# Groundwater Hydraulics

Institute for Fluid Mechanics and Environmental Physics in Civil Engineering, Universität Hannover

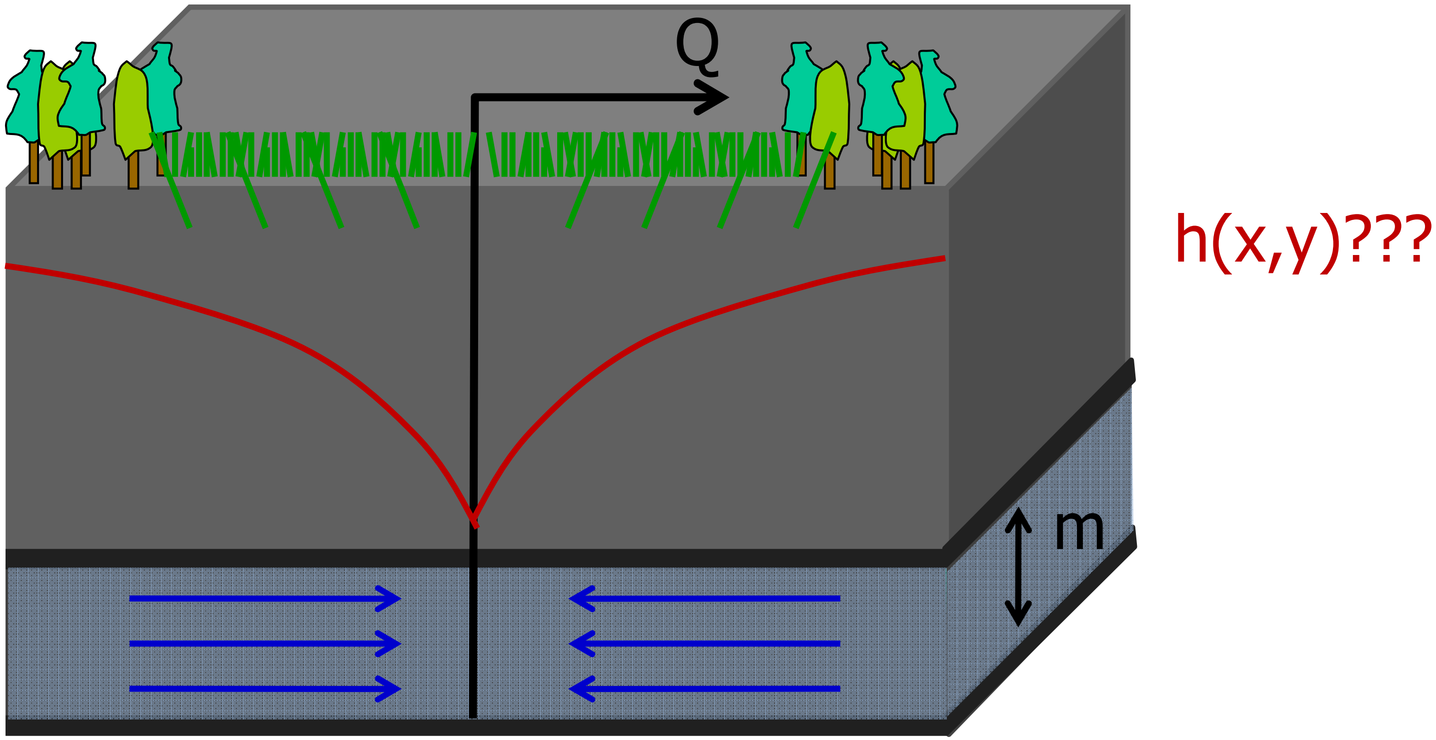
# Steady state well flow

# Radial flow – well hydraulics

## Steady state flow towards a well

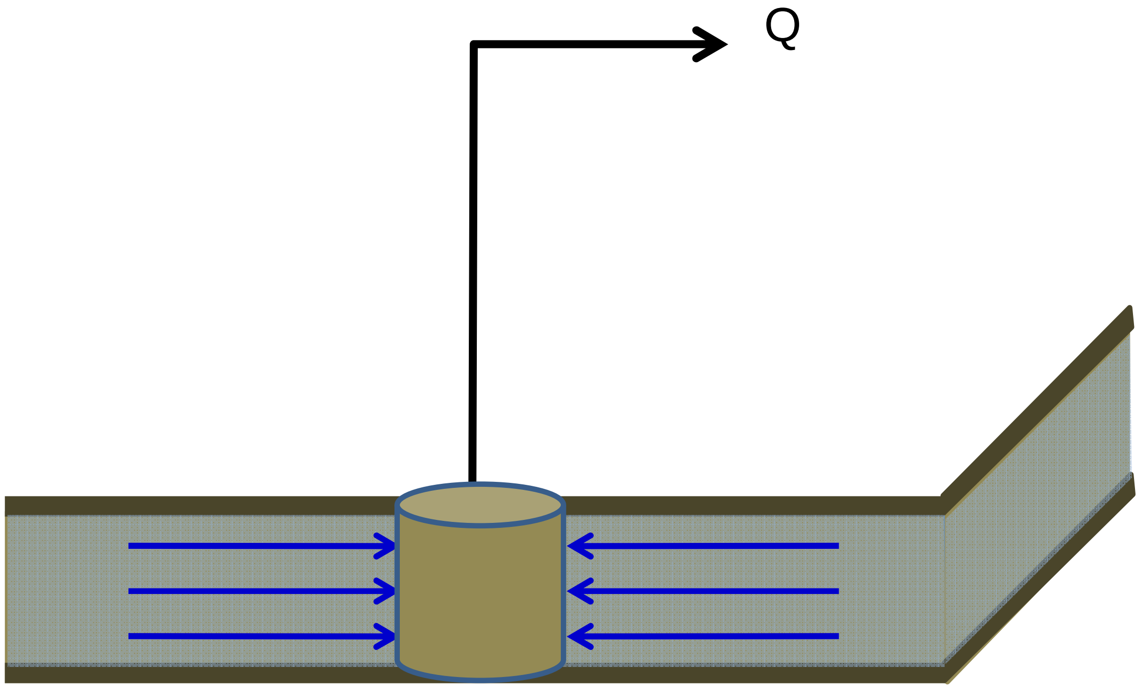
### Confined aquifer

Vertical cut



# Radial flow – well hydraulics

**Flow through a cylinder:**



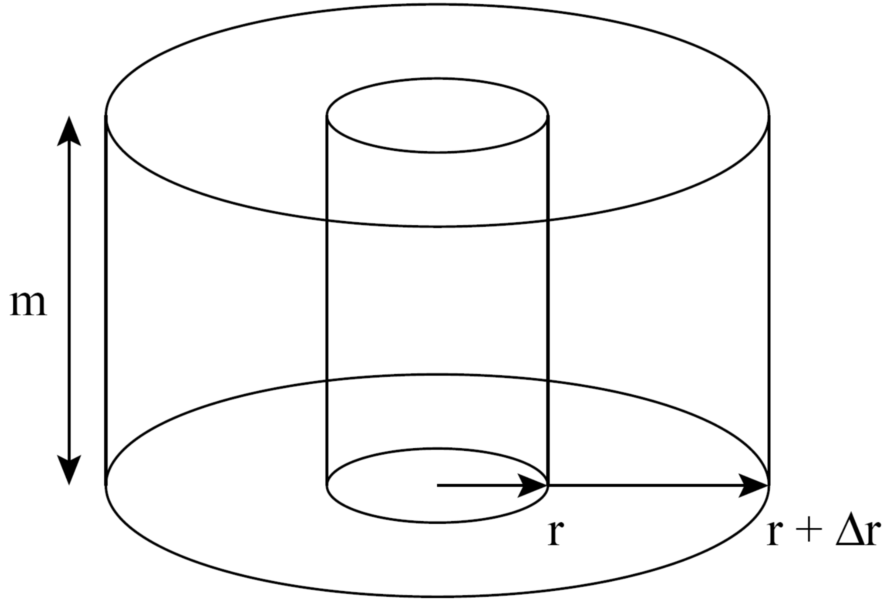
# Radial flow – well hydraulics

## Calculation of the head distribution:

$$Q_W = 2\pi r m v_f$$

$$v_f = -k_f \frac{\partial h}{\partial r}$$

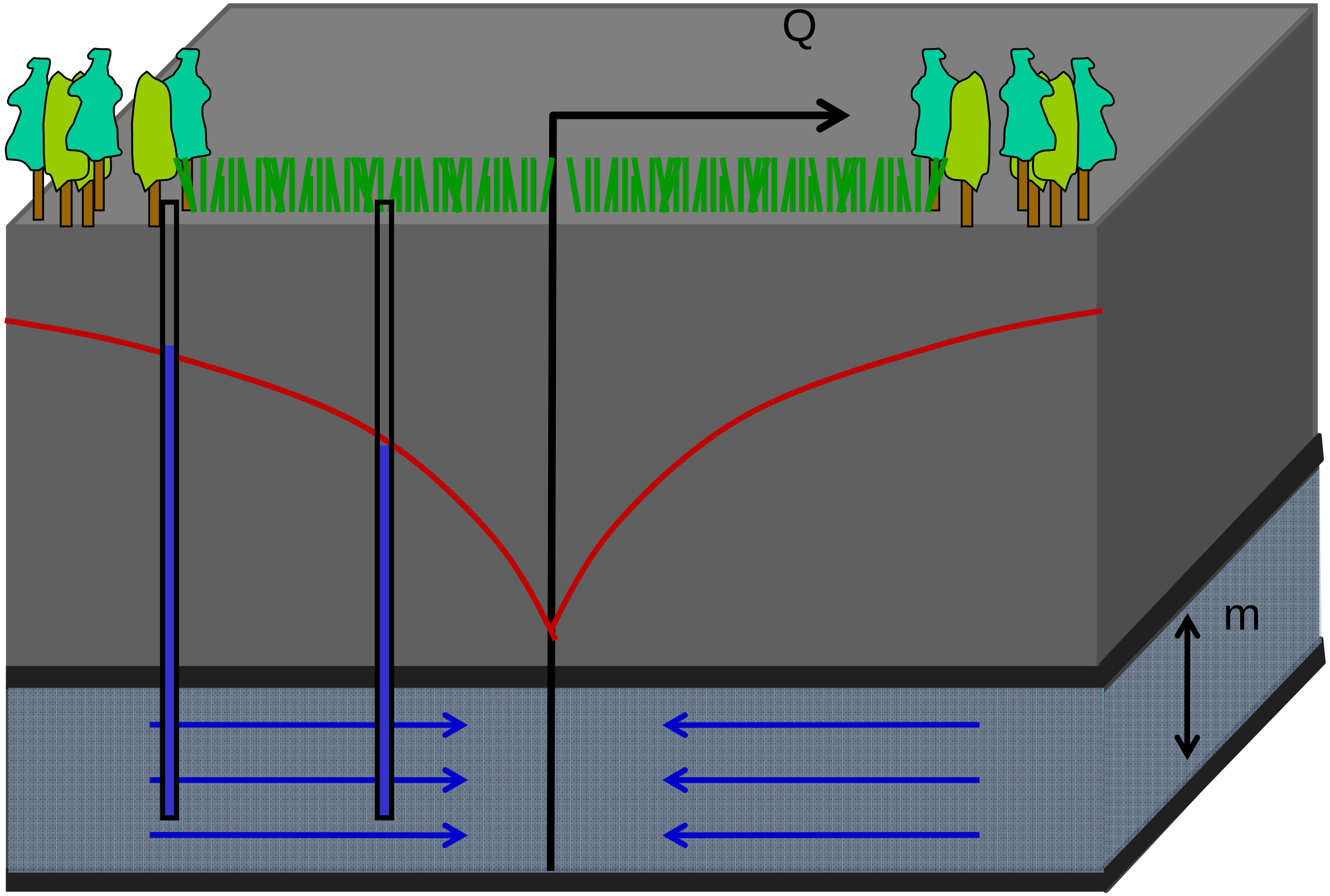
$$Q_W = -2\pi T r \frac{\partial h}{\partial r}$$




$$dh = -\frac{Q_W}{2\pi T}\frac{dr}{r} \qquad h = -\frac{Q_W}{2\pi T}\ln(r) + C_1$$

# Radial flow – well hydraulics

## Dupuit-Thiem (1906)



# Radial flow – well hydraulics

## Dupuit-Thiem (1906)

Observation of the piezometric head at to observation wells at distance $r_1$ and $r_2$

$$\frac{-Q_w}{2\pi T}\ln(r_1) = h(r_1) + C_1$$

$$\frac{-Q_w}{2\pi T}\ln(r_2) = h(r_2) + C_1$$

Taking the difference gives

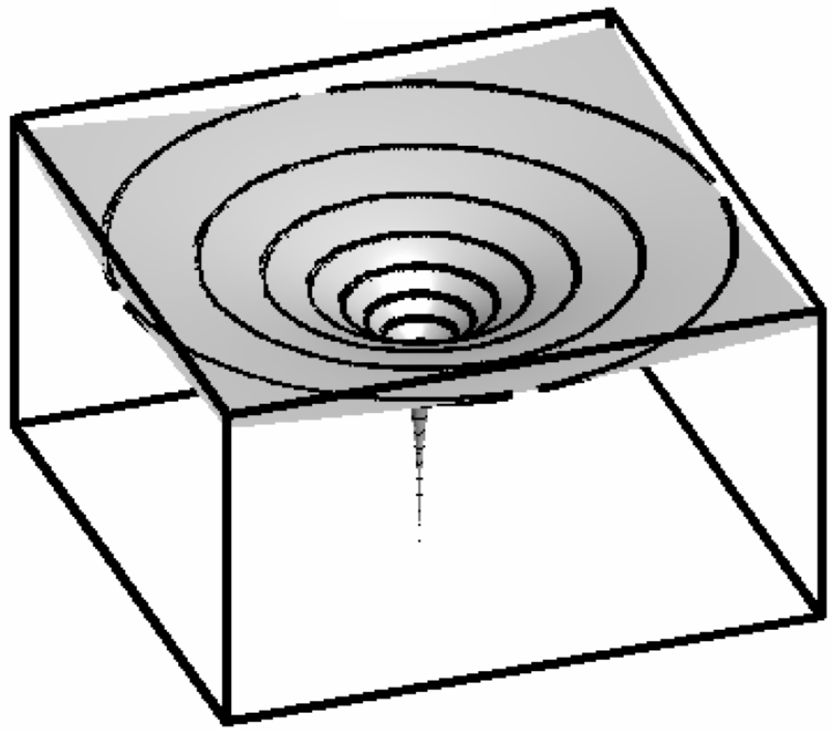
$$h(r_1) - h(r_2) = \frac{Q_w}{2\pi T}\ln\left(\frac{r_2}{r_1}\right)$$

Can be used to determine the transmissivity of an aquifer.

# Radial flow – well hydraulics

## Steady-State Drawdown

- Drawdown $s$ = head difference without and with pumping
- „Cone of depression“ for extraction well
- „Groundwater mounting“ for injection well
- Logarithmic radial profile (without recharge)



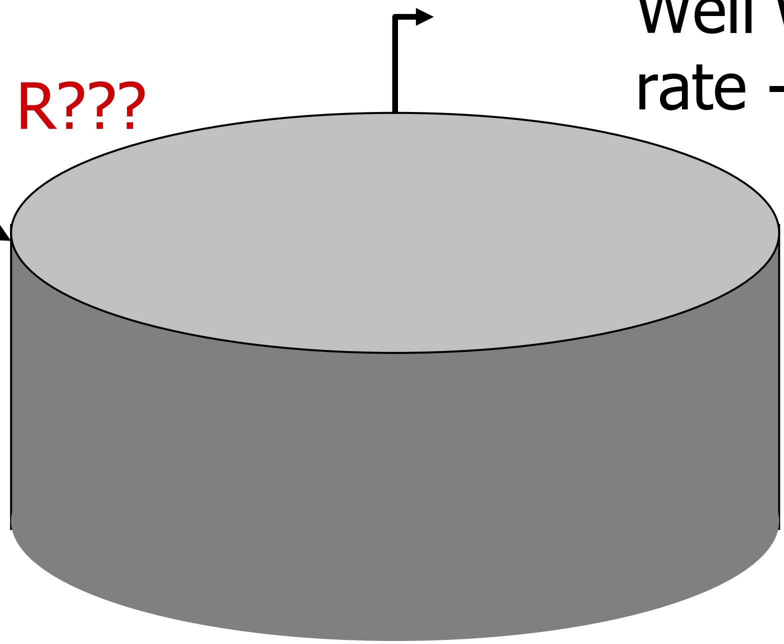
# Radial flow – well hydraulics

## Radius of influence

How could the outer boundary and the boundary condition be set?

Well with constant rate -> flux

2. boundary $h_0$ at R???



$$h(r) = h_0 + \frac{Q_w}{2\pi T} \ln\left(\frac{R}{r}\right)$$

# Radial flow – well hydraulics

## Radius of influence

- Distance at which no drawdown occurs
- Strictly speaking, no radius of influence possible at steady state without recharge.
- Empirical relationships:

**Table 5.1:** Approximations of the radius of influence $R$.

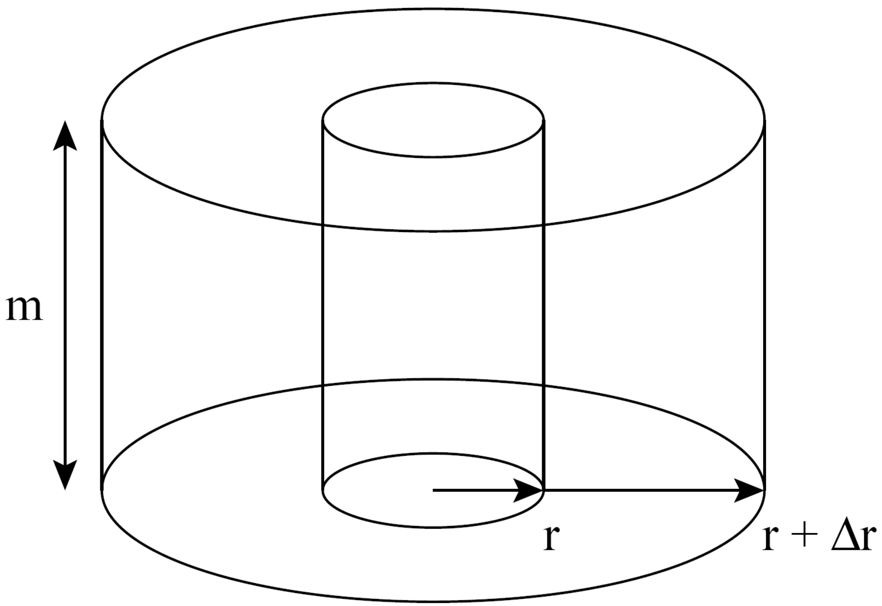
| | |
|---|---|
| Sichardt [1928] | $R \approx 3000 s_w \sqrt{K}$ |
| Kusakin [Strozodka 1977] | $R \approx 575 s_w \sqrt{K h_0}$ |
| Weber [1928] | $R \approx 3\sqrt{\frac{h_0 K t}{n_f}}$ |
| Lembke [1886, 1887] | $R \approx h_0 \sqrt{\frac{K}{2N}}$ |

$t\,[s]$: time of operation; $n_f\,[-]$: flow-effective porosity; $N\,[m/s]$: recharge rate; $h_0\,[m]$: original water table; $s_w\,[m]$: drawdown in the well; $K\,[m/s]$: hydraulic conductivity

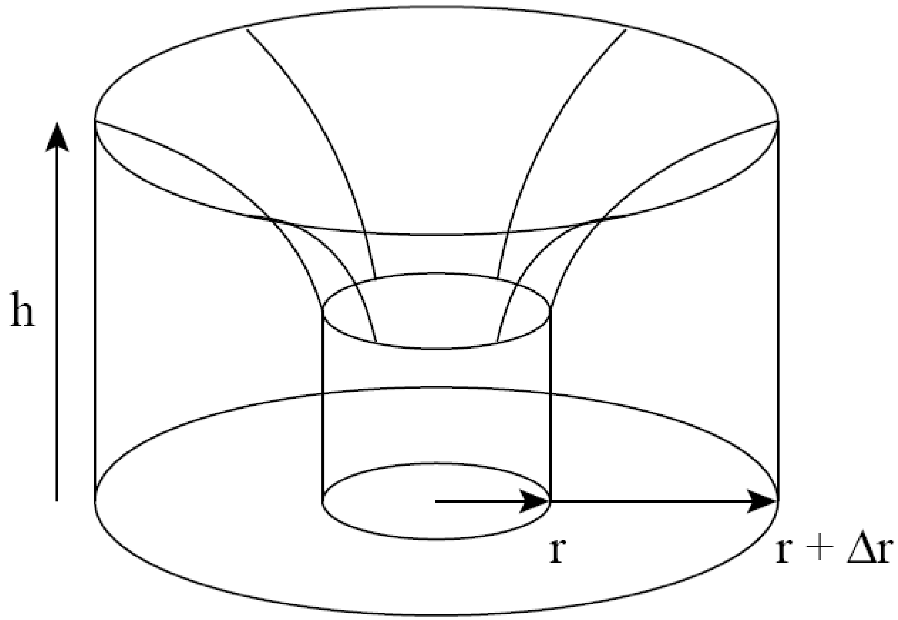
# Radial flow – well hydraulics

## Steady state flow towards a well

### Unconfined aquifer




confined




unconfined

# Radial flow – well hydraulics

## Steady state flow towards a well

**Unconfined aquifer**

Without recharge, similar to confined:

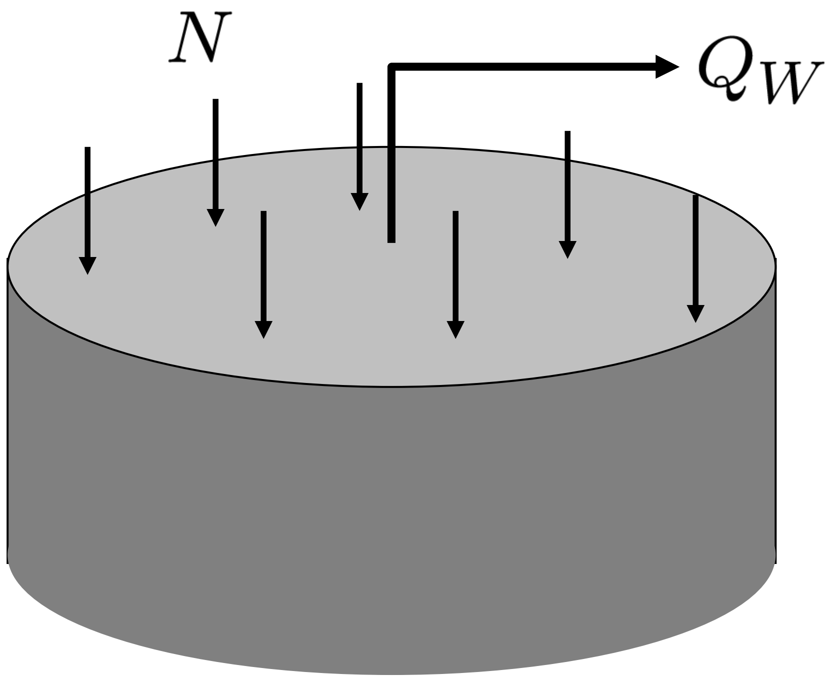
$$Q(r) = Q_W = -\pi r k_f \frac{\partial h^2}{\partial r}$$

$$h^2 = -\frac{Q_W}{\pi k_f} \ln(r) + C_1$$

Difference: $$h_1^2 - h_2^2 = \frac{Q_W}{\pi k_f} \ln\left(\frac{r_2}{r_1}\right)$$

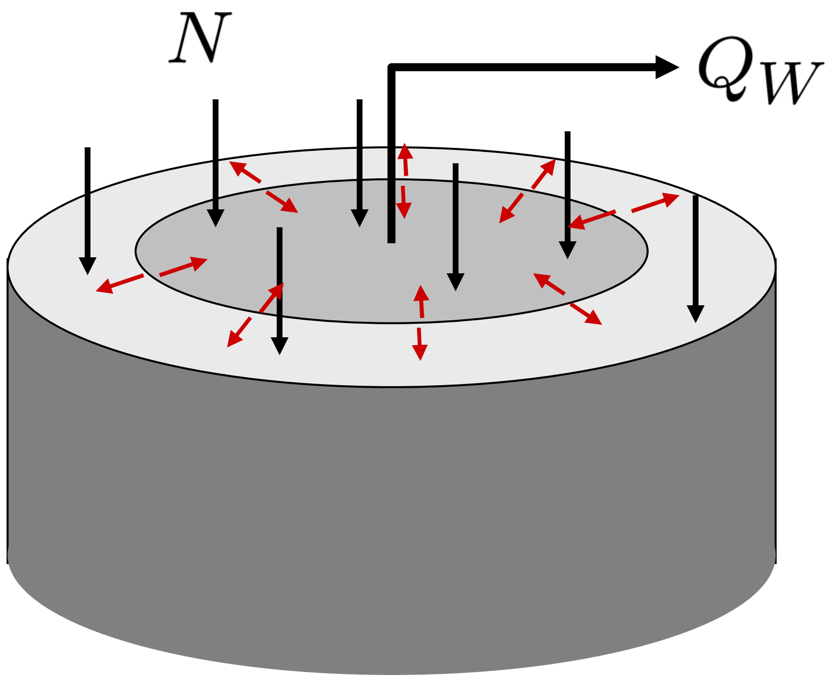
# Radial flow – well hydraulics

**Unconfined aquifer with recharge:**




$$Q(r) = Q_W + Q_N$$

# Radial flow – well hydraulics




**Radius of influence or capture zone:**

$$Q_W + Q_N = Q_W + \pi r_e^2 N = 0$$

$Q_W$ <0, $Q_N$ >0

$$r_e = \sqrt{\frac{-Q_W}{\pi N}}$$

$r_e > 0$ flow away from well

$r_e < 0$ flow towards well

# Radial flow – well hydraulics

- Compare head squared at two distances

$$h_1^2 - h_2^2 = \boxed{\frac{Q_w}{\pi K}\ln\left(\frac{r_2}{r_1}\right)} + \boxed{\frac{N}{2K}\left(r_2^2 - r_1^2\right)}$$

  – Logarithmic contribution from well

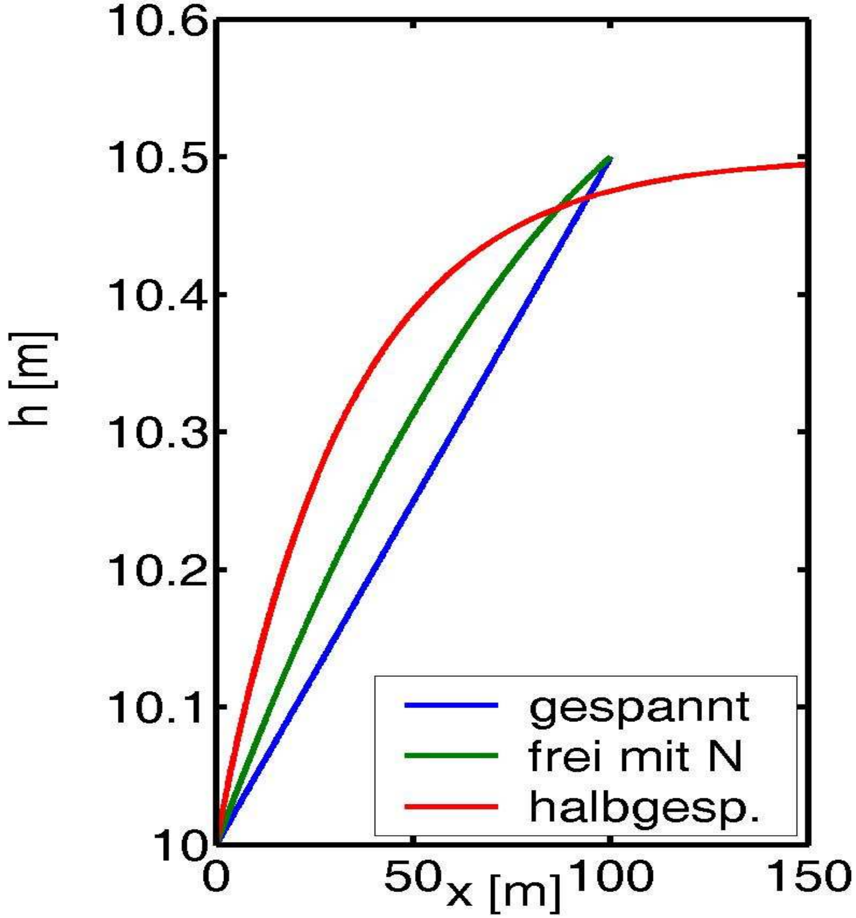
  – Quadratic contribution from recharge

- Pumping well + recharge: capture zone
- No recharge and only one observation well: Radius of influence

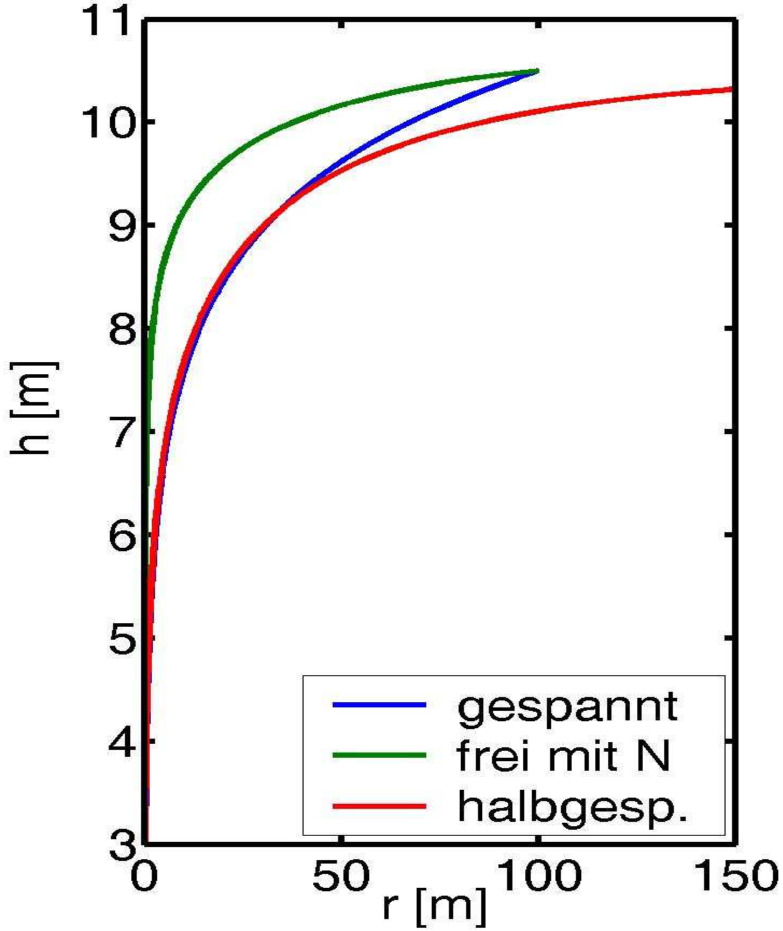
$$h_0^2 - h_1^2 = \frac{Q_w}{\pi K}\left(\ln(r_1) - \ln(R)\right)$$

# Radial flow – well hydraulics

X-symmetric




cylinder-symmetric



# Radial flow – well hydraulics

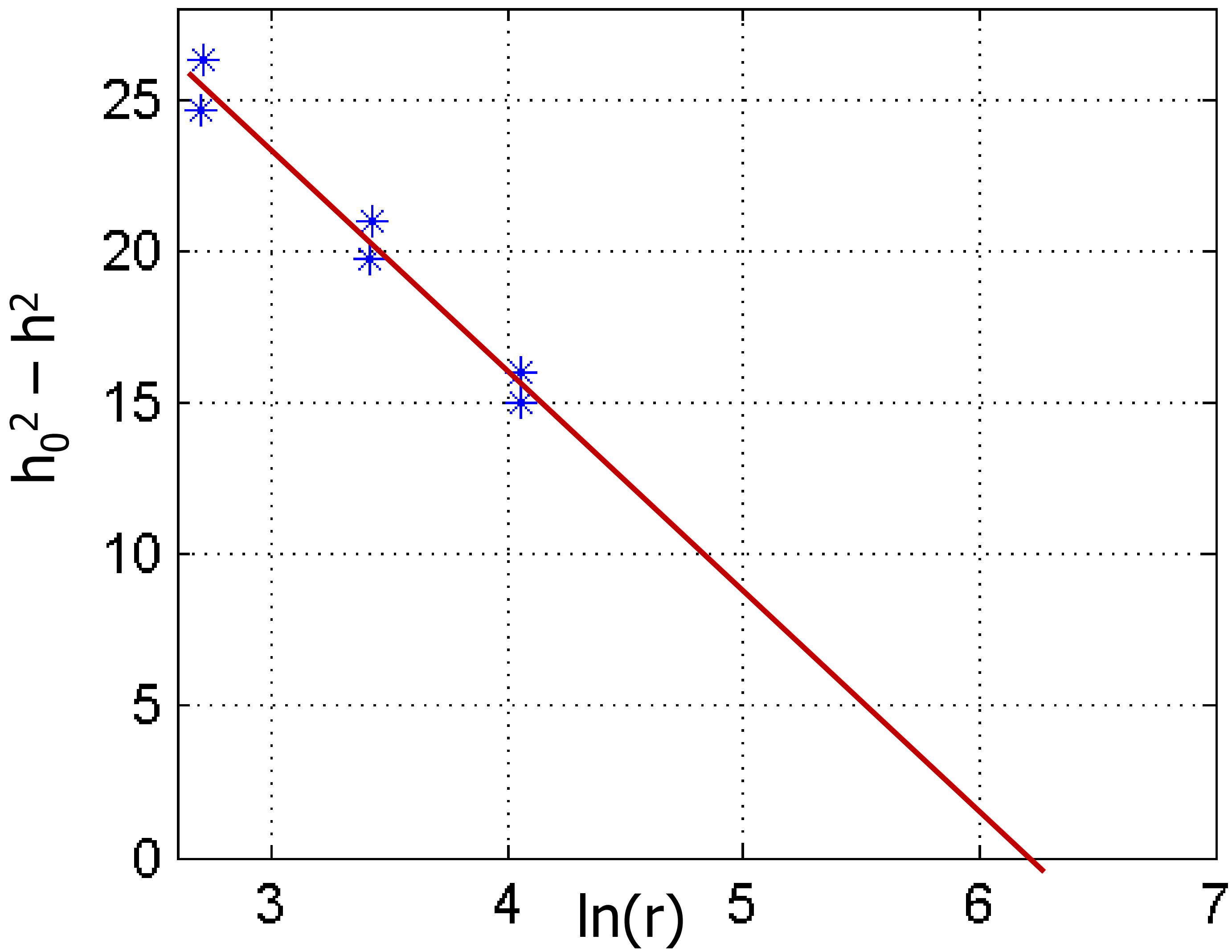
## Exercise # 11:

Consider a sandy-gravely unconfined aquifer with a well extracting water with a rate of $Q_w$ of 6.31 $10^{-2}$ $m^3/s$. The hydraulic head without pumping is 8.2 m. In quasi steady state, the following hydraulic heads have been measured:

| $r\ [m]$ | $h\ [m]$ | $r\ [m]$ | $h\ [m]$ |
|---|---|---|---|
| 15 | 6.40 | 14.95 | 6.53 |
| 30.7 | 6.80 | 30.6 | 6.89 |
| 57.7 | 7.16 | 57.9 | 7.23 |

- Perform a linear regression of the difference between the squares of the undisturbed and measured head $h_0^2$-$h^2$ and the logarithm of the radius ln(r) and determine the hydraulic conductivity kf and the radius of influence of the aquifer. Careful: The rate $Q_w$ has a negative sign if water is pumped from the well.
- Calculate for various radii of the well $r_w$ (0.1 m, 0.25 m, 0.6 m, 1m) Sirchardt's maximum capacity and the maximum extraction rate possible in the given formation with the given well.
- Discuss constructive measures to obtain larger effective well diameters.

# Radial flow – well hydraulics



# Radial flow – well hydraulics

Read the slope: $$\frac{\Delta(h_0^2 - h^2)}{\Delta \ln(r)} \approx \frac{10m^2}{1.4} = 7.14m^2$$

$$sl = \frac{Q_W}{\pi k_f} = \frac{-6.31\,10^{-2}\,m^3/s}{\pi k_f}$$

$$k_f = 0.0026m/s$$

Read the x-axis for y=0: $\ln(R) \approx 6.2$ $R = 490m$