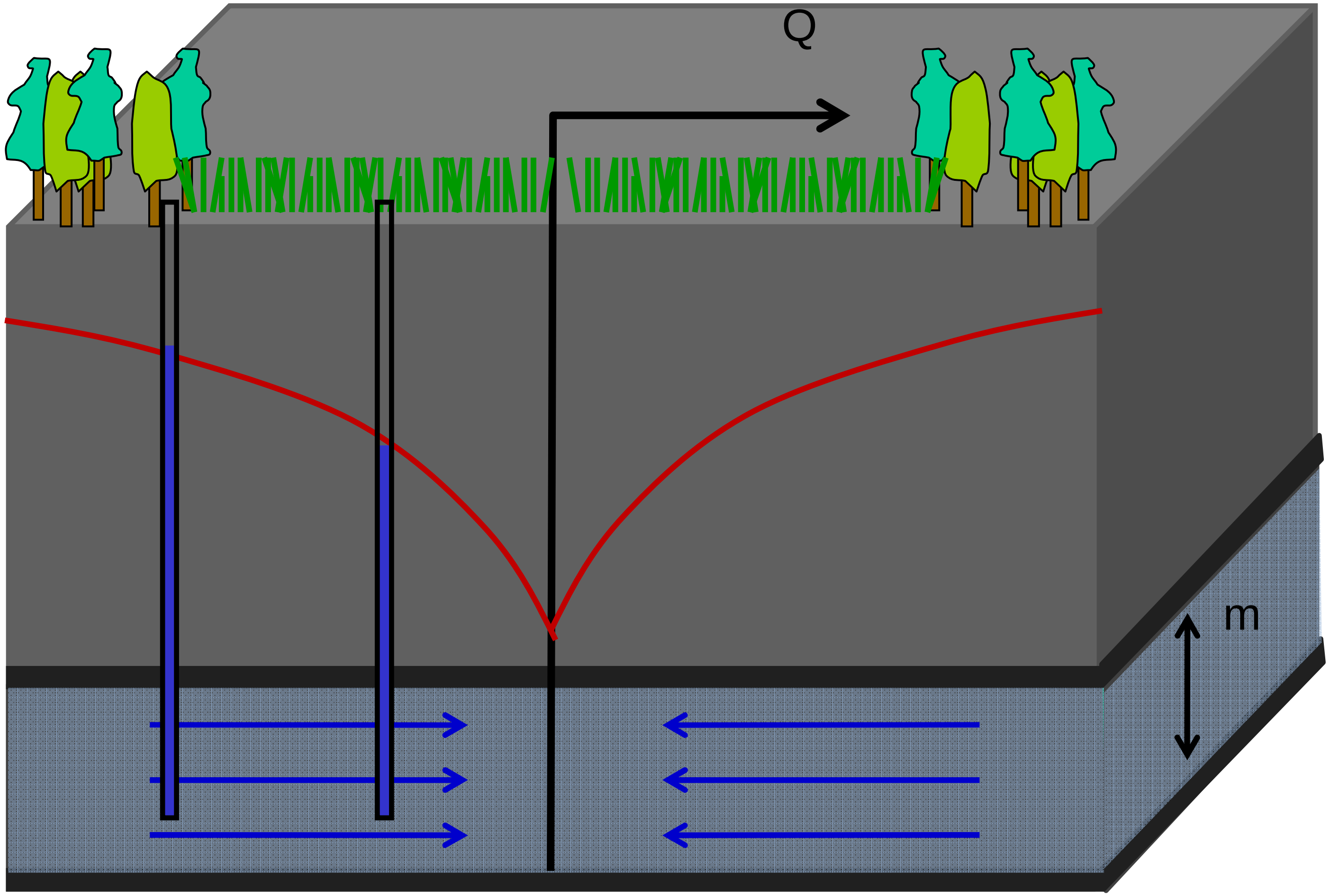
Groundwater Hydraulics

Institute for Fluid Mechanics and Environmental Physics in Civil Engineering, Universität Hannover

Steady state well flow

Radial flow – well hydraulics

Dupuit-Thiem (1906)



Radial flow – well hydraulics

Dupuit-Thiem (1906)

Observation of the piezometric head at to observation wells at distance r_1 and r_2

$$\frac{-Q_w}{2\pi T}\ln(r_1) = h(r_1) + C_1$$

$$\frac{-Q_w}{2\pi T}\ln(r_2) = h(r_2) + C_1$$

Taking the difference gives

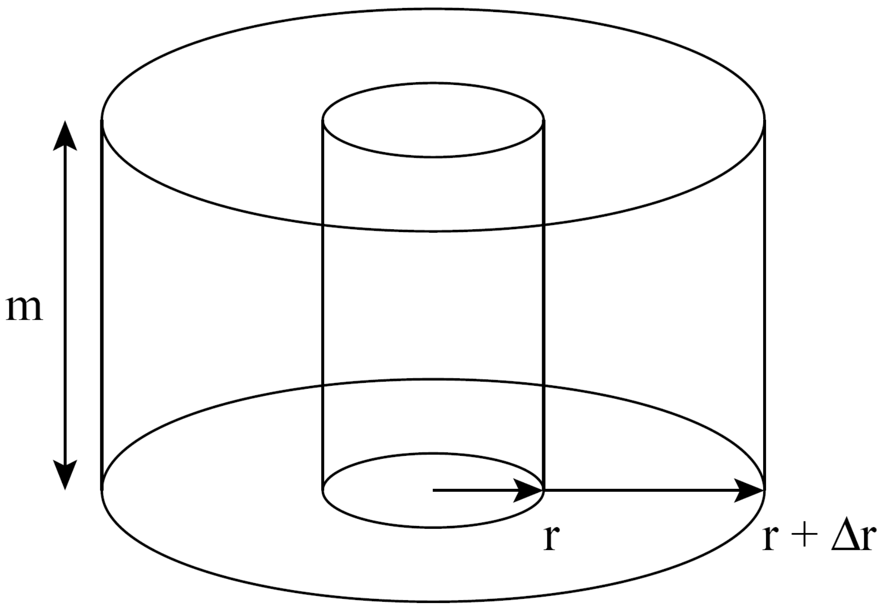
$$\boxed{h(r_1) - h(r_2) = \frac{Q_w}{2\pi T}\ln\left(\frac{r_2}{r_1}\right)}$$

Can be used to determine the transmissivity of an aquifer.

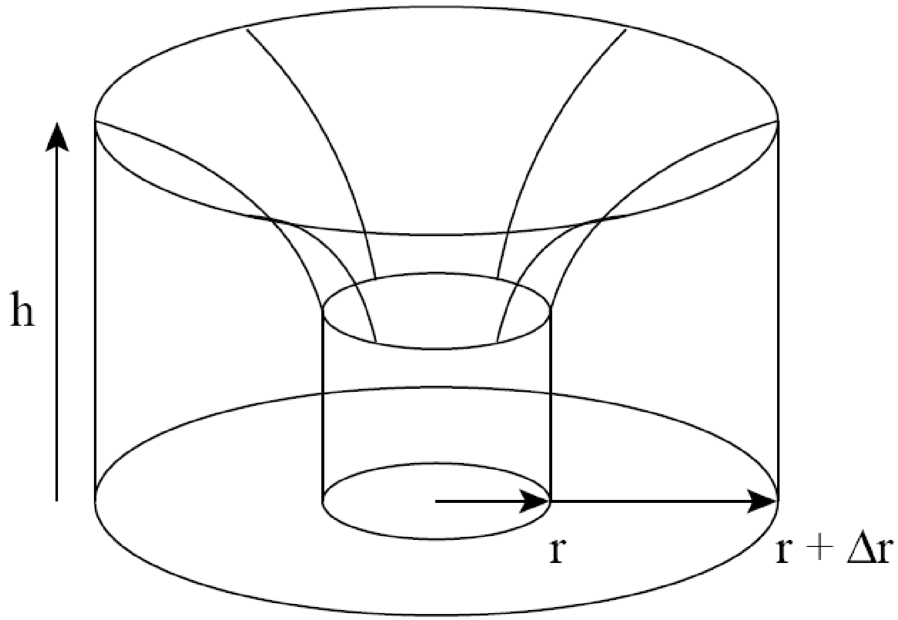
Radial flow – well hydraulics

Steady state flow towards a well

Unconfined aquifer



confined



unconfined

Radial flow – well hydraulics

Steady state flow towards a well

Unconfined aquifer

Without recharge, similar to confined:

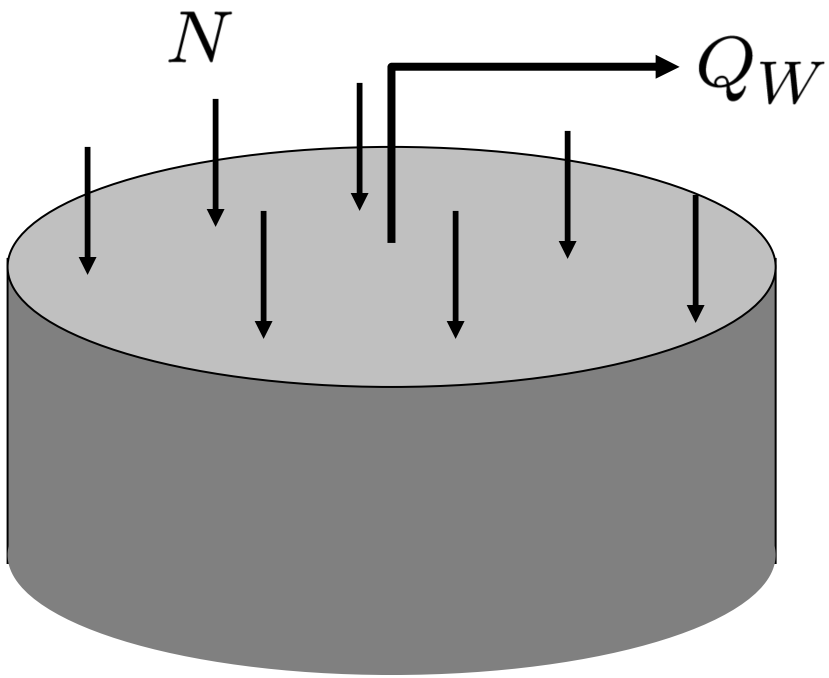
$$Q(r) = Q_W = -\pi r k_f \frac{\partial h^2}{\partial r}$$

$$h^2 = -\frac{Q_W}{\pi k_f} \ln(r) + C_1$$

Difference: $$h_1^2 - h_2^2 = \frac{Q_W}{\pi k_f} \ln\left(\frac{r_2}{r_1}\right)$$

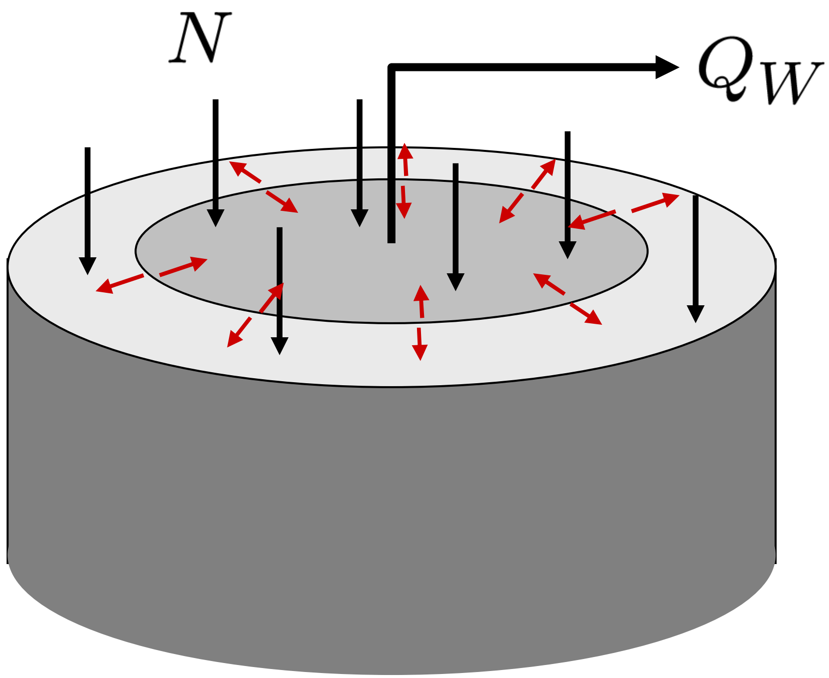
Radial flow – well hydraulics

Unconfined aquifer with recharge:



$$Q(r) = Q_W + Q_N$$

Radial flow – well hydraulics



Radius of influence or capture zone:

$$Q_W + Q_N = Q_W + \pi r_e^2 N = 0$$

Q_W <0 Q_N >0

$$r_e = \sqrt{\frac{-Q_W}{\pi N}}$$

$r_e > 0$ flow away from well

$r_e < 0$ flow towards well

Radial flow – well hydraulics

- Compare head squared at two distances

$$h_1^2 - h_2^2 = \boxed{\frac{Q_w}{\pi K}\ln\left(\frac{r_2}{r_1}\right)} + \boxed{\frac{N}{2K}\left(r_2^2 - r_1^2\right)}$$

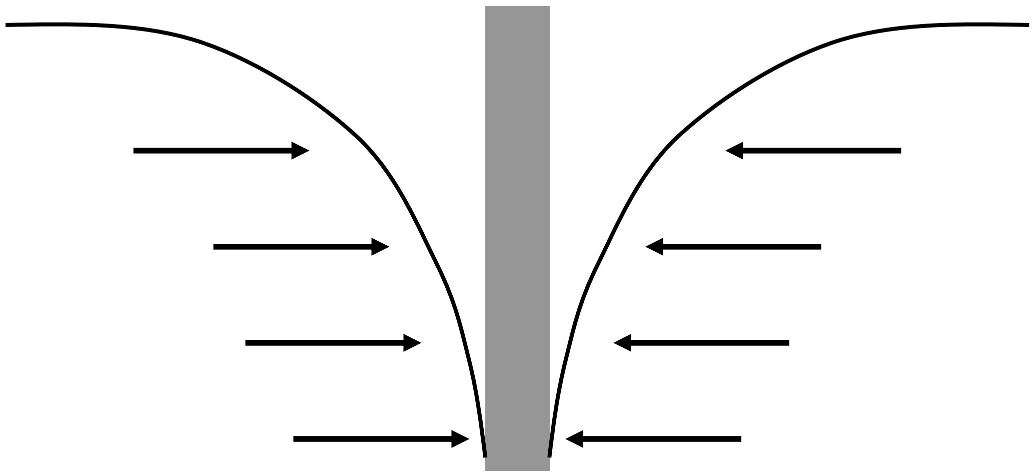
  - Logarithmic contribution from well
  - Quadratic contribution from recharge
- Pumping well + recharge: capture zone
- No recharge and only one observation well: Radius of influence

$$h_0^2 - h_1^2 = \frac{Q_w}{\pi K}\left(\ln\left(r_1\right) - \ln\left(R\right)\right)$$

Radial flow – well hydraulics

Maximum well capacity in an unconfined aquifer

At the well: large drawdown of the piezometric head

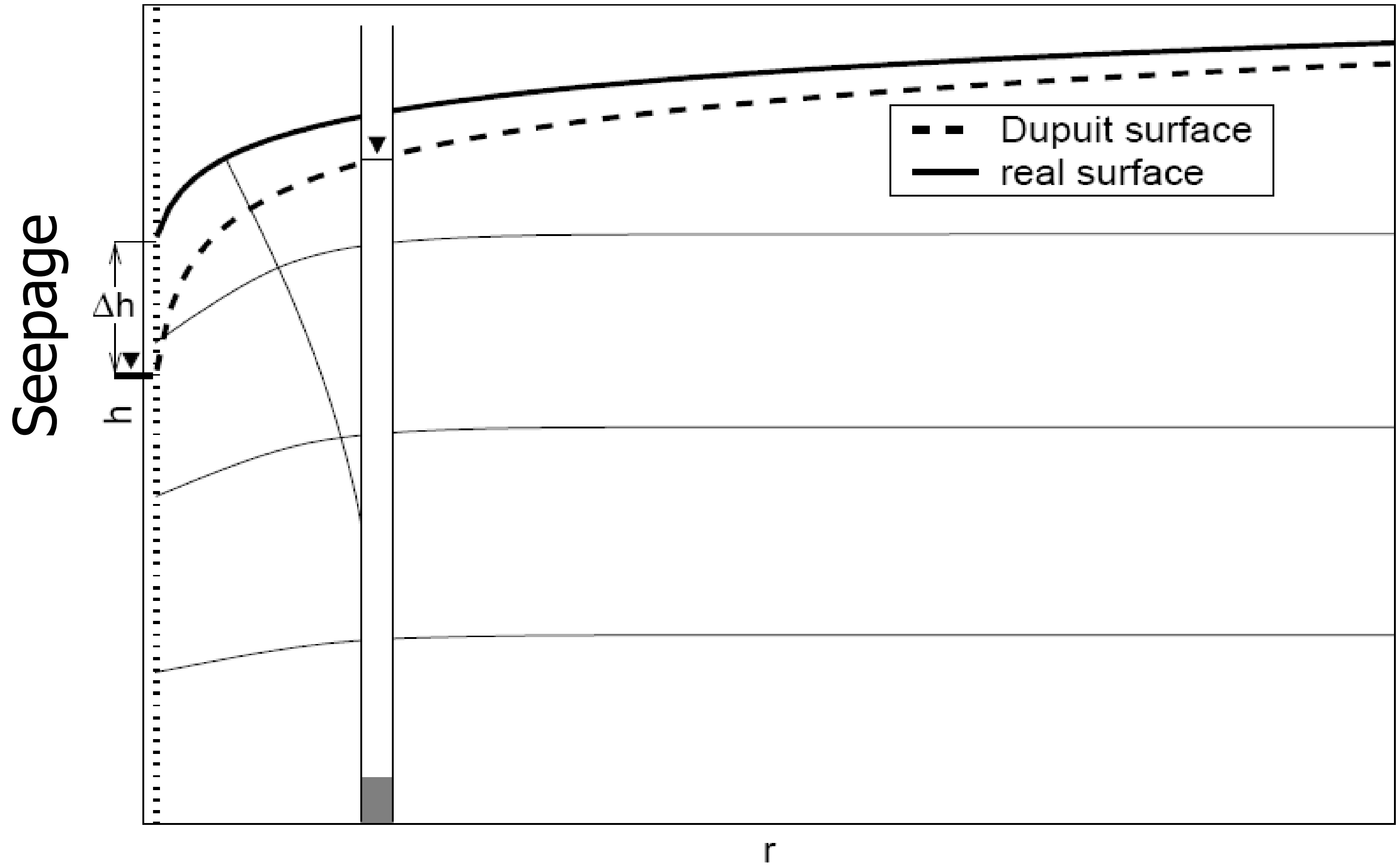


Dupuit assumption does no longer hold

-> “free seepage” in the well

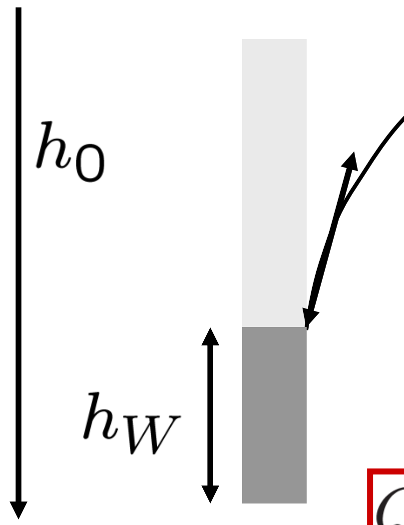
Radial flow – well hydraulics

Maximum well capacity in an unconfined aquifer



Radial flow – well hydraulics

Capacity of a well is limited



Maximum head gradient I_{max}

Sichard (1928): $I_{\text{max}} = \frac{1}{15\sqrt{k_f}}$ $[k_f] = \frac{m}{s}$

$$Q_{\text{capac}} = 2\pi h_W k_f I_{\text{max}}$$

Yield of the aquifer with a given radius of influence:

$$Q_{\text{yield}} = \frac{(h_0^2 - h_W^2)\pi k_f}{\ln\left(\frac{R}{r_W}\right)}$$

Radial flow – well hydraulics

h_W small -> Q_{capac} small, Q_{yield} large

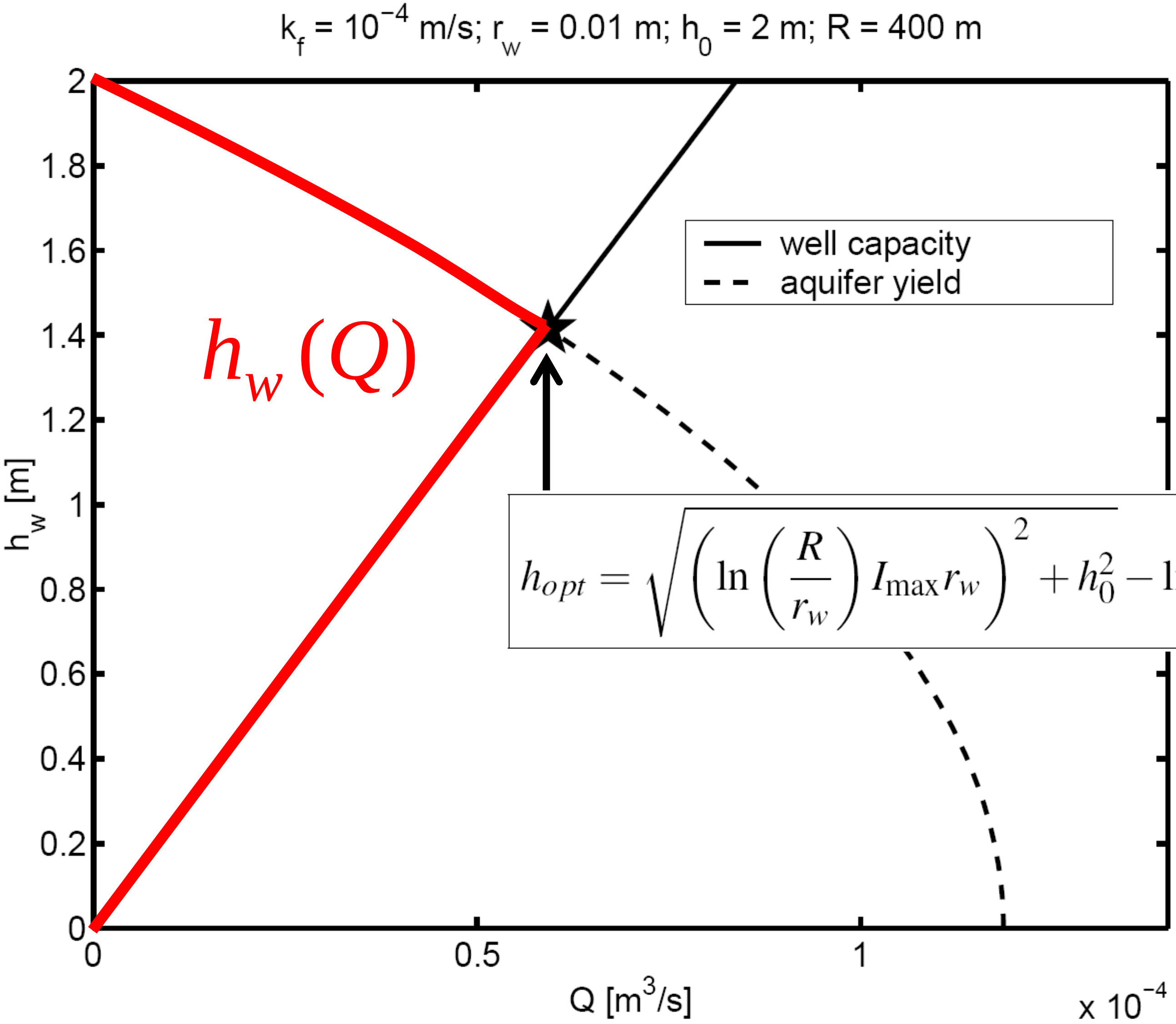
Hydraulic gradient gives much water, but the well cannot "take" it.

↕ ⟶ **Search for optimum**

h_W large -> Q_{capac} large, Q_{yield} small

The well can withdraw a large amount of water, but the hydraulic gradient cannot deliver it.

Radial flow – well hydraulics



$$h_{opt} = \sqrt{\left(\ln\left(\frac{R}{r_w}\right) I_{\max} r_w\right)^2 + h_0^2} - \ln\left(\frac{R}{r_w}\right) I_{\max} r_w$$

Radial flow – well hydraulics

Maximum Well Capacity in Unconfined Aquifers (Sichardt, 1928)

- Problem of well capacity exists only in unconfined aquifers
- The shallower the aquifer is, the more restrictive is the well capacity
- The smaller the well radius, the smaller is the well capacity
- Larger well diameter = higher drilling costs
- Shallow gravel aquifers: horizontal collector wells to obtain large well capacity

Radial flow – well hydraulics

Exercise # 11:

Consider a sandy-gravely unconfined aquifer with a well extracting water with a rate of Q_w of 6.31 10^{-2} m^3/s. The hydraulic head without pumping is 8.2 m. In quasi steady state, the following hydraulic heads have been measured:

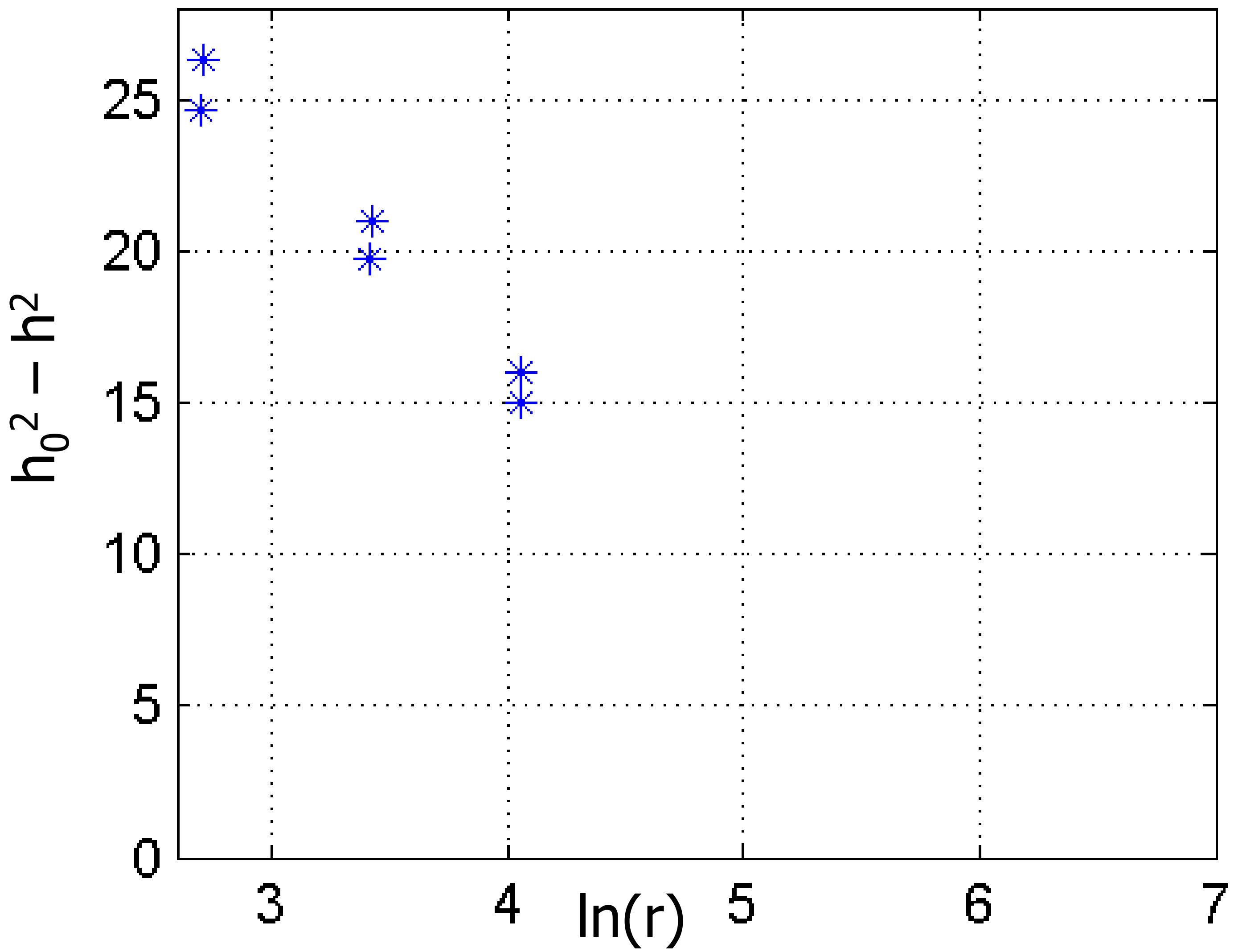
r [m]	h [m]	r [m]	h [m]
15	6.40	14.95	6.53
30.7	6.80	30.6	6.89
57.7	7.16	57.9	7.23

• Perform a linear regression of the difference between the squares of the undisturbed and measured head h_0^2-h^2 and the logarithm of the radius ln(r) and determine the hydraulic conductivity kf and the radius of influence of the aquifer. Careful: The rate Q_w has a negative sign if water is pumped from the well.

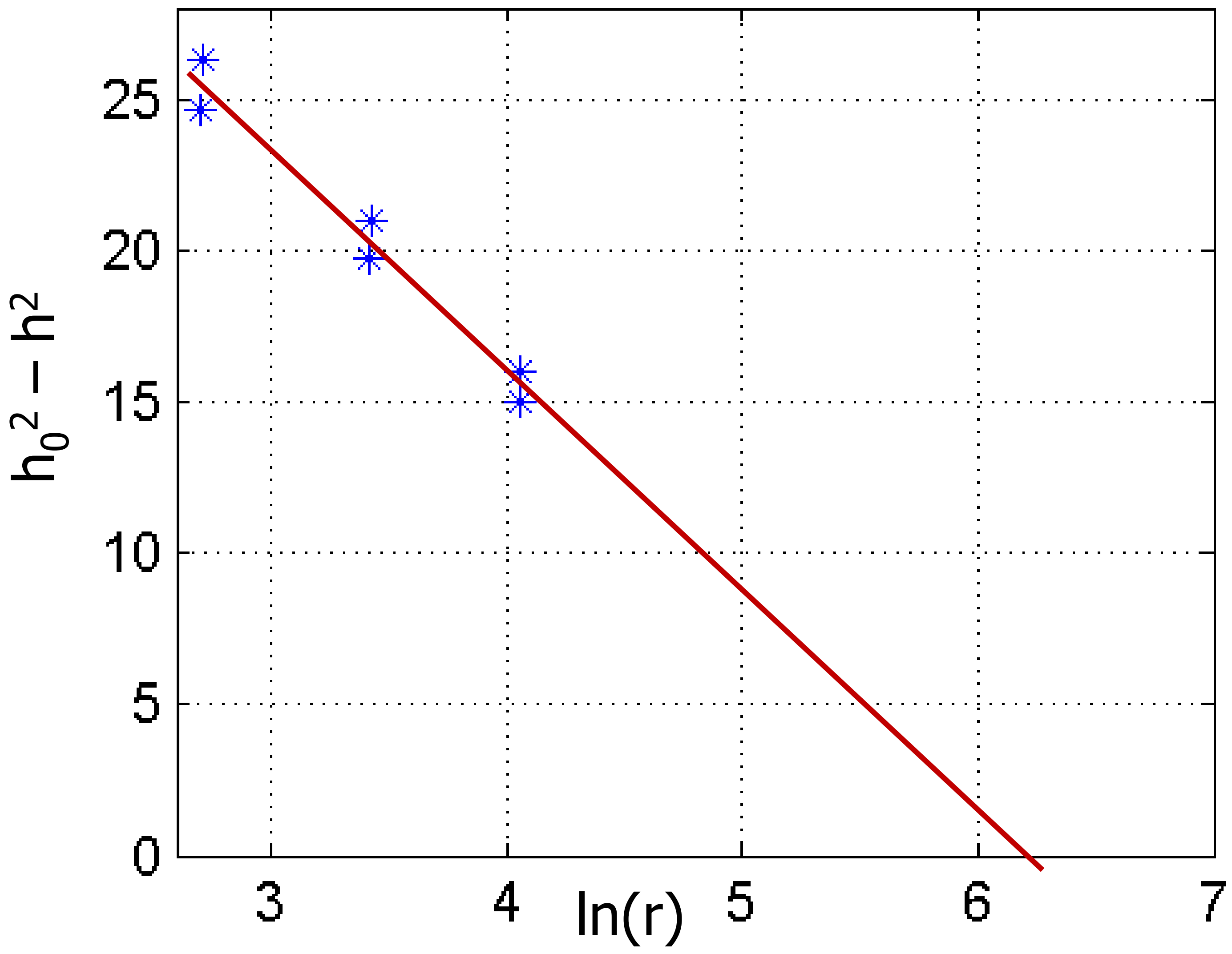
• Calculate for various radii of the well r_w (0.1 m, 0.25 m, 0.6 m, 1m) Sirchardt‘s maximum capacity and the maximum extraction rate possible in the given formation with the given well.

• Discuss constructive measures to obtain larger effective well diameters.

Radial flow – well hydraulics



Radial flow – well hydraulics



Radial flow – well hydraulics

Read the slope: $$\frac{\Delta(h_0^2 - h^2)}{\Delta \ln(r)} \approx \frac{10m^2}{1.4} = 7.14m^2$$

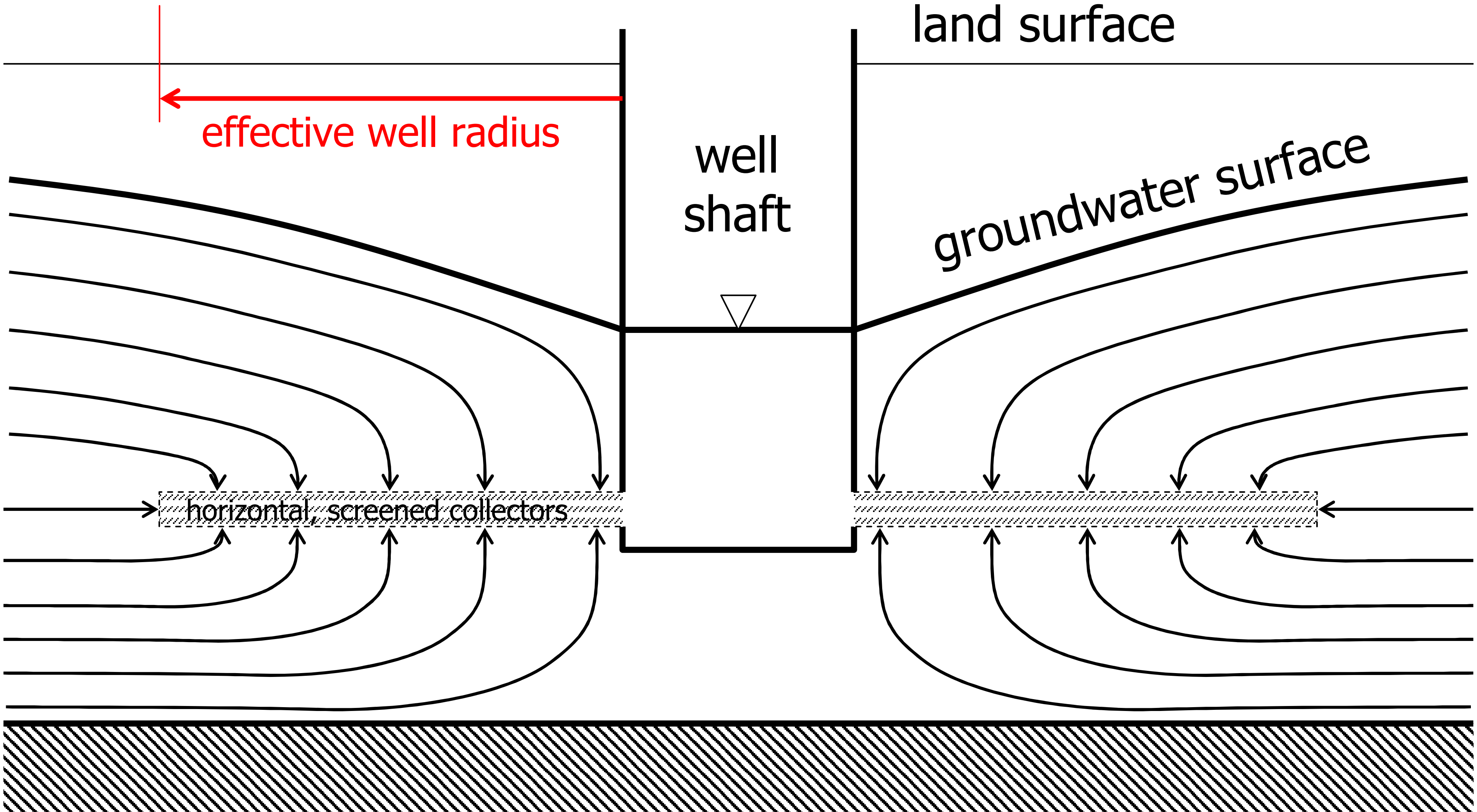
$$sl = \frac{Q_W}{\pi k_f} = \frac{-6.31\,10^{-2}\,m^3/s}{\pi k_f}$$

$$k_f = 0.0026m/s$$

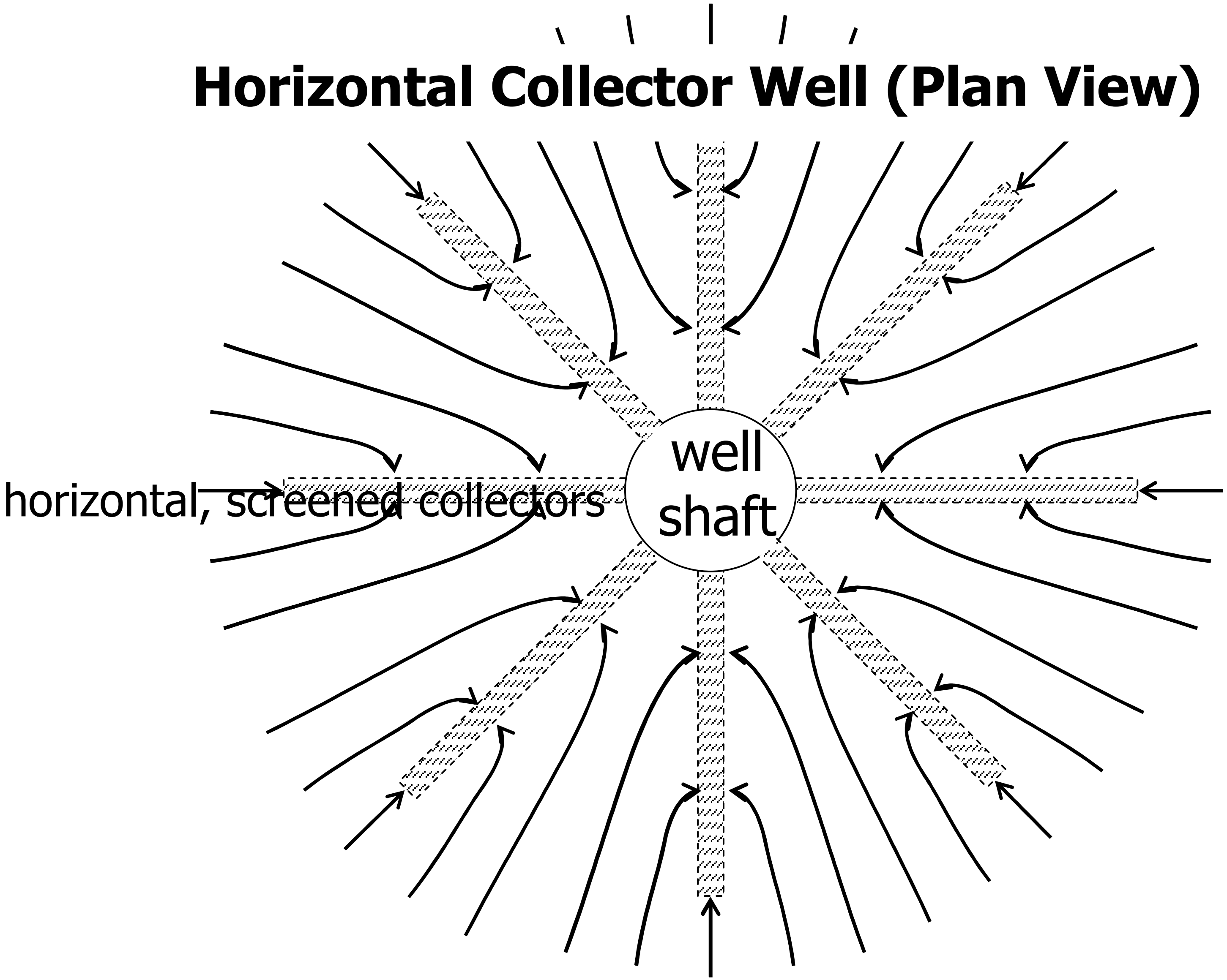
Read the x-axis for y=0: $\ln(R) \approx 6.2$ $R = 490m$

Radial flow – well hydraulics

Horizontal Collector Well (Vertical Cut)



Radial flow – well hydraulics



Regional groundwater flow

Regional groundwater flow

Consequences of Linearity

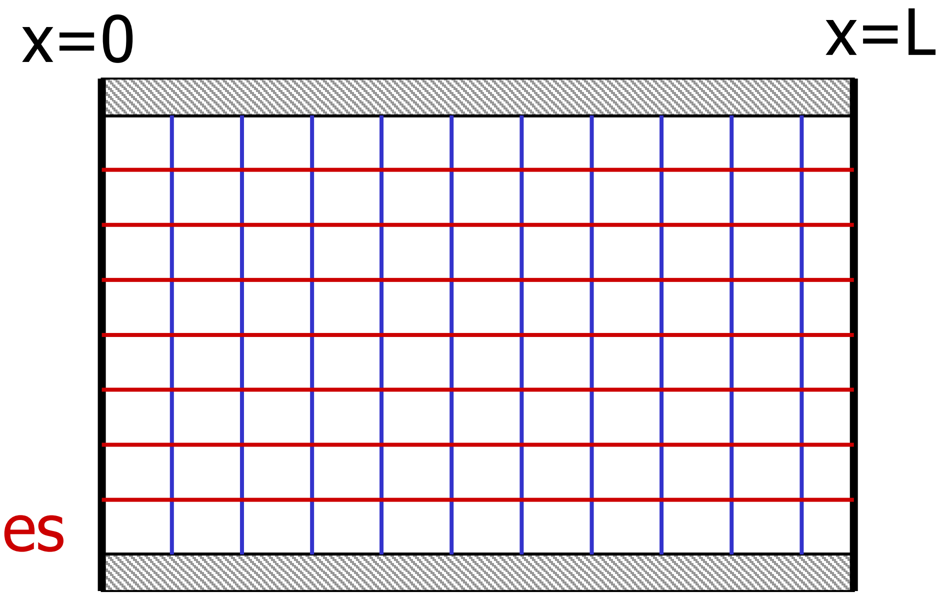
- Groundwater flow equation is linear in hydraulic head (at least in confined aquifers)
- Response to a particular stress (pumping) is also linear
- Responses to multiple stresses are additive
  - Sum up drawdowns related to individual wells
  - Subtract drawdown from head field without wells
  - Velocities are also additive

⇨ „Superposition"

Regional groundwater flow

Base case 1: Ambient groundwater flow (confined)

$$h = h_0 + \frac{h_L - h_0}{L} x = h_0 + I x \qquad \vec{q} = \begin{pmatrix} -k_f I \\ 0 \end{pmatrix}$$



Isolines h

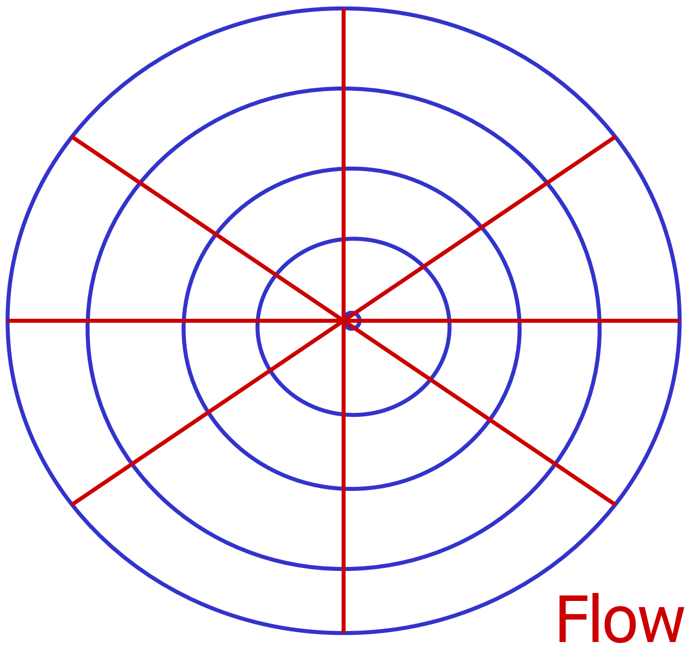
Streamlines

Flow goes along the red lines

Regional groundwater flow

Base case 2: Well flow (confined)

$$h = h_0 + \frac{Q}{2\pi T} \ln\left(\frac{R}{r}\right) \qquad \vec{q} = -k_f \vec{\nabla} h = -\frac{Q}{2\pi m(x^2+y^2)} \begin{pmatrix} x \\ y \end{pmatrix}$$



Streamlines

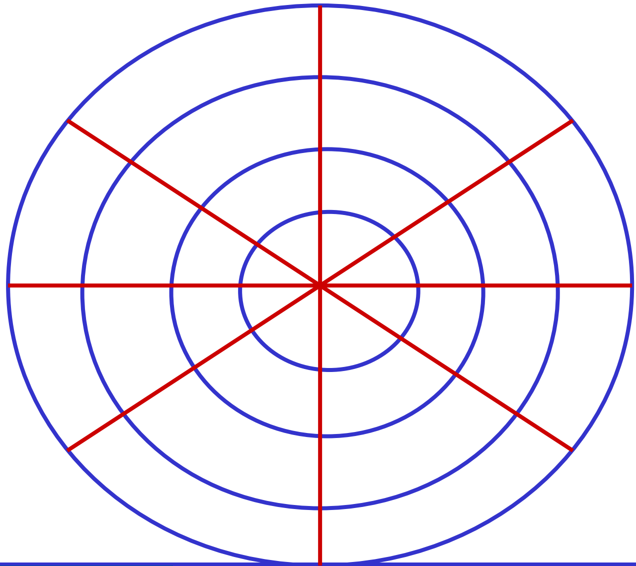
Isolines h

Flow goes along the red lines

Regional groundwater flow

Example: Injection well and extraction well

$$h = C_h - \frac{Q_1}{2\pi T} ln\left(\sqrt{(x-x_1)^2 + (y-y_1)^2}\right) - \frac{Q_2}{2\pi T} ln\left(\sqrt{(x-x_2)^2 + (y-y_2)^2}\right)$$



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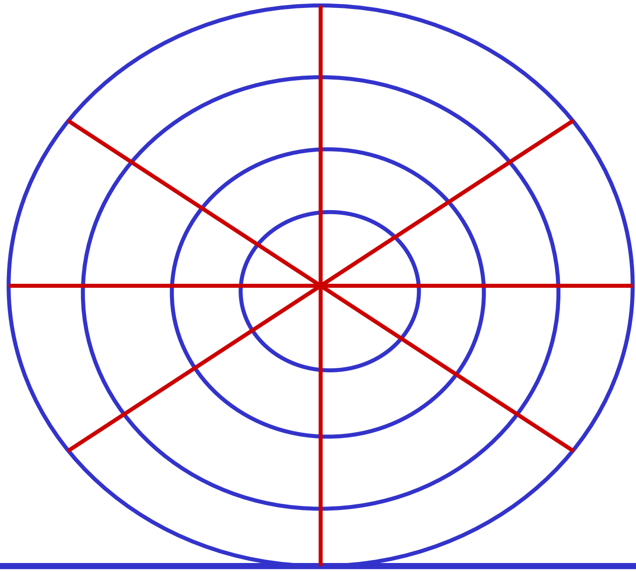
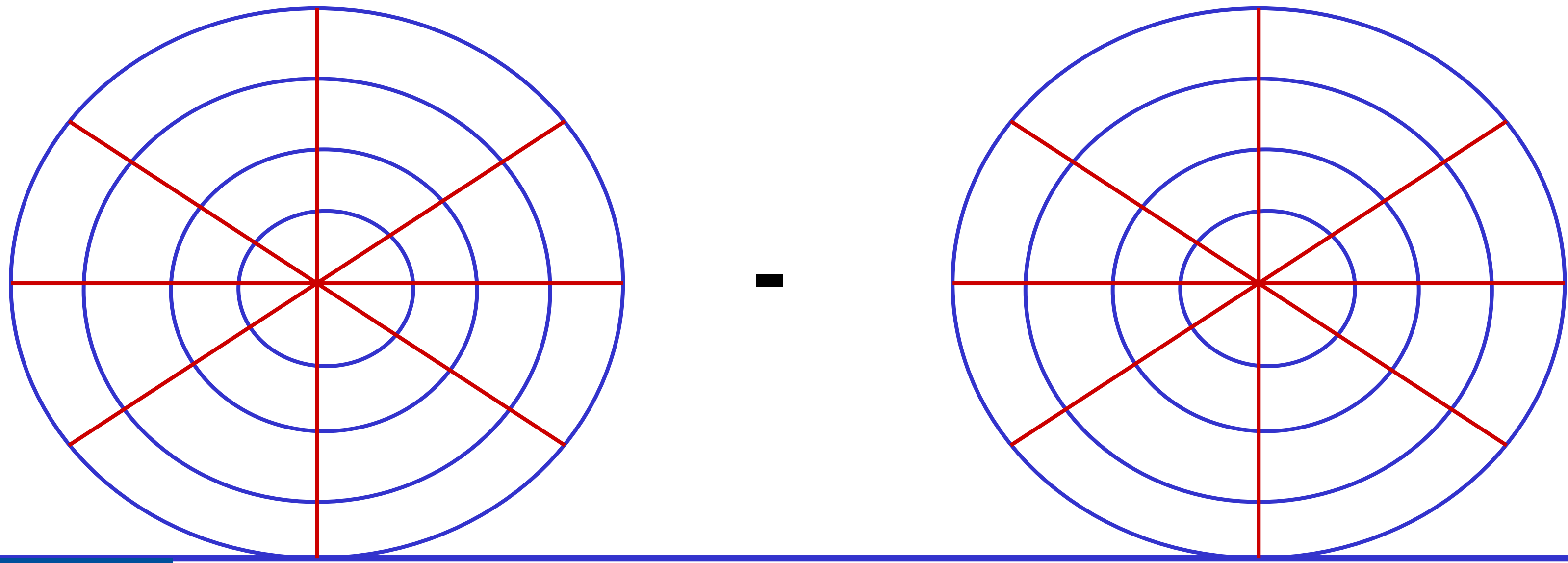


Regional groundwater flow

Example: Injection well and extraction well

$$q_x = \frac{Q_1}{2\pi m} \cdot \frac{x - x_1}{(x - x_1)^2 + (y - y_1)^2} + \frac{Q_2}{2\pi m} \cdot \frac{x - x_2}{(x - x_2)^2 + (y - y_2)^2}$$

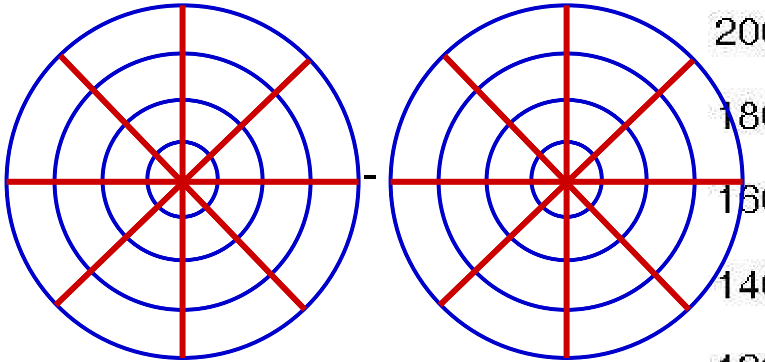
$$q_x = \frac{Q_1}{2\pi m} \cdot \frac{y - y_1}{(x - x_1)^2 + (y - y_1)^2} + \frac{Q_2}{2\pi m} \cdot \frac{y - y_2}{(x - x_2)^2 + (y - y_2)^2}$$



Regional groundwater flow

Example: Injection well and extraction well

2 times base case 2



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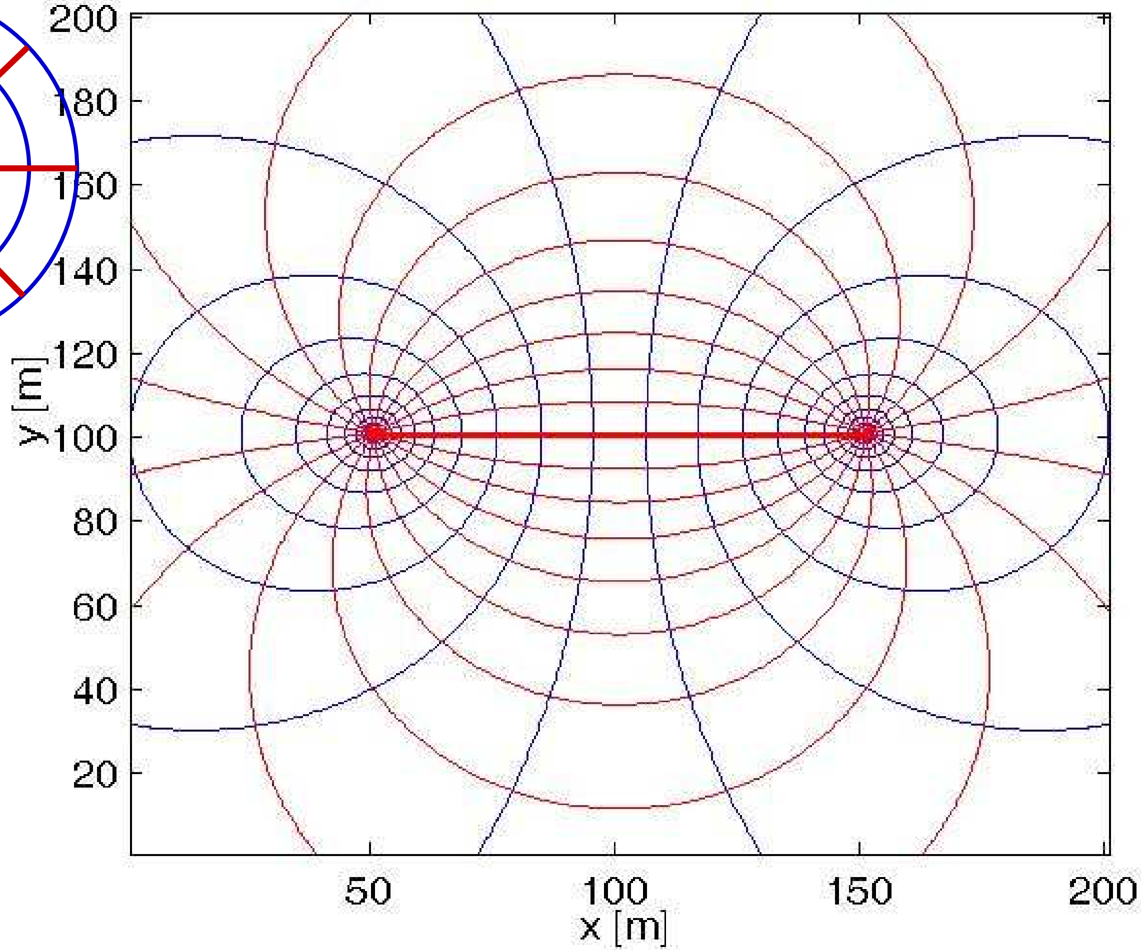
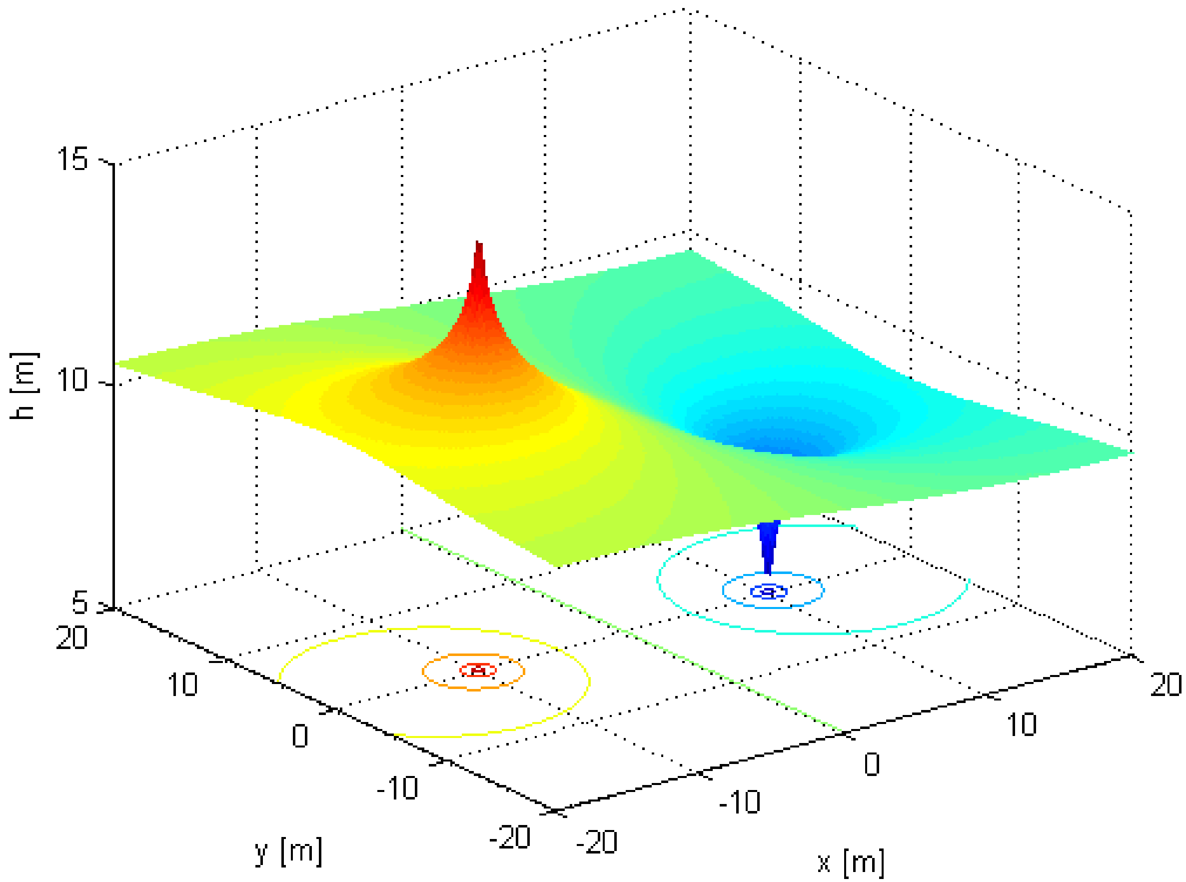


Isolines h

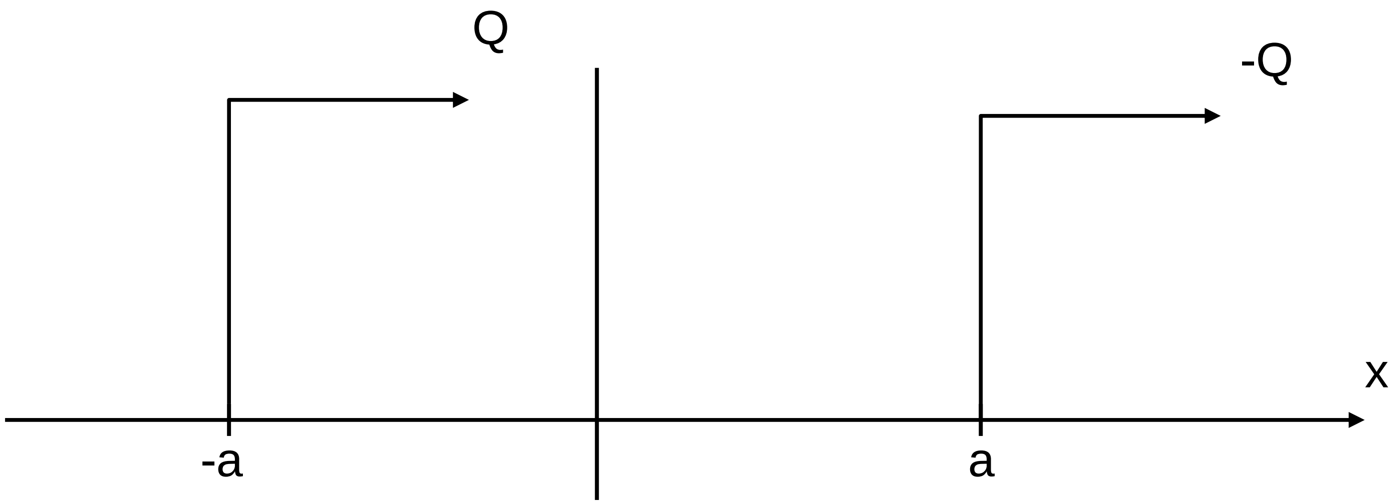
Streamlines

Regional groundwater flow

Example: Injection well and extraction well



Regional groundwater flow

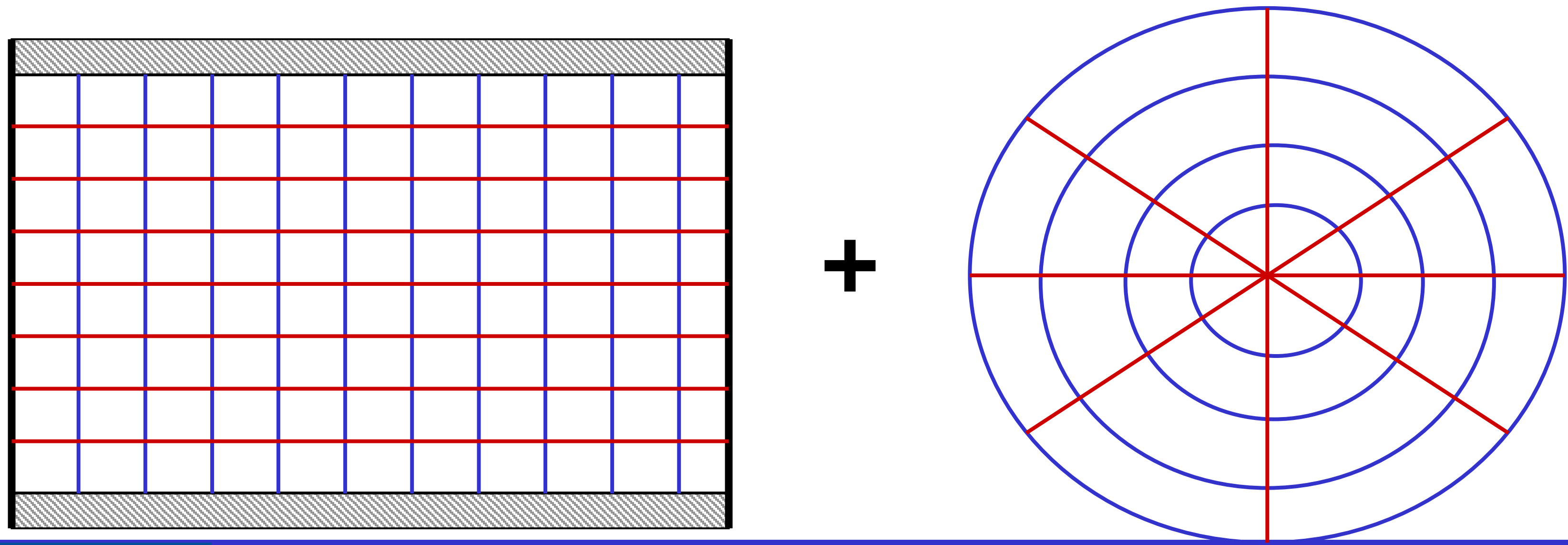


Drawdown $$s(x,y) = \frac{Q}{2\pi T} \ln \left(\frac{\sqrt{(x-a)^2+y^2}}{\sqrt{(x+a)^2+y^2}} \frac{R}{R} \right)$$

Regional groundwater flow

Examle: Ambient flow and well flow

$$h = C_h - Ix - \frac{Q}{2\pi T} \ln\left(\sqrt{(x - x_w)^2 + (y - y_w)^2}\right)$$

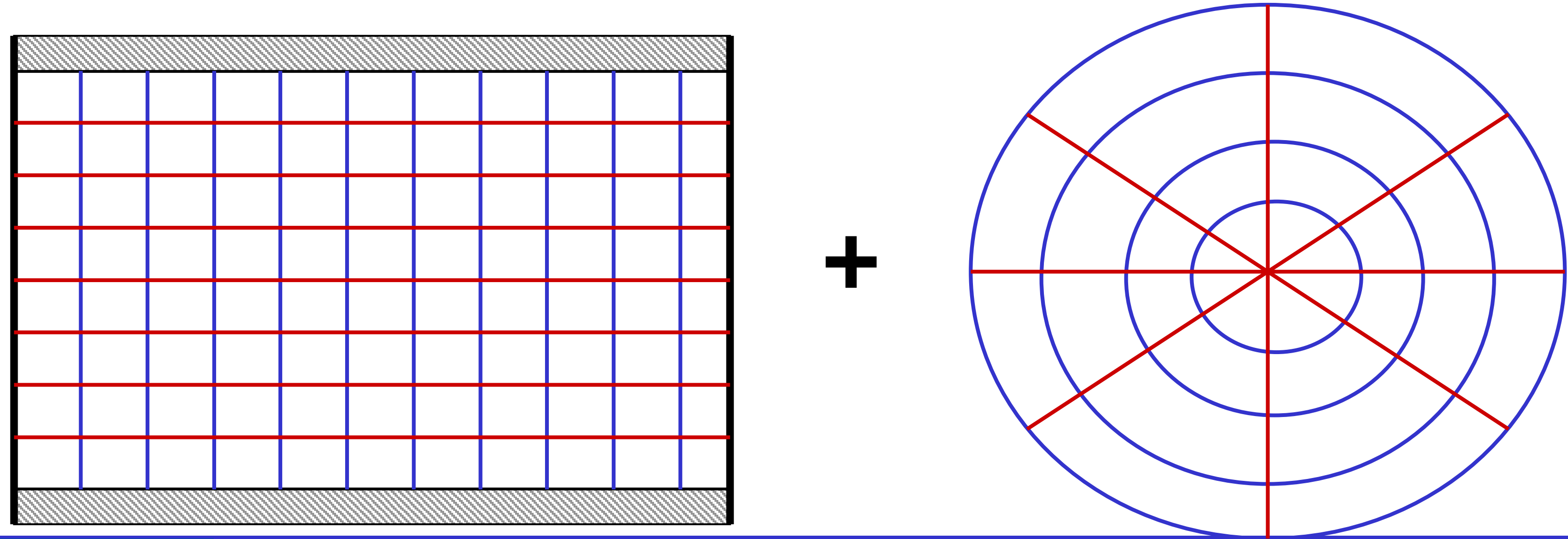


Regional groundwater flow

Examle: Ambient flow and well flow

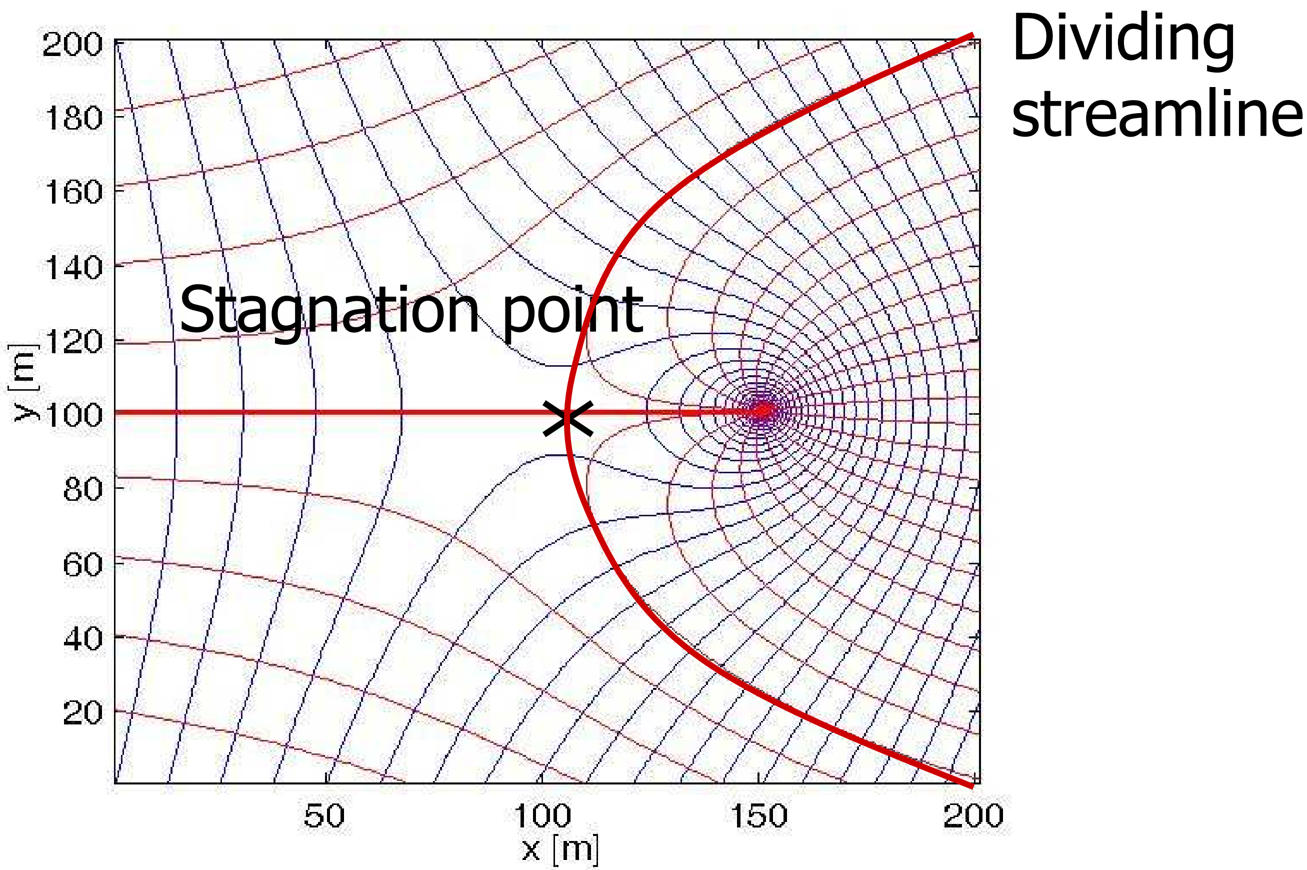
$$q_x = IK_f + \frac{Q}{2\pi m} \cdot \frac{x - x_w}{(x - x_w)^2 + (y - y_w)^2}$$

$$q_y = \frac{Q}{2\pi m} \cdot \frac{y - y_w}{(x - x_w)^2 + (y - y_w)^2}$$



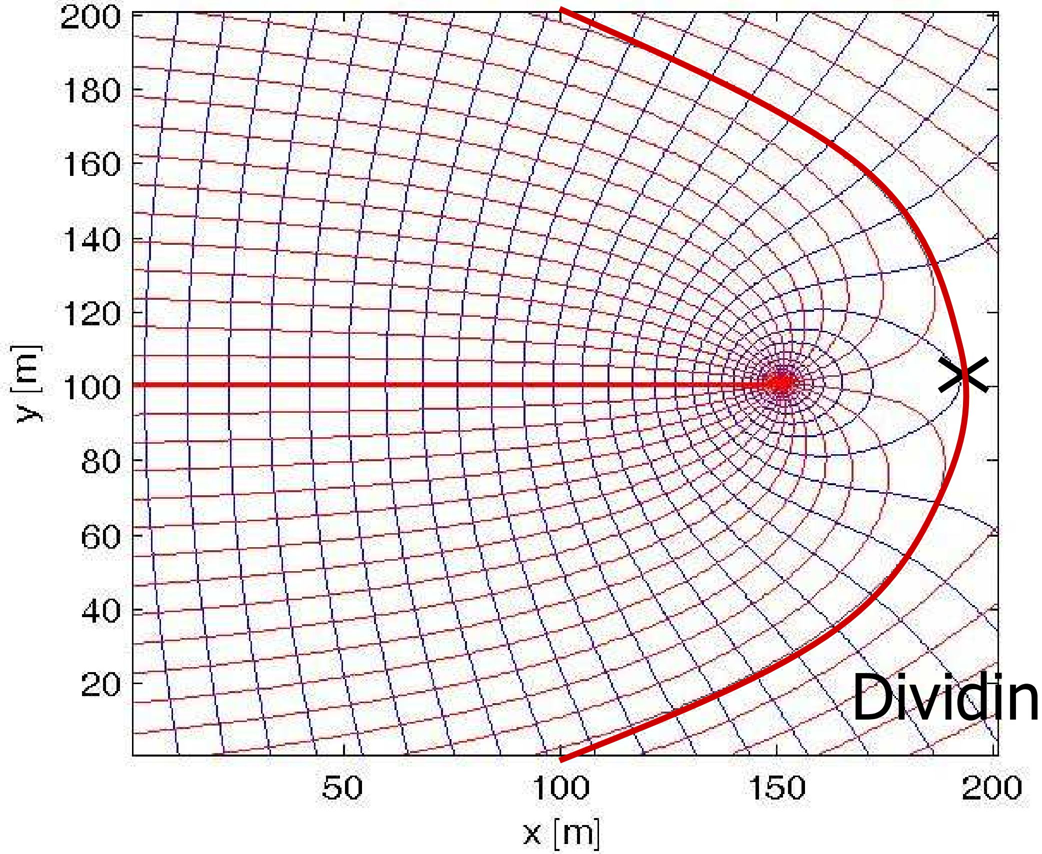
Regional groundwater flow

Ambient flow and injection well



Regional groundwater flow

Ambient flow and extraction well



Stagnation point

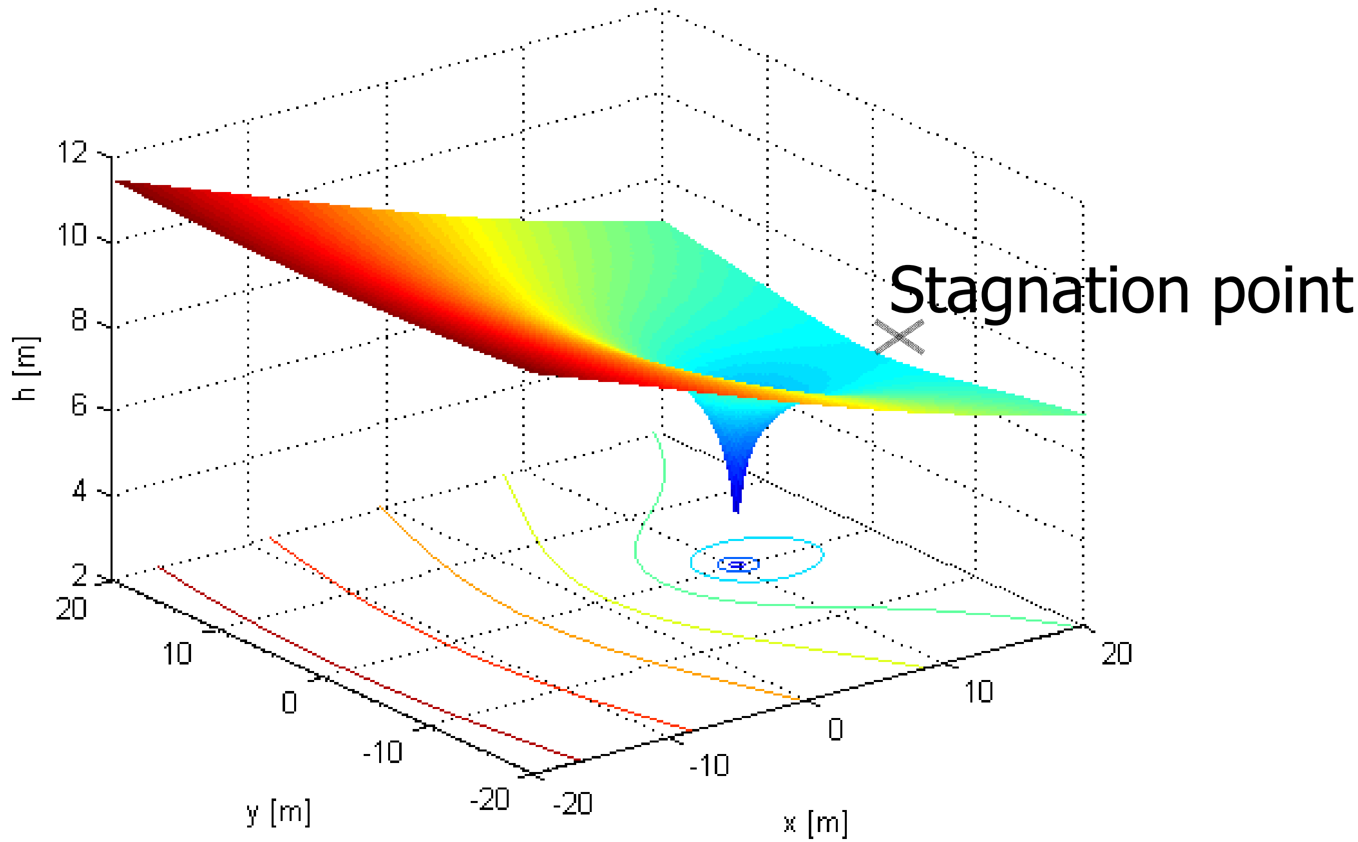
Streamlines

Isoline h

Dividing streamline

Regional groundwater flow

Ambient flow and extraction well



Regional groundwater flow

Characteristics of Flow Fields related to (Multiple) Wells in Ambient Flow

- Dividing streamlines:
  - Separate water bodies of different origin (injection wells, ambient flow) or different destination (extraction wells, ambient flow)
- Stagnation points:
  - Points with zero velocity
  - Crossing points of two dividing streamlines