Groundwater Hydraulics

Institute for Fluid Mechanics and Environmental Physics in Civil Engineering, Universität Hannover

Darcy‘s law

How does the water flow?

Darcy's law

Hydraulic conductivity:

$$q = \frac{Q}{A} = -k\frac{\Delta h}{\Delta s}$$

resp. (isotropic media)

$$q = -k\nabla h$$

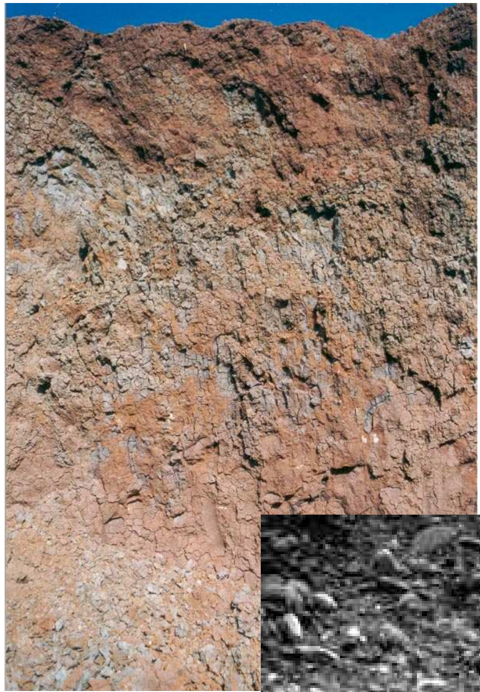
➢ velocity proportional to hydraulic gradient

➢ proportionality factor k called **hydraulic conductivity**

Darcy's law

Hydraulic conductivity:

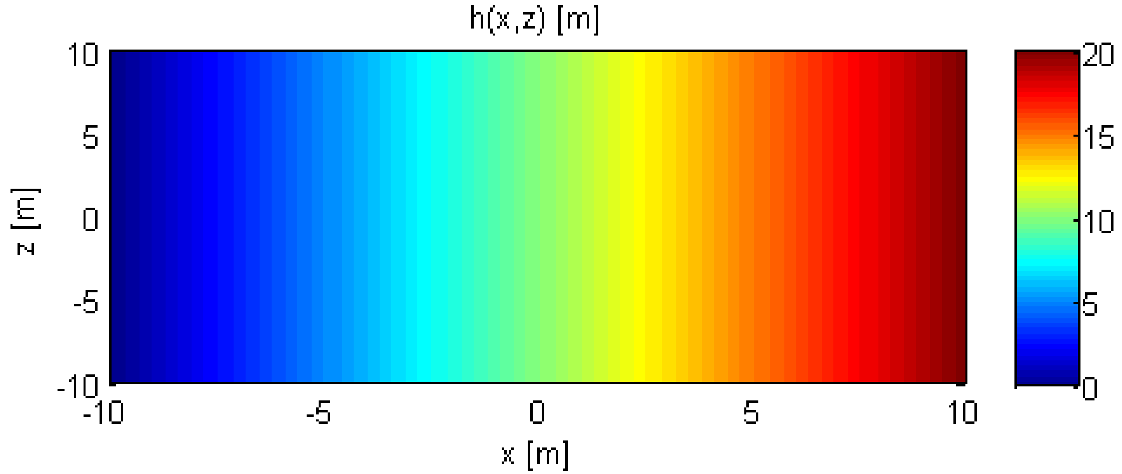
Soil Type	K [m/s]
pure gravel	1.0e-2 – 1.0e-1
coarse sand	~1.0e-3
medium sand	1.0e-4 – 1.0e-3
fine sand	1.0e-5 – 1.0e-4
silty sand	1.0e-7 – 1.0e-5
clayey silt	1.0e-9 – 1.0e-6
clay	<1.0e-9

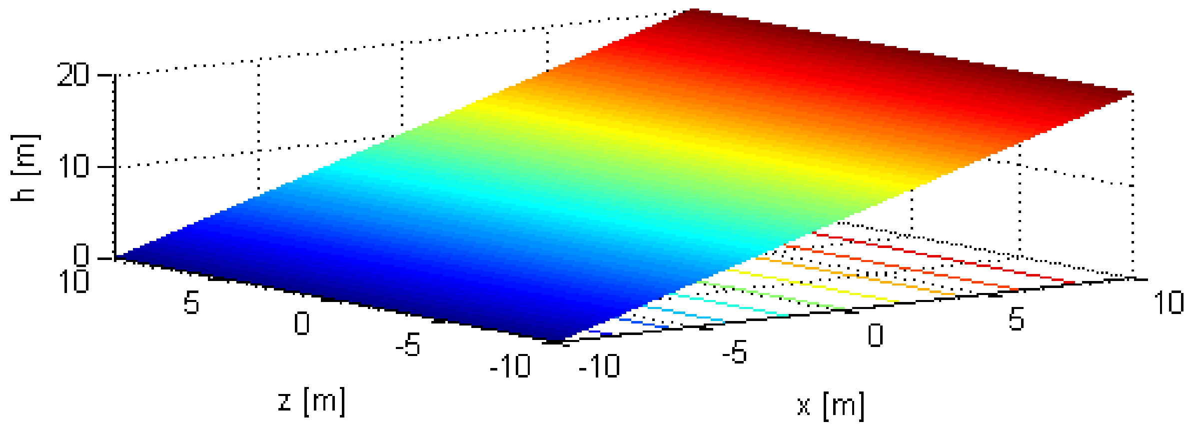


Darcy's law

The gradient

Example $h(x,z) = x + 10m$

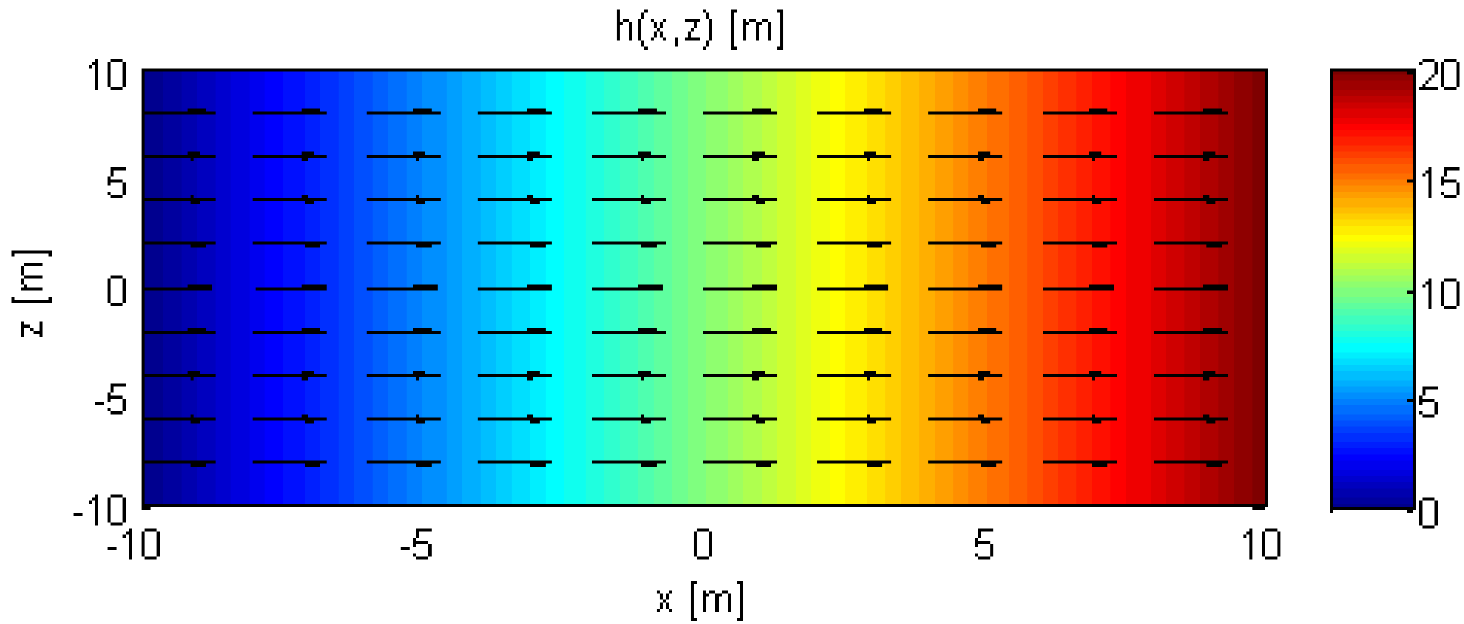




Darcy's law

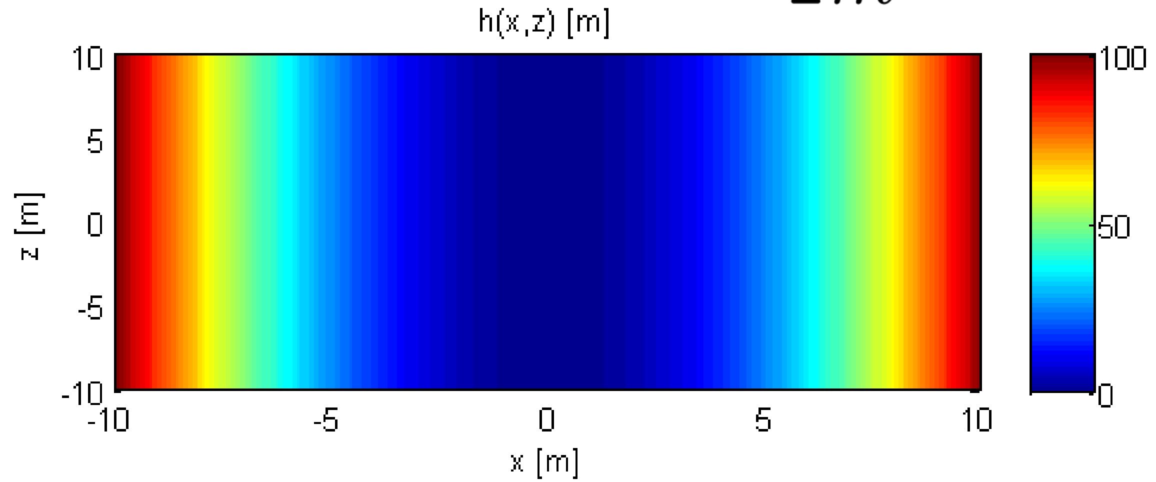
Example $h(x,z) = x + 10m$

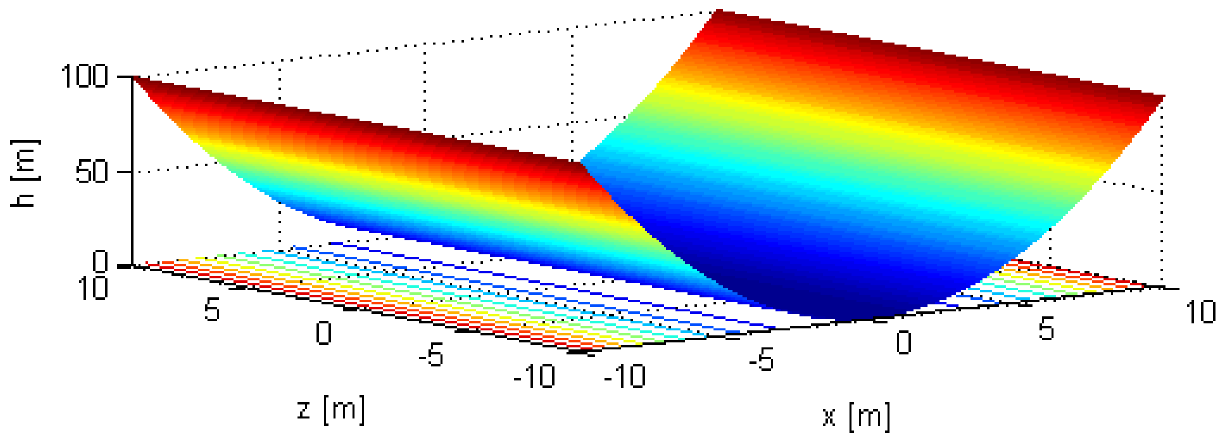
Gradient $\vec{\nabla} h = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$



Darcy's law

Example $$h(x,z) = \frac{x^2}{1m}$$

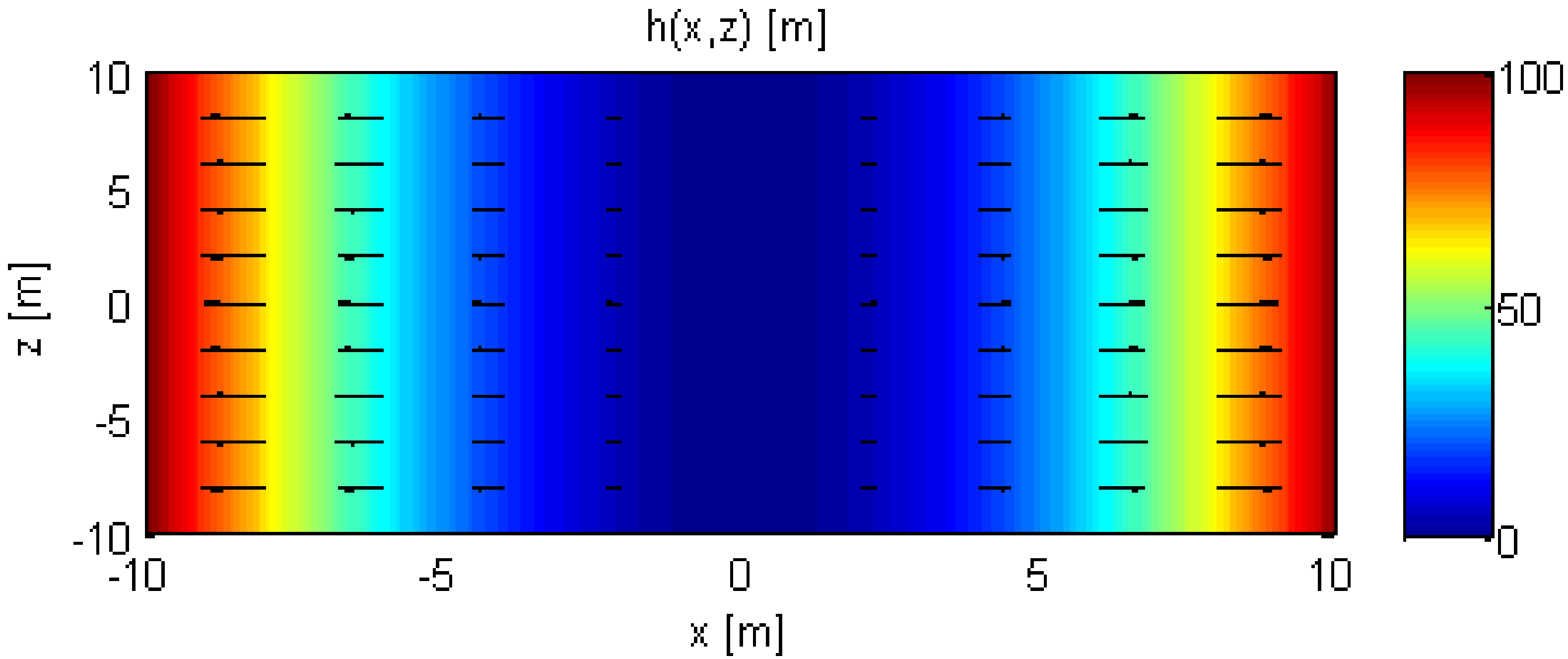




Darcy's law

Example $h(x,z) = \frac{x^2}{1m}$

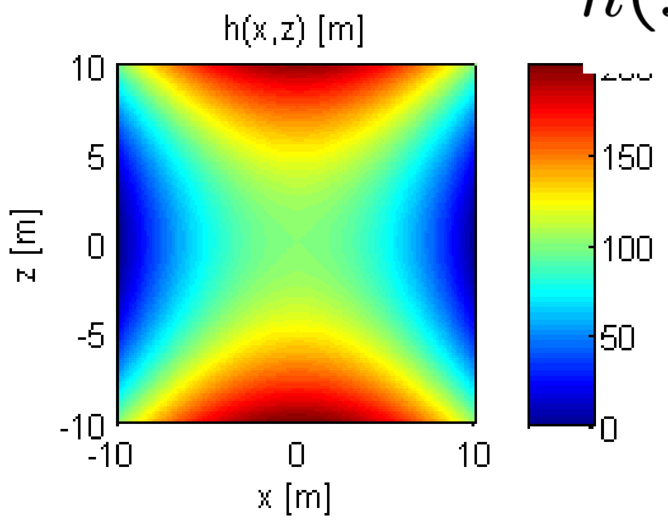
Gradient $\vec{\nabla} h = \frac{1}{1m}\begin{pmatrix} 2x \\ 0 \end{pmatrix}$

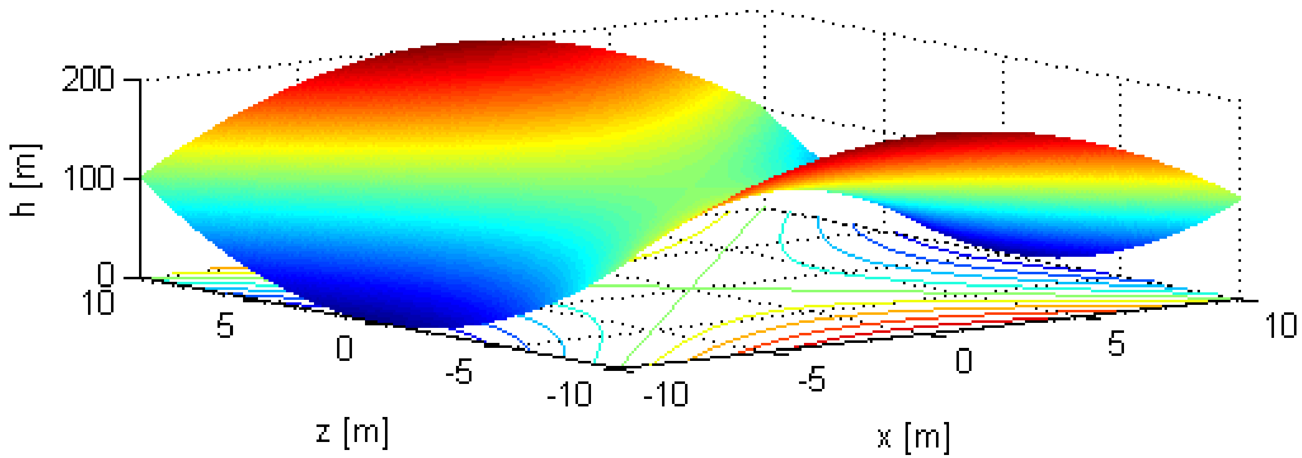


Darcy's law

Example

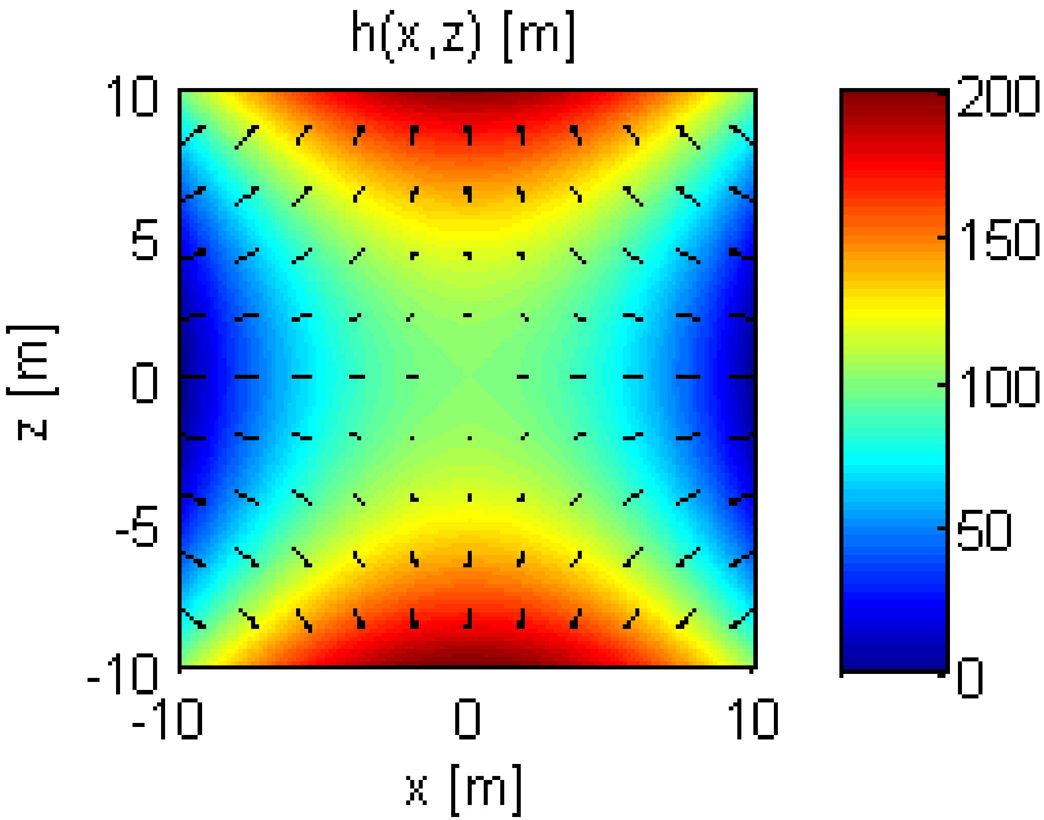
$$h(x,z) = 10m - \frac{x^2 - z^2}{1m}$$





Darcy's law

Example $h(x,z) = 10m - \frac{x^2 - z^2}{1m}$

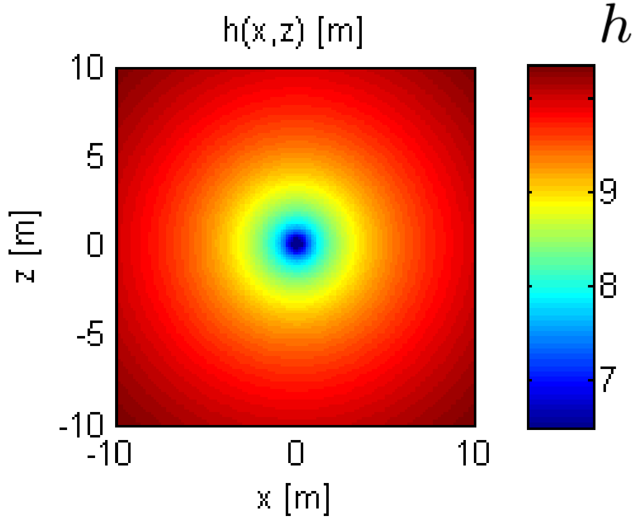


Gradient

$$\vec{\nabla} h = \frac{1}{1m} \begin{pmatrix} -2x \\ 2z \end{pmatrix}$$

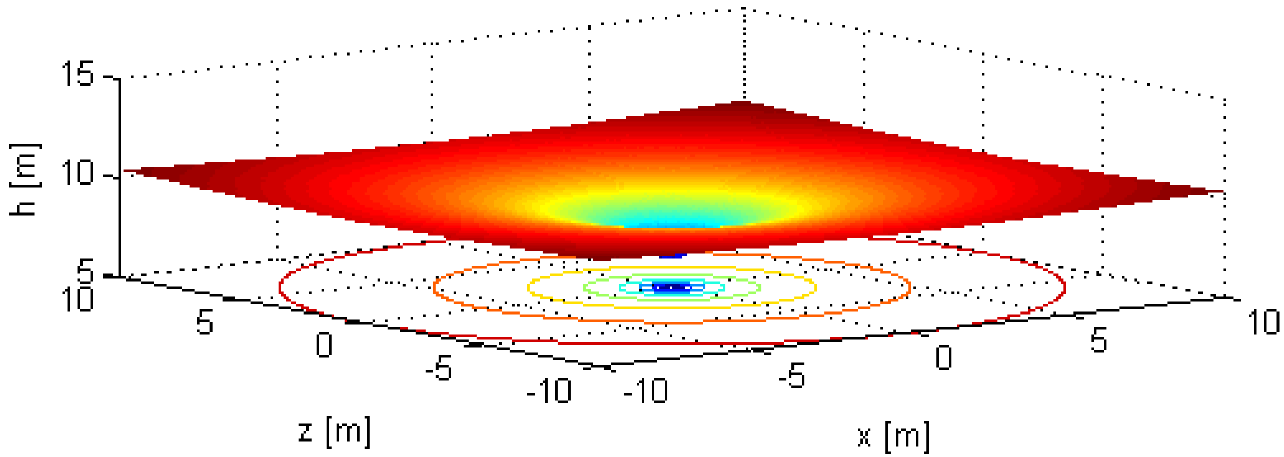
Darcy’s law

Example



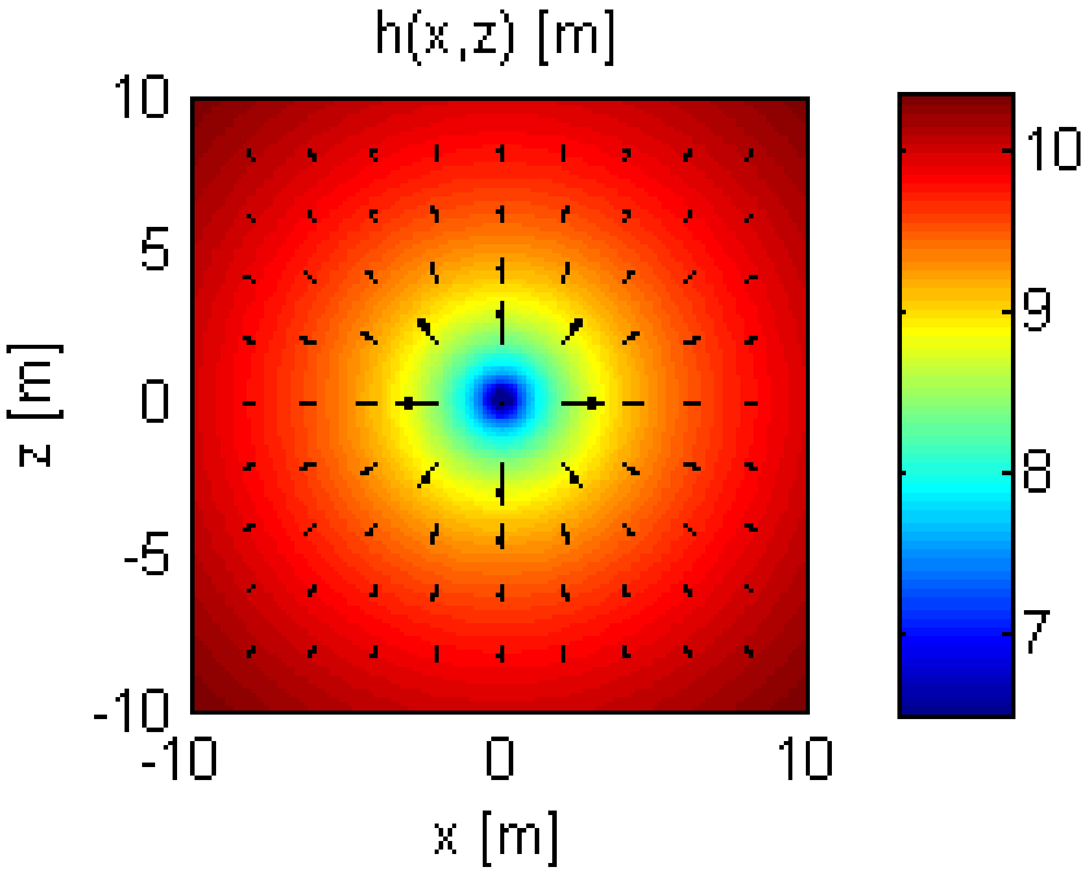
$$h(x,z) = 10m + 1m \ln\left(\frac{\sqrt{x^2 - z^2}}{10m}\right)$$
$$= 10m + 1m \ln\left(\frac{r}{10m}\right)$$

Piezometric head around a well



Darcy's law

Example $h(x,z) = 10m + 1m \ln\left(\frac{r}{10m}\right)$



Gradient

$$\vec{\nabla} h = 1m \begin{pmatrix} \frac{x}{r^2} \\ \frac{z}{r^2} \end{pmatrix}$$

Darcy’s law

Darcy‘s law: $\vec{q} = -K_f \vec{\nabla} h$

K_f: Hydraulic conductivity

$[K_f]$ = m/s

Empirical parameter,
Contains information about porous medium and fluid properties

Measurements:
- Soil samples → Laboratory
- Pump tests
- Inverse modeling of field experiments

Darcy's law

Empirical approaches to determine hydraulic conductivity:

- Hazen (1892)

$$k[m/s] = 0.0116 \cdot d_{10}^2 [mm]$$

- Kozeny (1927)

$$k[m/s] = \frac{d_w^2}{18} \cdot \frac{\varphi_f^3}{(1-\varphi_f)^2}$$

with d_w = effective pore diameter [mm]

$U = d_{60}/d_{10}$	d_w/d_{10}
$1.0 \leq U < 2.0$	1.3
$2.0 \leq U < 3.0$	1.75
$3.0 \leq U < 5.0$	2.05
$5.0 \leq U < 10$	2.35
$10 \leq U$	2.5

Darcy's law

Empirical approaches to determine hydraulic conductivity:

- Beyer (1964)

$$k[m/s] = C \cdot d_{10}^2[mm]$$

with C = non-uniformity and density coefficient

$U = d_{60}/d_{10}$ (range)	C (range)	(mean)
$1.0 \leq U < 2.0$	$1.05 \cdot 10^{-2} < C < 1.2 \cdot 10^{-2}$	$1.1 \cdot 10^{-2}$
$2.0 \leq U < 3.0$	$9.5 \cdot 10^{-3} < C < 1.05 \cdot 10^{-2}$	$1.0 \cdot 10^{-2}$
$3.0 \leq U < 5.0$	$8.5 \cdot 10^{-3} < C < 9.5 \cdot 10^{-3}$	$9.0 \cdot 10^{-3}$
$5.0 \leq U < 10$	$7.5 \cdot 10^{-3} < C < 8.5 \cdot 10^{-3}$	$8.0 \cdot 10^{-3}$
$10 \leq U < 20$	$6.5 \cdot 10^{-3} < C < 7.5 \cdot 10^{-3}$	$7.0 \cdot 10^{-3}$
$20 \leq U$	$C < 6.5 \cdot 10^{-3}$	$6.0 \cdot 10^{-3}$

Darcy’s law

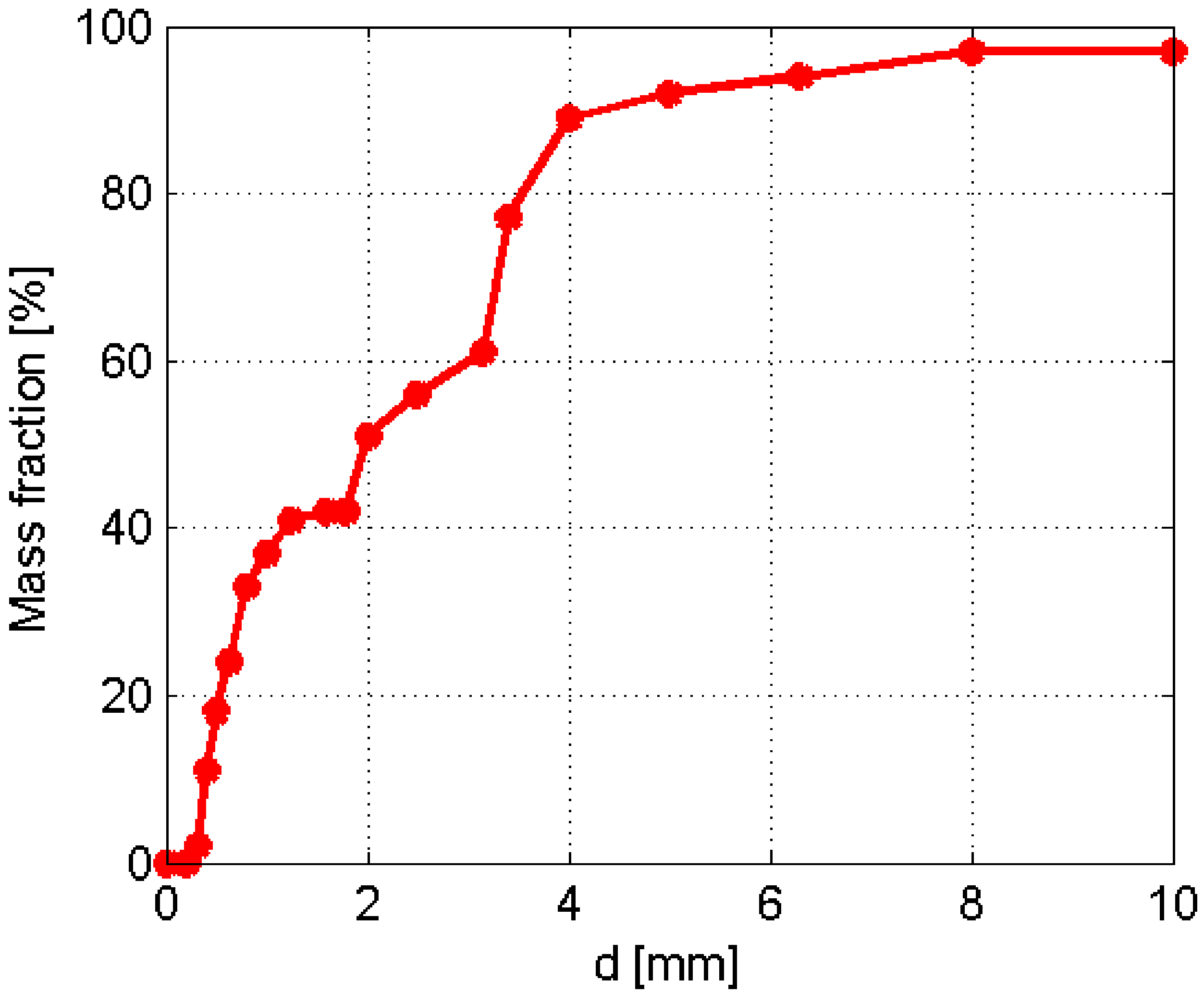
Exercise #2

The Filter Sand Dorfner Dorsilit Nr.0-7 has the following grainsize distribution:

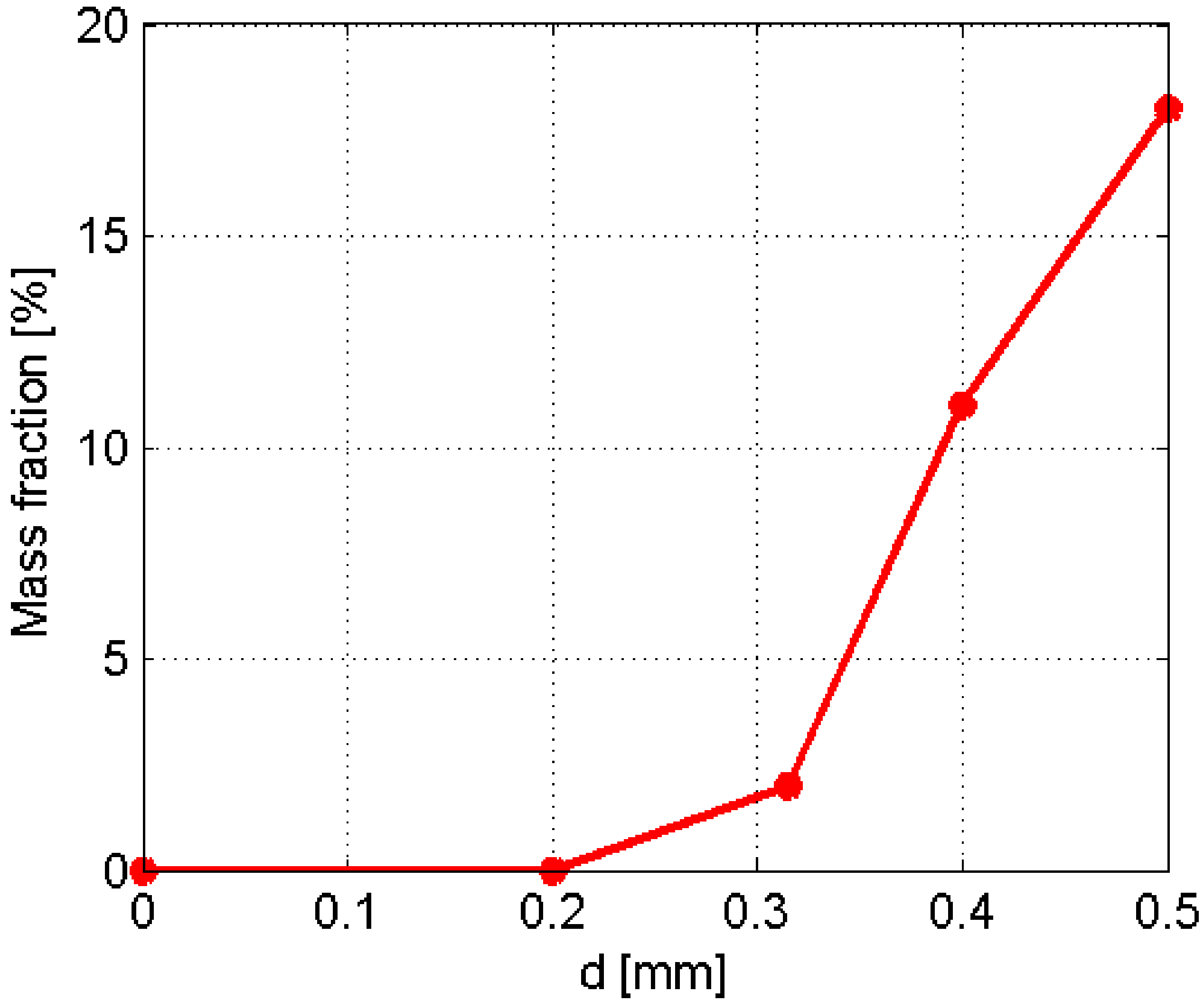
d [mm]	0.1-0.2	0.2-0.315	0.315-0.4	0.4-0.5	0.5-0.63	0.63-0.8
fraction [%]	0	2	9	7	6	9
d [mm]	0.8-1.0	1.0-1.25	1.25-1.6	1.8-2.0	2.0-2.5	2.5-3.15
fraction [%]	4	4	1	9	5	5
d [mm]	3.15-3.4	3.4-4.0	4.0-5.0	5.0-6.3	6.3-8.0	>8.0
fraction [%]	16	12	3	2	3	0

Estimate the hydraulic-conductivity value after Hazen, Kozeny and Beyer. The effective porosity is 40%.

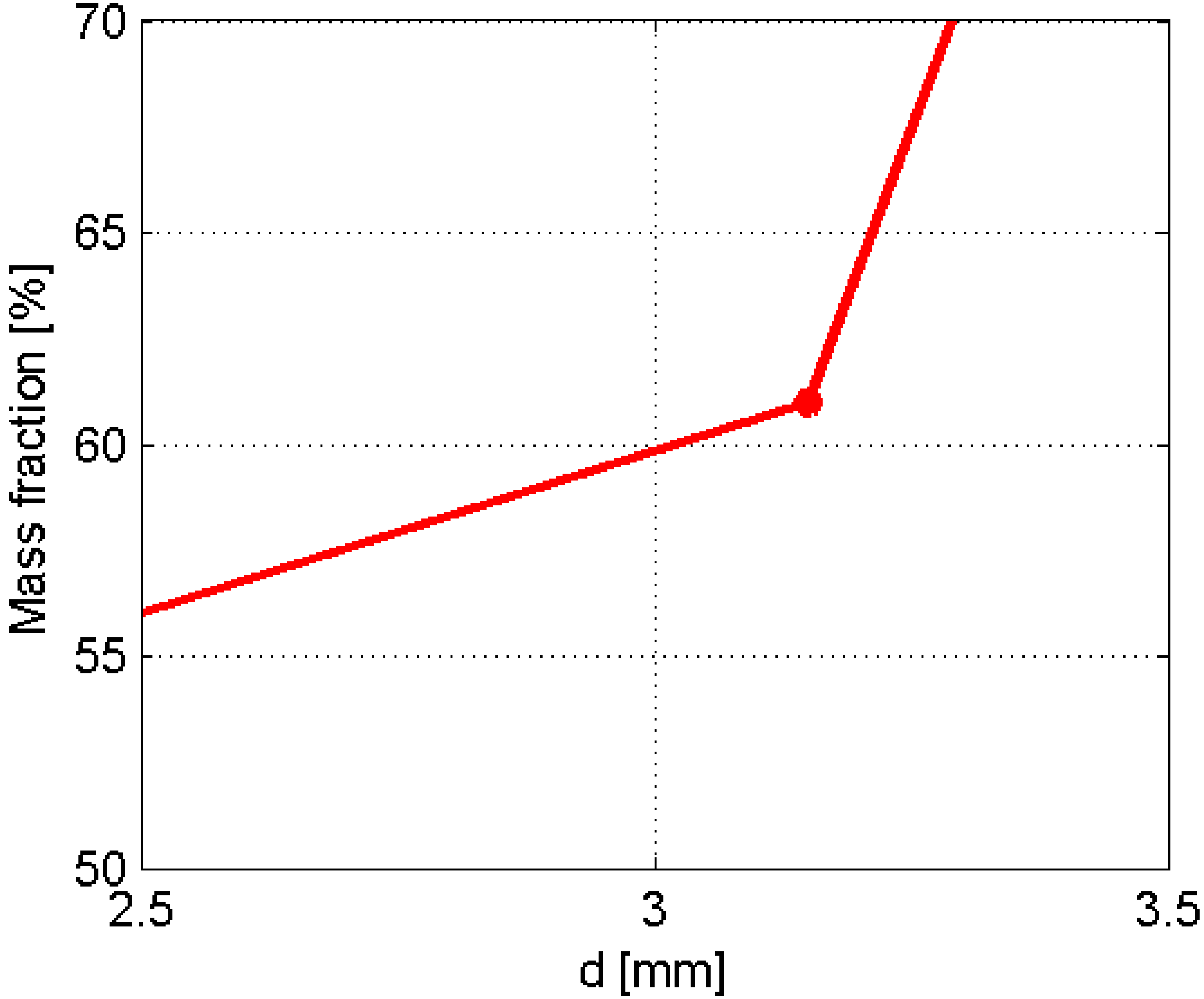
Darcy’s law



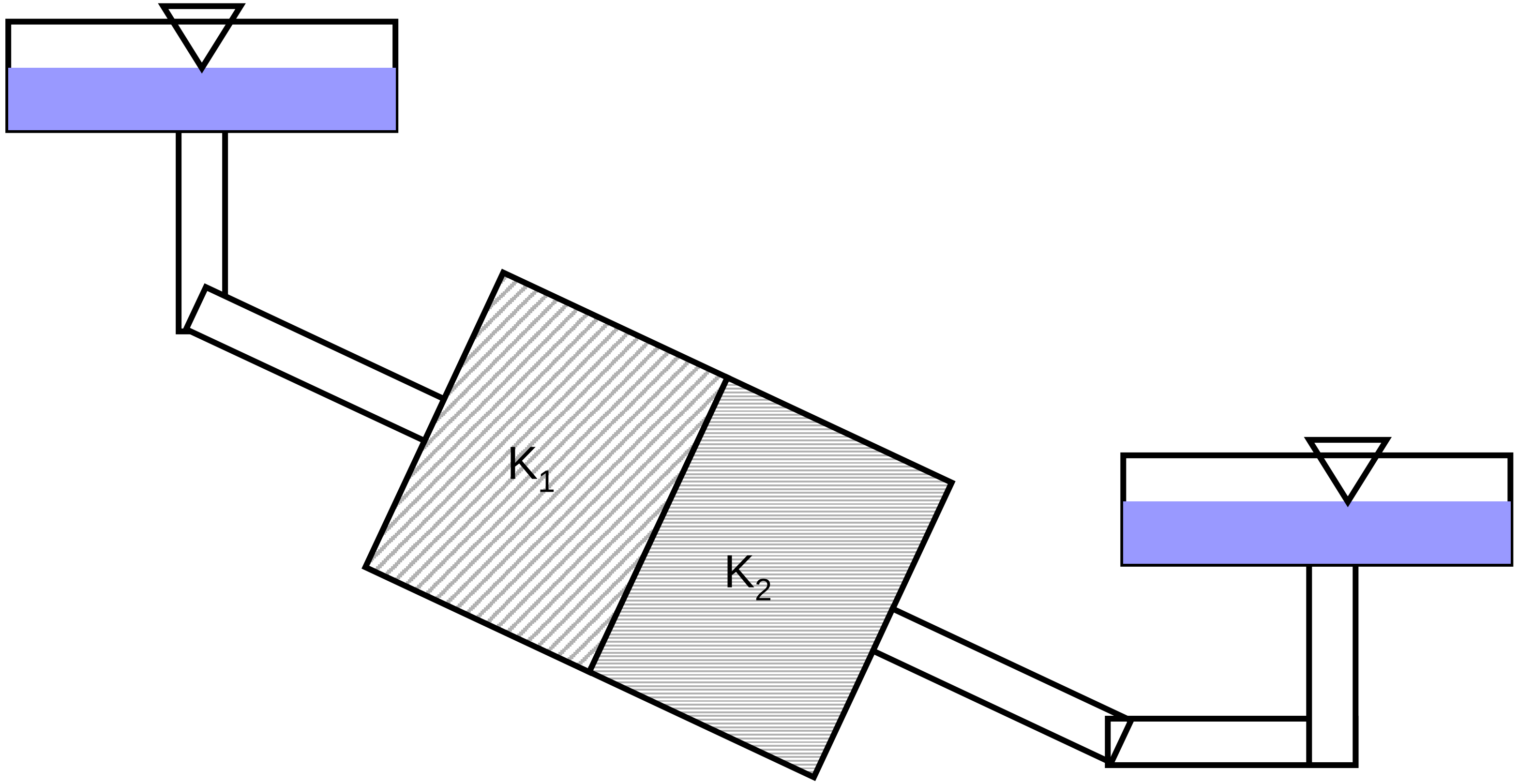
Darcy's law



Darcy's law



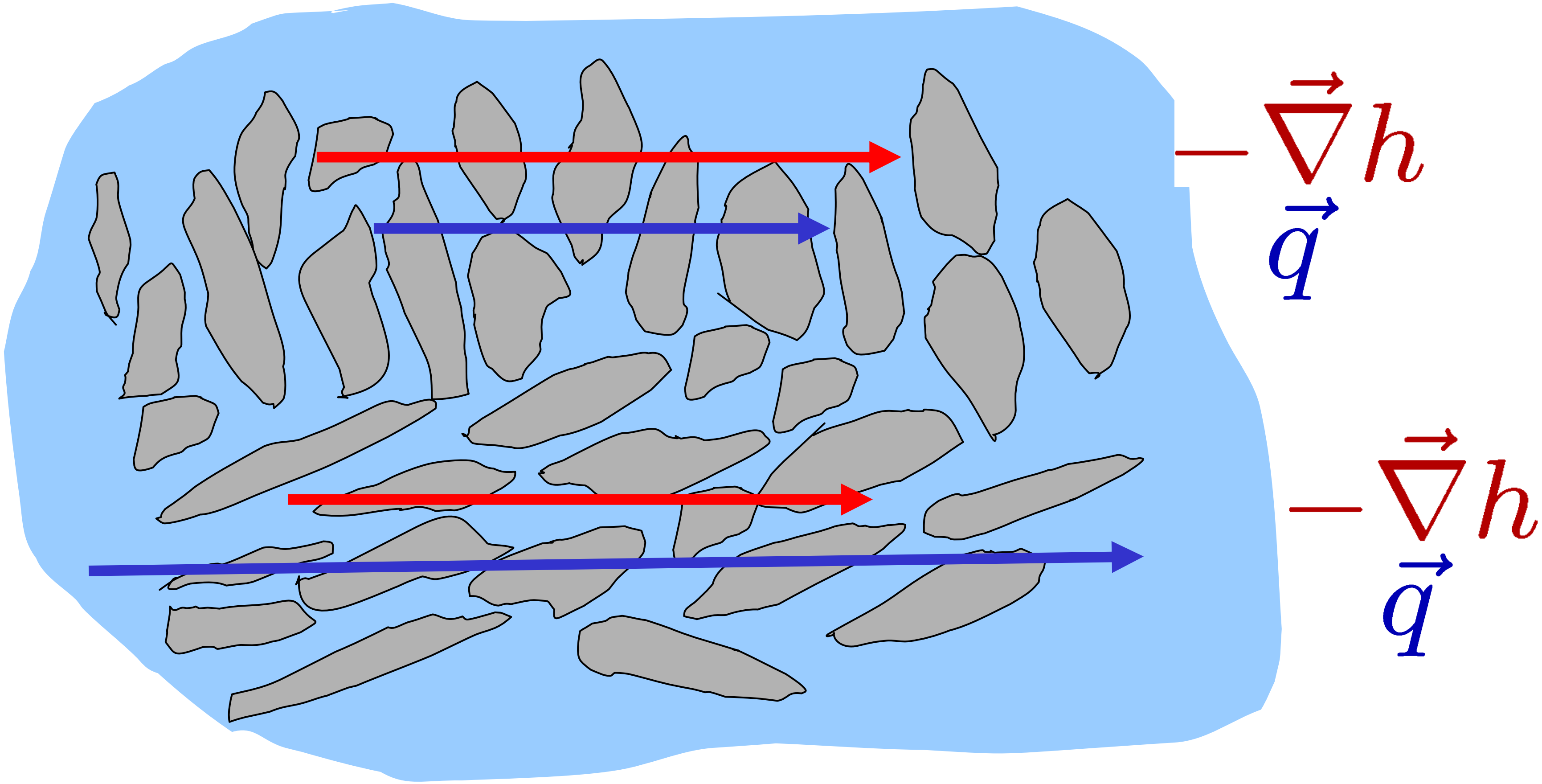
Darcy's law



Piezometric head?

Darcy's law

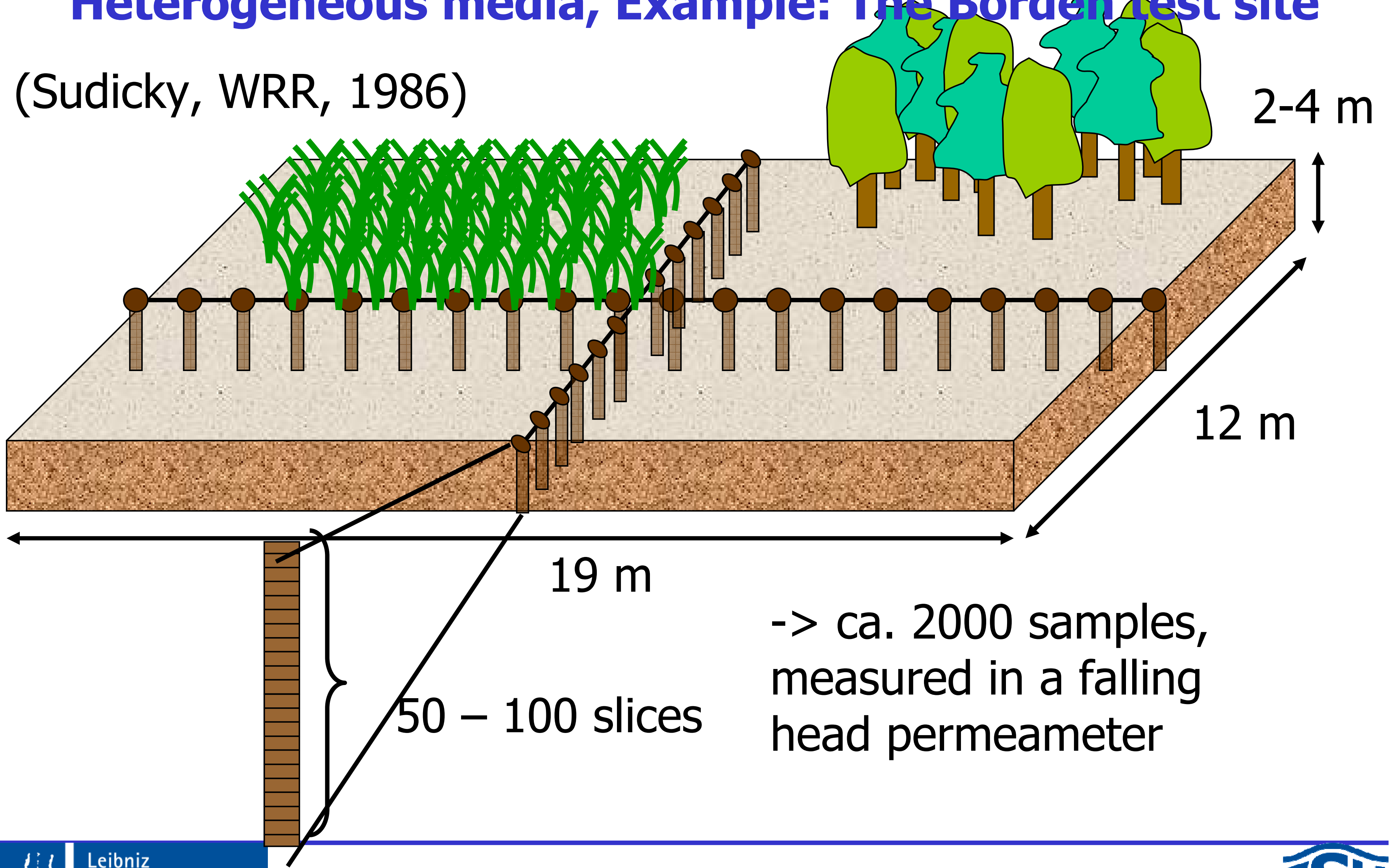
Heterogeous Medien



Darcy’s law

Heterogeneous media, Example: The Borden test site

(Sudicky, WRR, 1986)

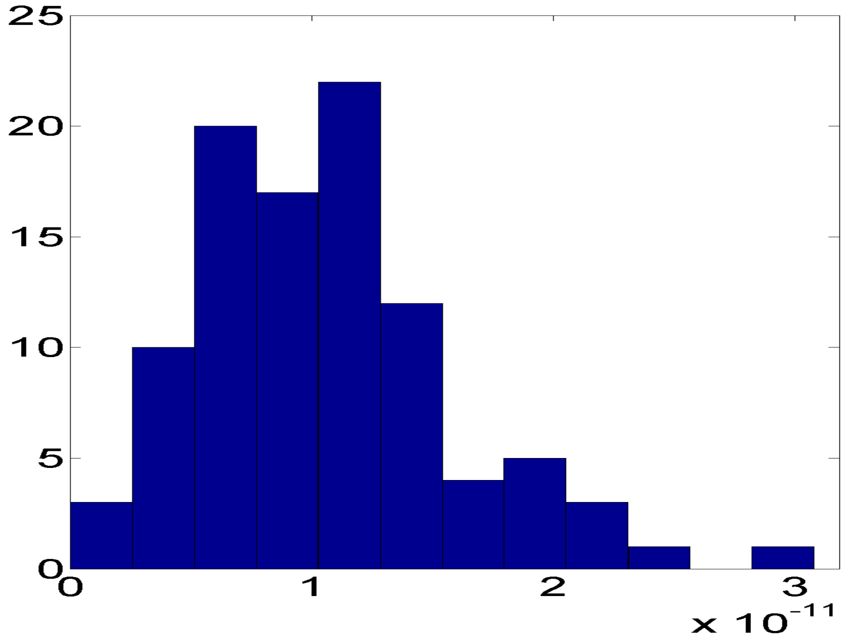


-> ca. 2000 samples, measured in a falling head permeameter

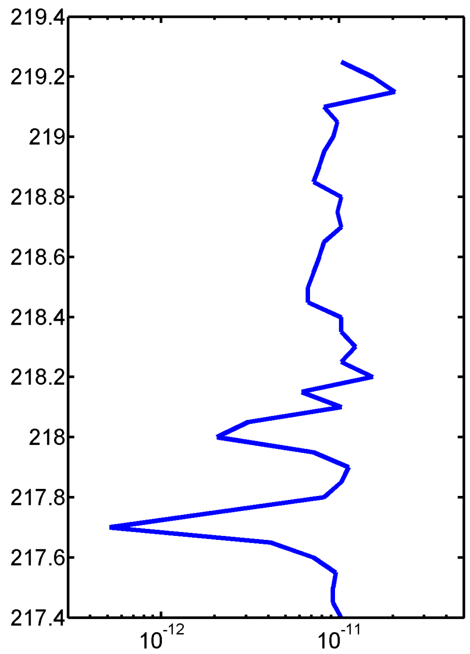
Darcy's law

Parameter distribution:

Frequency [%]



Permeability [m²]

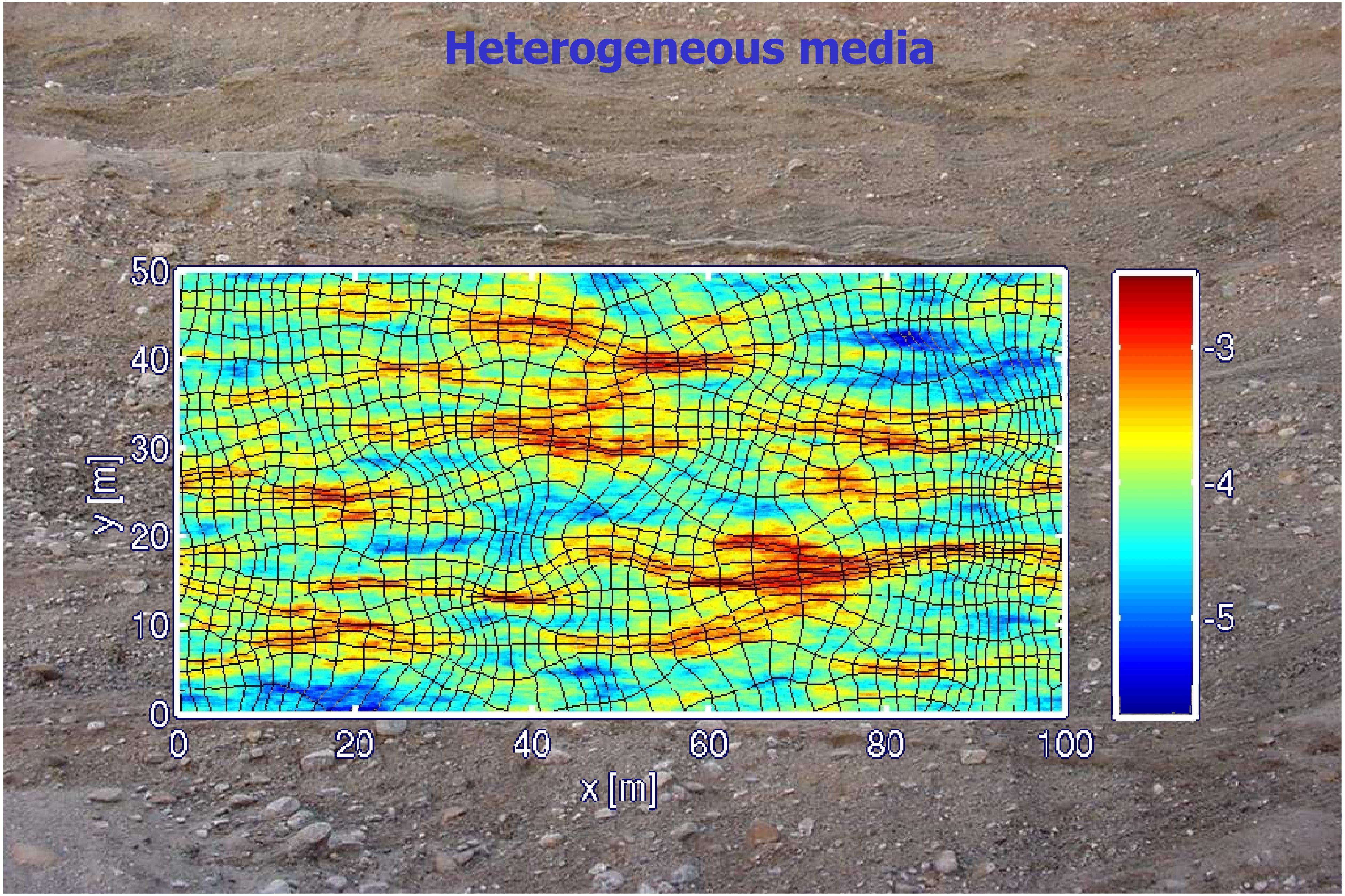


Permeability K [m²]

(replotted from: Sudicky, WRR, 1986)

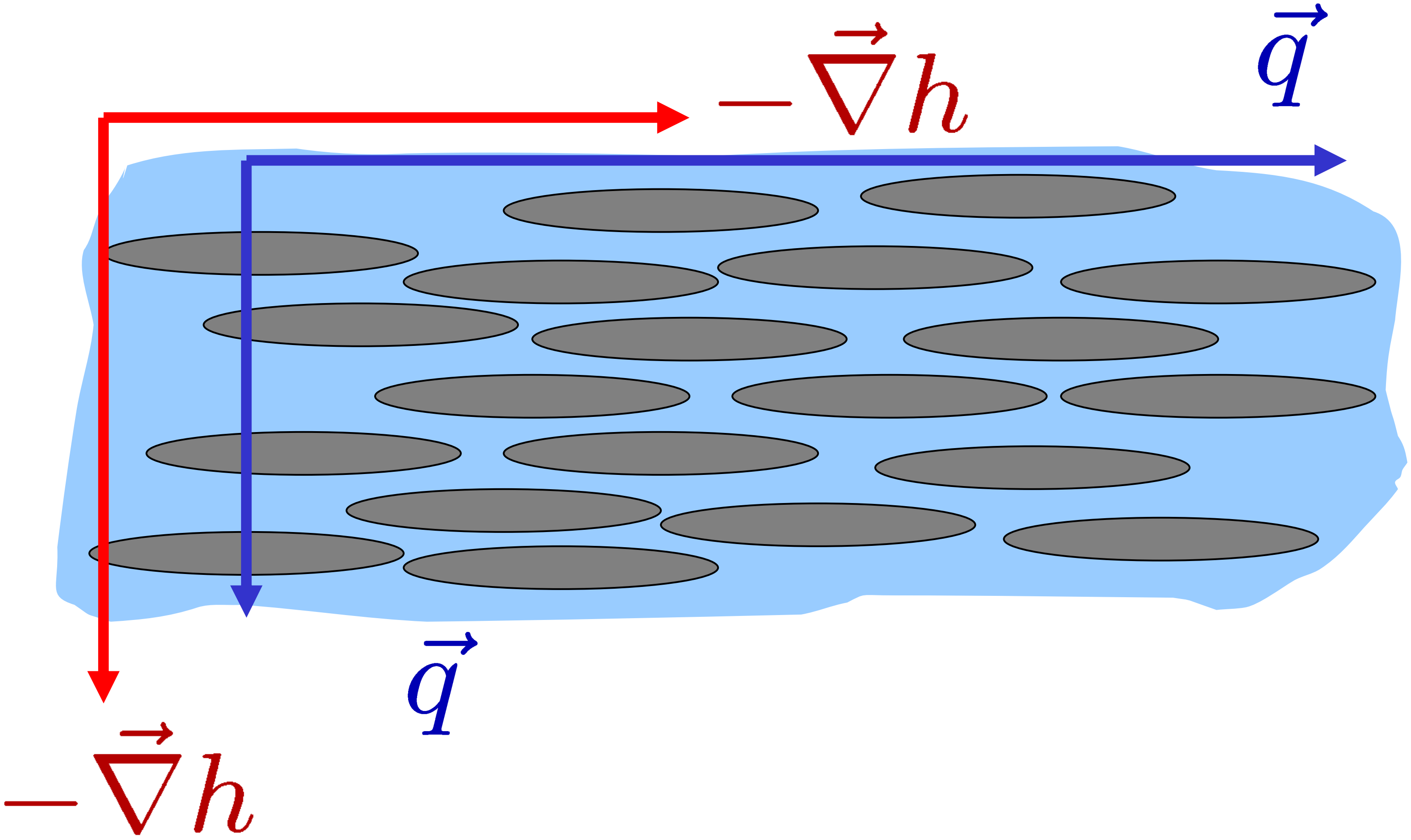
Darcy's law

Heterogeneous media



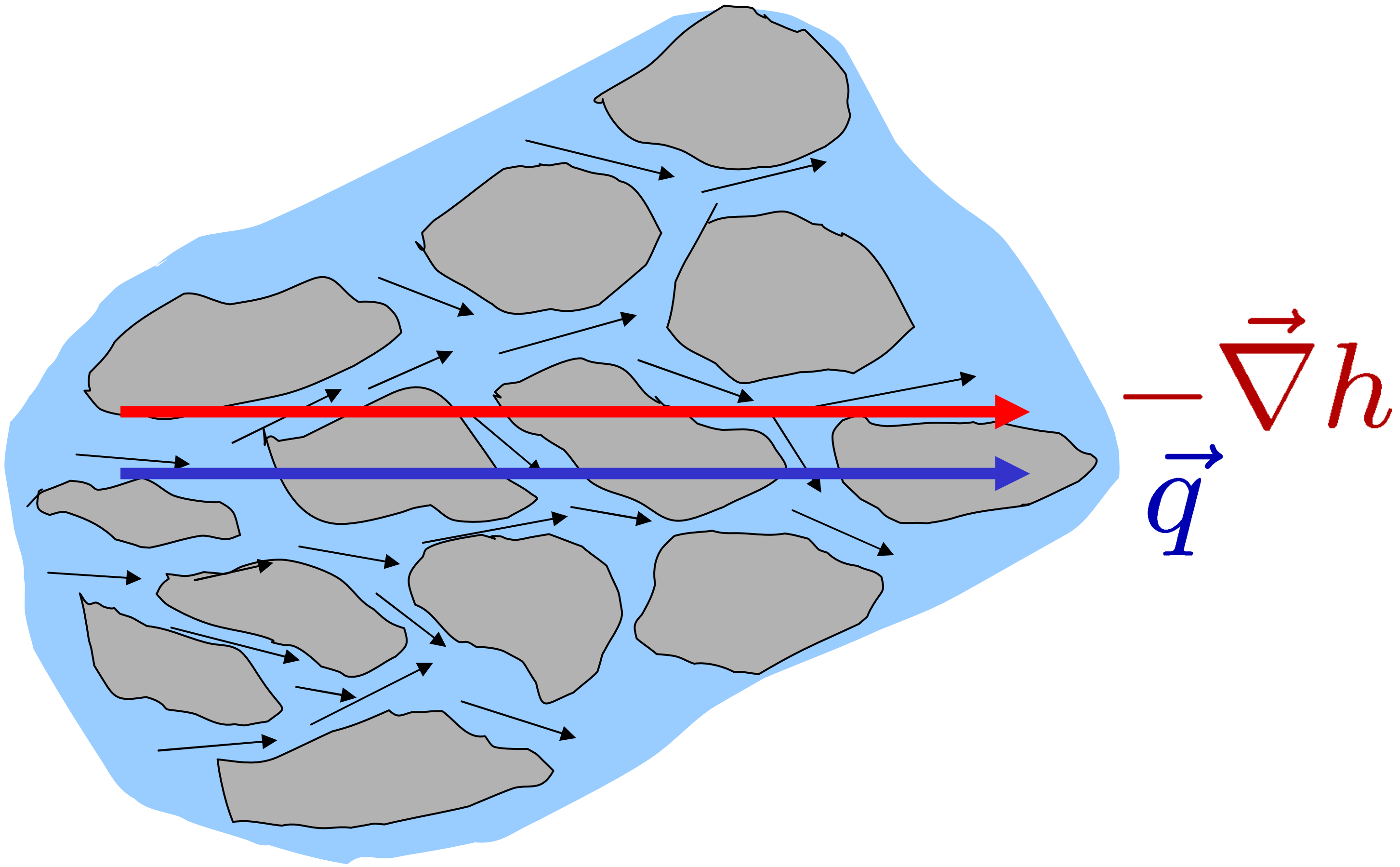
Darcy's law

Anisotropic media



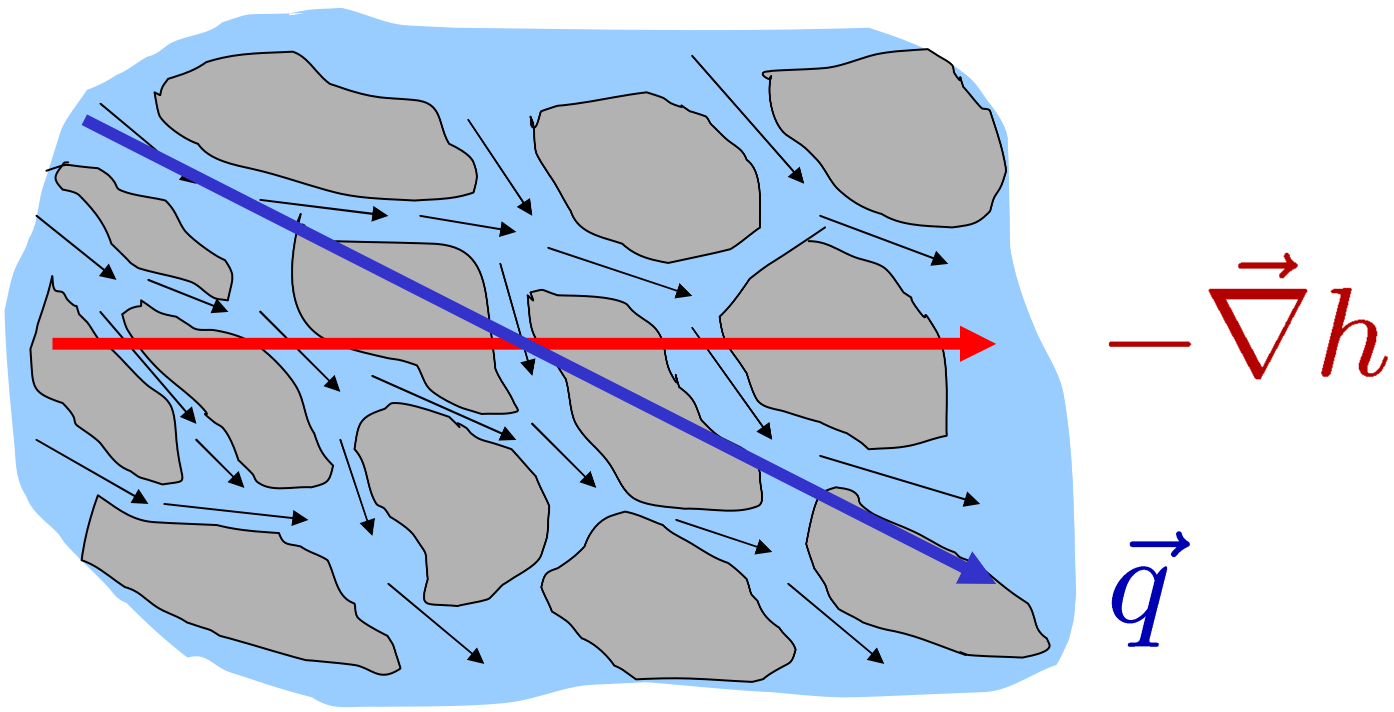
Darcy's law

Anisotropic media



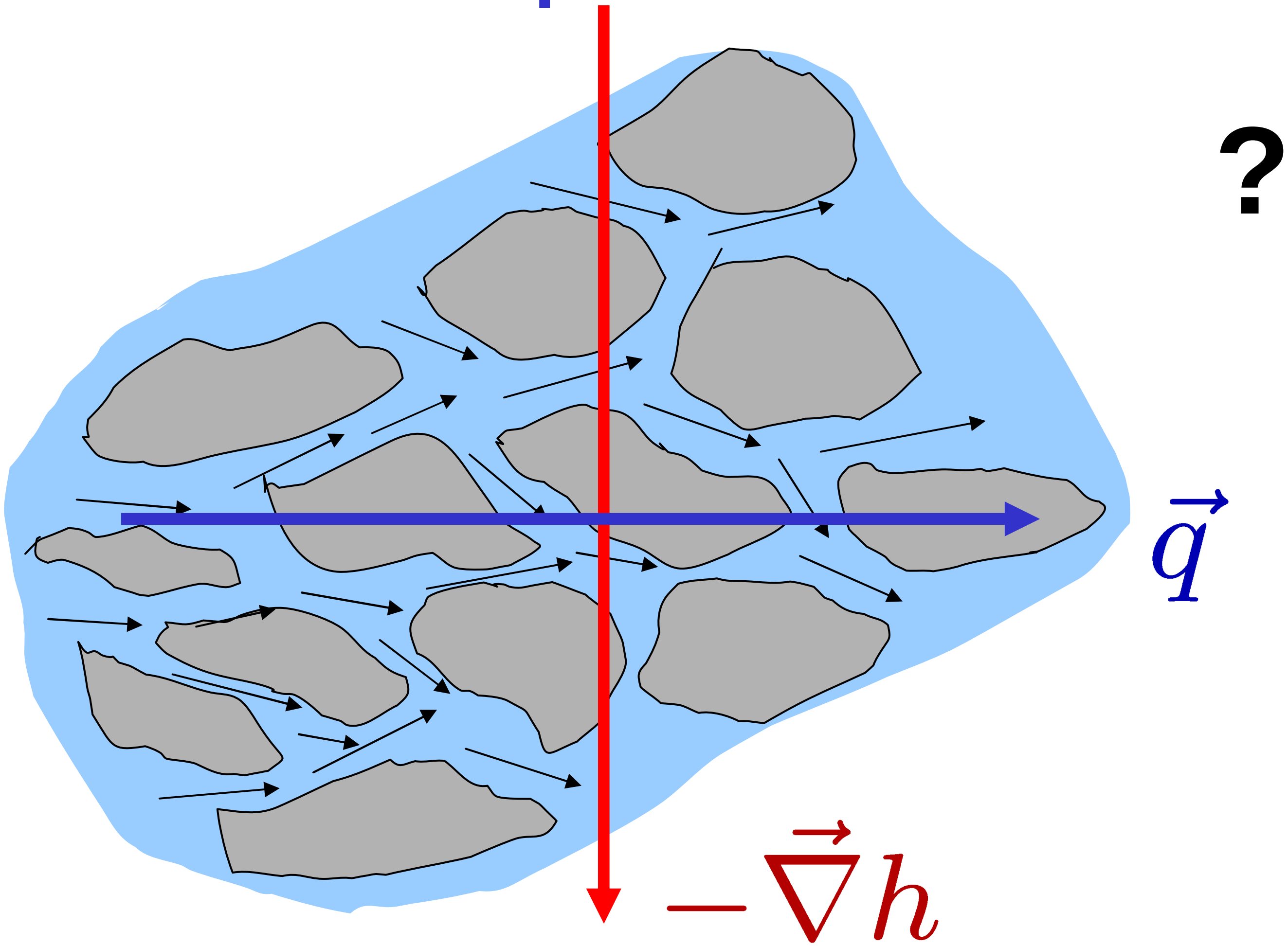
Darcy’s law

Anisotropic media



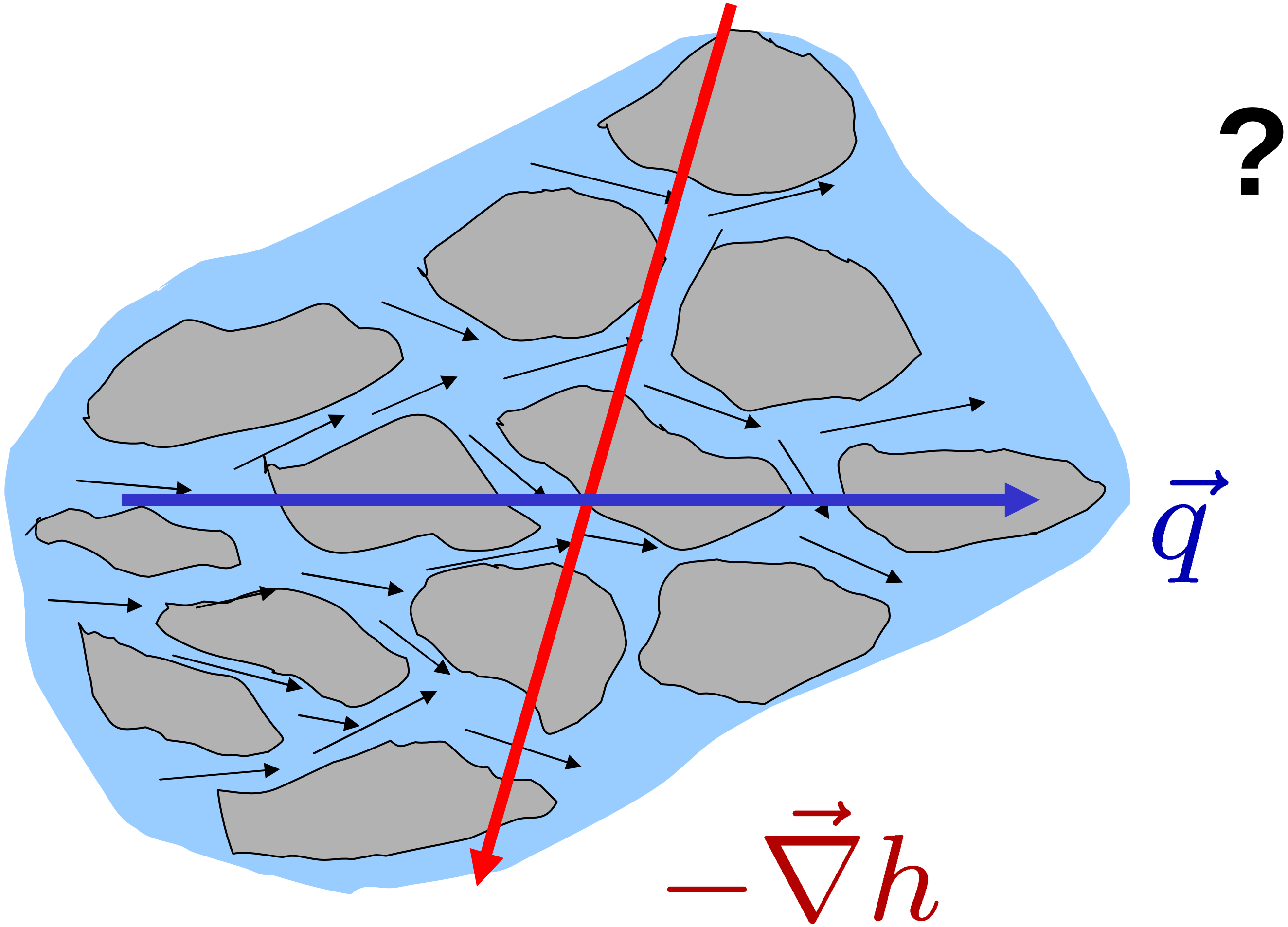
Darcy's law

Anisotropic media



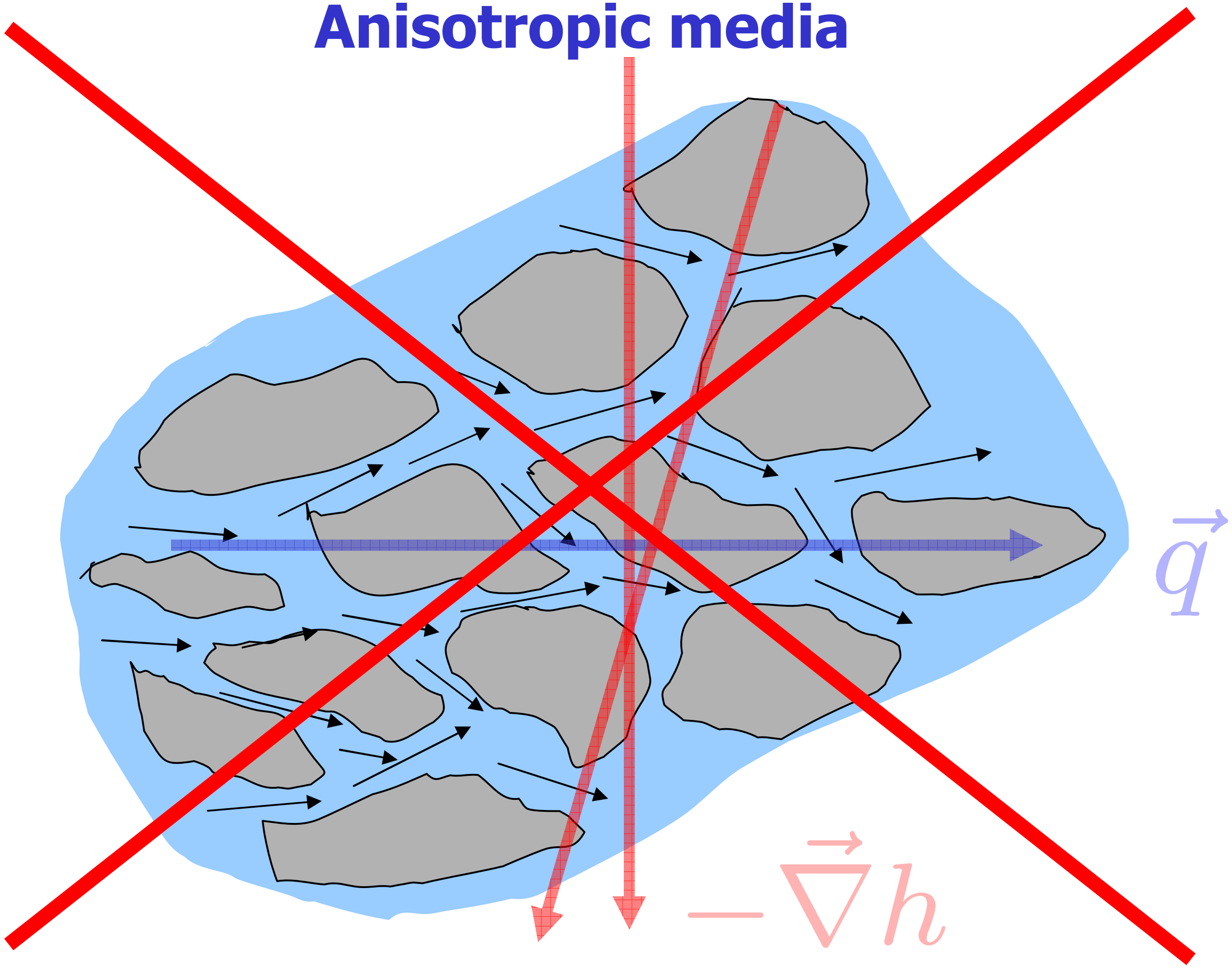
Darcy's law

Anisotropic media



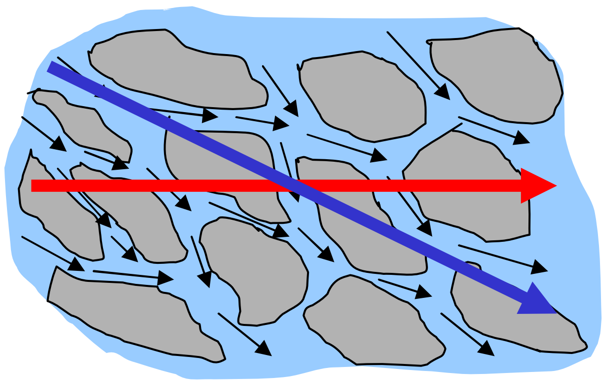
Darcy's law

Anisotropic media



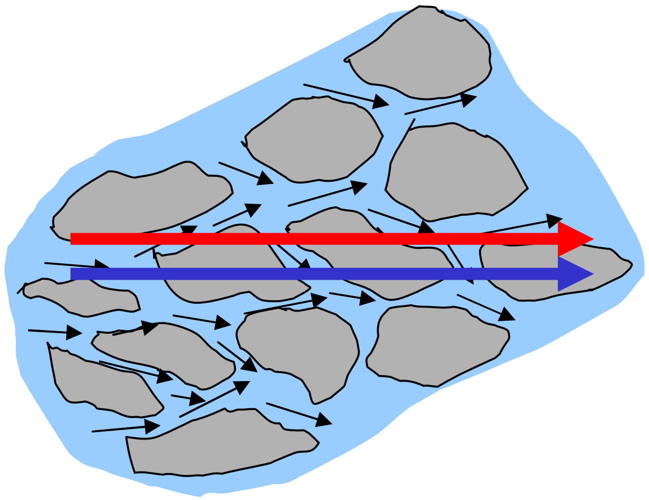
Darcy's law

Darcy equation: $\vec{q} = -\mathbf{K}_f \vec{\nabla} h$



Anisotropic (general)

$$\begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = -\begin{pmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{pmatrix} \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} h$$



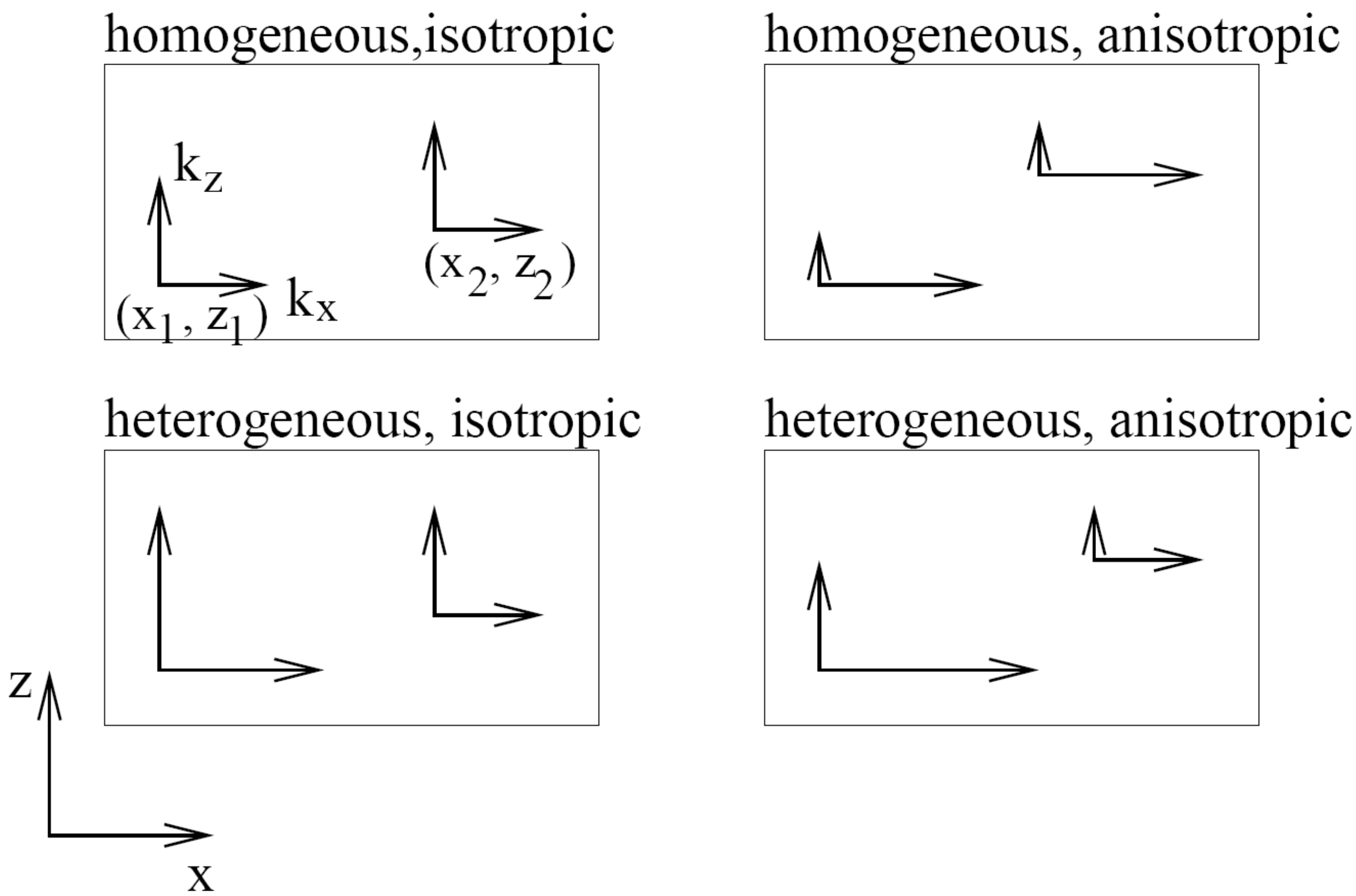
Anisotropic (in principal directions)

$$\begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = -\begin{pmatrix} K_{xx} & 0 & 0 \\ 0 & K_{yy} & 0 \\ 0 & 0 & K_{zz} \end{pmatrix} \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} h$$

K is positiv definit

Darcy’s law

Heterogeneity and anisotropy



Darcy's law

Exercise #3

A sand box with length L and constant width d is used for experimental modeling of quasi-2D flow in porous media. The boundary conditions at the left and the right are fixed by regulating water reservoirs, while the upper and lower boundary are impermeable.
The head difference controlled by the boundary condition is kept constant Δh=h1-h2 and the box is filled with two soils with k1 and k2. In case (a), it is valid that L1=L2=L and m1+m2=m. In case (b), it is valid that L1+L2=L and m1=m2=m.

Find for (a) and (b) the effective permeability that accounts in each case for the averaged effect of the soils.