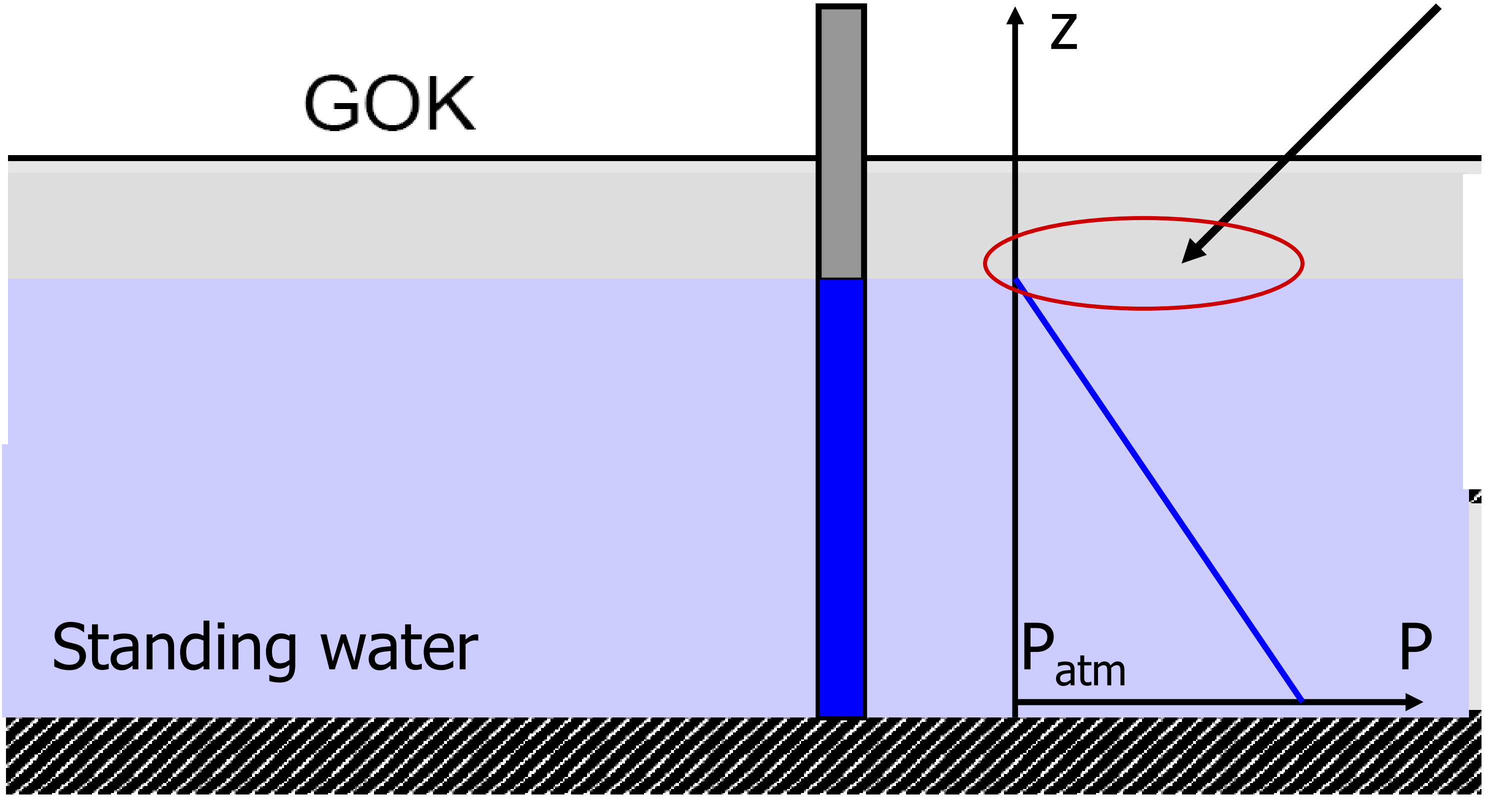
Groundwater Hydraulics

Institute for Fluid Mechanics and Environmental
Physics in Civil Engineering, Universität Hannover

Hydrostatics

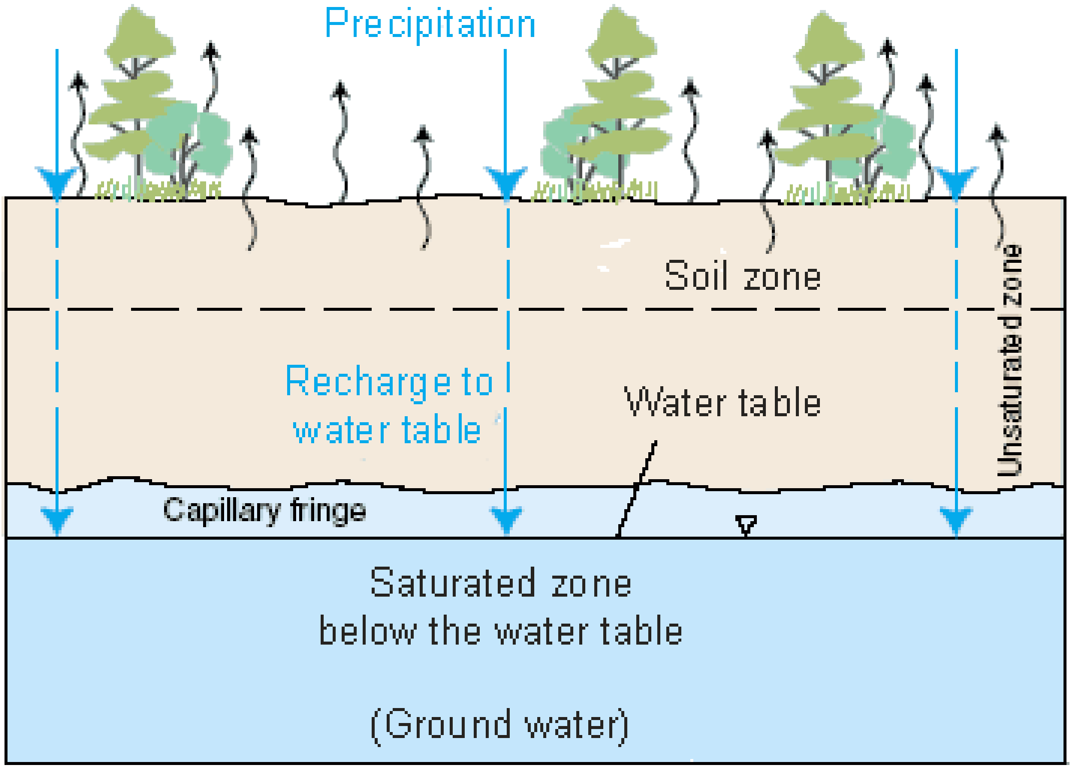
Unconfined aquifer

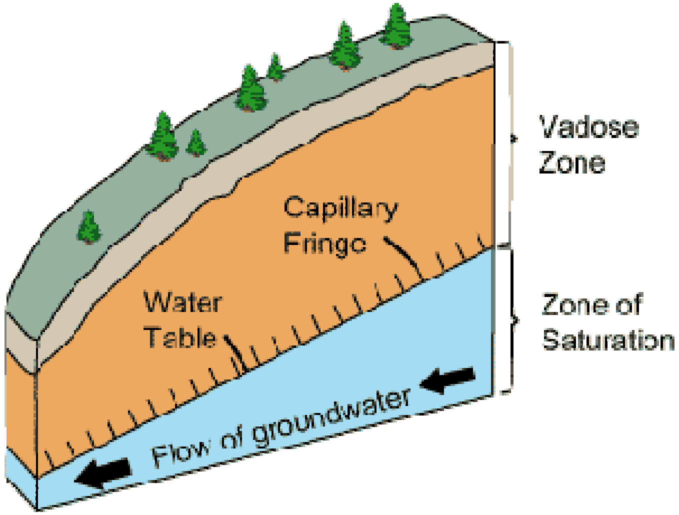
What happens here?



Hydrostatics

Aquifer – unsaturated zone





USGS

Hydrostatics

Aquifer – unsaturated zone

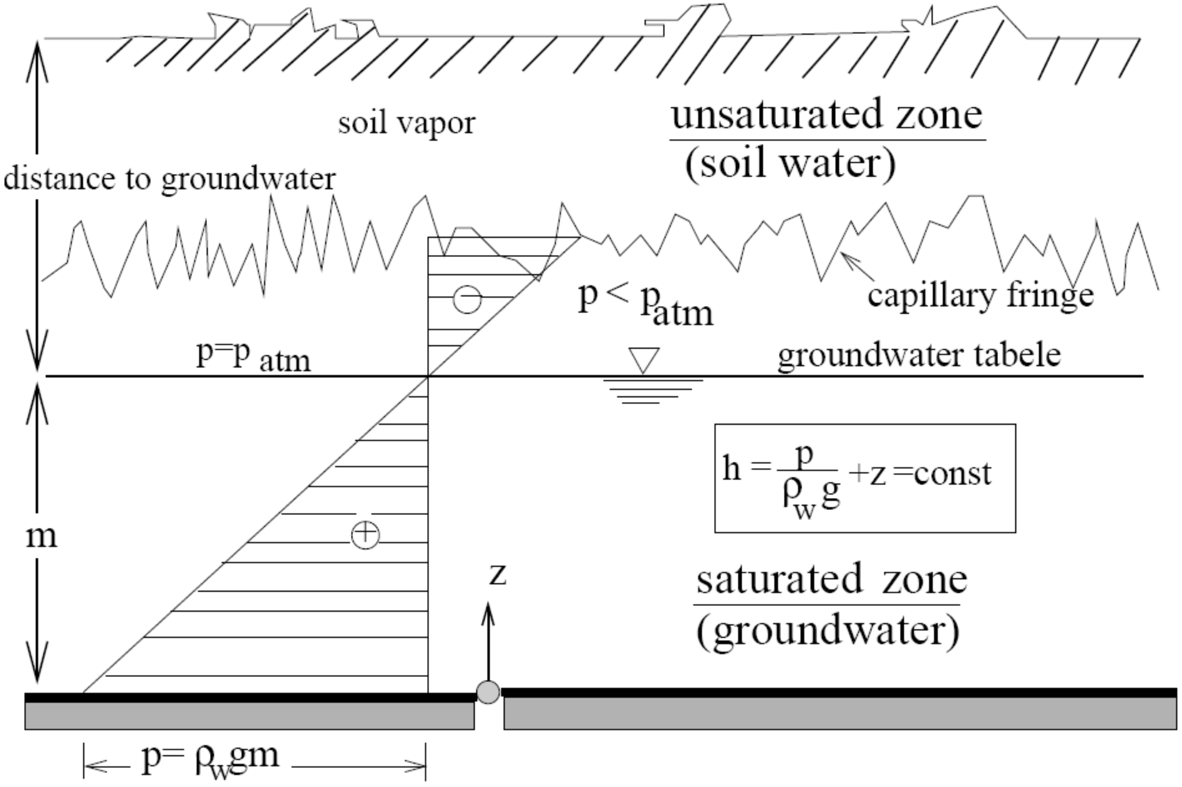
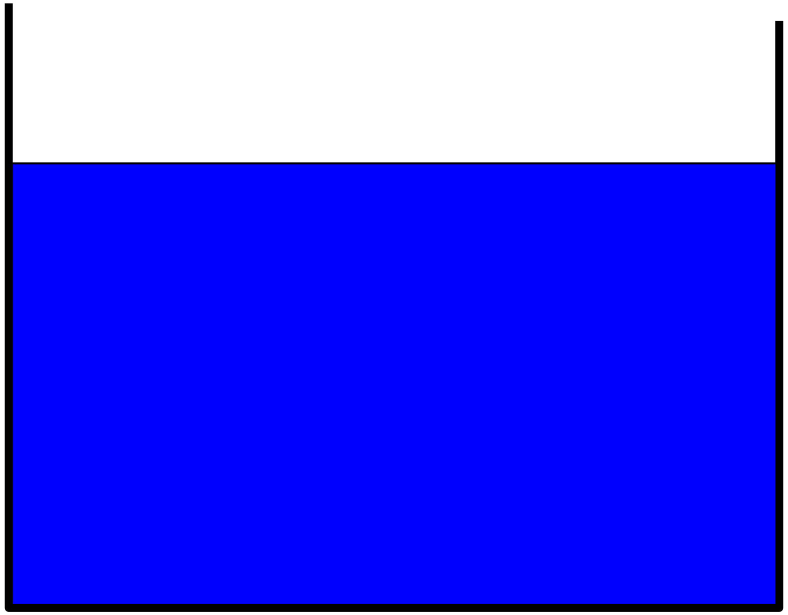


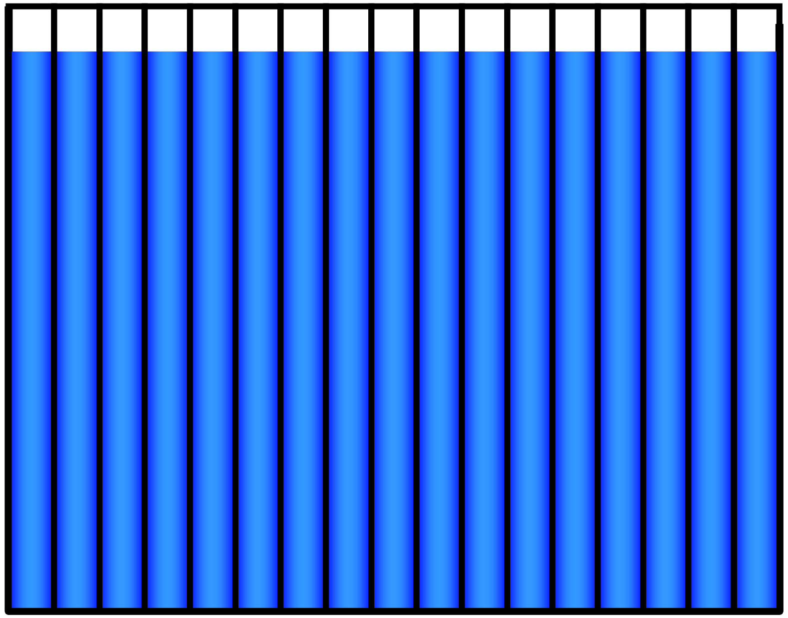
Figure 3.6: Profile of water potentials in the subsurface under hydrostatic conditions.

Hydrostatics

Simplified picture: Straw model



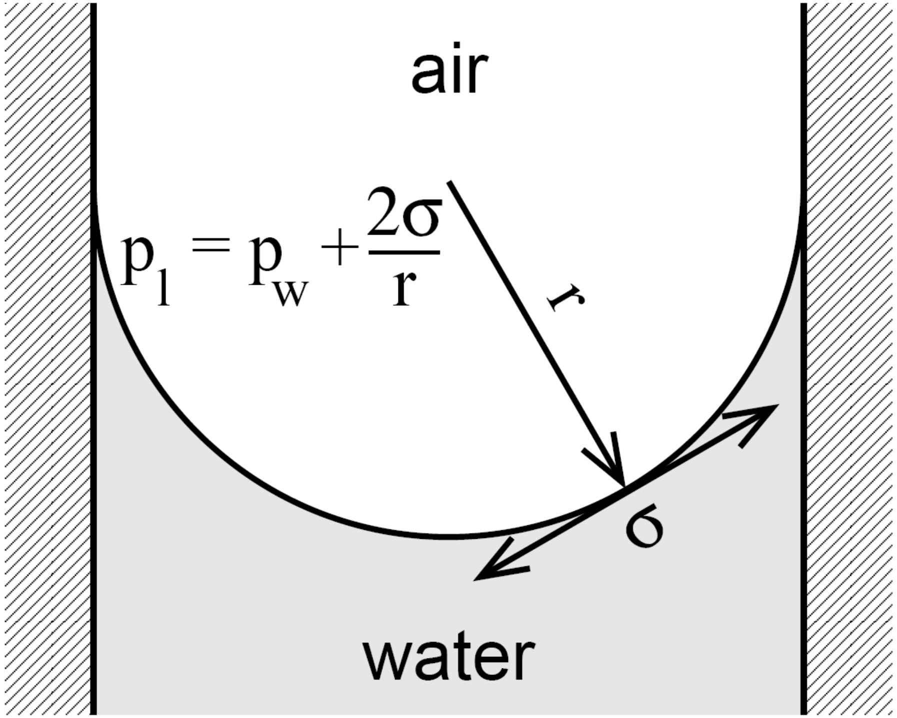
Standing water without porous medium



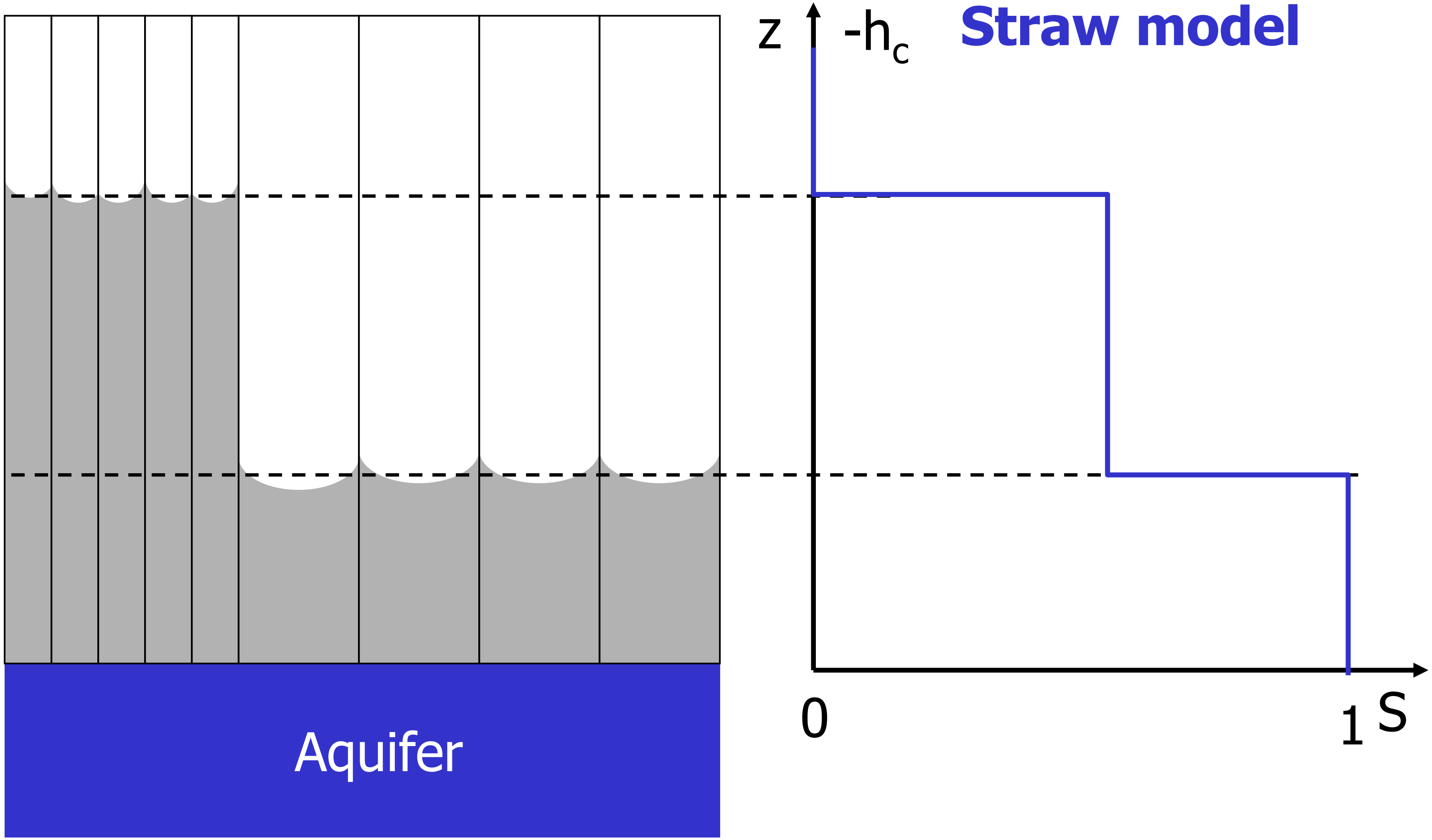
Standing water with porous medium

Hydrostatics

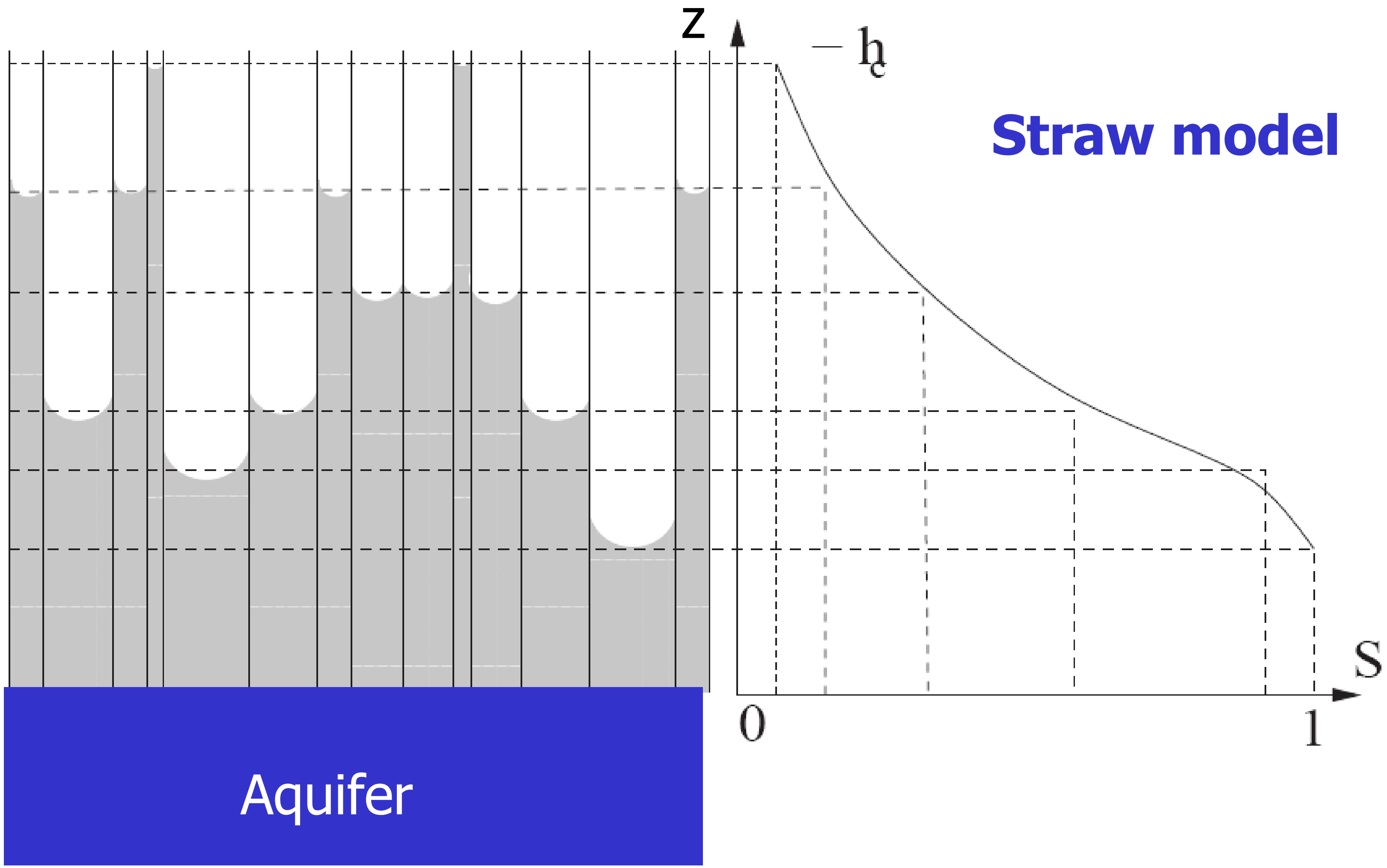
Capillary forces



Hydrostatics



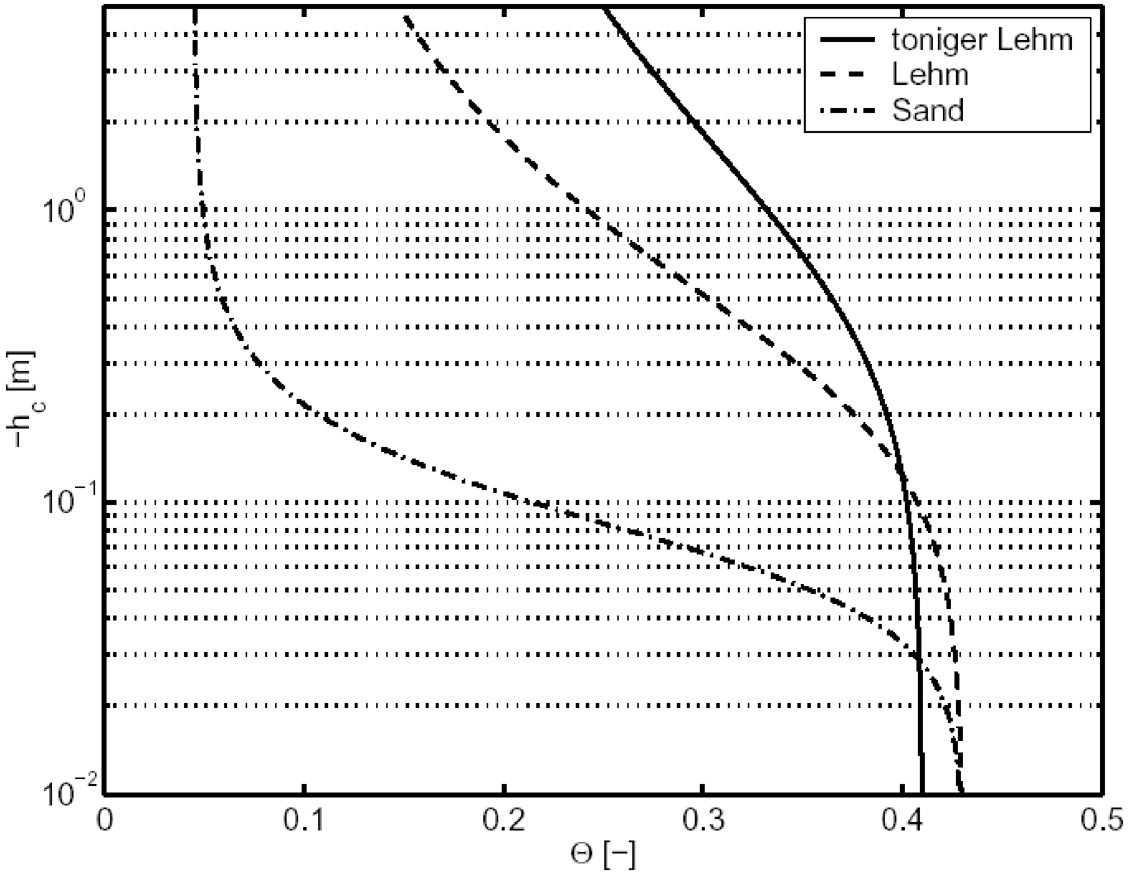
Hydrostatics



Straw model

Hydrostatics

Retention curves in soils



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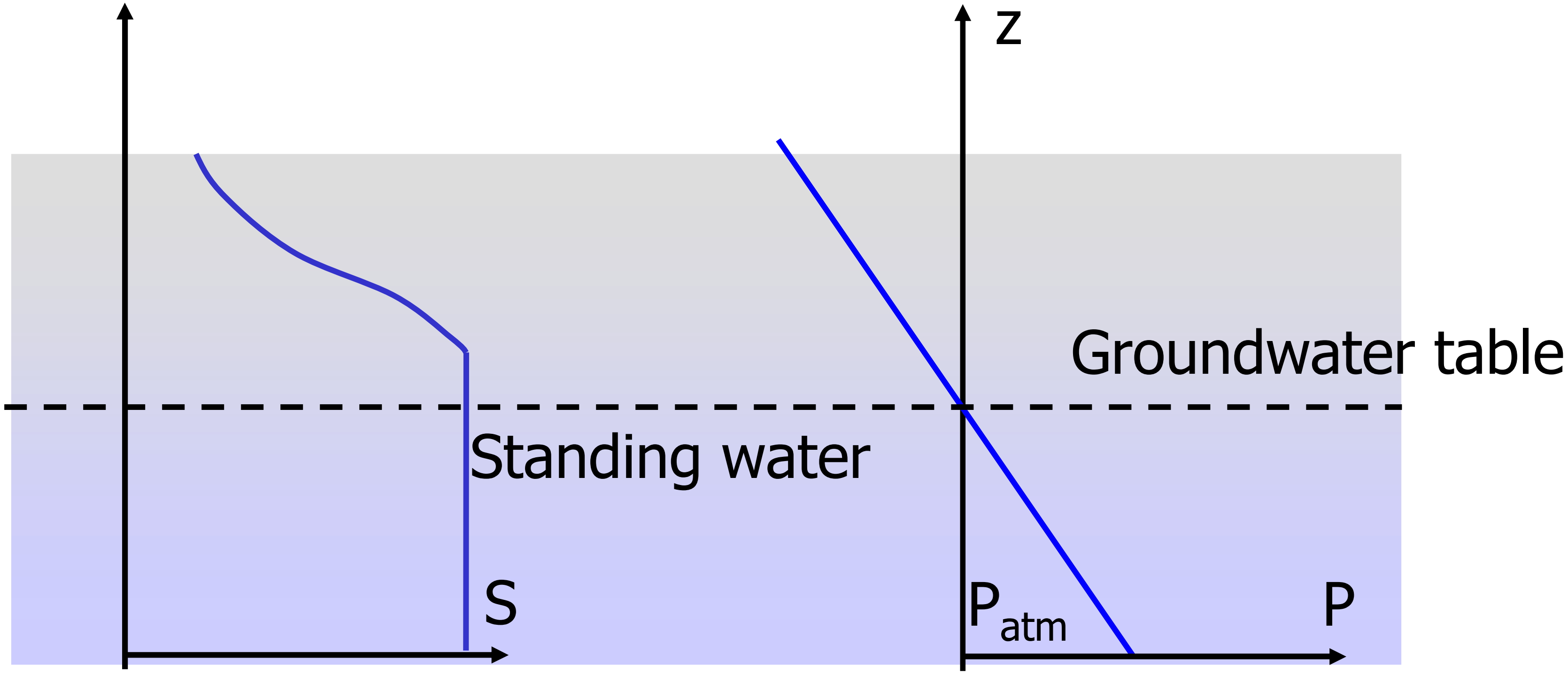
Hydrostatic pressure and water content profile



Effective Saturation S_e

- Gravity cannot completely drain the soil
- Residual water content Θ_r even at infinite capillary head
- Porosity is the maximum volumetric water content Θ_s
- Entrapped gas may be immobile ⇨ even smaller maximum water content Θ_s

$$S_e = \frac{\Theta - \Theta_r}{\Theta_s - \Theta_r}$$

Hydrostatics

Soil Retention Curve in Unsaturated Soils

- Water content as function of capillary head
- Van Genuchten (1980) parameterization:

$$S_e(h_c) = \left(1 + (\alpha h_c)^N\right)^{\frac{1-N}{N}}$$

- α, N, Θ_r, and Θ_s are soil parameters depending on soil type

Hydrostatics

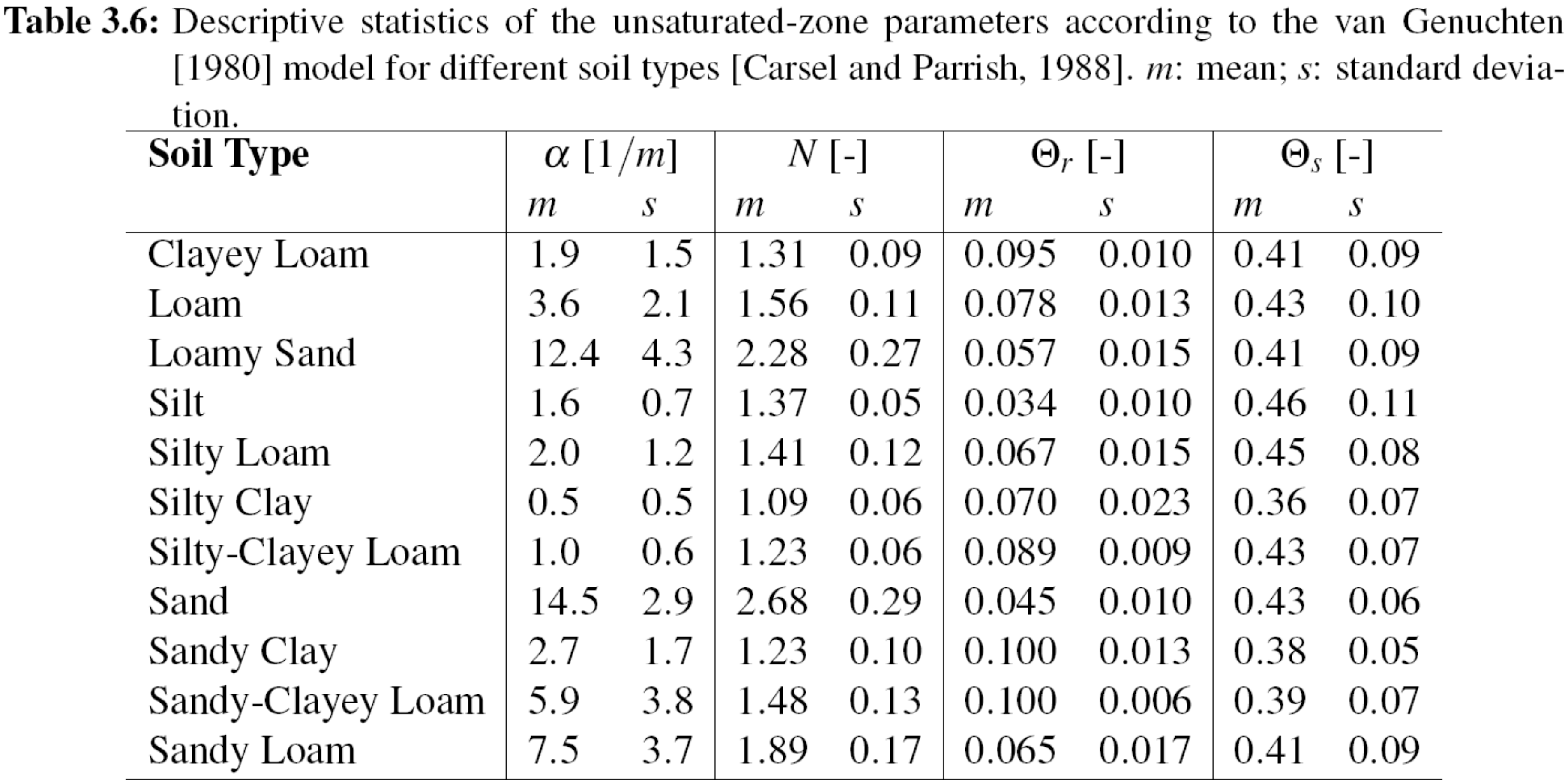
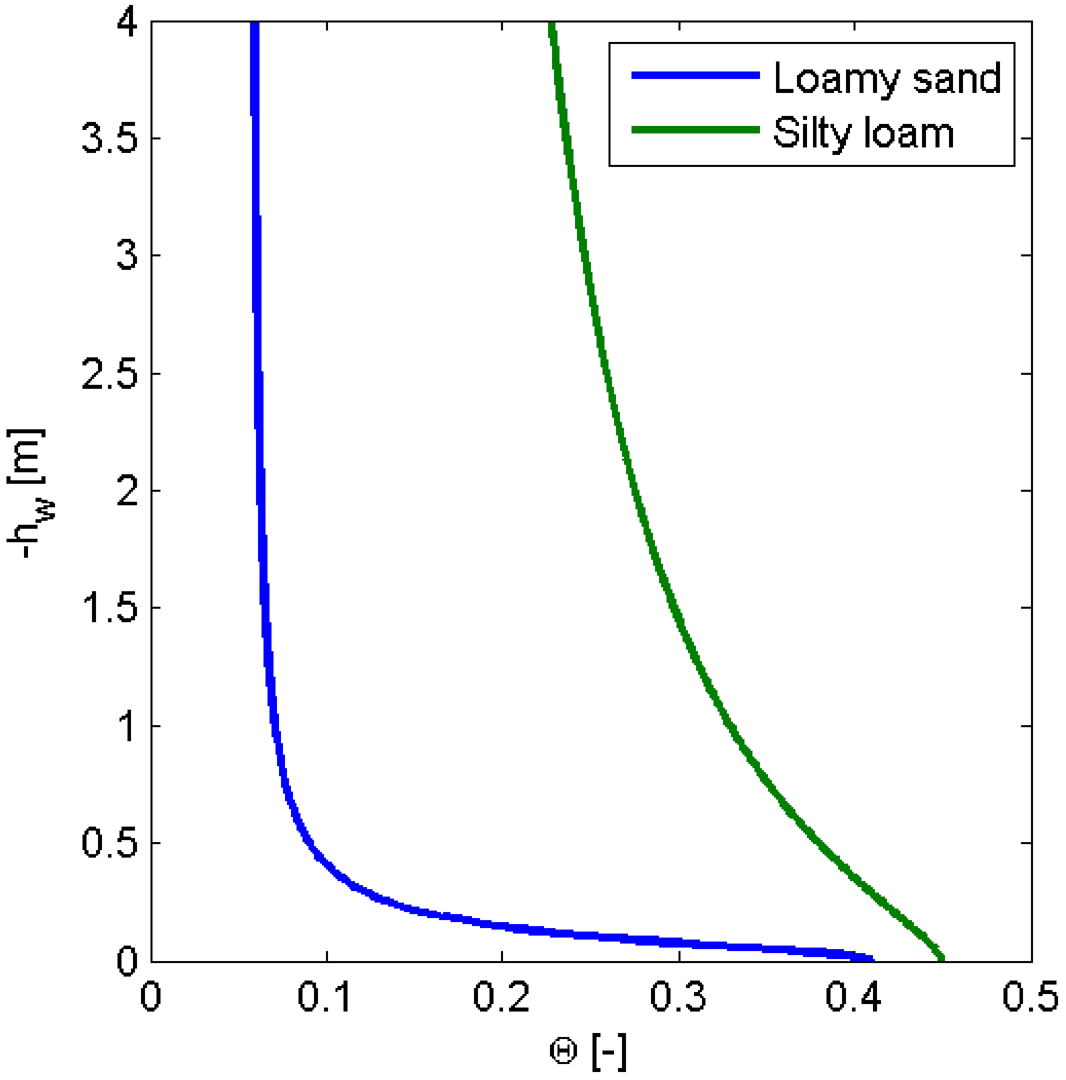


Table 3.6: Descriptive statistics of the unsaturated-zone parameters according to the van Genuchten [1980] model for different soil types [Carsel and Parrish, 1988]. m: mean; s: standard deviation.

Soil Type	α $[1/m]$		N [-]		Θ_r [-]		Θ_s [-]	
	m	s	m	s	m	s	m	s
Clayey Loam	1.9	1.5	1.31	0.09	0.095	0.010	0.41	0.09
Loam	3.6	2.1	1.56	0.11	0.078	0.013	0.43	0.10
Loamy Sand	12.4	4.3	2.28	0.27	0.057	0.015	0.41	0.09
Silt	1.6	0.7	1.37	0.05	0.034	0.010	0.46	0.11
Silty Loam	2.0	1.2	1.41	0.12	0.067	0.015	0.45	0.08
Silty Clay	0.5	0.5	1.09	0.06	0.070	0.023	0.36	0.07
Silty-Clayey Loam	1.0	0.6	1.23	0.06	0.089	0.009	0.43	0.07
Sand	14.5	2.9	2.68	0.29	0.045	0.010	0.43	0.06
Sandy Clay	2.7	1.7	1.23	0.10	0.100	0.013	0.38	0.05
Sandy-Clayey Loam	5.9	3.8	1.48	0.13	0.100	0.006	0.39	0.07
Sandy Loam	7.5	3.7	1.89	0.17	0.065	0.017	0.41	0.09

Hydrostatics



Hydrostatics

Exercise #1

Nutrients dissolved in water are carried to upper parts of plants by tiny tubes partly because of the capillary effect. Determine how high the water solution will rise in a tree in a 0.005-mm-diameter tube as a result of the capillary effect. The surface tension at 20° C is 0.073 N/m.

(Cengel and Cibala, Fluid Mechanics)

?

Darcy‵s law

How does the water flow?

Darcy’s law

Darcy’s law:

Flow equations: Navier Stokes equations

$$\rho \boldsymbol{g} - \nabla p + \mu \nabla^2 \boldsymbol{v} = \rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v}\right)$$

for porous media: not so simple!!!

Darcy’s approach (1856): relate pressure head to flow velocity with an empirical approach

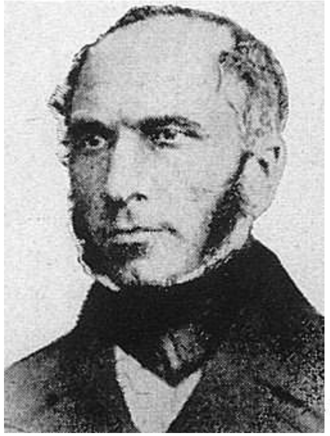
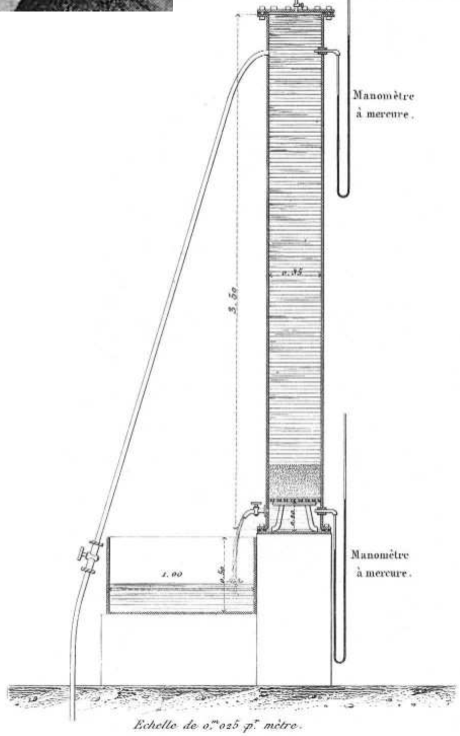


Fig. 3.

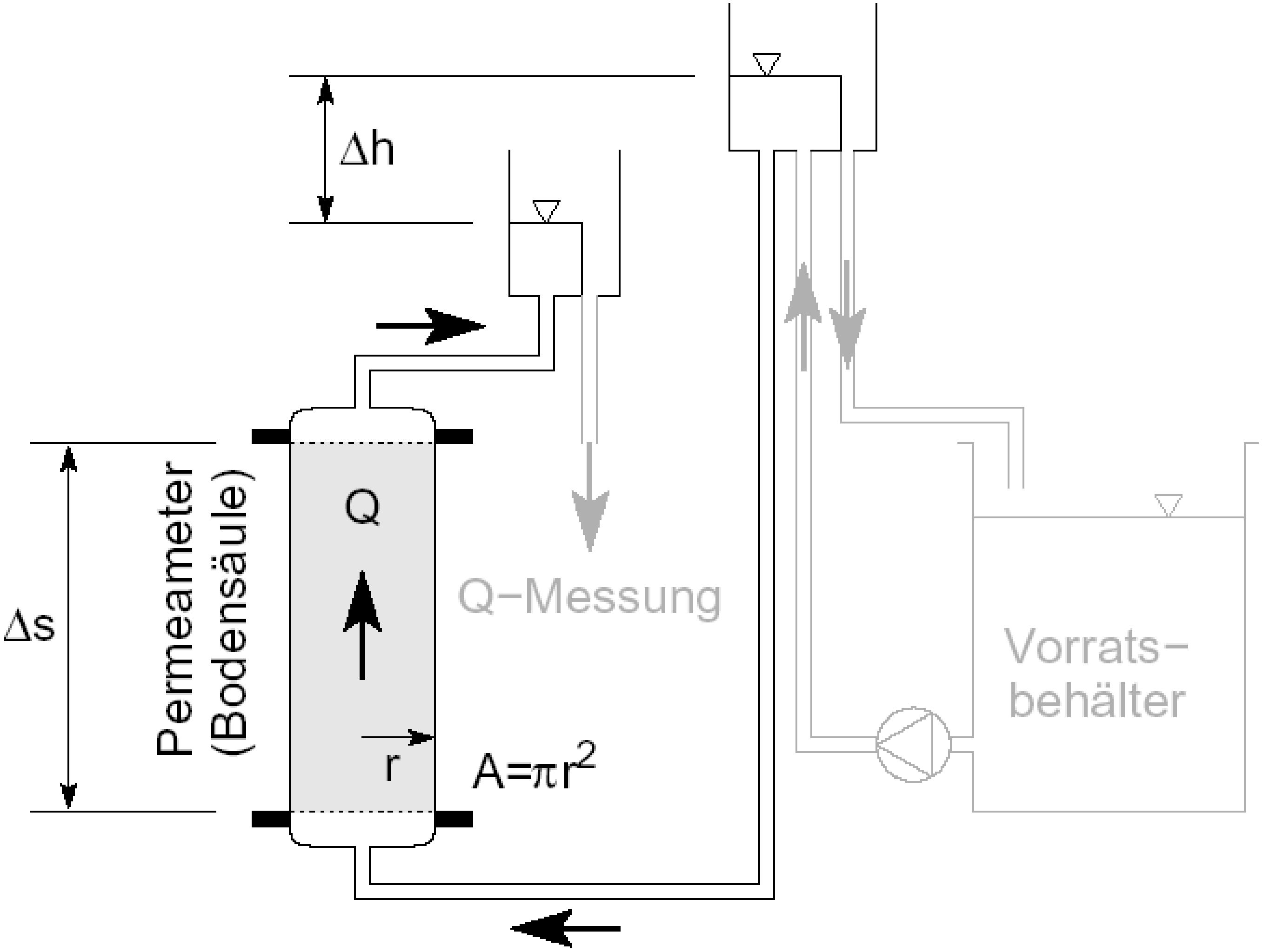
Appareil destiné à déterminer la loi de l'écoulement de l'eau à travers le sable.



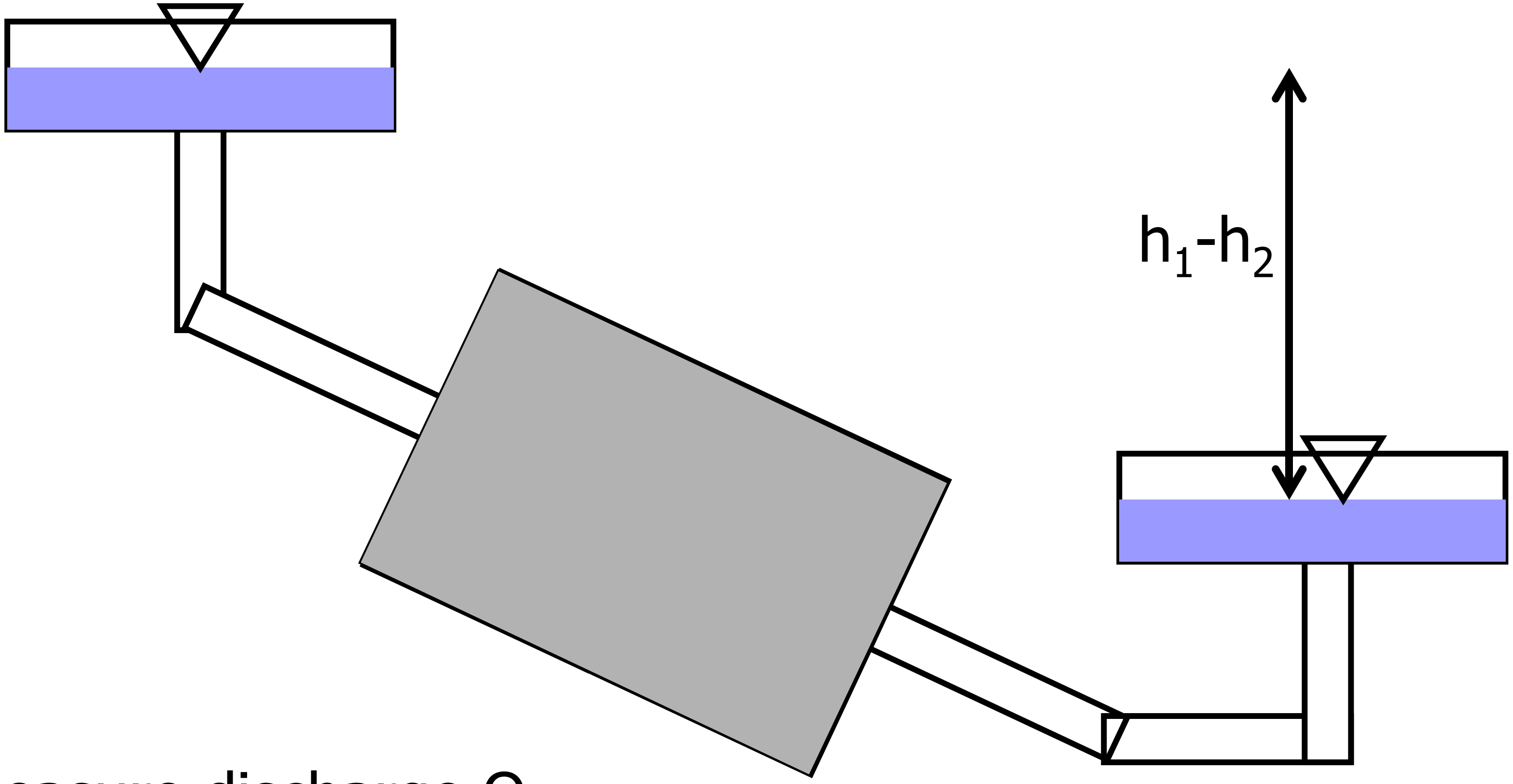
Echelle de 0.m025 p.r mètre.

Darcy’s law

Darcy Experiment



Darcy’s law



- measure discharge Q
- measure cross-section area of the probe
- measure head difference in the reservoirs

Darcy's law

Hydraulic conductivity:

$$q = \frac{Q}{A} = -k\,\frac{\Delta h}{\Delta s}$$

resp. (isotropic media)

$$q = -k\nabla h$$

- ➢ velocity proportional to hydraulic gradient
- ➢ proportionality factor k called **hydraulic conductivity**