Groundwater Hydraulics

Institute for Fluid Mechanics and Environmental Physics in Civil Engineering, Universität Hannover

Continuity equation

Mass balance over an infinitesimal small control volume

$$\frac{\partial(\rho n_f)}{\partial t} + \vec{\nabla} \cdot (\rho \vec{q}) = \rho W_0$$

Continuity equation

Water has a small compressibility:

$$\frac{1}{\rho}\frac{\partial(\rho n_f)}{\partial t} + \vec{\nabla} \cdot \vec{q} = W_0$$

Write the flow equation in terms of piezometric head:

$$\frac{1}{\rho}\frac{d(\rho n_f)}{dh}\frac{\partial h}{\partial t} - \vec{\nabla} \cdot \mathbf{K}_f \vec{\nabla} h = W_0$$

Specific storage coefficient

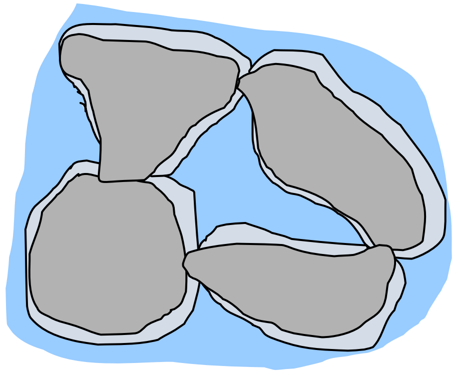
$$S_0 = \frac{n_f}{\rho}\frac{d\rho}{dh} + \frac{dn_f}{dh}$$

Continuity equation

Specific storage coefficient: $$S_0 = \frac{dn_f}{dh} + \frac{n_f}{\rho_w}\frac{d\rho_w}{dh}$$

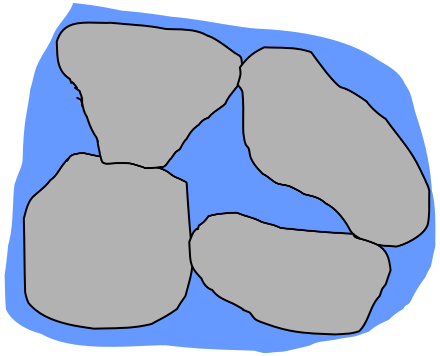
Compressibility of pore space:

$$S_{0,g} = 10^{-6} \rightarrow 10^{-2}\frac{1}{m}$$



Compressibility of water:

$$S_{0,w} \approx 10^{-6}\frac{1}{m}$$



Storage large for entrapped gas (z.B. CO_2)

Continuity equation

Specific storage coefficient

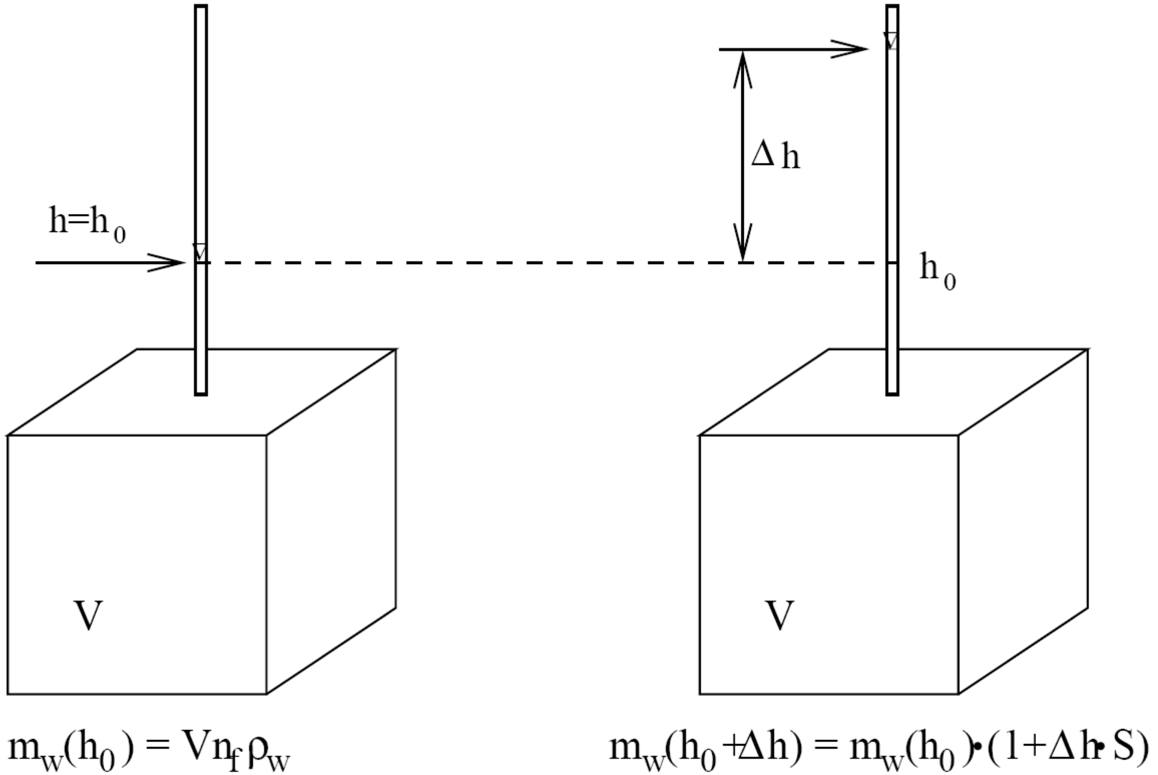


Figure 4.8: The specific storage coefficient S_0 describes the relative change of mass of the stored water due to a change in the hydraulic head h.

Continuity equation

Groundwater flow equation in general

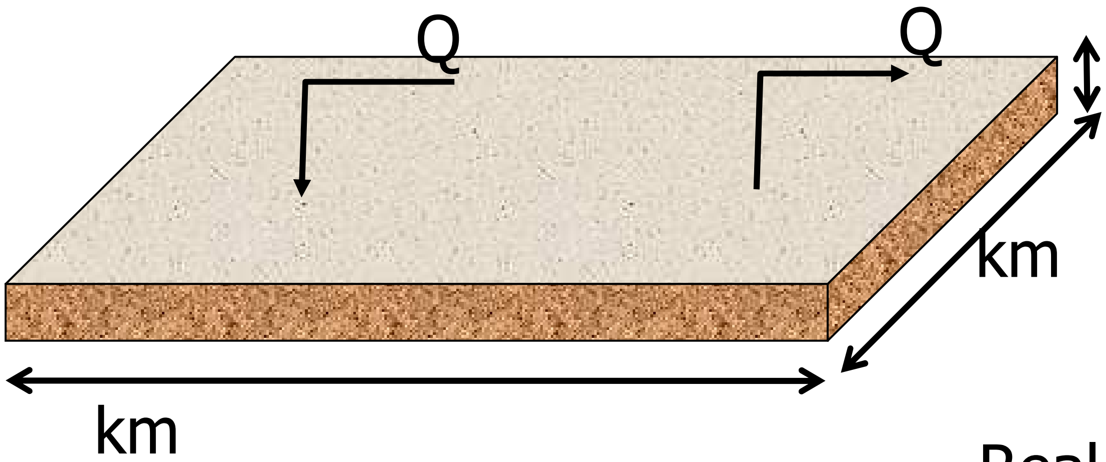
$$S_0 \frac{\partial h}{\partial t} - \vec{\nabla} \cdot \mathbf{K}_f \vec{\nabla} h = W_0$$

Continuity equation

Depth averaged description on large scales

- Groundwater flow
- Well flow

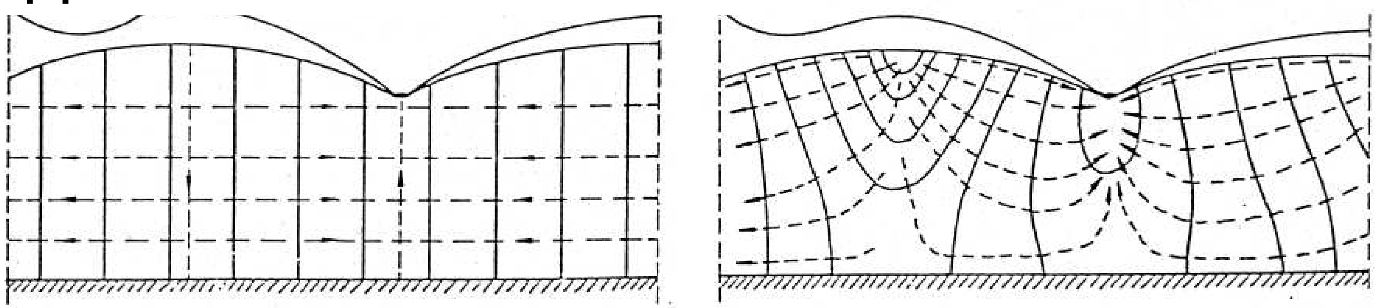
} Often mainly horizontal flow on large scales



Vertical movement of water negligible

Approximated streamlines

Real streamlines



Continuity equation

Dupuit Approximation

$$h = h(x,y,t) \qquad \vec{v}_f = \begin{pmatrix} v_{f,x}(x,y,t) \\ v_{f,y}(x,y,t) \\ 0 \end{pmatrix}$$

Reduction from 3d to 2d

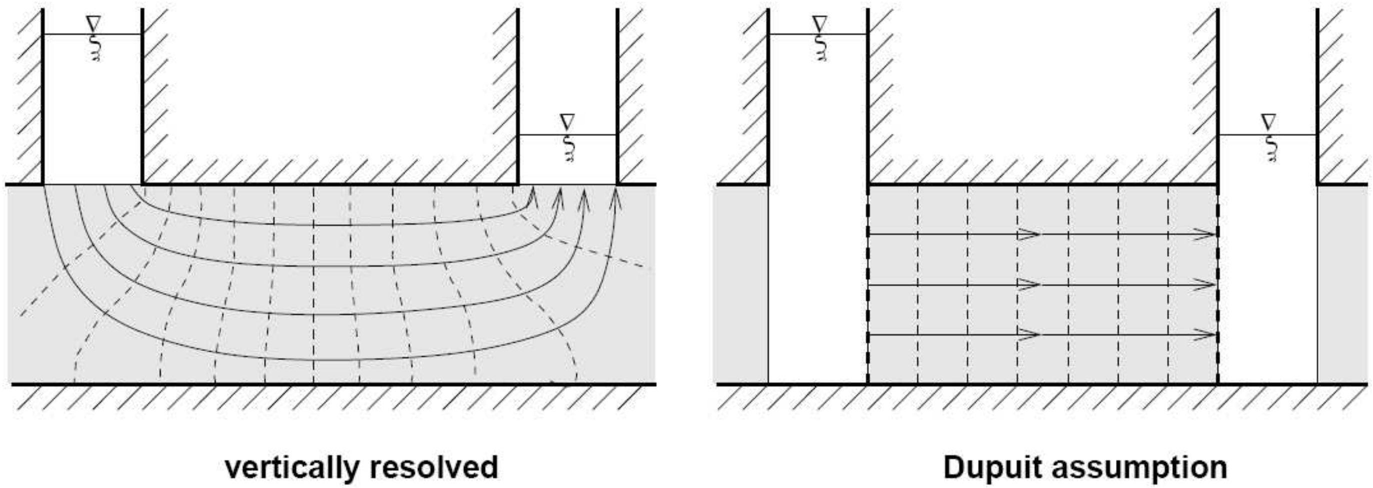
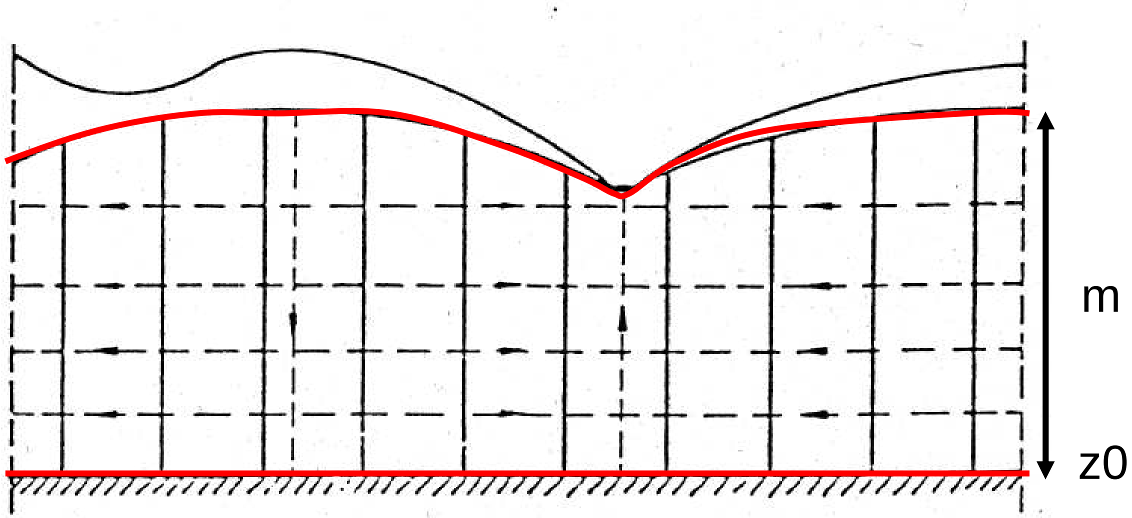


Figure 4.9: Illustration of the Dupuit assumption for the example of groundwater flow between two rivers in perfect hydraulic contact to the aquifer.

Continuity equation

Averaging over depth

$$\int_{z_0}^{z_0+m} \left(S_0 \frac{\partial h}{\partial t} - \vec{\nabla} \cdot \left(k_f \vec{\nabla} h \right) - W_0 \right) dz = 0$$



Continuity equation

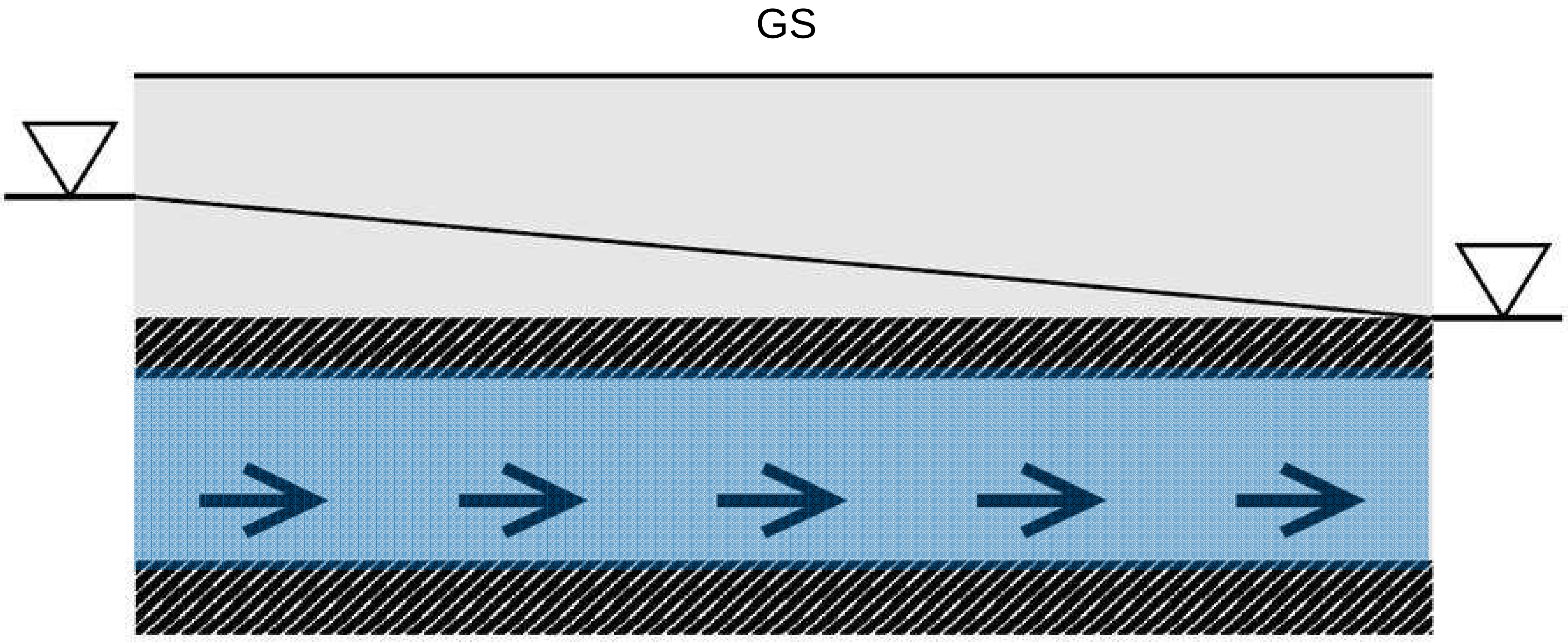
Exercise #4

Discuss for which applications a depth-integrated description is valid:

1. Calculation of the capture zone of a production well,
2. Calculation of drawdown in the direct vicinity of a fully screened well
   a) Under confined conditions,
   b) In a thick phreatic aquifer,
   c) In a shallow phreatic aquifer
3. Calculation of drawdown in the direct vicinity of an incomplete well,
4. Calculation of the groundwater flow field for remediation of a spill at a gas station
5. Calculation of transport in the remediation of that spill.

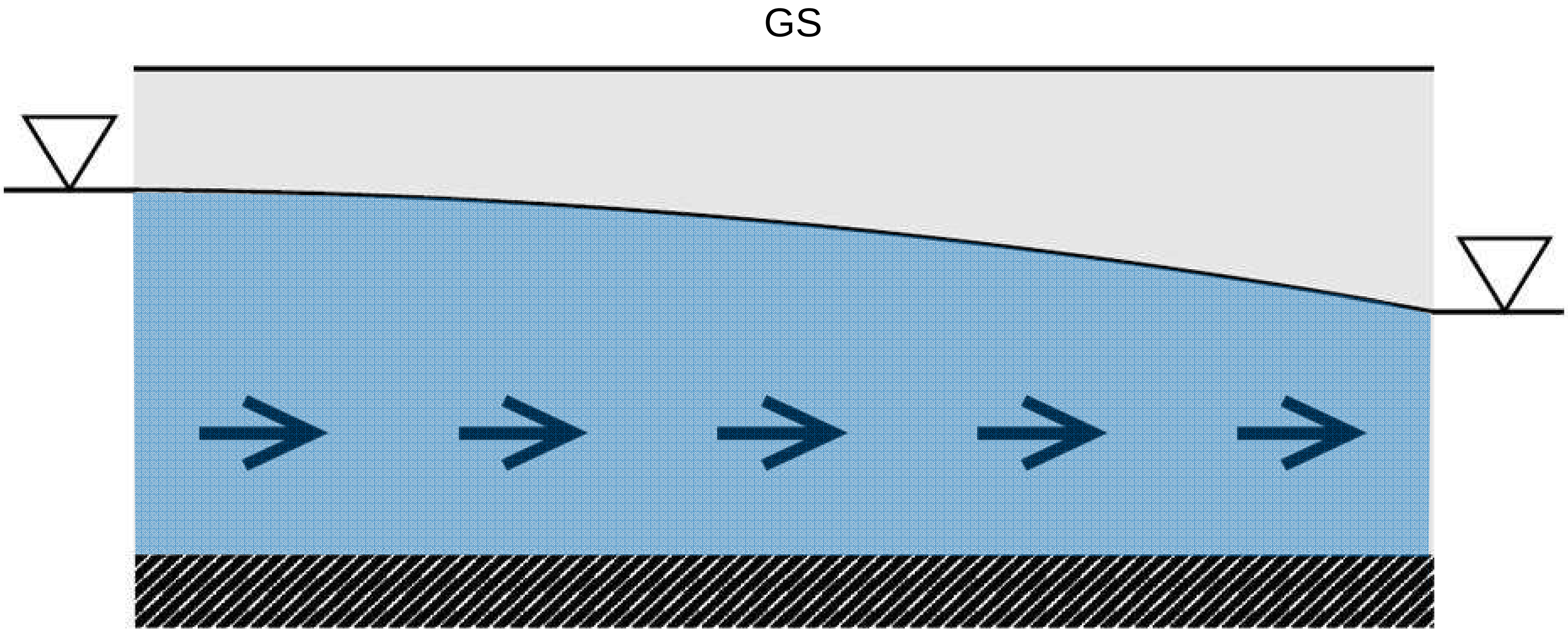
Continuity equation

Reminder aquifer types, confined aquifer:



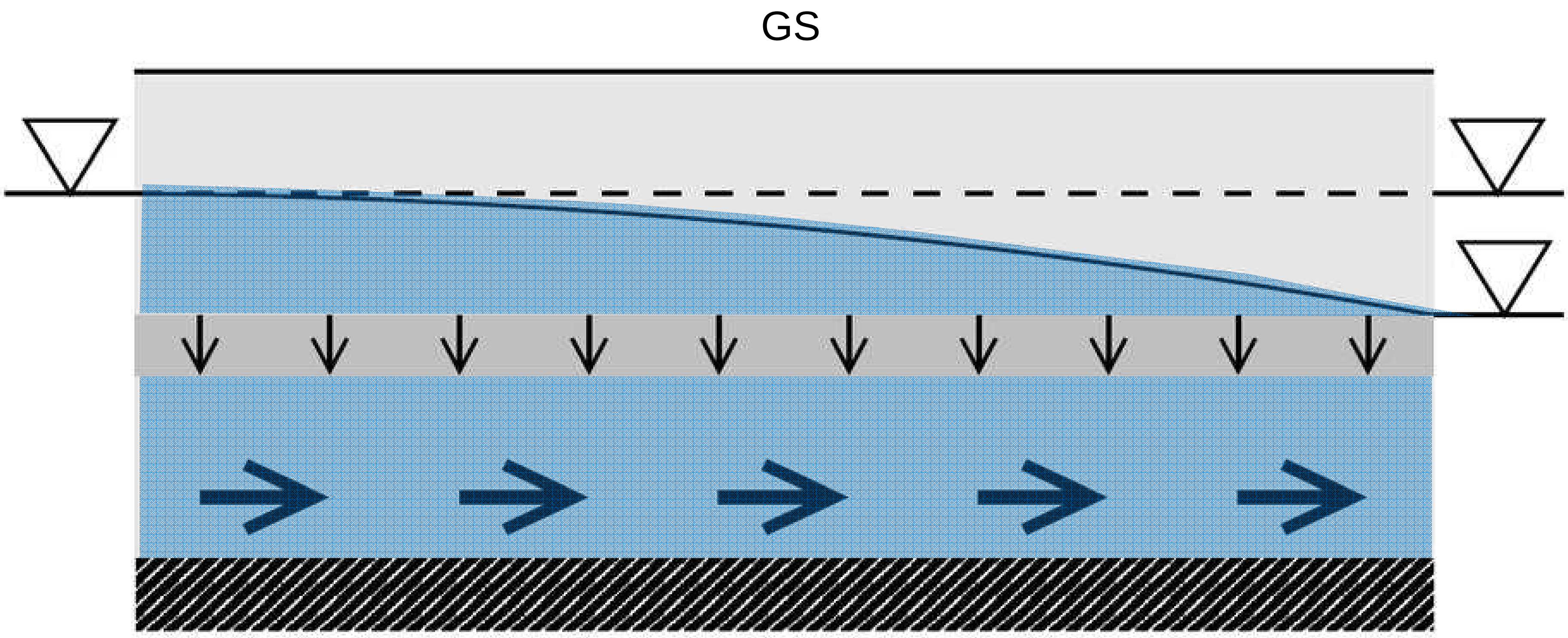
Aquifer types

Reminder aquifer types, unconfined (phreatic) aquifer:



Continuity equation

Reminder aquifer types, leaky (semi-confined) aquifer:



Continuity equation

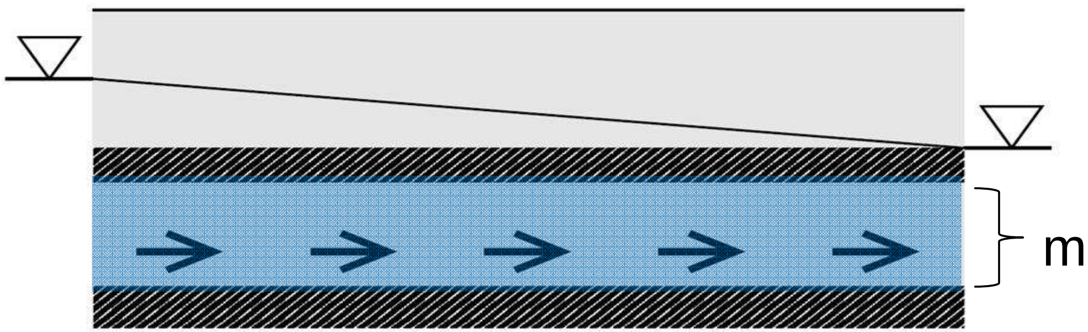
Integration of S0, Kf:

General groundwater flow equation

$$S_0 \frac{\partial h}{\partial t} - \vec{\nabla} \cdot \left(K_f \vec{\nabla} h \right) = W_0$$

becomes:

$$S \frac{\partial h}{\partial t} - \vec{\nabla} \cdot \left(T \vec{\nabla} h \right) = W$$



Storage coefficient S:
depth-integrated S_0

$$S = \int_0^m S_0(x,y)\,dz$$

Transmissivity Tensor **T**:
depth-integrated **K**f

$$\boldsymbol{T} = \int_0^m \boldsymbol{K}(x,y)\,dz$$

Source/sink W:
depth-integrated W_0

$$W = \int_0^m W_0\,dz$$

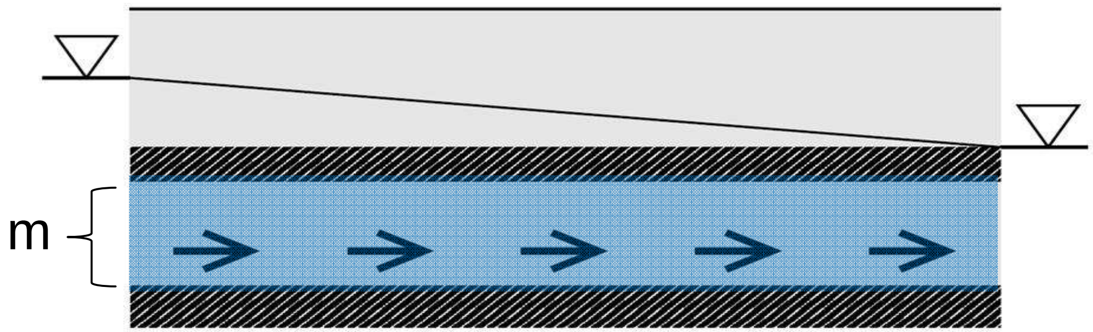
Continuity equation

Integrated groundwater flow equation: Confined aquifers

m constant and independent of h:

$$S = \overline{S_0} \cdot m$$

$$\boldsymbol{T} = \overline{K} \cdot m$$



ie, homogeneous, isotropic aquifer with constant depth m:

$$S\frac{\partial h}{\partial t} - \vec{\nabla} \cdot \left(T\vec{\nabla}h\right) = 0$$

$$S\frac{\partial h}{\partial t} - T\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial x^2}\right) = 0$$