deformed non-rigid shafts are considered in the paper. Necessary conditions for optimal control of technological system under consideration are investigated using the Pontryagin maximum principle. The influence of changes in moments of cutting forces on parameters of parts handling and transient process time are determined. The values of tensile forces and shaft deflections along the sections in the process of handling are determined. Graphic dependencies characterizing the parameters of processing to control technological system of grinding of non-rigid shafts based on the change in their elastically strained state are presented. Developed mathematical models and optimization of parameters in control of elastically strained state of processed parts ensure an increase in precision of handling and in surface quality of the part by an order of magnitude in comparison with previous developments.

1. Introduction

These guidelines, written in the style of a submission to *J. Phys.: Conf. Ser.*, show the best layout for your paper using Microsoft Word. If you don't wish to use the Word template provided, please use the following page setup measurements. Creation of competitive products in machine-building is to the greatest extent ensured by the precision of its manufacturing; the most important from the point of view of required precision of the product is the process of parts manufacturing. An increase in precision requirements for manufacturing parts is explained by the tendency to improve the quality of modern competitive machines and by the aim to improve technical characteristics of machine, improve its reliability, durability, geometric and dynamic precision parameters; all that is impossible without consideration of dynamic properties of technological system [1].

Circular external grinding is one of the most common processes used in the finishing stage of processing of shafts (stepped, crank, distributive ones) and its study is quite relevant.

Macrogeometric indices of the surface in processes of circular cutting grinding have not been sufficiently studied, taking into account the creation of modern software packages for research and modelling of dynamic systems. The most important aspect of such studies is the possibility of observing transient processes in the cycle of cutting grinding and accompanying dynamic phenomena, which makes it possible to detect conditions for eliminating or reducing their negative influence on the quality of processed surface.

Control of grinding of low-rigidity shafts processing allows, with corresponding mathematical model, to increase the precision of dimensions and shapes of processed products, to improve technical and economic parameters of processing and to improve the reliability of normal operation of technological system (TS). Therefore, one of the primary tasks is to develop a mathematical description of TS, which functions in dynamic regimes.

2. Construction of kinematic scheme and dynamic model

The most common is the grinding of external cylindrical surfaces, which is most often done on circular grinding machines.

The scheme of this grinding is shown in Figure 1: here, the grinding wheel 1 rotates at high speed, and processed shaft 2 rotates at a speed 60-100 times less than the speed of the grinding wheel [2-4].

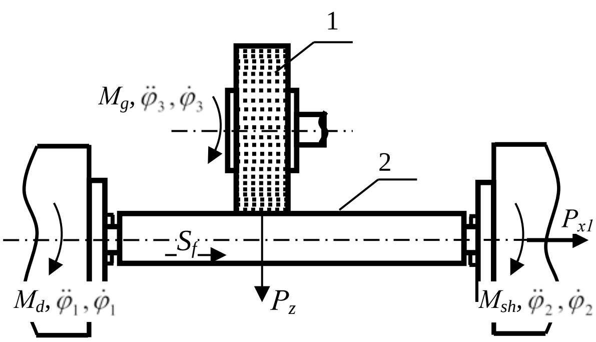

Figure 1. Kinematic scheme of circular external grinding: 1– grinding wheel; 2 – processed shaft.

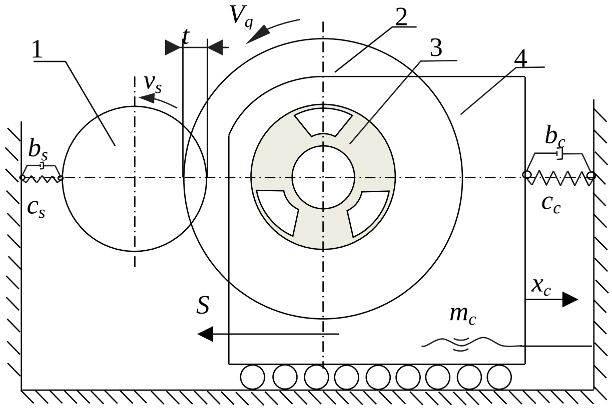

Figure 2. Circular grinding machine: 1– processed shaft; 2 – grinding wheel; 3 – grinding head; 4 – grinding machine bed.

For modelling the process of forming the macro relief of a grinding part, it is necessary to have a model of a circular grinding machine with a cutting and an external action in the form of rough surface of the part. The grinding machine is shown in Figure 2.

3. Development of mathematical model and solution of the problem of optimal control of the process of parts grinding

To achieve the goal, a mathematical model of technological system (TS) of grinding-polishing process for handling the parts of low rigidity is built using the Lagrange equation of the second kind [5-7].

$$\left.\begin{aligned} j_1\ddot{\varphi}_1 &= M_d - b_{sh}(\dot{\varphi}_1 - \dot{\varphi}_2) - c_{sh}(\varphi_1 - \varphi_2) \\ j_2\ddot{\varphi}_2 &= b_{sh}(\dot{\varphi}_1 - \dot{\varphi}_2) + c_{sh}(\varphi_1 - \varphi_2) - k_{kf}\cdot j_3\ddot{\varphi}_2 \\ j_3\ddot{\varphi}_3 &= k_{rf}\cdot\left(b_c\cdot\dot{x}_c + c_c(\Delta x_c - x_c)\right) - \frac{j_2\ddot{\varphi}_2}{r_{sh}}\cos\dot{x}_c t \end{aligned}\right\} \quad (1)$$

where j_1, j_2, j_3 – inertial moments of rotating mass TS, $N{\cdot}m{\cdot}s^2$; $\ddot{\varphi}_1, \ddot{\varphi}_2, \ddot{\varphi}_3$ – angular accelerations of TS rotating mass in the processing, s^{-2}; $\dot{\varphi}_1, \dot{\varphi}_2, \dot{\varphi}_3$ – angular rates of TS rotating mass in the processing, s^{-1}; φ_1, φ_2 – angular displacements of TS rotating mass in the processing, *rad*; b_s, b_c – coefficients of viscous resistance of processed shaft and machine bed, $N{\cdot}m{\cdot}s/rad$; c_s, c_c – rigidity coefficients of processed shaft and machine bed, $N{\cdot}m/rad$; $M_{sh} = j_2\ddot{\varphi}_2$, $N{\cdot}m$; k_{kf}; k_{rf} – coefficients of kinetic friction and rolling friction; $x_c, \dot{x}_c$ – displacement and acceleration of grinding machine bed; M_d, M_g – driving moments of the processed shaft and the grinding wheel, $N{\cdot}m$; r_{sh} – radius of shaft, *m*.

The cutting force in the process of circular grinding can be divided into three components: tangential P_z, radial P_y and axial P_x. [8].

There are similar empirical formulas to determine the forces P_y and P_x. However, to simplify and accelerate the calculations, it is recommended to take the values of forces P_y and P_x according to the following relations:

$$P_y=(0.25\text{-}.0.5)\cdot Pz, \quad P_x=(0.1\text{-}.0.25)\cdot Pz,$$

$$P_y = 0.375 \cdot P_z = 0.375 \cdot 46.74 = 17.53\,\text{N},$$

$$P_x = P_c = 0.175 \cdot P_z = 0.175 \cdot 46.74 = 8.18\,\text{N},$$

At constructing models of elastic lines of parts of low rigidity, and processing them in elastically strained state, bending moments along X axis are taken as the most significant factors, since elastic strains along this axis exert a dominant influence on the errors of shape in longitudinal direction.

For required rigidity, corresponding moment of inertia of processed shaft is determined by solving the conjugate system of the Pontryagin maximum principle.

At modeling it is necessary to consider the interrelation of parameters providing real results on precision of processing that in turn, leads to the solution of a problem of optimum control of technological process.

The main purpose of control of operation process of TS is to determine the best transient processes so that the energy expended during the transient process be minimal, i.e., it is required to select such a control *u(t),* which translates the parameters of motion of a grinding wheel and machined shaft into the given value at minimum time. Then, periodic speed performance in the form of minimization of the functional is taken as the main criterion for evaluating the process of functioning [9,10]

$$J(\varphi_0, u(t), \varphi(t)) = \int_{t_0}^{T} f^0(\varphi(t), u(t), t)dt) \;. \tag{2}$$

At conditions

$$\varphi_i(0) = \varphi_0(0), \qquad \dot{\varphi}_i(0) = \dot{\varphi}_0(0). \tag{3}$$

$$\varphi_i(t) = \varphi_0(t), \quad \dot{\varphi}_i(t) = \dot{\varphi}_0(t), \quad 0 \le t \le T \quad (i = \overline{1,n}) \tag{4}$$

$$\dot{\varphi}(t) = f(\varphi(t), u(t), t), \tag{5}$$

$$u \in U, \quad t_0 \le t \le T, \tag{6}$$

where $f(\ldots)$ – continuously differentiable with its derivatives; $u(t)$ – sectional continuous function on an interval $[t_0,T]$.

To investigate necessary conditions for optimal control of considered TS, the Pontryagin maximum principle is used [9,10].

To formulate the maximum principle, the Hamilton-Pontryagin function for TS is introduced

$$H = (\varphi, u, t, \psi_i, \psi_0) = -f^0(\varphi, u, t) + \langle \psi, u \rangle \tag{7}$$

and conjugate system

$$\left.\begin{aligned} \frac{d\psi_1}{dt} &= -\frac{\partial H_v}{\partial y_1} = -j_2^{-1} c_v \psi_2, \\ \frac{d\psi_2}{dt} &= -\frac{\partial H_v}{\partial y_2} = -\psi_1 + j_2^{-1} b_v \psi_2 \end{aligned}\right\} \tag{8}$$

with limited control $|u| \le 1$.

To solve the problem under consideration the following necessary condition should be satisfied:

$$H(\varphi_i(t), u(t), t, \psi_i, \psi_0) = \max_{u \in U} H(\varphi_i(t), u, t, \psi_i(t), \psi_0)\,. \tag{9}$$

To define optimal control on the basis of (7), the following function is formed as

$$\left.\begin{array}{l}\varphi_1 = y_1, \dot{\varphi}_1 = y_2, \dot{y}_2 = u_\partial - j_1^{-1}\left[b_{sh}(y_2 - y_4) + c_{sh}(y_1 - y_3)\right] \\ \varphi_2 = y_3, \dot{\varphi}_1 = y_4, \dot{y}_4 = j_2^{-1}\left[b_{sh}(y_2 - y_4) + c_{sh}(y_1 - y_3)\right] - u_c \\ \varphi_3 = y_5, \dot{\varphi}_5 = y_6, \dot{y}_6 = u_g - u_{sh}\end{array}\right\}. \quad (10)$$

So, if $f^0 \equiv 1$, so $J(\phi_0, u(t), \phi(t)) = T - t_0$. In this case the task (2)–(6) is called the problem of operating speed.

The object under consideration is a stationary system and problem (4) means that f and U do not depend explicitly on time, i.e.

$$f(t, y, u) = f(y, u), \qquad U(t) = U\ . \quad (11)$$

If stationary problem (4), (11) has an optimal control u(t) and an optimal trajectory $\varphi_0(t)$, then there exists a nonzero vector of conjugate variables $(\psi_1(t),\ \psi_2(t)),\ \ \psi(t) \in R^n$ satisfying conditions (9), that is, the maximum condition is satisfied (7)

$$\psi_0(t) = const \le 0\,. \quad (12)$$

So as the conjugate system (8) is congeneric in relation to ψ_i, constant in an equation (9) can be chosen liberally so that

$$\psi_0(t) = -1 \qquad 0 \le t \le T\ . \quad (13)$$

Then the boundary value problem of the maximum principle is written in the form

$$\left.\begin{array}{l}\varphi_1 = y_1, \dot{\varphi}_1 = y_2, \dot{y}_2 = sign\,\psi_2 - j_1^{-1}\left[b_{sh}(y_2 - y_4) + c_{sh}(y_1 - y_3)\right] \\ \varphi_2 = y_3, \dot{\varphi}_2 = y_4, \dot{y}_4 = j_2^{-1}\left[b_{sh}(y_2 - y_4) + c_{sh}(y_1 - y_3)\right] - sign\,\psi_2 \\ \varphi_3 = y_5, \dot{\varphi}_3 = y_6, \dot{y}_6 = sign\,\psi_2 - sign\,\psi_2\end{array}\right\}. \quad (14)$$

We compose the Hamilton-Pontryagin function, which has the form

$$\left.\begin{array}{l}H_1 = \psi_0 + \psi_1 y_2 + \psi_2 \dot{y}_2 \\ H_2 = \psi_0 + \psi_1 y_4 + \psi_2 \dot{y}_4 \\ H_3 = \psi_0 + \psi_1 y_6 + \psi_2 \dot{y}_6\end{array}\right\}. \quad (15)$$

Hence it is clear that the condition (9) separates the function $u = sign\psi_2,\ \ \psi_2 \ne 0$. The boundary problem (10), (14) consists of

$$H_i = -f^0 u + \psi_2(t) u_d\ . \quad (16)$$

In this case

$$u_i = sign\,\psi_2(t) = \begin{cases}1, & \psi_2(t) > 1 \\ -1, & \psi_2(t) < 1\end{cases},\ \ i = 2,4,\ldots,2n \quad (17)$$

is, the control $u_i(t)$ can have one switching point only.

To determine auxiliary functions (8), a conjugate system with a variation of design parameters b_i, c_i, j_i is studied by a numerical method.

Systems (1), (8), (14) are solved using numerical Runge-Kutta method [11]. The control $u_k(t)$, which delivers the maximum of function (9), is defined in the region (17). The processing of results of the solution of system (8) has shown that the change in moments of inertia and elastic-dissipative forces dramatically changes the function of the variables $\psi_1, \dot{\psi}_1, \psi_2, \dot{\psi}_2$, that is, the motion of processed shaft.

Therefore, to increase the precision of dimensions and shapes of shafts to be processed, it is necessary to determine variables of the conjugate system that ensure normal functioning of TS.

4. Discussion of experimental results

Results of numerical solutions of system (1), presented in table 1 and on figures. 3, 4 make it possible to determine optimum values of parameters of non-rigid shafts processing.

Table 1. Values of parameters of processed shaft functioning

T, s	$\dot{\varphi}_1$, s^{-1}	$\ddot{\varphi}_1$ s^{-2}	M_d, Nm	$\dot{\varphi}_2$, s^{-1}	$\ddot{\varphi}_2$, s^{-2}	M_{sh}, Nm	$\dot{\varphi}_3$, s^{-1}	$\ddot{\varphi}_3$, s^{-2}	M_g, Nm	n_{sh}, rev/min	n_{g}, rev/min
0	0	40.47	22.97	0	-60.76	-0.6	0	117.22	66.93	0	0
0.1	3.87	38.33	21.75	3.87	60.49	0.6	11.65	118.33	67.57	36.99	111.34
0.2	7.74	38.33	21.75	7.74	60.48	0.6	23.3	118.33	67.57	73.98	222.68
0.3	11.61	38.33	21.75	11.61	60.52	0.6	34.96	118.33	67.57	110.97	333.98
0.4	15.48	38.33	21.75	15.48	60.48	0.6	46.6	118.33	67.57	147.97	445.15
0.5	19.36	38.33	21.75	19.36	60.38	0.6	58.25	118.33	67.57	184.96	556.52
0.6	23.23	38.33	21.75	23.23	60.38	0.6	69.2	118.33	67.57	221.95	668.01
0.7	27.1	38.33	21.75	27.1	60.38	0.6	81.6	118.33	67.57	258.95	779.54
0.8	30.97	38.33	21.75	30.97	60.38	0.6	93.26	118.33	67.57	295.94	891.06
0.9	34.86	38.33	21.75	34.86	60.53	0.6	104.93	118.33	67.57	333.06	1002.6
1	38.75	38.33	21.75	38.75	60.53	0.6	116.61	118.33	67.57	370.24	1114.11

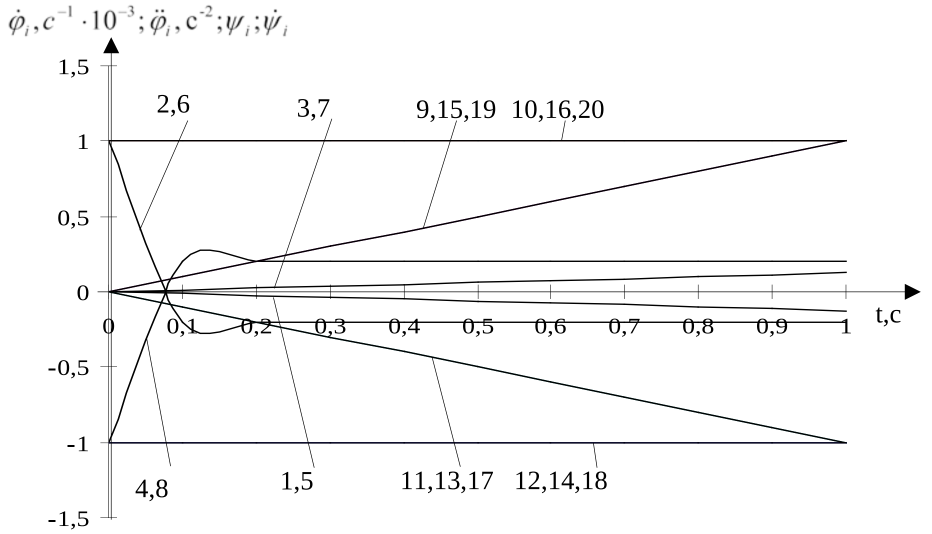

Figure 3. Graphs of the motion parameters change for the processed shaft and the grinding wheel in the transient process, obtained by solving (14) of the maximum principle boundary problem: 1,5,9 - angular velocity $\dot{\varphi}_1, \dot{\varphi}_2, \dot{\varphi}_3$, 2,6,10 - angular accelerations $\ddot{\varphi}_1, \ddot{\varphi}_2, \ddot{\varphi}_3$ and auxiliary functions 13-ψ_1,14-$\dot{\psi}_1$, 17-ψ_2, 18-$\dot{\psi}_2$ by u(t)= +1; 3-$\dot{\varphi}_1$, 7-$\ddot{\varphi}_2$ 11-$\dot{\varphi}_3$ – angular velocity, 4-$\ddot{\varphi}_1$, 8-$\ddot{\varphi}_2$, 12-$\ddot{\varphi}_3$ – angular accelerations and auxiliary functions – 15-ψ_1,16-$\dot{\psi}_1$, 19-ψ_2, 20-$\dot{\psi}_2$ by u(t)= -1.

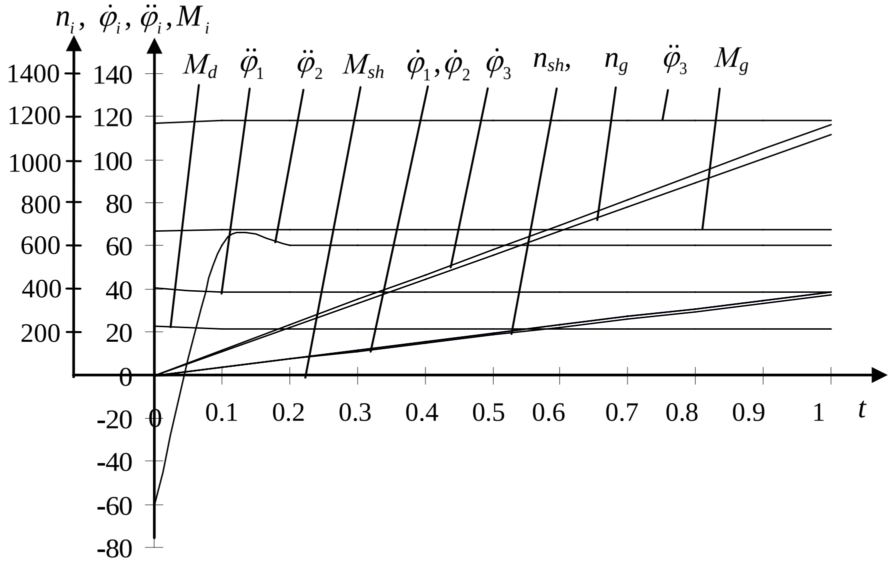

Figure 4. The characterization of the changing parameters of the TS in the shaft grinding process.

The influence of inertia moments and elastic and dissipative forces to changing of in-processing moving shaft were investigated. Change in the moment of inertia of headstock and tailstock of TS significantly affects angular velocities and accelerations of processed shaft [4,5]. To reduce the range of changes in angular velocities and accelerations, a variation in rigidity coefficients and viscous resistance of processed shaft is carried out. As rigidity coefficient increases, the coefficient of viscous resistance of processed shaft increases too. The amplitude of angular velocity of vibrations decreases significantly. This indicates that the amplitude and frequency of vibrations of angular velocities and accelerations of the shaft depend on the moment of inertia and elastic-dissipative forces.

5. Determination of elastic deflections and the character of changes in shaft processing precision of TS functioning

The most expedient direction of problem solution is a control of technological systems by machining of non-rigid parts in an elastically deformed state on the basis of scientifically grounded technological methods of influencing the workpiece.

When controlling TS processing of parts of low rigidity based on the change in their elastically deformed state, separate force or a combination of regulated force actions are used as control actions aimed to compensate force factors from the cutting process: bending moments on supports; control of flexure-torsion forces of strain [3].

To evaluate the possibilities of the method and establish theoretical regularities of behaviour of the detail in longitudinal-transverse bending, the equation of elastic line of a non-rigid shaft is solved [6].

The equations of elastic line in the I and II sections have the form

$$\left.\begin{aligned} y_I &= y_0' kx + y_0''(1-\cos kx) + y_0'''(kx - \sin kx) \\ y_{II} &= y_0' kx + y_0''(1-\cos kx) + y_0'''(kx - \sin kx) + f(x) \end{aligned}\right\} \quad (18)$$

Initial parameters are determined, given the function of the effect of lateral load [6].

Finally, equations of deflections along sections take the form

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$$\left.\begin{aligned} y_I(x) = -\frac{P_y}{P_{x1}}\left[A(1-\cos kl)-(1-\cos\alpha kl)\right]x+\frac{M}{P_{x1}}[B(1-\cos kl)-\sin kl]kx- \\ -\frac{M}{P_{x1}}(1-\cos kx)+\left(\frac{P_y\cdot A}{P_{x1}}-\frac{M\cdot B}{P_{x1}}\right)(kx-\sin kx) \\ y_{II}(x)=y_I(x)-\frac{P_y}{P_{x1}}(x-a)+\frac{P_y}{k\cdot P_{x1}}\sin k(x-a) \end{aligned}\right\}$$

where $A=\dfrac{kl(\alpha-1+\cos\alpha kl)-\sin\alpha kl}{kl\cos kl-\sin kl}$; $B=\dfrac{kl\sin kl+\cos kl-1}{kl\cos kl-\sin kl}$, $\alpha=\dfrac{l-a}{l}$; l - is the length of the part; a is the coordinate of lateral load application.

Based on results of numerical solutions of system (1) and design scheme, the calculations of deflections and precision of shaft processing are presented in the Table 2 and on Figures 5, 6.

The results of the computational experiment are obtained for the following values of rigidity coefficients c_c=2398826.26, viscous resistance b_c =615.32, m_c=560 kg, M_d=66.853, k_{kf} =0.8, k_{rf} =0.004 and Δx_c=0.0006148.

Table 2. Results of calculation of deflections and precision of shaft processing

T, s	$\dot{x}_c$, s^{-1}	$\ddot{x}_c$, s^{-2}	P_c, N	y_0, μm	y_I, μm	y_{II}, μm	y_{def}, μm
0	0	0.01	6.08	-130	3200	3200	3000
0.1	0.001	0.0094	5.3	1.56	-48	-48	-46.6
0.2	0.0019	0.0073	4.12	1.51	-47	-47	-45.4
0.3	0.0025	0.0041	2.318	1.52	-47	-47	-45.6
0.4	0.0029	0.0016	0.093	1.5	-46,7	-46.7	-45.2
0.5	0.0029	-0.004	-2.28	1.65	-50	-50	-48.8
0.6	0.0026	–	-4.54	1.65	-50	-50	-48.8
0.7	0.0019	–	-6.41	1.65	-50	-50	-48.8
0.8	0.0011	–	-7.67	1.47	-46	-46	-44.5
0.9	0.0001	–	-8.17	1.7	-52	-52	-50
1	–	-0.014	-7.85	1.7	-52	-52	-50

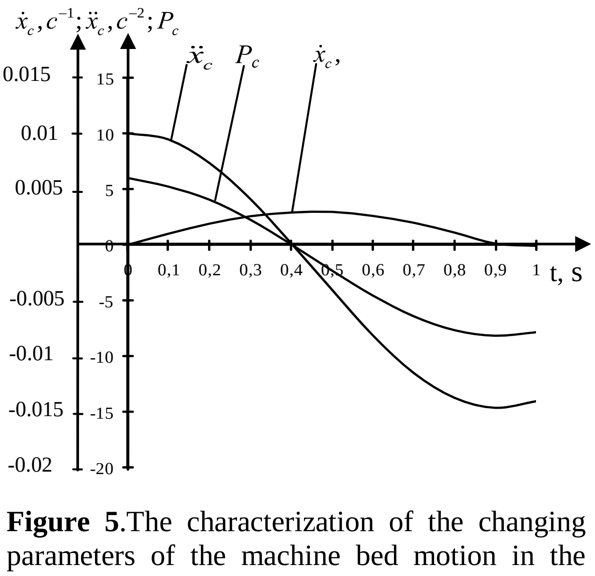

Figure 5.The characterization of the changing parameters of the machine bed motion in the shaft grinding process.

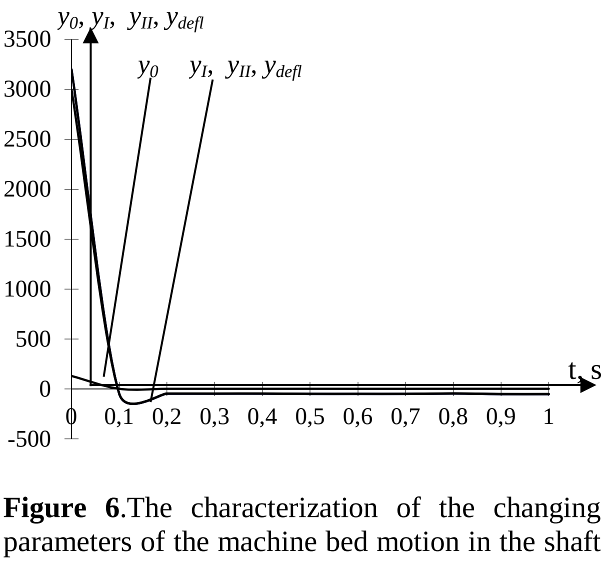

Figure 6.The characterization of the changing parameters of the machine bed motion in the shaft grinding process.

As seen from the graph, the character of changes in velocity, acceleration, displacement corresponds to the character of cutting force changing in along the processed line.

An increase in rigidity due to stretching leads to a decrease in elastic deformations of processed shaft and decrease in transient process; as a result the precision of the shape and quality of processed surface increase.

6. Conclusion

Thus, for given values of moments of inertia of rotating masses and rigidity coefficients and viscous resistance of processed shaft, transient processes for processed shaft are obtained by solving the boundary value problem based on the Pontryagin maximum principle. The variation of in-processing shaft coefficients of rigidity, viscous resistance and stretching forces were performed to reduce the range of angular velocity and accelerations change. Rigidity increase at the expense of stretching led to reduction of in-processing shaft deformation and reduction of transition process.

Mathematical models have been developed to control the precision of processing a shaft of low rigidity, and optimum parameters for an elastically deformed state of a part have been obtained. The regularities of changes in elastic axis of the shaft under the action of tensile forces and bending moments are established. Values of tensile forces, bending moments and deflections of the shaft along sections are determined under processing. As a result of research, the precision of processing is increased by an order of magnitude in comparison with previous developments.

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