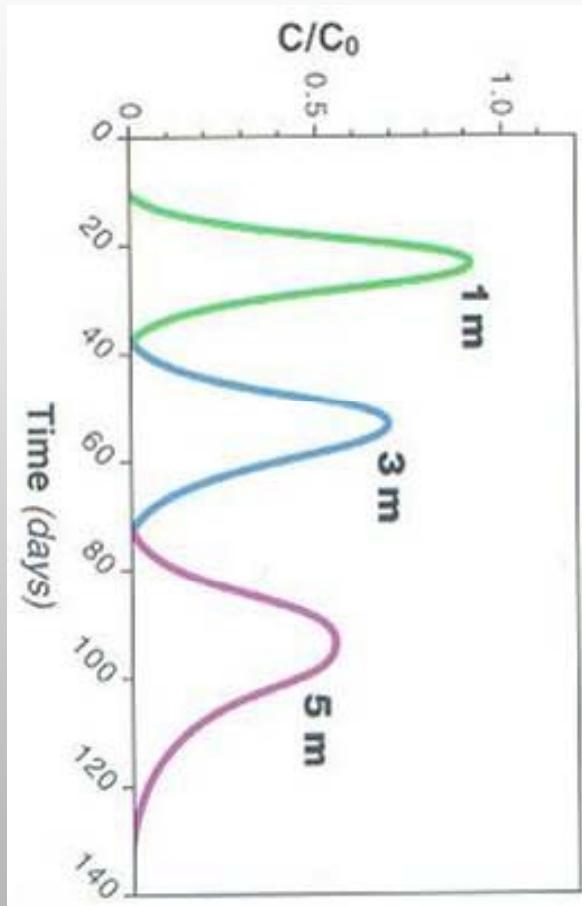
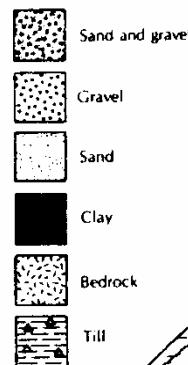


# Measuring and modeling of solute transport in soils



- Prediction of the solute transport in soils: fertilizer, salts, pesticides, trace elements
- Influence of different parameter on solute transport
- Determination of solute transport parameter
- Insight into the plant nutrient dynamics in the soil: uptake of nutrients by roots
- Influence of plant uptake on concentrations
- Leaching of plant nutrients from the rooting zone e.g. nitrate
- Development and optimization of the agricultural practice:
  - Minimizing of fertilizer losses
  - prevent the salinization due to irrigation
- Insight into the transport of environmental pollutants: prediction of their behavior in the environment



## Region

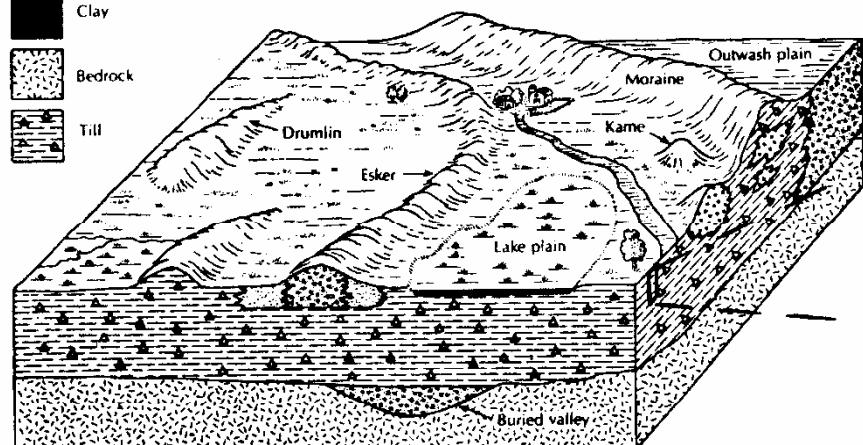
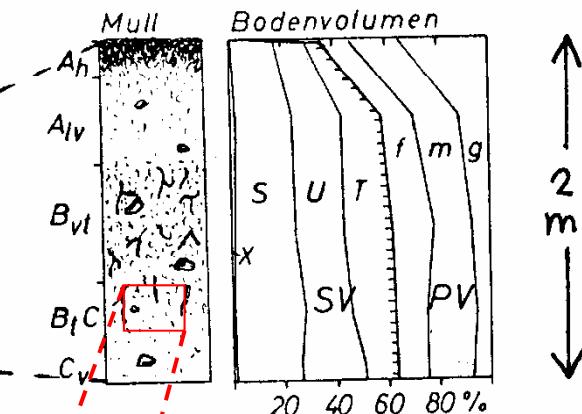


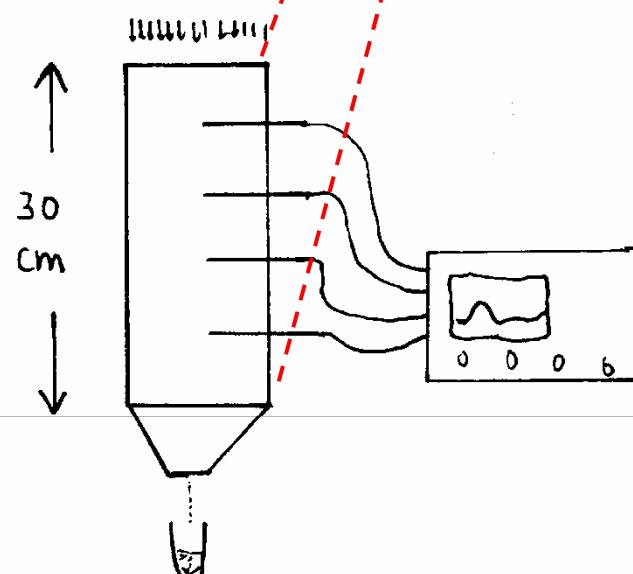
FIGURE 8.1 Distribution of sediments in a glaciated terrane.

## Profile



## Soil texture composition

## Column



## The measurement problem or the difference between a lab column and a field

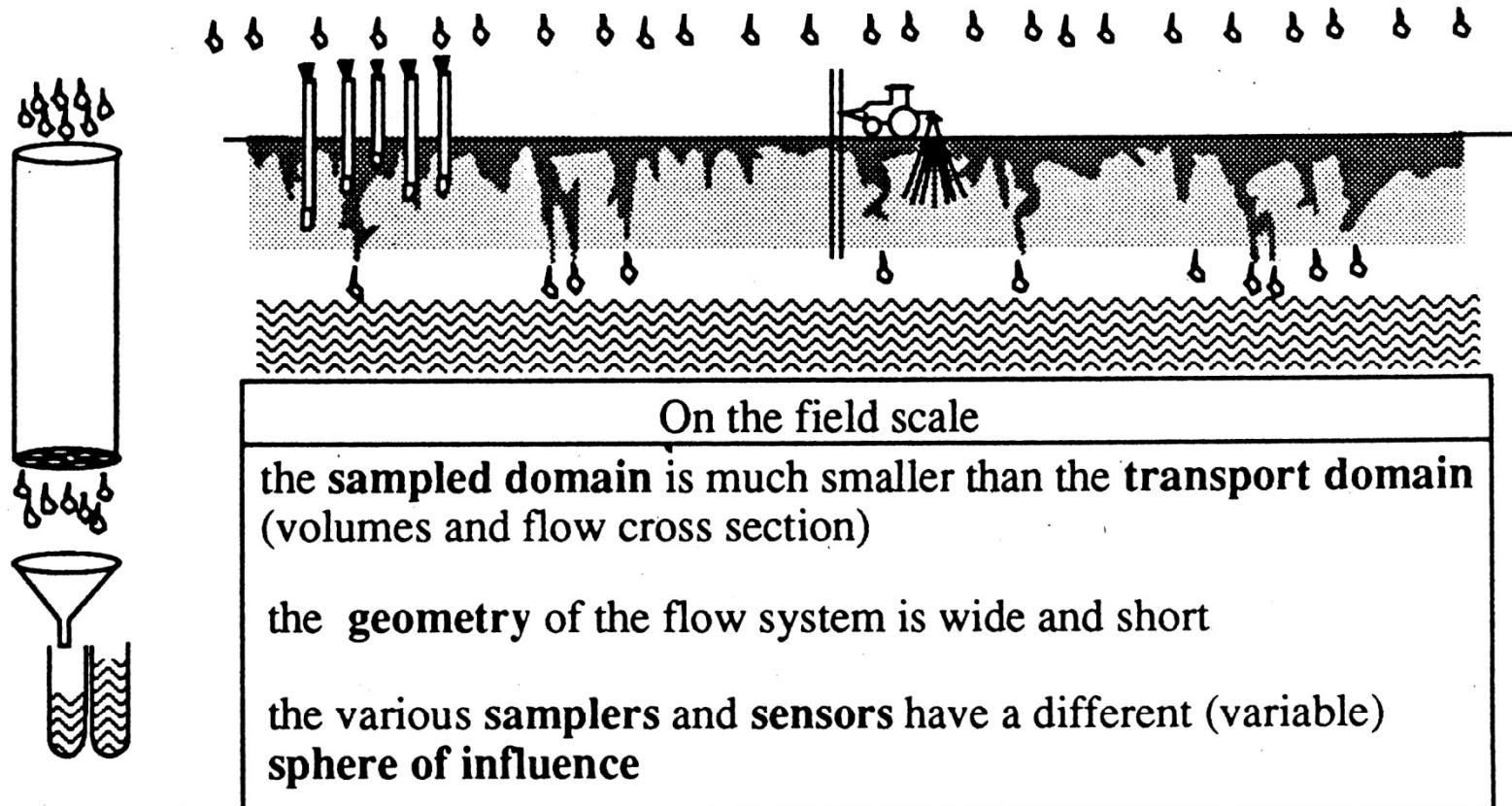
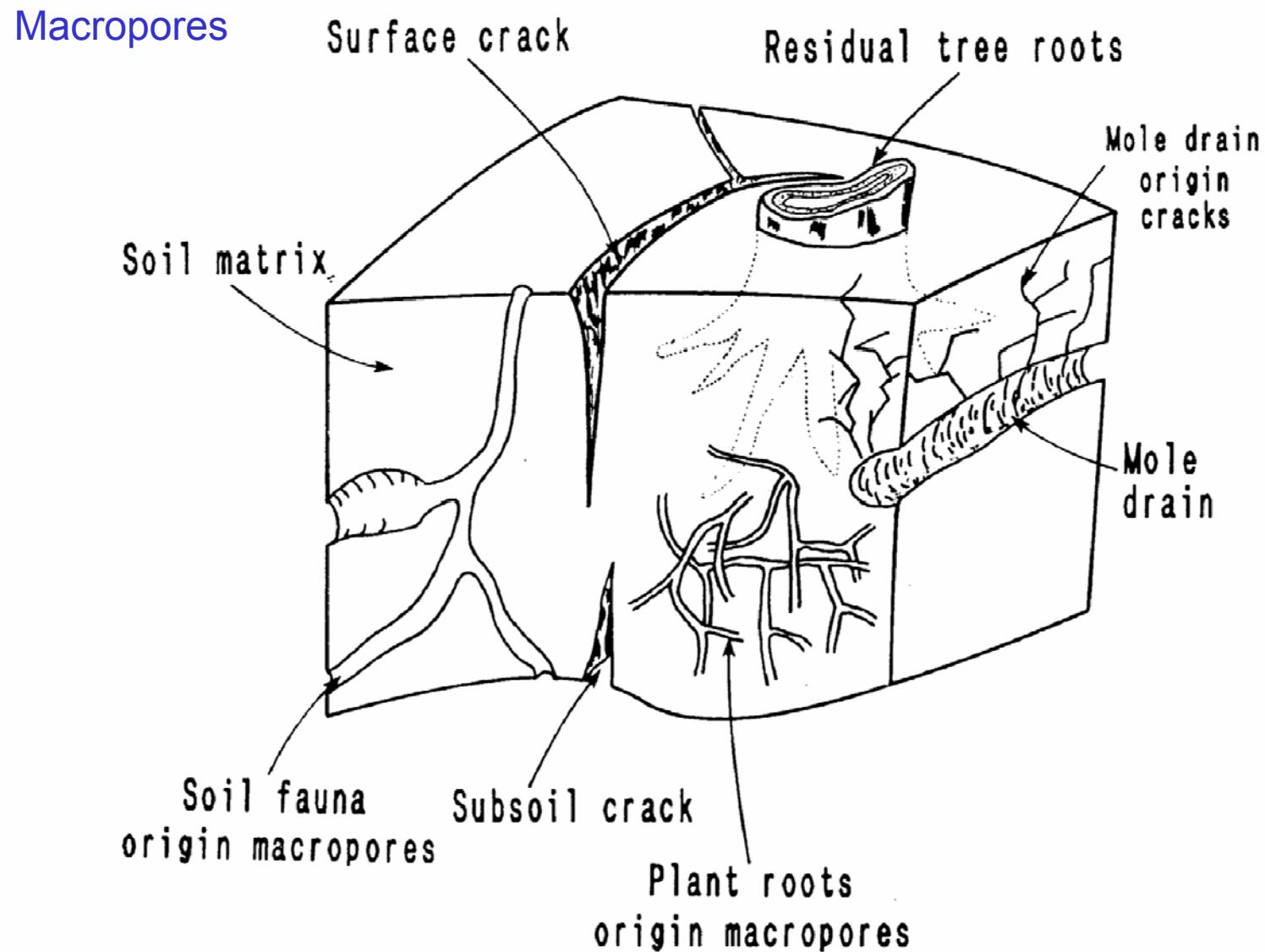
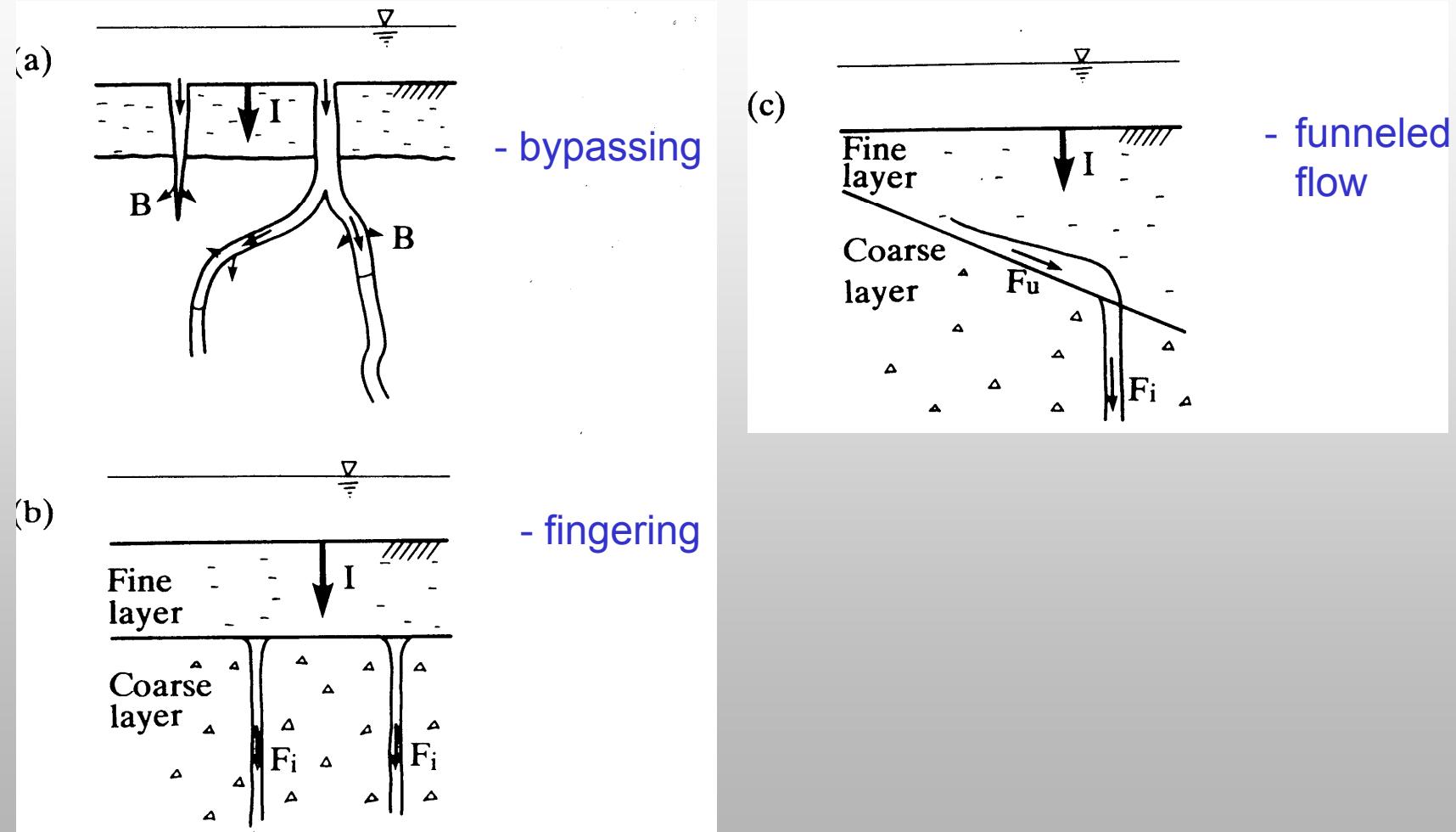


Fig.1 A field (soil) and a laboratory soil column are two very different systems. The scaling-up from lab scale to field scale is a problem which still waits to be resolved.

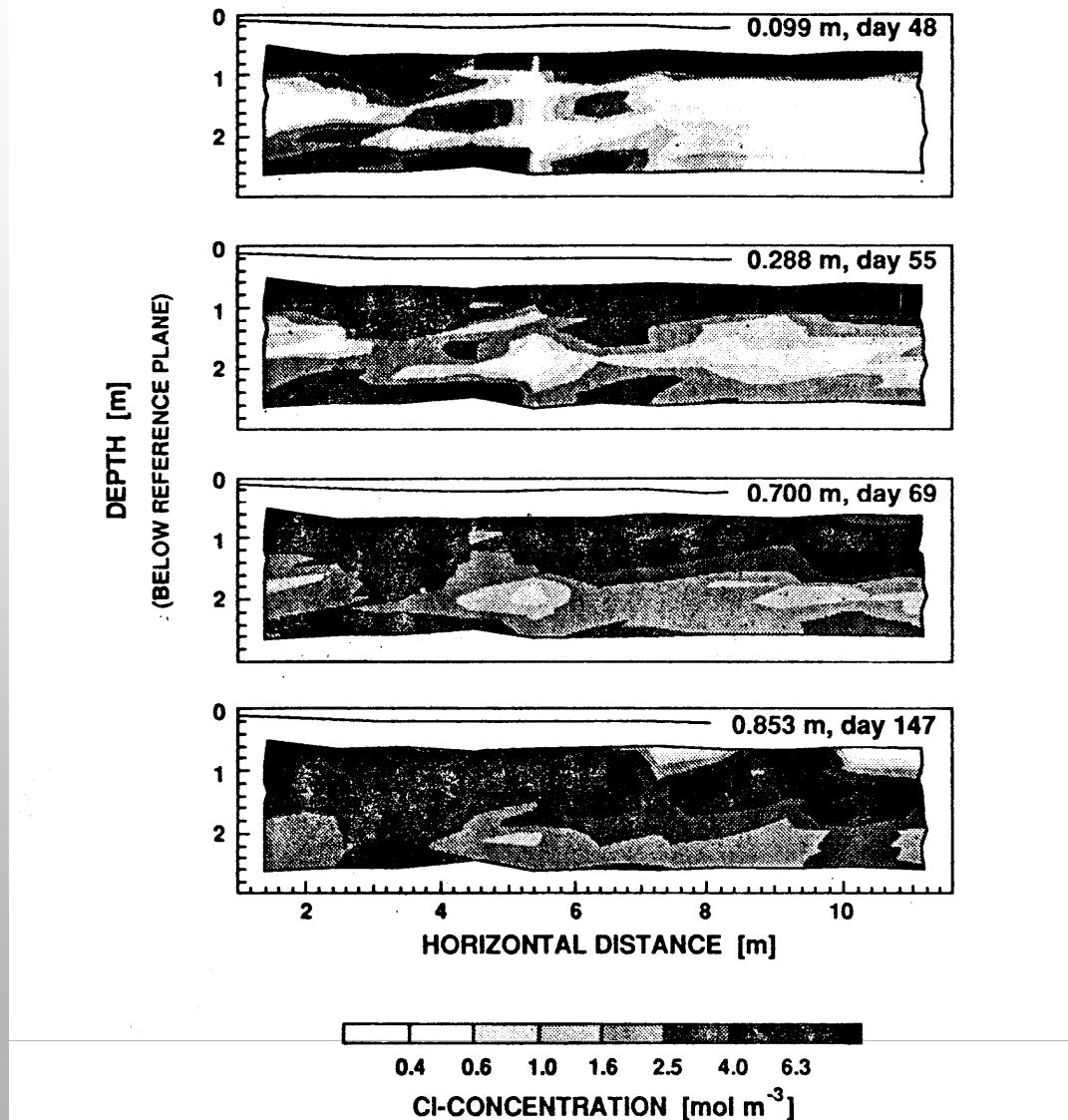


**Figure 9.22** Exaggerated macropores in soils.

### Preferential flow



## ROTH ET AL.: CHLORIDE TRANSPORT THROUGH AN UNSATURATED FIELD SOIL



6. Spatial distribution of the tracer after a cumulative infiltration of 0.099 m (beginning of irrigation), 0.288 m (one third of the irrigation), 0.700 m (end of irrigation) and 0.853 m (end of the experiment).

Preferential flow



Gimmi et al., Soil Physics  
Institute of Terrestrial Ecology, ETH Zurich



Infiltration experiment with:  
- ERT-measurements  
- soil physical measurements



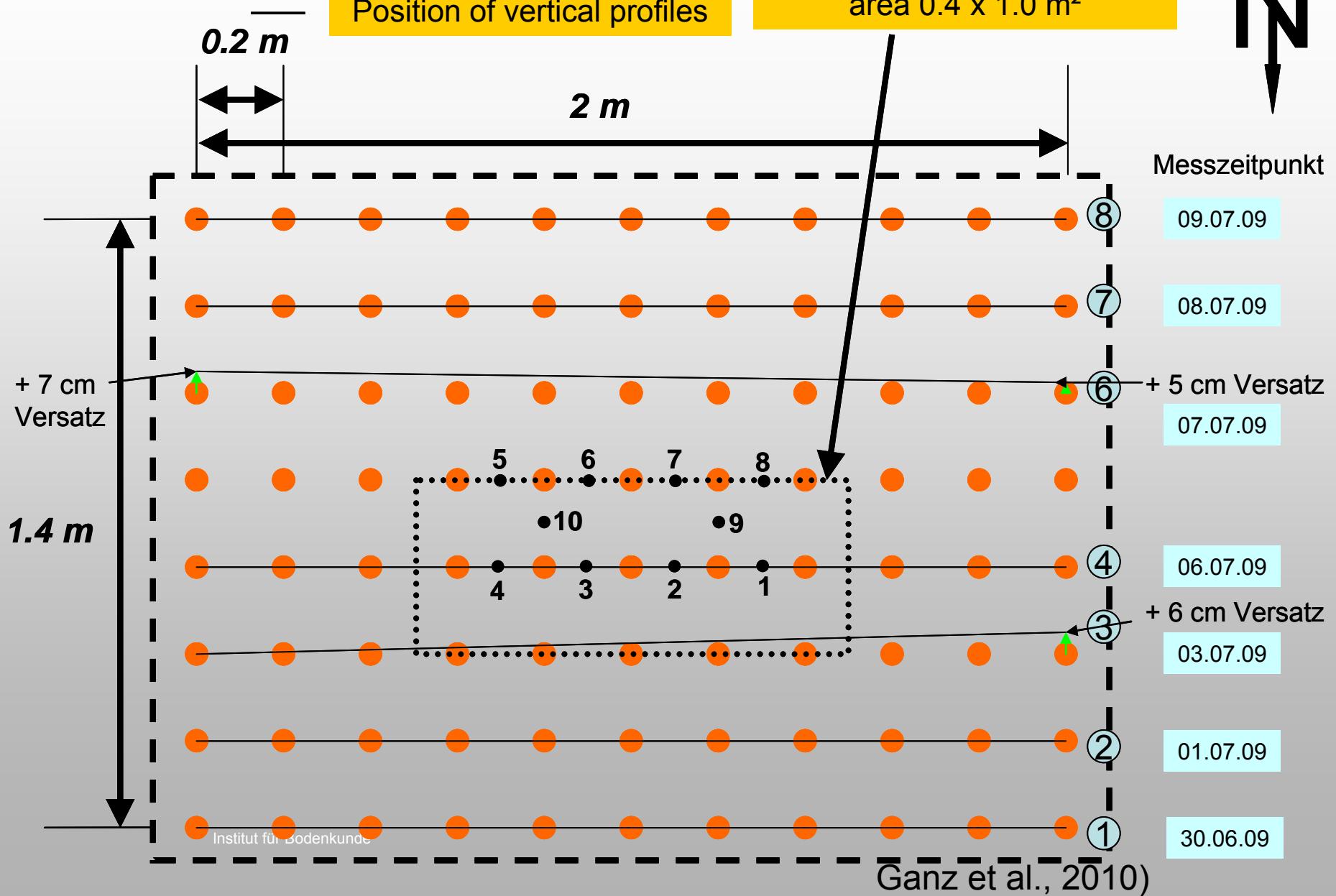
## Solute transport

● ERT-Elektroden

● Tensiometer

Position of vertical profiles

Water application  
area  $0.4 \times 1.0 \text{ m}^2$



Tracer experiment 2009:

- water application area  $0.40 * 1.00$  m
- Brilliant Blue
- water application amount 200 mm in 8 h.

Vertical profile 2  
( 40 cm from center)



Vertical profile 3  
( 20 cm from center)



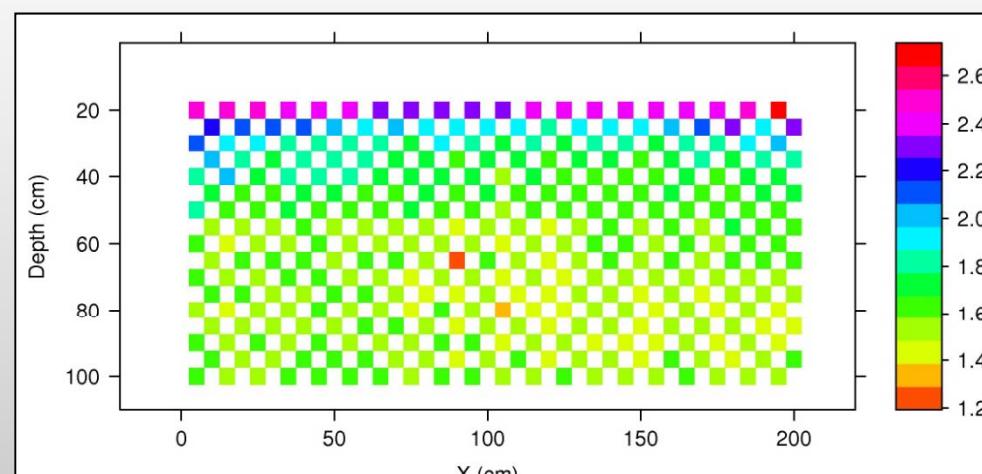
Vertical profile 4  
(center)



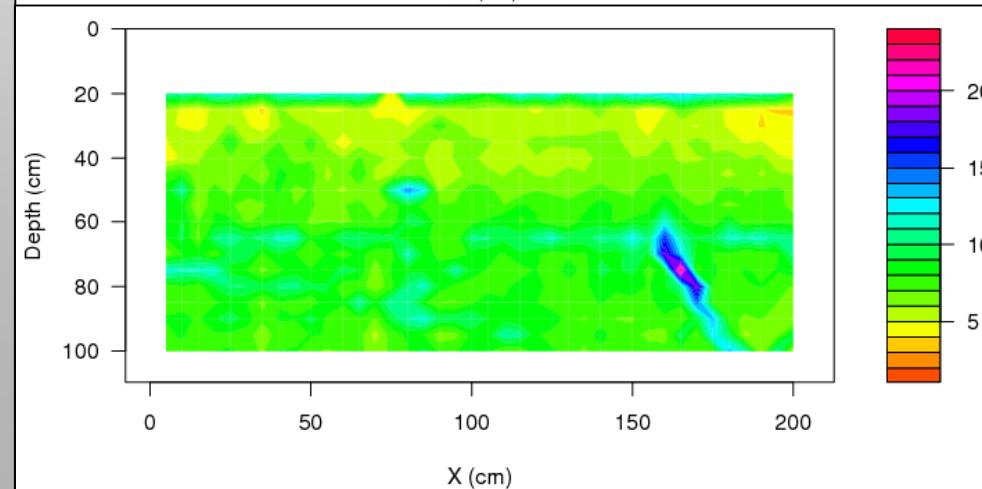
### Vertical profile 4 (center)



matric potential (pF)

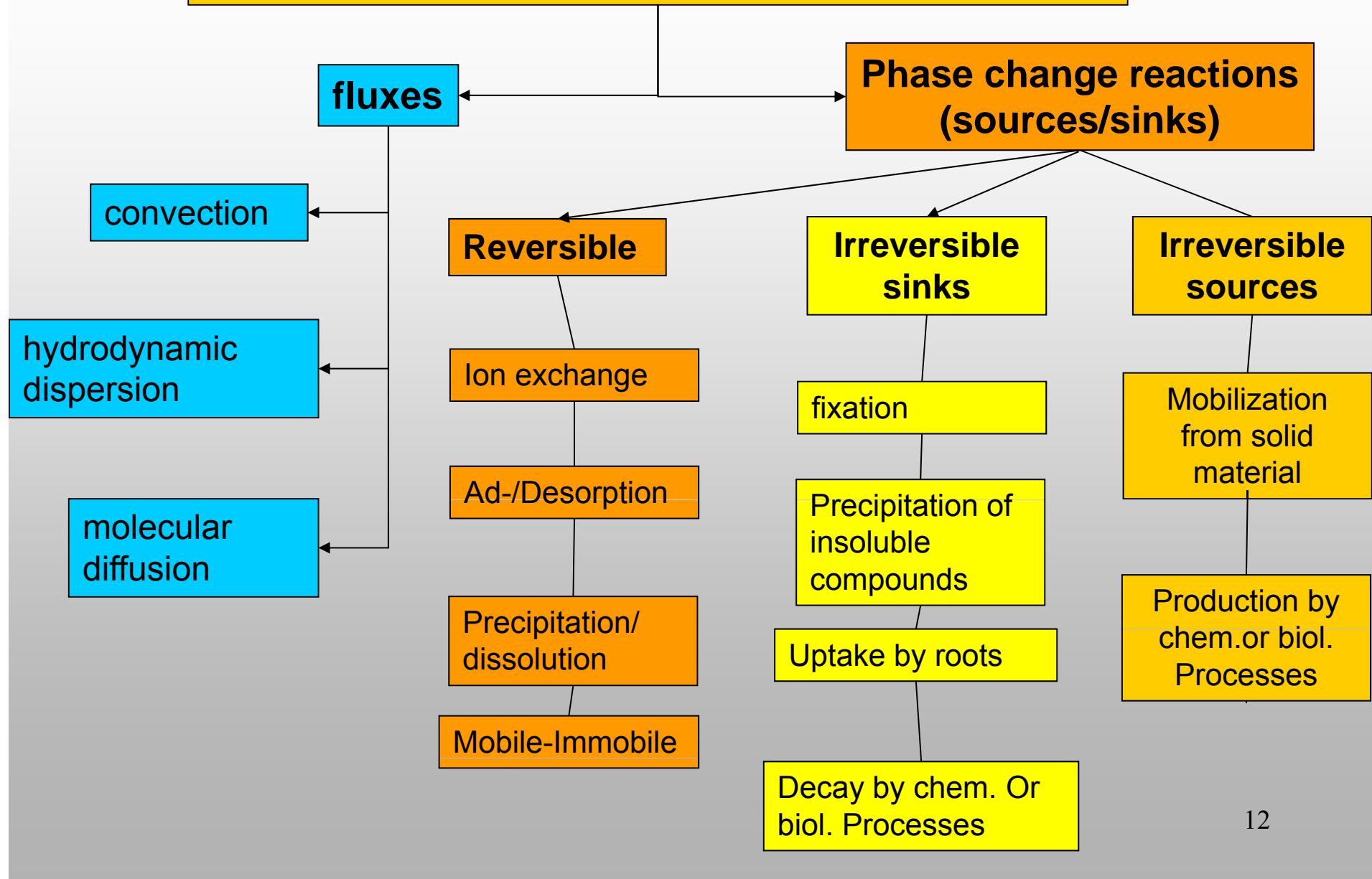


Water content (%-vol.)



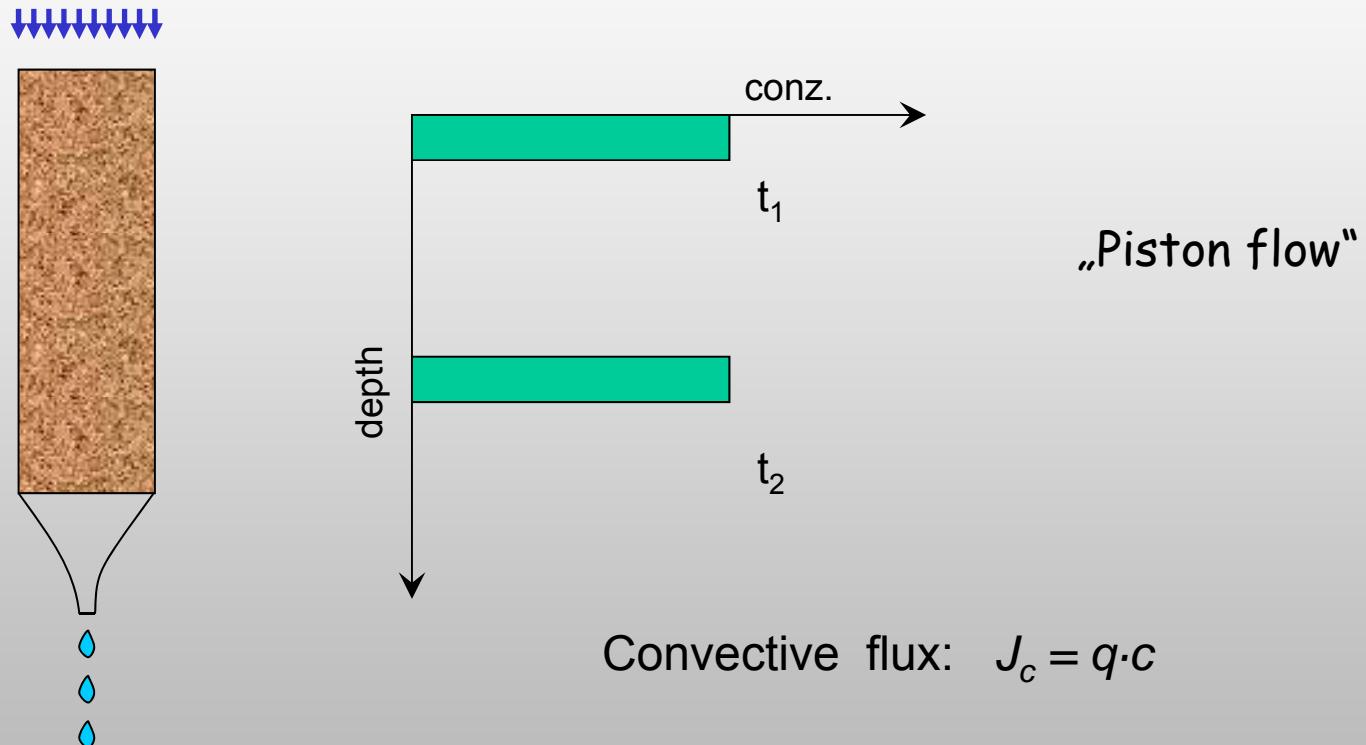
(Ganz et al. , 2010)

## Transport mechanism and processes in soils

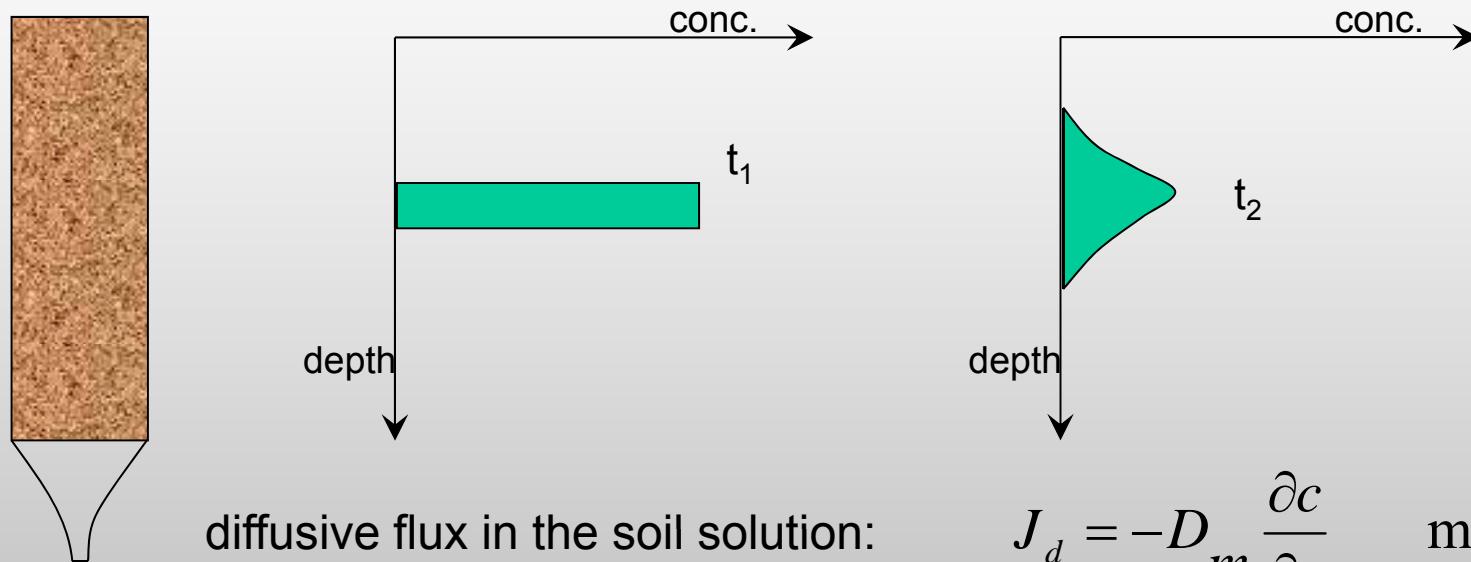


## Mass flow or transport by convection

In water dissolved solutes are transported with the water



Diffusion: thermal movement of molecules of in water solved solutes  
(Brownian motion)



diffusive flux in the soil solution:

$$J_d = -D_m \frac{\partial c}{\partial z} \quad \text{mg.cm}^{-2}.\text{d}^{-1}$$

$D_m$  = molecular diffusion coefficient  
in the oil solution

$$\left[ \text{cm}^2.\text{d}^{-1} \right]$$

$\frac{\partial c}{\partial z}$  = concentration gradient

$$\left[ \text{mg.cm}^{-4} \right]$$

The molecular diffusion coefficient is less as in free water :

- Tortuosity - water content - Pore diameter - elektrochemical effects

tortuosity faktor  $\tau$

$$\tau = f(\theta)$$

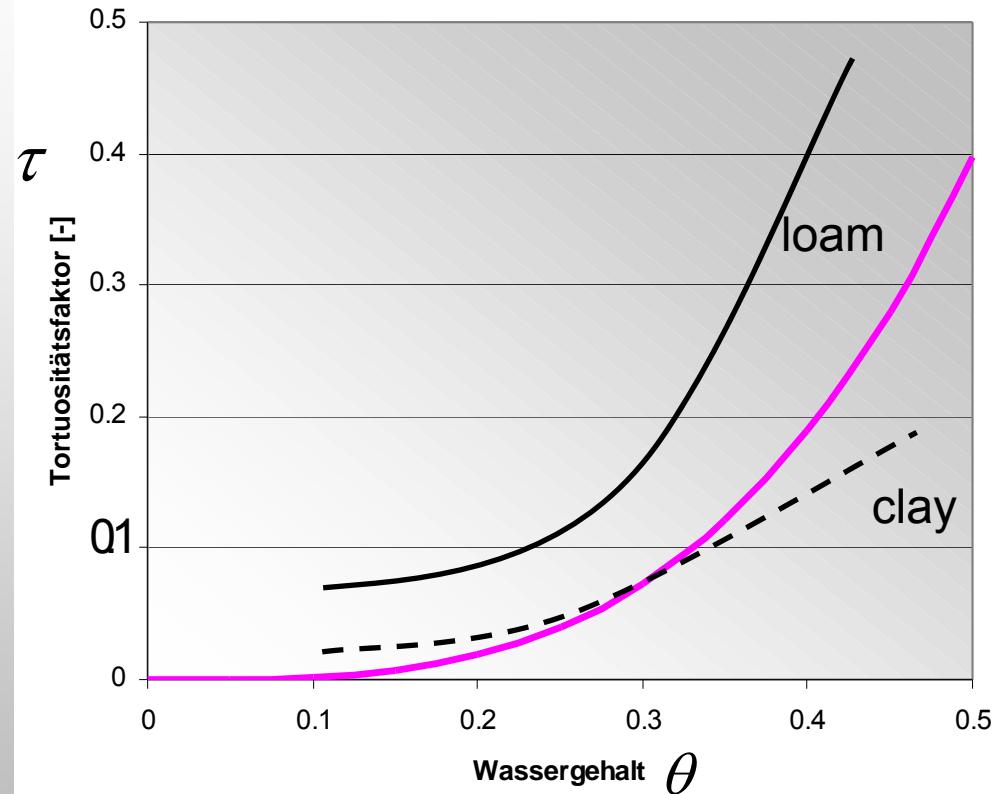
$$D_m = \tau \cdot D_o$$

$D_o$  = molecular diffusion coefficient  
in free water

For Cl<sup>-</sup>, NO<sub>3</sub><sup>-</sup>:  $D_o = 1.6 \text{ cm}^2 \cdot \text{d}^{-1}$

$$\tau = \frac{\theta^{10/3}}{n^2}$$

Millington und Quirk (1961)



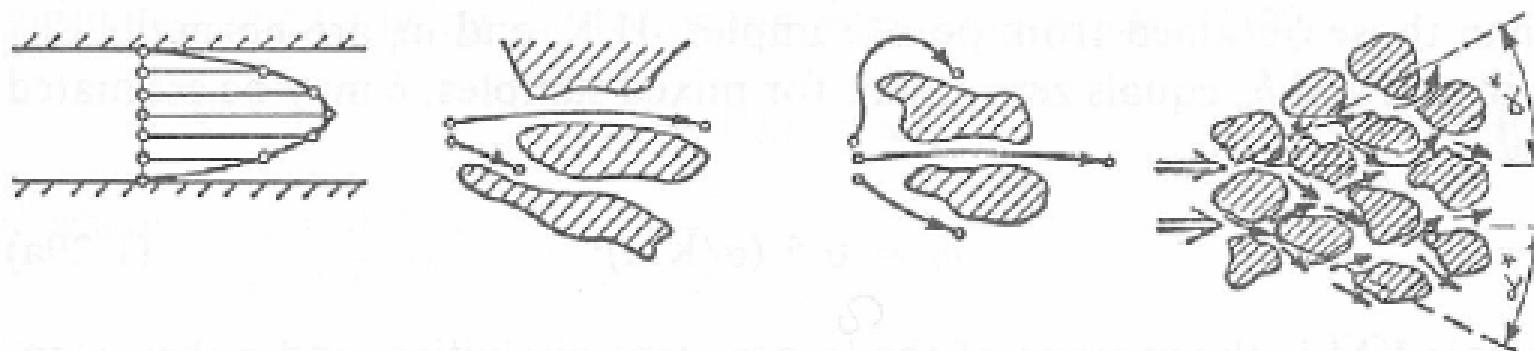
Olsen and Kemper (1968):  $\tau = a \cdot e^{b \cdot \theta}$

with  $b = 10$  and  
clay:  $a = 0.001$ <sup>15</sup>  
loam:  $a = 0.005$

### Hydrodynamic dispersion:

Due to the irregular velocity distribution of the water movement in soil pores and the different local velocities and flow paths .

**Only active with water movement!!**



longitudinal dispersion

transversal dispersion

## Hydrodynamic dispersion:

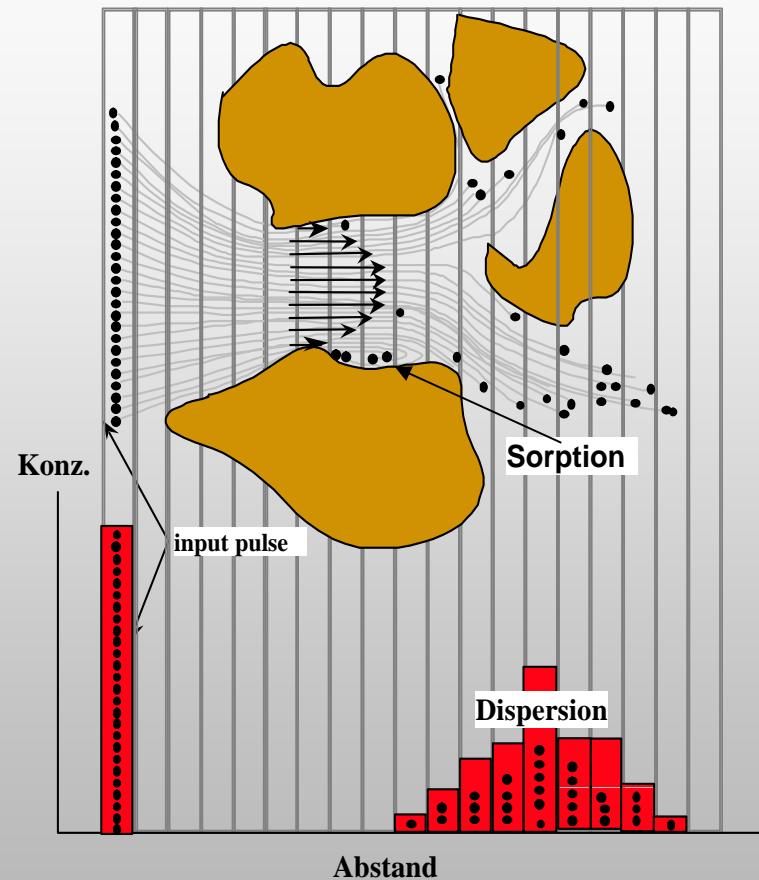
Dispersive flux in the soil solution :

$$J_{hd} = -D_{hd} \cdot \frac{\partial c}{\partial z} \quad \text{mg.cm}^{-2}.\text{d}^{-1}$$

$D_{hd}$  is a function of the mean porewater velocity :

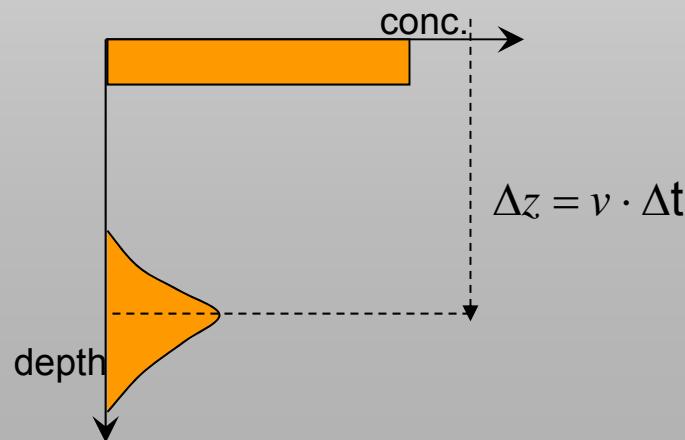
$$D_{hd} = \lambda \cdot v \quad \text{cm}^2.\text{d}^{-1}$$

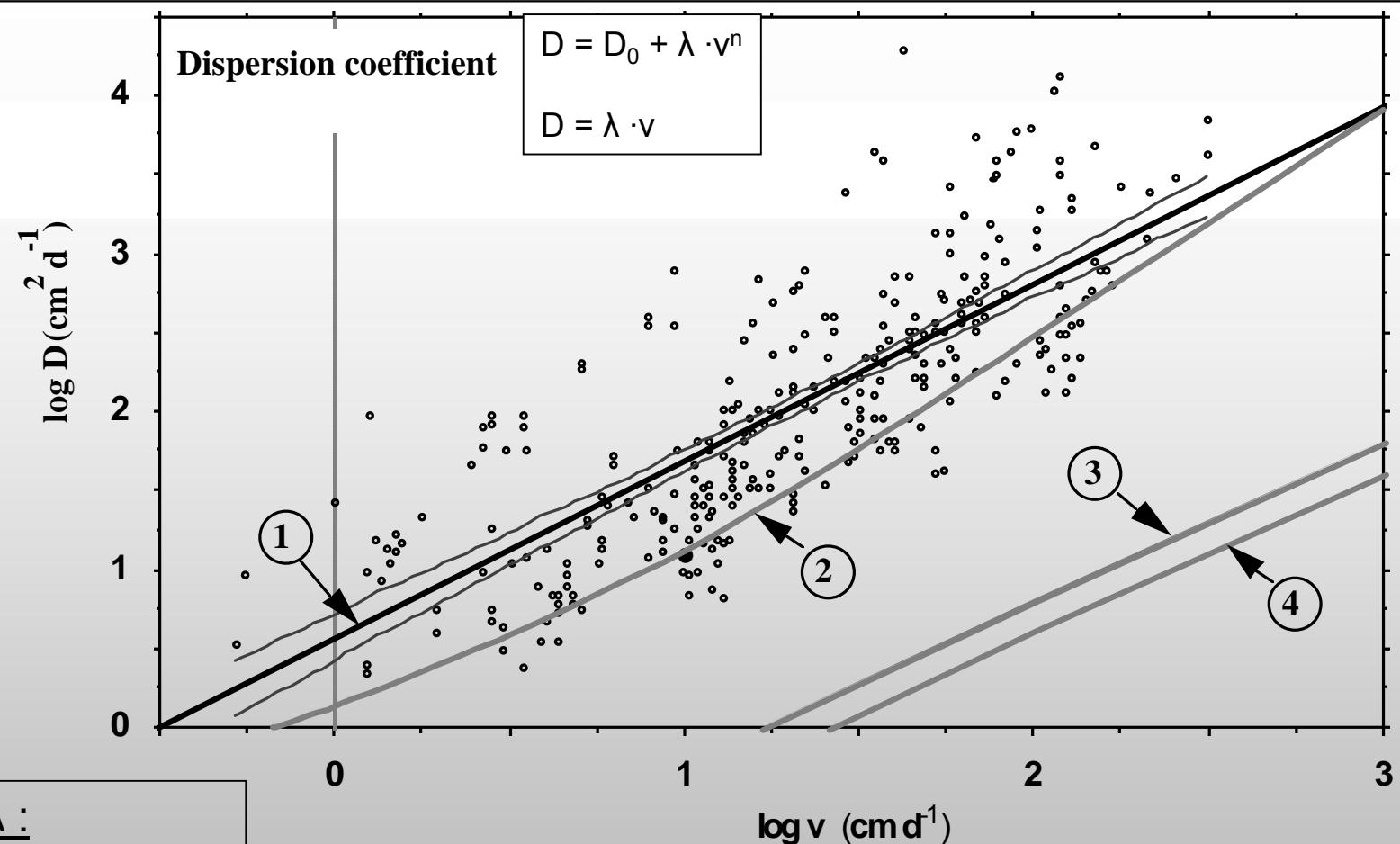
$\lambda$  = dispersivity [cm]



## Notation:

$q$	= filter velocity or darcy velocity	$\text{cm}^3 \cdot \text{cm}^{-2} \cdot \text{d}^{-1}$
$v$	= $q/\theta$ mean pore water velocity	$\text{cm} \cdot \text{d}^{-1}$
$v_m$	= $q/\theta_m$ effective pore water velocity	$\text{cm} \cdot \text{d}^{-1}$
$\theta$	= water content (mit $\theta = \theta_m + \theta_{im}$ )	$\text{cm}^3 \cdot \text{cm}^{-3}$
$\theta_m, \theta_{im}$	= mobile water content, resp. immobile water content	$\text{cm}^3 \cdot \text{cm}^{-3}$





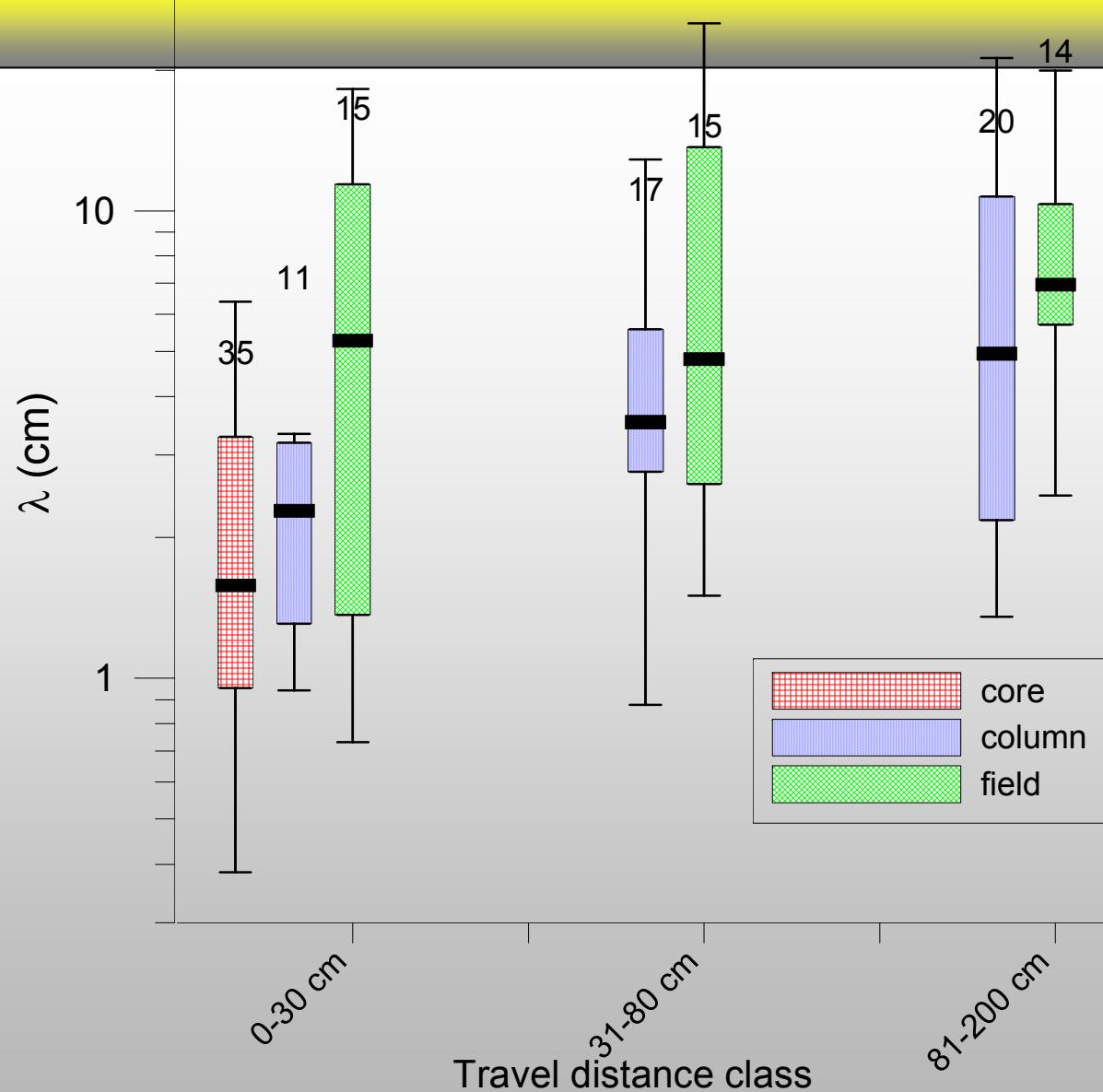
Dispersivity  $\lambda$ :

Lab column: 0.01-1.0 cm

Field soils: 0.5 – 20 cm

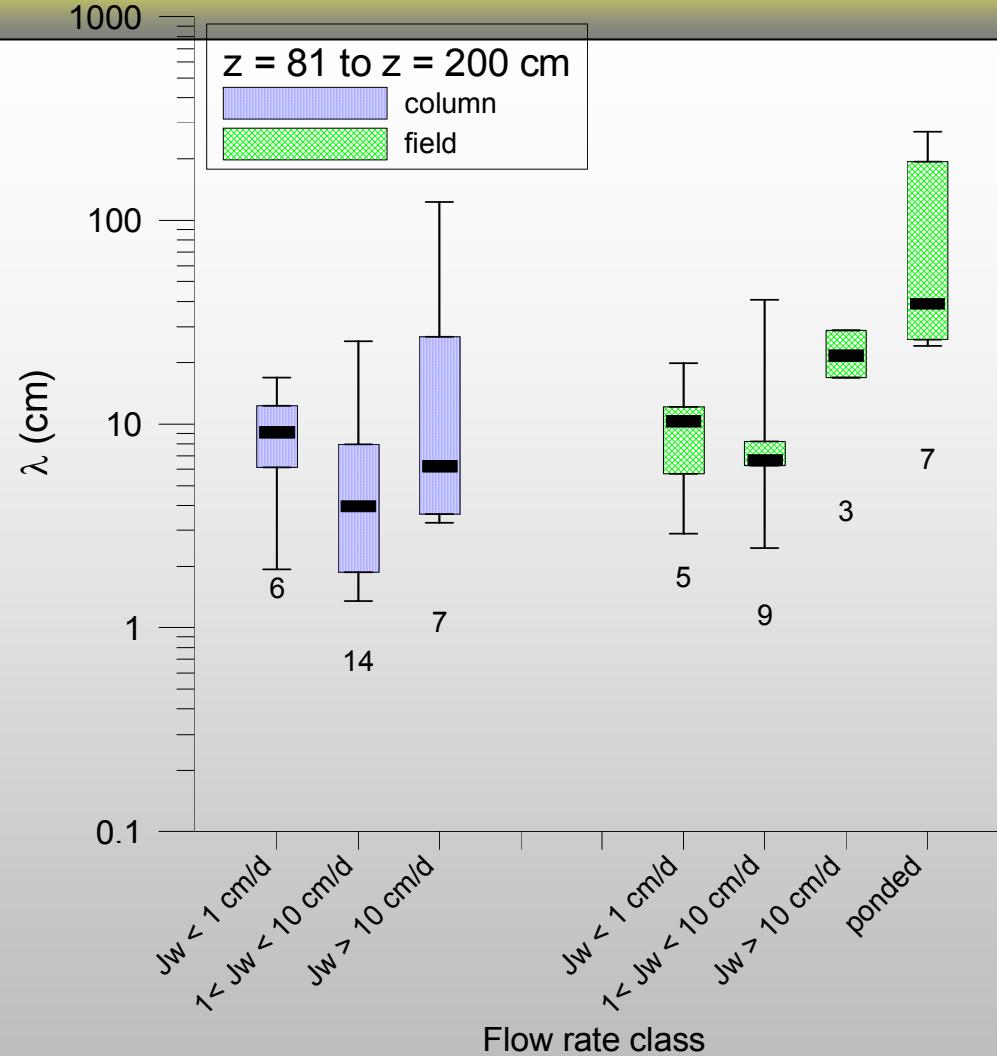
curve 1 $D = 0.60 + 2.93 v^{1.11}$ (Nielsen et al. 1976) curve 2 $D = 0.62 + 0.545 v^{1.355}$ (Kirda et al. 1973)	}	field experiments
curve 3 $D = 0.04 v$ (Bertsch et al. 1978) curve 4 $D = 0.063 v$ (Bertsch et al. 1978)		

laboratory experiments 19



(Vanderborght & Vereecken, 2007)

The dispersivity  $\lambda$  as a function of the transport distance (0-30 cm, 30-81 cm and 81-200 cm) of core, column and field experiments



(Vanderborght & Vereecken, 2007)

The dispersivity  $\lambda$  as a function of 4 flow rate classes for a transport distance of 81 to 200 cm in column and field experiments

### Apparent or effective dispersion coefficient

Diffusive and hydromechanical caused dispersive fluxes cannot be separated macroscopically and thus are taken together:

diffusive-dispersive flux in the soil:

$$J_{dd} = -(D_m + D_{hd}) \cdot \frac{\partial c}{\partial z} = -D_e \cdot \frac{\partial c}{\partial z} \quad \text{mg.cm}^{-2}.\text{d}^{-1}$$

$D_e$  = „effective“ or „apparent“ diffusion coefficient  
or just „dispersion coefficient“

### Convective-dispersive flux:

Total solute flux in the soil is the sum of the convective and the diffusive-dispersive flux:

$$J_{cd} = J_{dd} + J_c$$

$$J_{cd} = \underbrace{-(D_m + D_{hd}) \cdot \frac{\partial c}{\partial z}}_{\text{diffusive-dispersive}} + q \cdot c \quad \text{mg.cm}^{-2} \cdot \text{d}^{-1}$$

convective

### Solute transport in a soil column:

- volumetric water content  $\theta$   $\text{cm}^3 \cdot \text{cm}^{-3}$

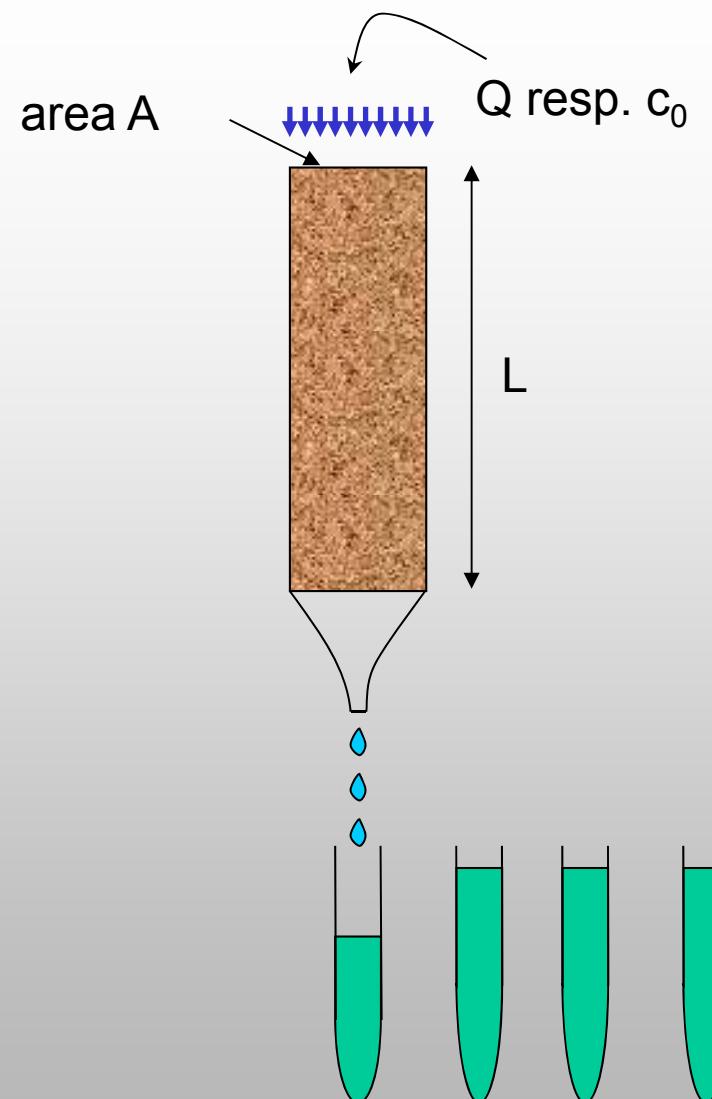
- column length  $L$   $\text{cm}$

- column area  $A$   $\text{cm}^2$

- irrigation amount  $Q$   $(Q = q \cdot A)$   $\text{cm}^3 \cdot \text{d}^{-1}$

- solute concentration  $c_0$   $\text{mg} \cdot \text{cm}^{-3}$

- Pore water velocity  $v$   $(v = q/\theta)$   $\text{cm} \cdot \text{d}^{-1}$



### Definition of pore volume:

$$1 \text{ PV} = A \cdot L \cdot \theta \quad \text{cm}^3$$

The exchange of 1 PV takes  $t_{PV}$  days

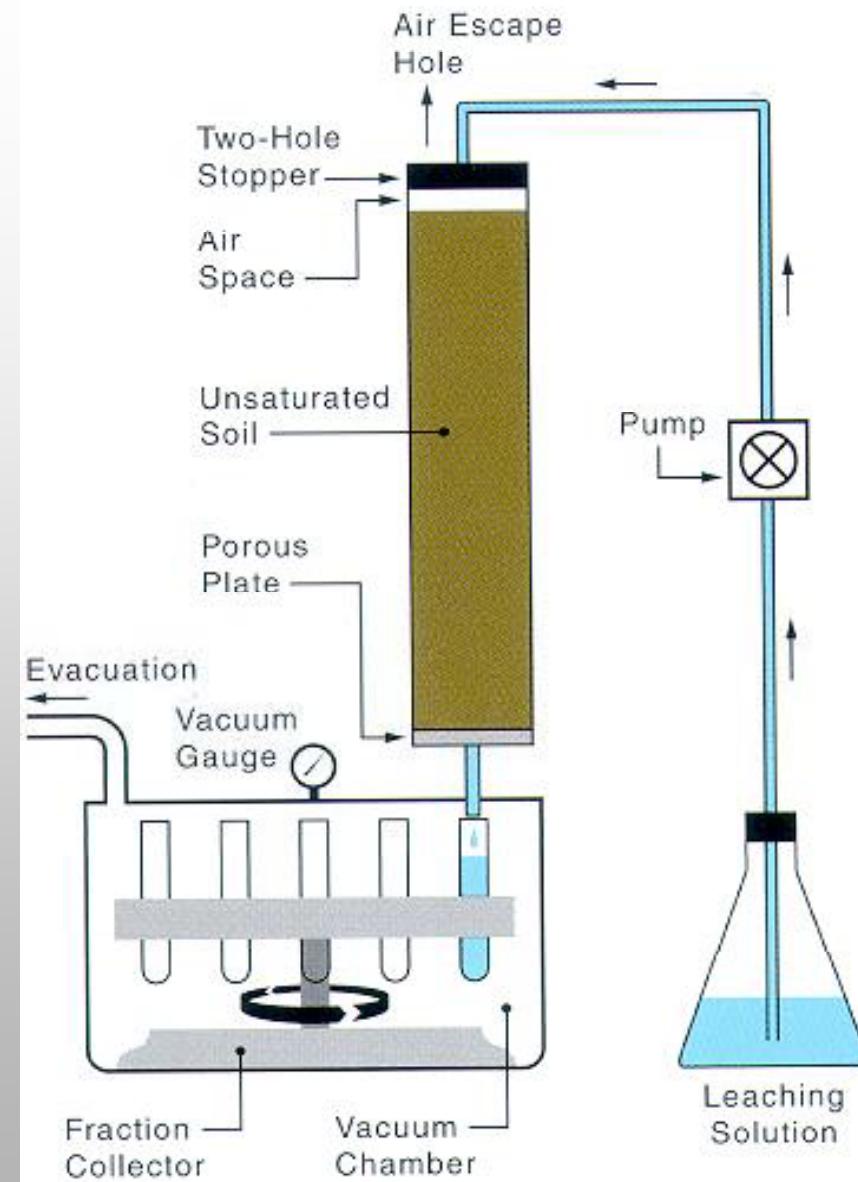
### Break through curves:

= concentration in time

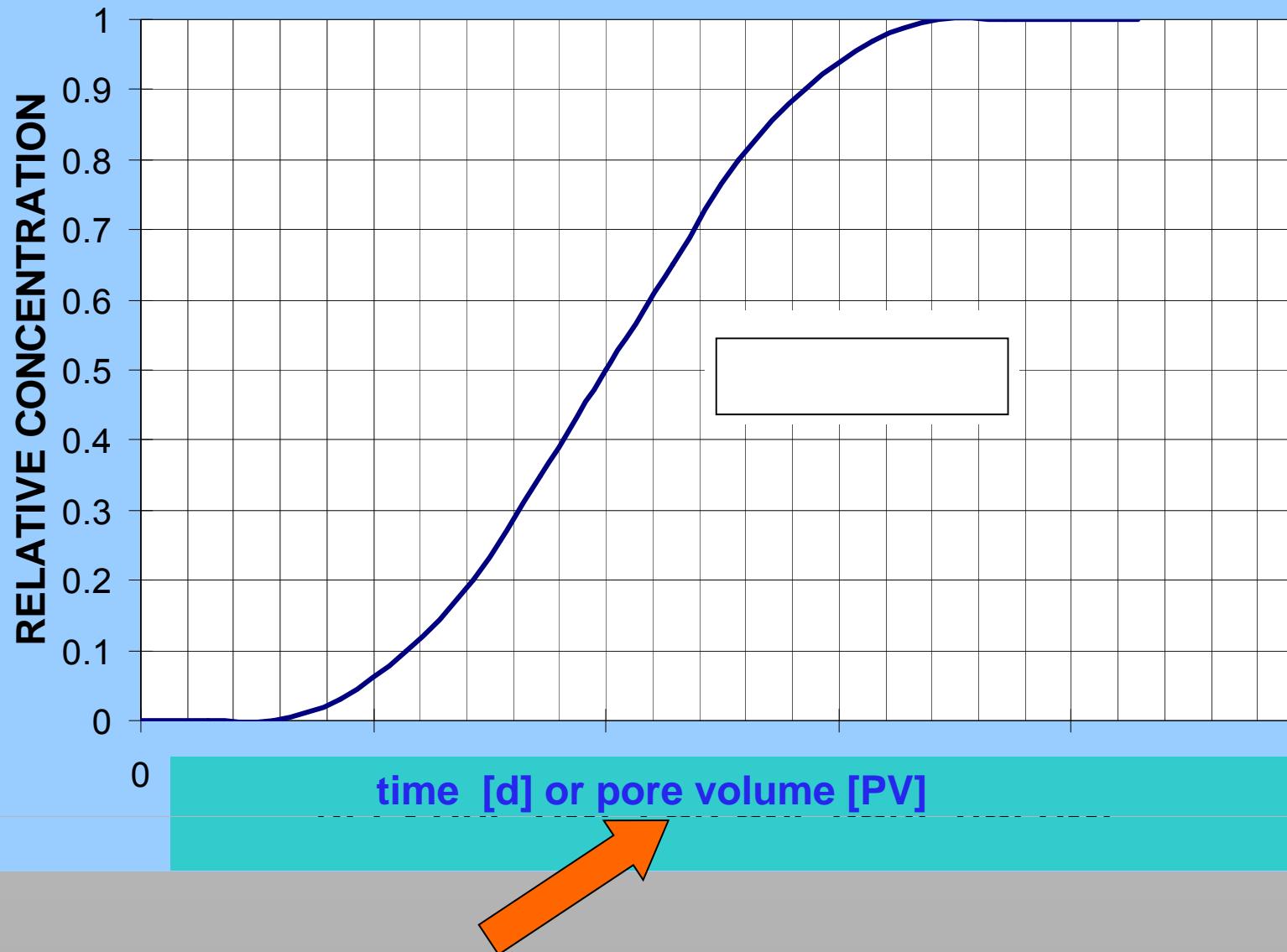
- at the end of a soil column

or

- in a chosen depth



Breakthrough curve : convection and diffusion-dispersion



- relative concentration

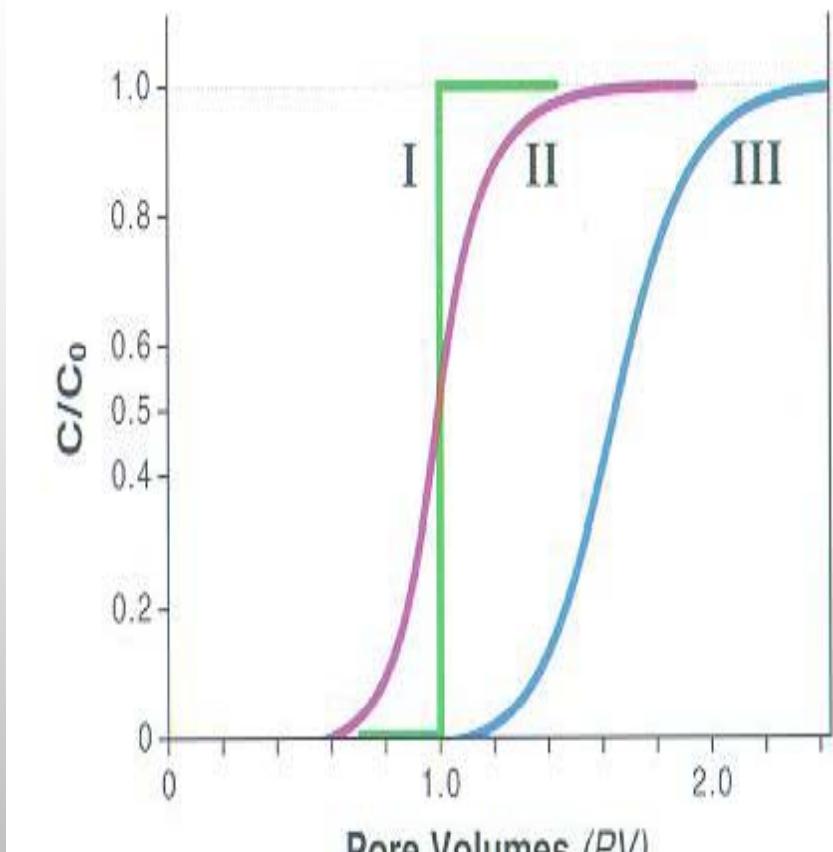
$$c/c_0$$

- Pore volume

(= dimensionless time)

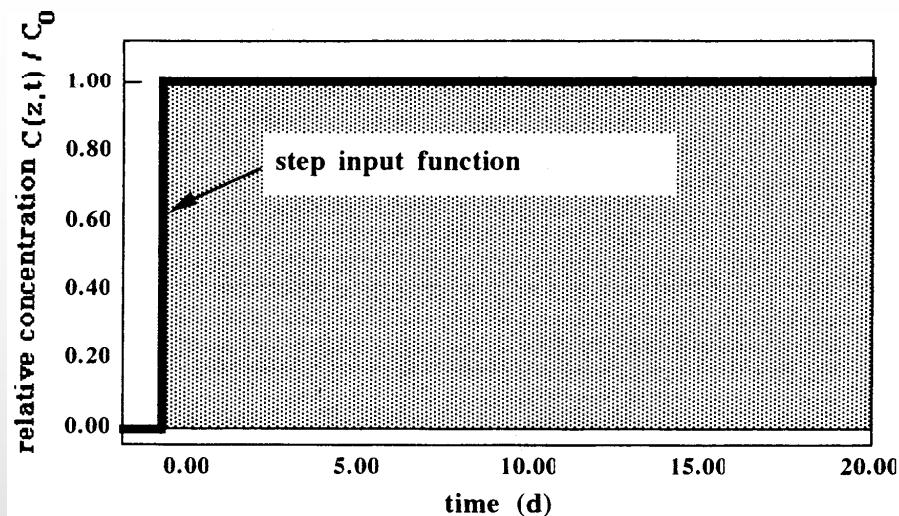
= time definition in multiple times  
needed to exchange one pore volume  
of soil solution

$$PV = t / t_{PV} = \frac{t}{L/v} = \frac{vt}{L}$$



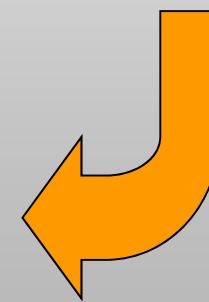
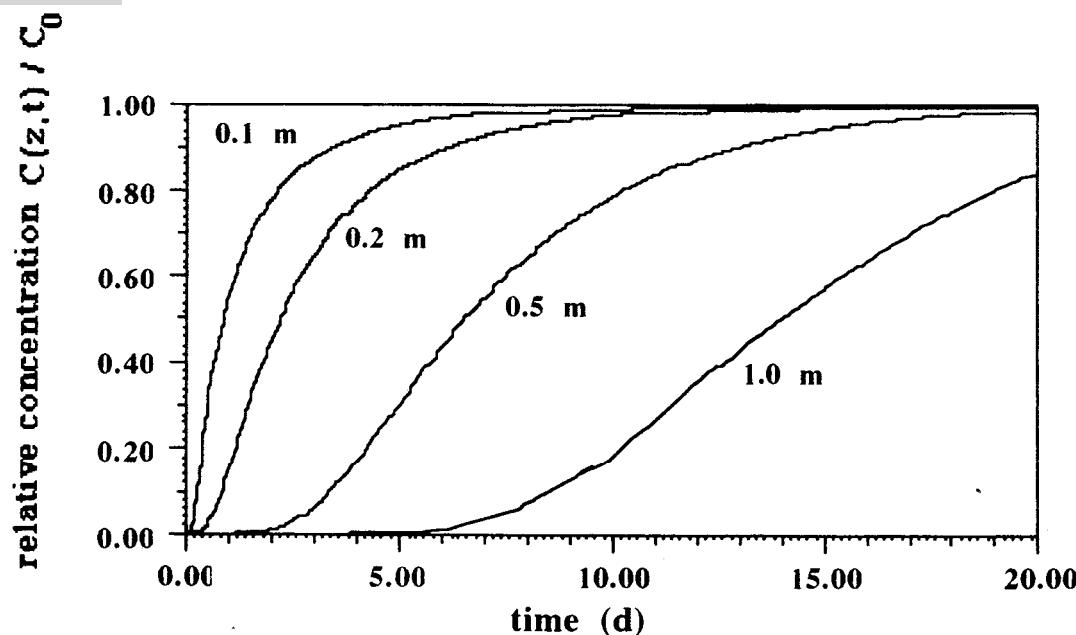
- Break through curves:
- I : only convection („piston flow“)
  - II : convection and diffusion-dispersion
  - III : convection and diffusion-dispersion and retardation

Breakthrough curves in different depths for a step input:



$$q = 0.02 \text{ m d}^{-1} \quad \theta = 0.30 \text{ m}^3 \text{m}^{-3} \quad D = 0.005 \text{ m}^2 \text{d}^{-1}$$

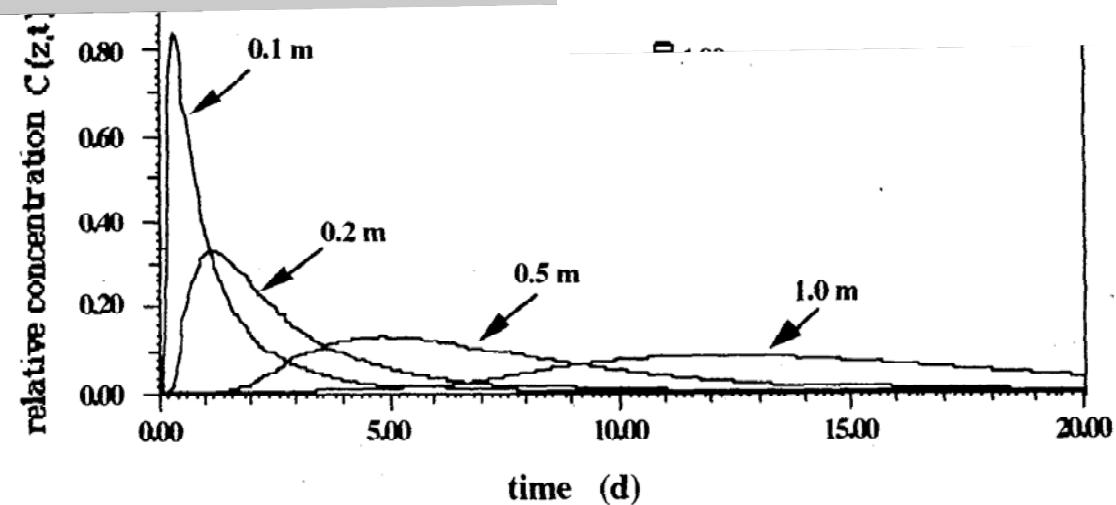
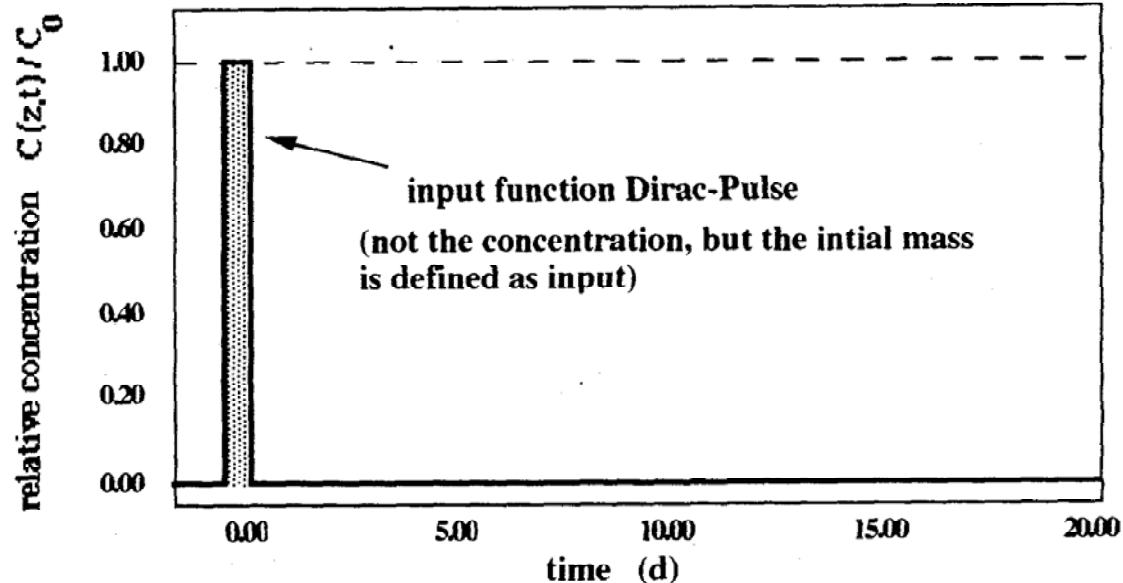
$$v = 0.067 \text{ m d}^{-1} \quad \lambda = D/v = 0.075 \text{ m}$$



Breakthrough curves in different depths :

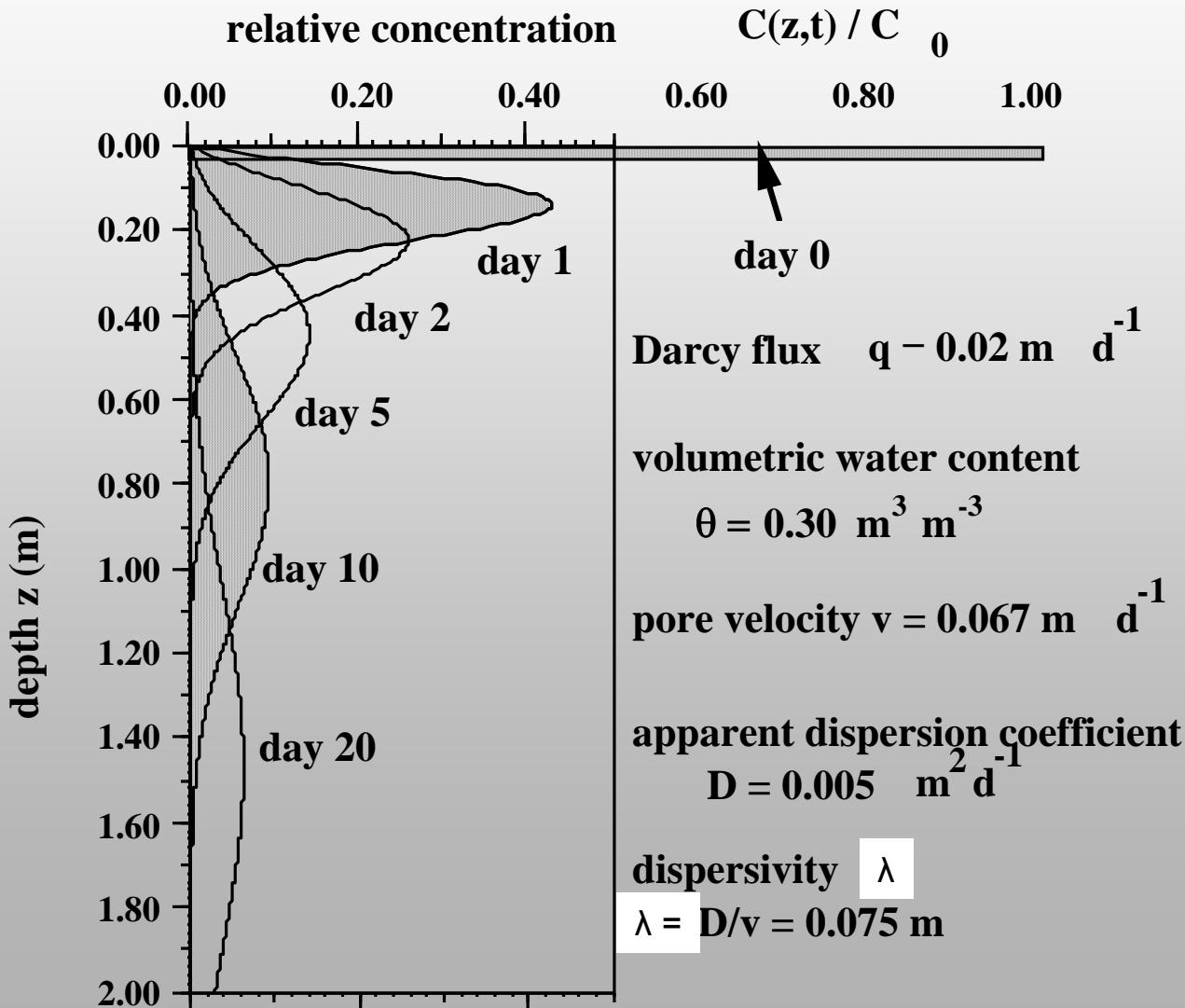
$$q = 0.02 \text{ m d}^{-1} \quad \theta = 0.30 \text{ m}^3 \text{ m}^{-3} \quad D = 0.005 \text{ m}^2 \text{ d}^{-1}$$

$$v = 0.067 \text{ m d}^{-1} \quad \lambda = D/v = 0.075 \text{ m}$$

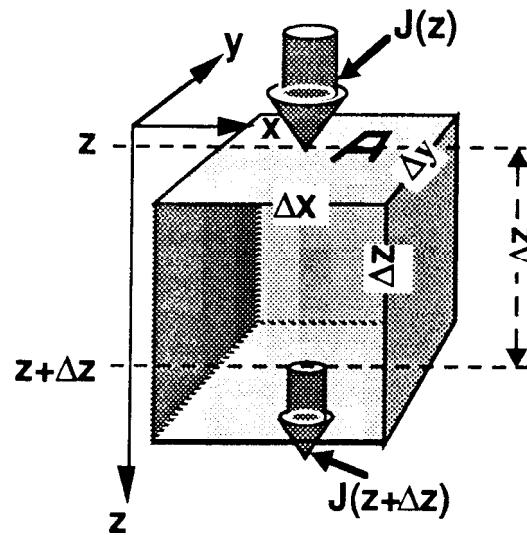


**displacement of a solute pulse**       $C_t(z)$

**concentration profiles**



### Derivation of the partial differential equation for solute transport in soils :



solute content in the soil :  $C_{tot} = \theta \cdot c + \rho \cdot S$

$\theta$	= volumetric water content	$\text{cm}^3 \cdot \text{cm}^{-3}$
$c$	= concentration in the soil solution	$\text{mg} \cdot \text{cm}^{-3}$
$\rho$	= bulk density of the soil	$\text{g} \cdot \text{cm}^{-3}$
$S$	= adsorbed solute amount	$\text{mg} \cdot \text{g}^{-1}$

### Mass balance : Mass conservation

solute inflow – outflow = Change in solute content

$$J_{in} = J_z \cdot \Delta x \cdot \Delta y \cdot \Delta t$$

$$J_{out} = J_{z+\Delta z} \cdot \Delta x \cdot \Delta y \cdot \Delta t$$

in solution:  $\Delta(\theta \cdot c) \cdot \Delta x \cdot \Delta y \cdot \Delta z$

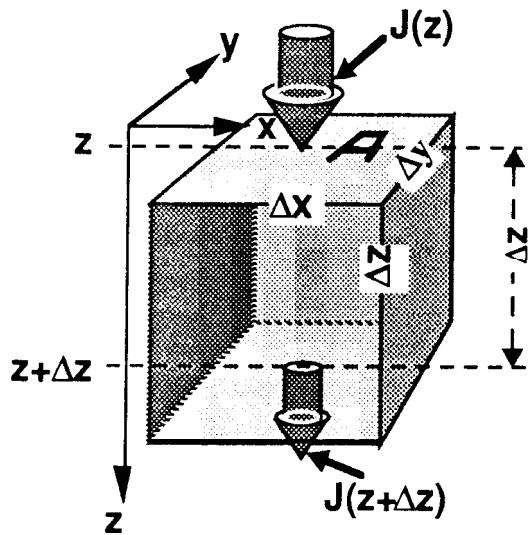
at the soil matrix:  $\Delta(\rho \cdot S) \cdot \Delta x \cdot \Delta y \cdot \Delta z$

$$\frac{\Delta C_{tot}}{\Delta t} = -\frac{\Delta J}{\Delta z}$$



$$\frac{\partial C_{tot}}{\partial t} = -\frac{\partial J}{\partial z}$$

partial Differential equation for solute transport in soils :



$\theta$	= volumetric water content	$\text{cm}^3 \cdot \text{cm}^{-3}$
$c$	= concentration in the soil solution	$\text{mg} \cdot \text{cm}^{-3}$
$\rho$	= bulk density of the soil	$\text{g} \cdot \text{cm}^{-3}$
$S$	= adsorbed solute amount	$\text{mg} \cdot \text{g}^{-1}$

$$\frac{\partial C_{tot}}{\partial t} = - \frac{\partial J}{\partial z} \quad (1)$$

$$C_{tot} = \theta \cdot c + \rho \cdot S \quad (2)$$

$$J = -(D_m + D_{hd}) \cdot \frac{\partial c}{\partial z} + q \cdot c \quad \text{mg} \cdot \text{cm}^{-2} \cdot \text{d}^{-1} \quad (3)$$

(2) and (3) in (1):

$$\frac{\partial \theta c}{\partial t} + \frac{\partial \rho S}{\partial t} = \frac{\partial}{\partial z} \left( D_e \frac{\partial c}{\partial z} \right) - \frac{\partial q c}{\partial z} \quad _{32}$$

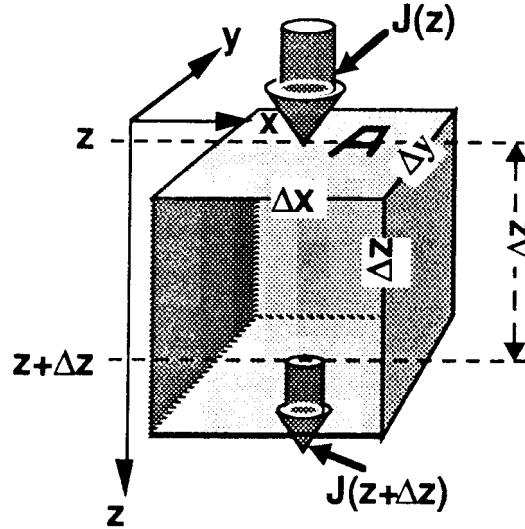
Additional processes:

uptake by plants :

$$\frac{\partial \theta c}{\partial t} + \frac{\partial \rho S}{\partial t} = \frac{\partial}{\partial z} \left( D_e \frac{\partial c}{\partial z} \right) - \frac{\partial qc}{\partial z} - Q_{\text{uptake}}$$

decay/production

$$\frac{\partial \theta c}{\partial t} + \frac{\partial \rho S}{\partial t} = \frac{\partial}{\partial z} \left( D_e \frac{\partial c}{\partial z} \right) - \frac{\partial qc}{\partial z} - \underbrace{\mu_w \cdot \theta \cdot c - \mu_s \cdot \rho \cdot S}_{\text{1. order decay}}$$



Mobile-immobile

1. order decay

partial differential equation in soils:

$$\frac{\partial \theta c}{\partial t} + \frac{\partial \rho S}{\partial t} = \frac{\partial}{\partial z} \left( D_e \frac{\partial c}{\partial z} \right) - \frac{\partial qc}{\partial z} + Q - \mu_w \cdot \theta \cdot c - \mu_s \cdot \rho \cdot S + \gamma_w \cdot \theta + \gamma_s \cdot \rho$$

$\theta$	= volumetric water content	$\text{cm}^3 \cdot \text{cm}^{-3}$
$c$	= concentration in the soil solution	$\text{mg} \cdot \text{cm}^{-3}$
$\rho$	= bulk density of the soil	$\text{g} \cdot \text{cm}^{-3}$
$S$	= adsorbed solute amount	$\text{mg} \cdot \text{g}^{-1}$
$D_e$	= Dispersion coefficient	$\text{cm}^2 \cdot \text{d}^{-1}$
$q$	= Darcy-Water flux	$\text{cm} \cdot \text{d}^{-1}$
$\mu$	= 1. order decay rate	$\text{d}^{-1}$
$\gamma$	= 0. order production rate	$\text{mg} \cdot \text{cm}^{-3} \cdot \text{d}^{-1}$
$Q$	= sources/sinks	$\text{mg} \cdot \text{cm}^{-3} \cdot \text{d}^{-1}$
$z$	= depth	cm
$t$	= time	d

## Sorption

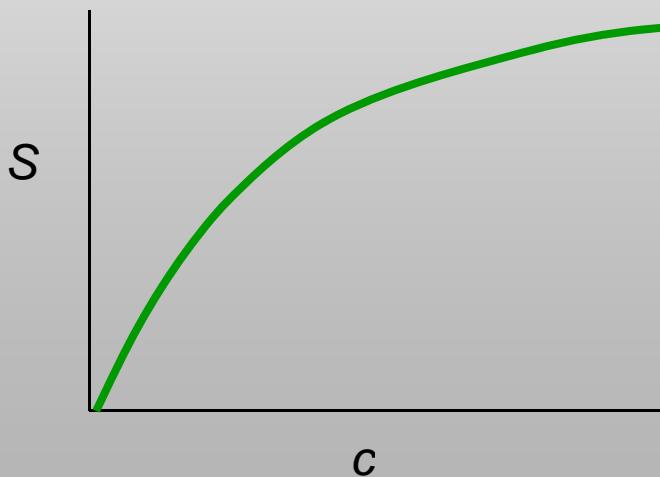
- linear  $\frac{\partial S}{\partial C} = \text{const.}$

$$S = k \cdot c$$

- non linear  $\frac{\partial S}{\partial C} = f(C)$

e.g.. Freundlich

$$S = k \cdot c^n$$



Stationary flow:

- $q(z,t) = \text{constant}$
- $\theta(z,t) = \text{constant}$

and

- no production or decay
- no sources and sinks
- linear sorption :  $S = k.c$  [mg.g<sup>-1</sup>]

$$\theta \frac{\partial c}{\partial t} + \rho k \frac{\partial c}{\partial t} = \frac{\partial}{\partial z} \left( D_e \frac{\partial c}{\partial z} \right) - q \frac{\partial c}{\partial z} \quad \text{with } D = \frac{D_e}{\theta}$$

$$(1 + \frac{\rho k}{\theta}) \frac{\partial c}{\partial t} = D \left( \frac{\partial^2 c}{\partial z^2} \right) - \frac{q}{\theta} \cdot \frac{\partial c}{\partial z} \quad R \cdot \frac{\partial c}{\partial t} = D \left( \frac{\partial^2 c}{\partial z^2} \right) - v \cdot \frac{\partial c}{\partial z}$$

Example:  $k = 2$   
 $\rho = 1.5$   
 $\theta = 0.4$

$$R = \text{retardation factor} = 1 + \frac{\rho k}{\theta} \quad [-]$$

$$v = \text{pore water velocity} \quad 36 \text{[cm.d}^{-1}\text{]}$$

## Sorption: general expression

$$S = \frac{k \cdot c^n}{1 + \eta c}$$

- linear

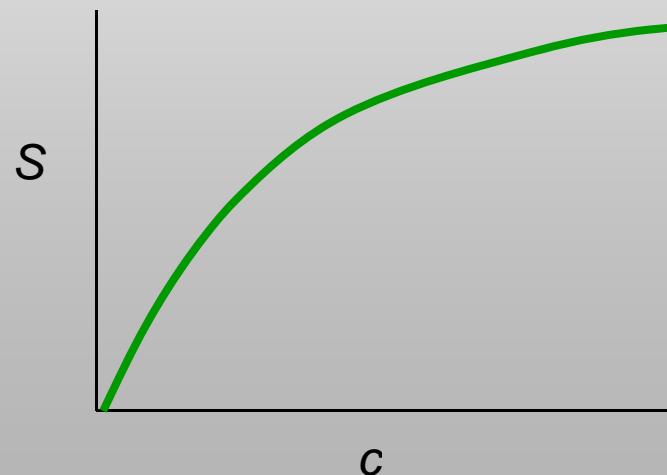
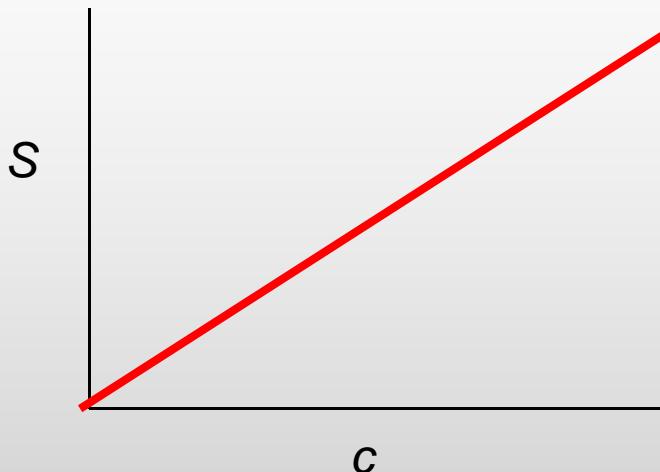
$$\frac{\partial S}{\partial C} = \text{const.}$$

$$S = k \cdot c$$

- non linear  $\frac{\partial S}{\partial C} = f(C)$

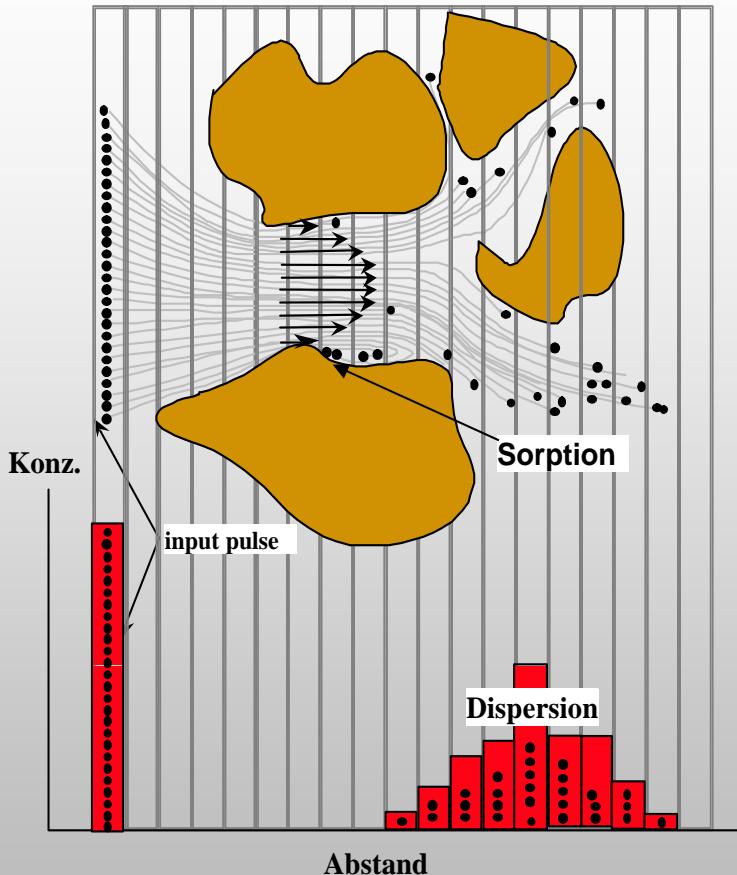
e.g.. Freundlich

$$S = k \cdot c^n$$



## Important transport processes

- Konvektion: gelöste Moleküle werden mit dem *Wasser* transportiert
- Diffusion: Brown'sche Bewegung
- Hydrodynamische Dispersion: Unterschiedliche lokale Fliessgeschwindigkeiten
- Sorption an der Bodenmatrix
- Abbau (mikrobiell)
- Lösung/Fällung



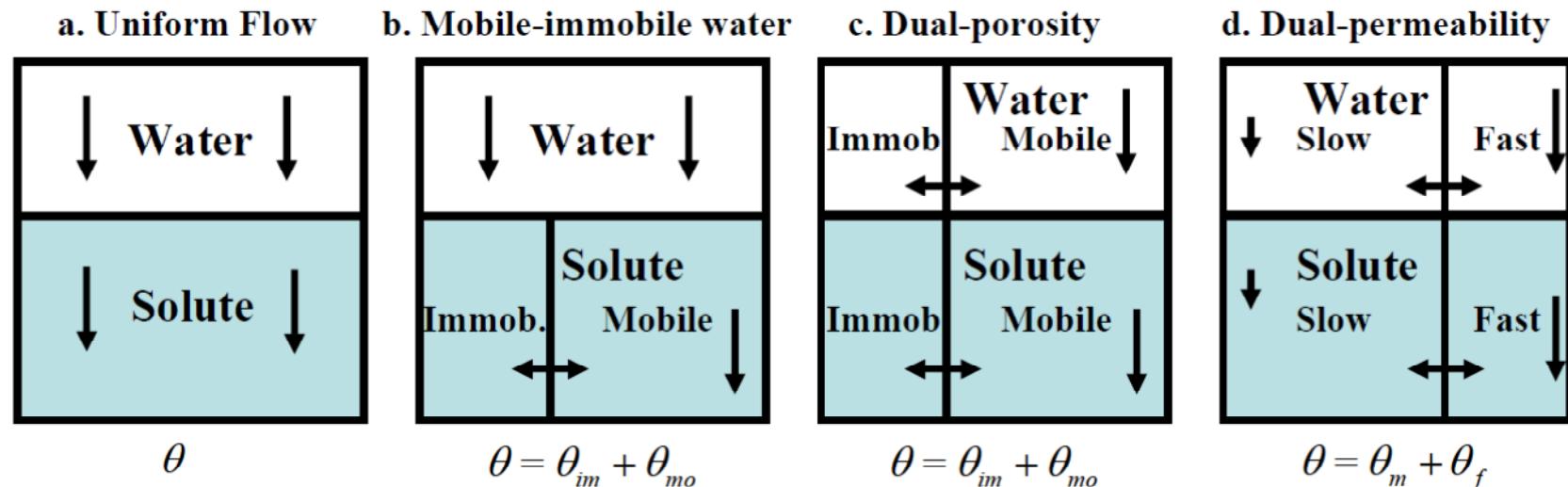


Figure 22.10. Conceptual models of water flow and solute transport ( $\theta$  is the water content,  $\theta_{mo}$  and  $\theta_{im}$  in (b) and (c) are water contents in the mobile and immobile flow regions, respectively, and  $\theta_m$  and  $\theta_f$  in (d) are water contents in the matrix and macropore (fracture) regions, respectively).

(Simunek and Van Genuchten, 2006)

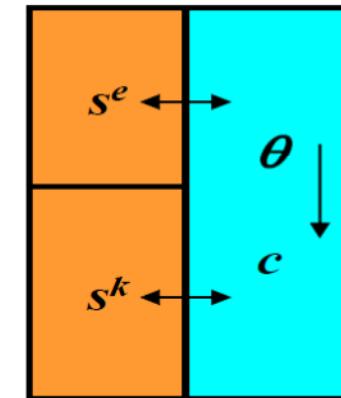
# Nonequilibrium Two-Site Adsorption Model

$$S = S^e + S^k$$

$s^e$  Type-1 sites with instantaneous sorption

$s^k$  Type-2 sites with kinetic sorption

$$\frac{\partial s^k}{\partial t} = \alpha[(1-f)K_d c - s^k]$$



$f$  fraction of exchange sites assumed to be at equilibrium