Groundwater exercise:

Task1: Soil Characteristics

A soil sample with the volume of **V_g = 100cm³** is taken. The weight of the bulk is measured **m_g = 198g**. After drying at 105 °C to a constant mass, the soil weighed **m_{tr} = 176g**. In a measuring cylinder filled with water the dried sample displaces a volume of **V_{tr} = 61cm³.**

Determine the following soil properties

a) The soil porosity, n
b) The bulk density, ρ_{tr}
c) The particle density, ρ
d) The gravimetric water content, Θ_m
e) The volumetric water content, Θ_V
f) The soil water content Θ of a soil with a thickness of 30cm
g) The degree of saturation, s

$$n = \frac{V_P}{V_g} \cdot 100 = \frac{V_g - V_{tr}}{V_g} \cdot 100 = \frac{100 - 61}{100} \cdot 100 = 39\%$$

$$\rho_{tr} = \frac{m_{tr}}{V_g} = \frac{176g}{100cm^3} = 1{,}76 \frac{g}{cm^3}$$

$$\rho = \frac{m_{tr}}{V_{tr}} = \frac{176g}{61cm^3} = 2{,}89 \frac{g}{cm^3}$$

$$\Theta_m = \frac{m_g - m_{tr}}{m_{tr}} \cdot 100 = \frac{198 - 176}{176} = 12{,}5\%$$

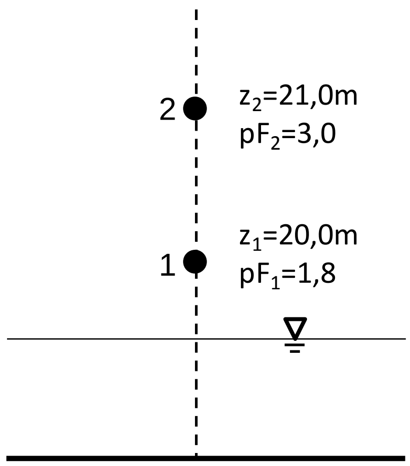
$$\Theta_V = \frac{V_w}{V_g} \cdot 100 = \frac{m_w / \rho_w}{V_g} \cdot 100 = \frac{\left(m_g - m_{tr}\right) / 1 \frac{g}{cm^3}}{V_g} \cdot 100 = \frac{198 - 176}{100} \cdot 100 = 22\%$$

$$\Theta = \Theta_V \cdot h \cdot 0{,}01 = 22\% \cdot 300mm \cdot 0{,}01 = 66mm$$

$$s = \frac{\Theta_V}{n} = \frac{22\%}{39\%} = 0{,}564$$

Task2: Potential Head

According to the figure below, by using the hydraulic gradient, determine whether water vertically penetrates through the soil or rises by the capillary forces. The geodetic heights z (above sea level) of the points is known and the water potential of the points is given in the figure.



'pF' is the decadic logarithm of the pressure in centimeters of water column.

<u>water potential</u>:

Point 1: $pF_1=1,8 \triangleq \Psi_1 = 10^{1,8} = 63,1\text{hPa} = 63,1\text{cm water column}$

Point 2: $pF_2=3,0 \triangleq \Psi_2 = 10^{3,0} = 1000\text{hPa} = 1000\text{cm water column}$

<u>Potential Head (energy levels):</u>

Point 1:

$$H_1 = z_1 - \frac{\Psi_1}{\rho_W \cdot g} = 20\text{m} - \frac{63{,}1\text{hPa}}{1000\frac{\text{kg}}{\text{m}^3}\cdot 10\frac{\text{m}}{\text{s}^2}} = 20\text{m} - 0{,}63\text{m} = 19{,}37\text{m}$$

$$1\text{Pa} = 1\frac{\text{N}}{\text{m}^2} = 1\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{m}^2}, \quad \Psi = \rho_W \cdot g \cdot h$$

Point 2: $$H_1 = z_1 - \frac{\Psi_1}{\rho_W \cdot g} = 21\text{m} - \frac{1000\text{hPa}}{1000\frac{\text{kg}}{\text{m}^3}\cdot 10\frac{\text{m}}{\text{s}^2}} = 21\text{m} - 10{,}00\text{m} = 11{,}00\text{m}$$

Water moves from higher to lower potential -> 1 to 2, rises by the capillary forces

Hydraulic gradient: $$I_{hy} = \frac{19{,}37 - 11{,}0}{1} = 8{,}37$$

Task 3: Available water capacity

Determine for a medium sand, loam and clay the available water capacity (AWC) and the root zone available water capacity (RZAWC). Use the field capacity, permanent wilting point and the effective rooting depth of crops for your calculations.

<u>Available water capacity from table 6.5:</u>
Sand: 10 mm/dm
Loam: 25,5 mm/dm
Clay: 22 mm/dm

<u>Effective rooting depth (ρ_b3) from table 6.6:</u>
Sand: 5-9dm -> 7dm
Loam: 10dm
Clay: 10-11dm -> 10,5dm

<u>Root zone available water capacity:</u>
Sand: 10mm/dm*7dm=70mm
Loam: 25,5mm/dm*10dm=255mm
Clay: 22mm/dm*10,5dm=231mm

Task 4: Water Availability

A farmer wants to know for how long he can manage his field area, with a sandy loam soil and winter wheat as the crop, without irrigating during the growing season when no rainfall occurs.

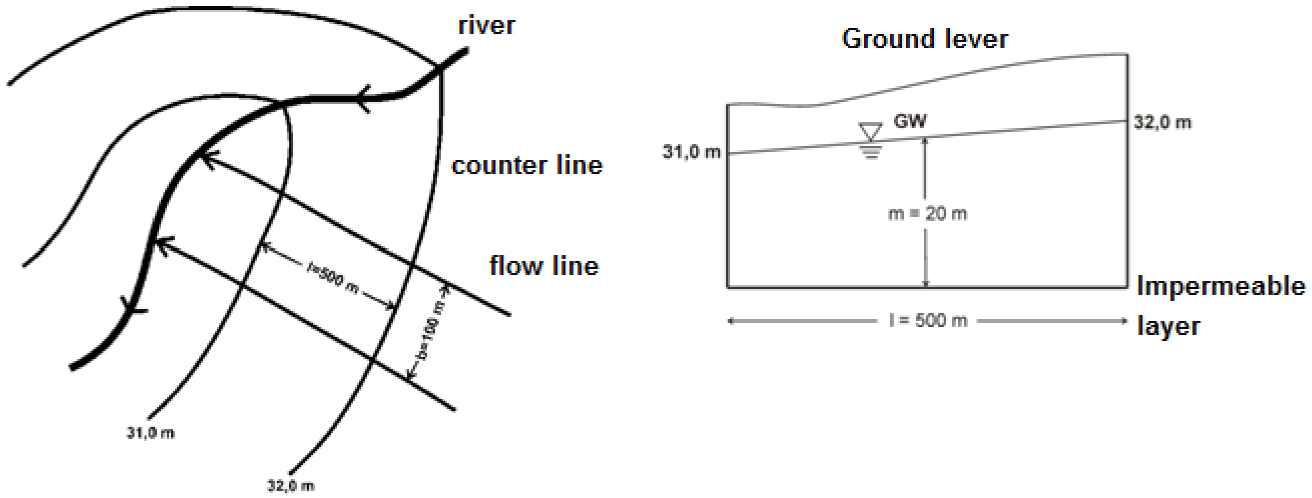
Assumptions: actual evaporation 7mm / d
at the beginning, the soil is said to be at field capacity
vadose zone > 5m

In order to prevent yield losses, the water content should not drop below 50% of the root zone available water capacity (RZAWC). In this case, how long can irrigating be waived?

<u>Available water capacity from table 6.5:</u> 21,0mm/dm
<u>Effective rooting depth from table 6.6:</u> 10dm
RZAWC = 21*10=210mm
50% RZAWC =210/2=105mm
waiving time: D=105mm/7mm/d=15 days

Task 5: Groundwater flow

Predominantly the groundwater flows horizontally to the river and feeds it as the base flow. This discharge can be estimated using water table contour lines. How big is the runoff according to the following sketches between the two given flow lines? The hydraulic conductivity of the aquifer is $k_f = 1.0 \cdot 10^{-5}$ m • s^{-1}. Determine also the flow time between the two water table contour lines (500 m) with the porosity of n = 20%



Longitudinal and cross-sectional view of the flow setting

Discharge:

$$Q = v_f \cdot A = k_f \cdot I_{hy} \cdot A$$

$$k_f = 1 \cdot 10^{-5} \, m/s$$

$$I_{hy} = \frac{\Delta h}{l} = \frac{32-31}{500} = 0{,}002 \approx 2‰$$

$$v_f = 1 \cdot 10^{-5} \, m/s \cdot 0{,}002 = 2 \cdot 10^{-8} \, m/s$$

$$A = m \cdot b = 20m \cdot 100m = 2000m^2$$

$$Q = v_f \cdot A = 2 \cdot 10^{-8} \, m/s \cdot 2000m^2 = 4 \cdot 10^{-5} \, m^3/s = 3{,}5 \, m^3/d$$

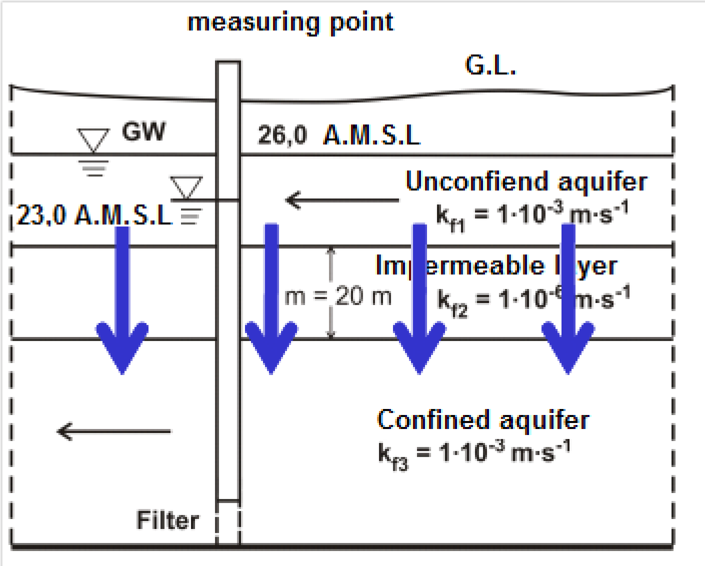
Flow time:

$$v_a = \frac{v_f}{n} = \frac{2 \cdot 10^{-8} \, m/s}{0{,}2} = 1 \cdot 10^{-7} \, m/s = 3{,}15 \, m/a$$

$$v_a = \frac{s}{t} \Rightarrow t = \frac{s}{v_a} = \frac{500m}{1 \cdot 10^{-7} \, m/s} = 5 \cdot 10^{9} s = 158{,}5a$$

Task 6: Groundwater System

A groundwater system consists of three layers of rock. The middle layer is an aquitard with a significantly lower permeability than the adjacent layers. The flow in the upper and lower layers exists, and the piezometric heads are known in the figure. How big is the flow through the aquitard?



Longitudinal section of the GW system

$$Q = v_f \cdot A = k_f \cdot I_{hy} \cdot A$$

$$k_{f2}(\text{Hemmer}) = 1 \cdot 10^{-6}\ \text{m}/\text{s}$$

A (horizontal area for vertical flow) : 1 m^2

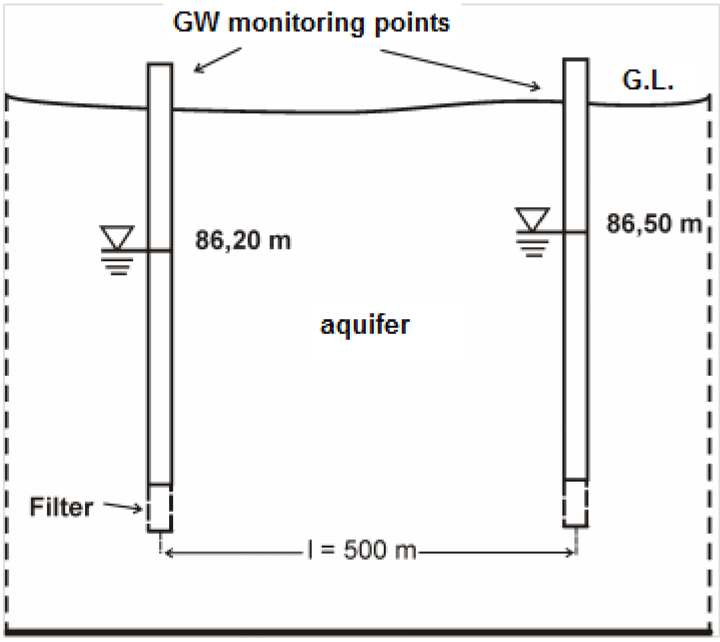
$$I_{hy} = \frac{\Delta h}{\Delta l} = \frac{26-23}{20} = 0{,}15 \hat{=} 15\%$$

$$Q = k_f \cdot I_{hy} \cdot A = 1 \cdot 10^{-6}\ \text{m}/\text{s} \cdot 0{,}15 \cdot 1{,}0\text{m}^2 = 1{,}5 \cdot 10^{-7}\ \text{m}^3/\text{s} = 1{,}3 \cdot 10^{-2}\ \text{m}^3/\text{d} = 13\ \text{l}/\text{d} \quad \text{per m}^2$$

Flow direction: from unconfined aquifer to confined aquifer (from higher to lower potential)!

Task 7: Groundwater flow time

Determine the time that the groundwater flows in a relatively homogeneous aquifer (drawing in longitudinal section) between the two monitoring points! Hydraulic conductivity of $k_f = 3 \cdot 10^{-3}$ m • s^{-1} and the effective porosity of $n_f = 0.15$ (-) have been estimated.



Hydraulic gradient: $I_{hy} = \Delta h / l = (86{,}50 - 86{,}20) / 500 = 6{,}0 \cdot 10^{-4}$ (-)

Filter velocity: $v_f = k_f \cdot I_{hy} = 3 \cdot 10^{-3} \cdot 6{,}0 \cdot 10^{-4} = \cdot 1{,}8 \cdot 10^{-6}$ m•s^{-1}

Pore water velocity: $v_a = v_f / n_f = 1{,}8 \cdot 10^{-6} / 0{,}15 = 1{,}2 \cdot 10^{-5}$ m•s^{-1}

Flow time: $t = l / v_a = 500 / 1{,}2 \cdot 10^{-5} = 4{,}17 \cdot 10^{7}$ s $= 1{,}16 \cdot 10^{4}$ h = 482 days

Formulas:

Porosity(n): $$n = \frac{V_P}{V_g} \cdot 100 \quad [\%] \quad (6.1)$$

Bulk density (ρ_{tr}): $$\rho_{tr} = \frac{m_{tr}}{V_g} \quad [g/cm^3] \quad (6.2)$$

Particle density (ρ): $$\rho = \frac{m_{tr}}{V_{tr}} \quad [g/cm^3] \quad (6.3)$$

Gravimetric water content (Θ_m): $$\Theta_m = \frac{m_g - m_{tr}}{m_{tr}} \cdot 100 = \frac{m_w}{m_{tr}} \cdot 100 \quad [\%] \quad (6.4)$$

Volumetric water content (Θ_V): $$\Theta_V = \frac{V_w}{V_g} \cdot 100 \quad [\%] \quad (6.5)$$

Soil water content in a thickness of soil (h): $$\Theta = \Theta_V \cdot h \cdot 0{,}01 \quad [mm] \quad (6.6)$$

h Soil thickness [mm]

Degree of saturation (s): $$s = \frac{\Theta_V}{n} \quad (6.7)$$

Tab. 6.5 FC and PWP for different soil types

Soil Type	Field Capacity F.C. (mm.dm^{-1})	Permanent Wilting Point
Sand	13,5	3,5
Loamy sand	21,0	3,0
Sandy loam	25,5	4,5
Loam	36,0	10,5
Clay	40,0	18,0
Peat soil	74,0	40,0

Tab. 6.6 The effective rooting depth for field crops (AD-HOC-AG Boden, 2005, Auszug)

Soil Type (symbols)	Effective root zone Re (dm)		
	Range of dry bulk density ρ_b (g·cm^{-3})		
	$\rho_b 1 - \rho_b 2$	$\rho_b 3$	$\rho_b 4 - \rho_b 5$
gS, gSms, gSfs	7	5	5
Ss, mS, fS, mSgs, mSfs	8	6	6
Sl2, Su2, Su3, Su4	9	7	6
Sl3, St2	10	8	7
Sl4, St3,Slu	13	9	8
Ls2, Ls3, Ls4, Lt2, Lt3, Lts, Uu, Us, Tu2, Tl, Tt	13	10	8
Uls, Ut2, Ut3, Ut4, Lu, Tu3, Tu4	14	11	9
For grasslands, 2 dm should be subtracted from the values in the table. For deciduous forests, they need to be multiplied by 1.5. Due to insufficient investigation results for Ts2-Ts4 soil types here no value is given.			

Filter velocity $$v_f = k_f \cdot I_{hy} = \frac{\Delta h}{\Delta l} = \frac{Q}{A_{ges}} \quad (6.17)$$

Pore water velocity $$v_a = \frac{s}{t} = \frac{v_f}{n} \quad (6.22)$$

Soil Triangle:

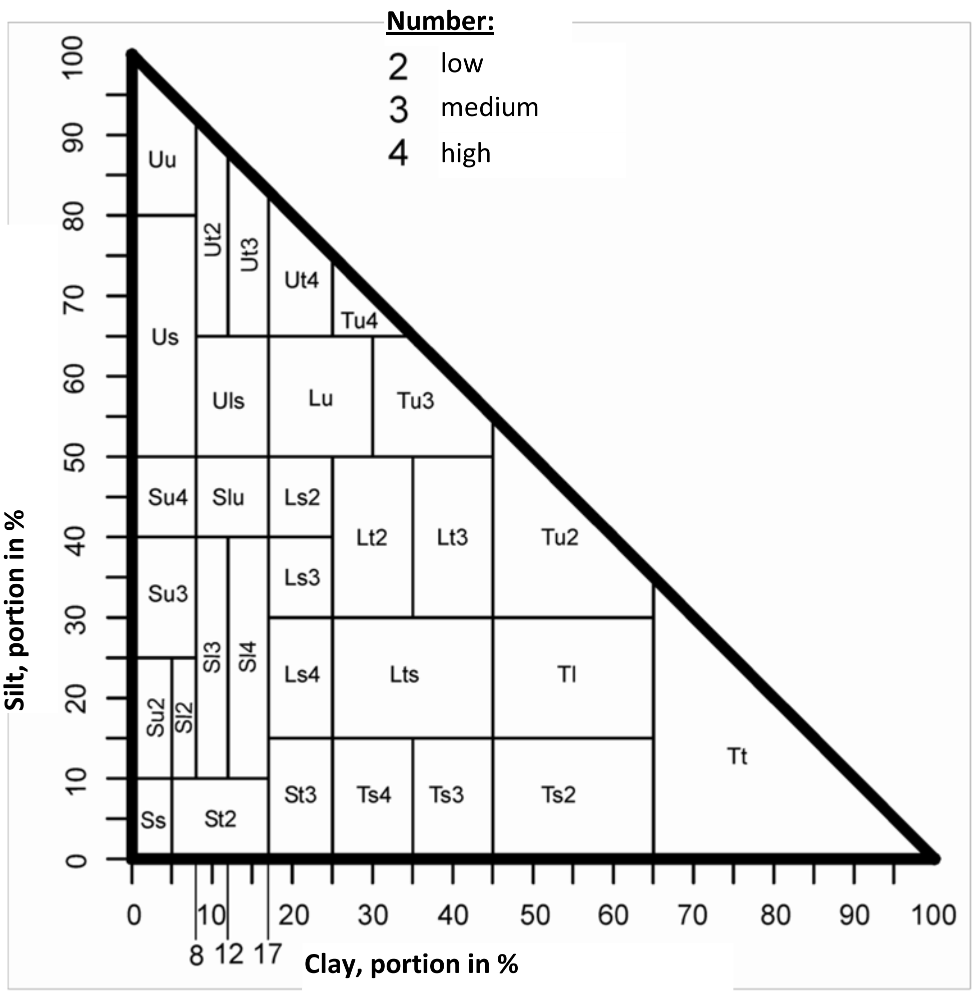


Fig. 6.6 Soil texture triangle for fine soils