Groundwater Hydraulics

Institute for Fluid Mechanics and Environmental Physics in Civil Engineering, Universität Hannover

Continuity equation

Integrated groundwater flow equation summary:

Confined aquifers:

$$S\frac{\partial h}{\partial t} - \vec{\nabla} \cdot \left(T\vec{\nabla}h\right) = 0$$

Unonfined aquifers:

$$n_f\frac{\partial h}{\partial t} - \frac{1}{2}\vec{\nabla} \cdot \left(\overline{K_f}\vec{\nabla}h^2\right) = N$$

Leaky aquifers:

$$\frac{S}{T}\frac{\partial h}{\partial t} - \frac{\partial^2 h}{\partial x^2} - \frac{\partial^2 h}{\partial y^2} - \frac{h_a - h}{\lambda_a^2} - \frac{h_b - h}{\lambda_b^2} = 0$$

1d solutions of groundwater flow

Goal:

- Estimation of simple scenarios
- Estimation of time scales

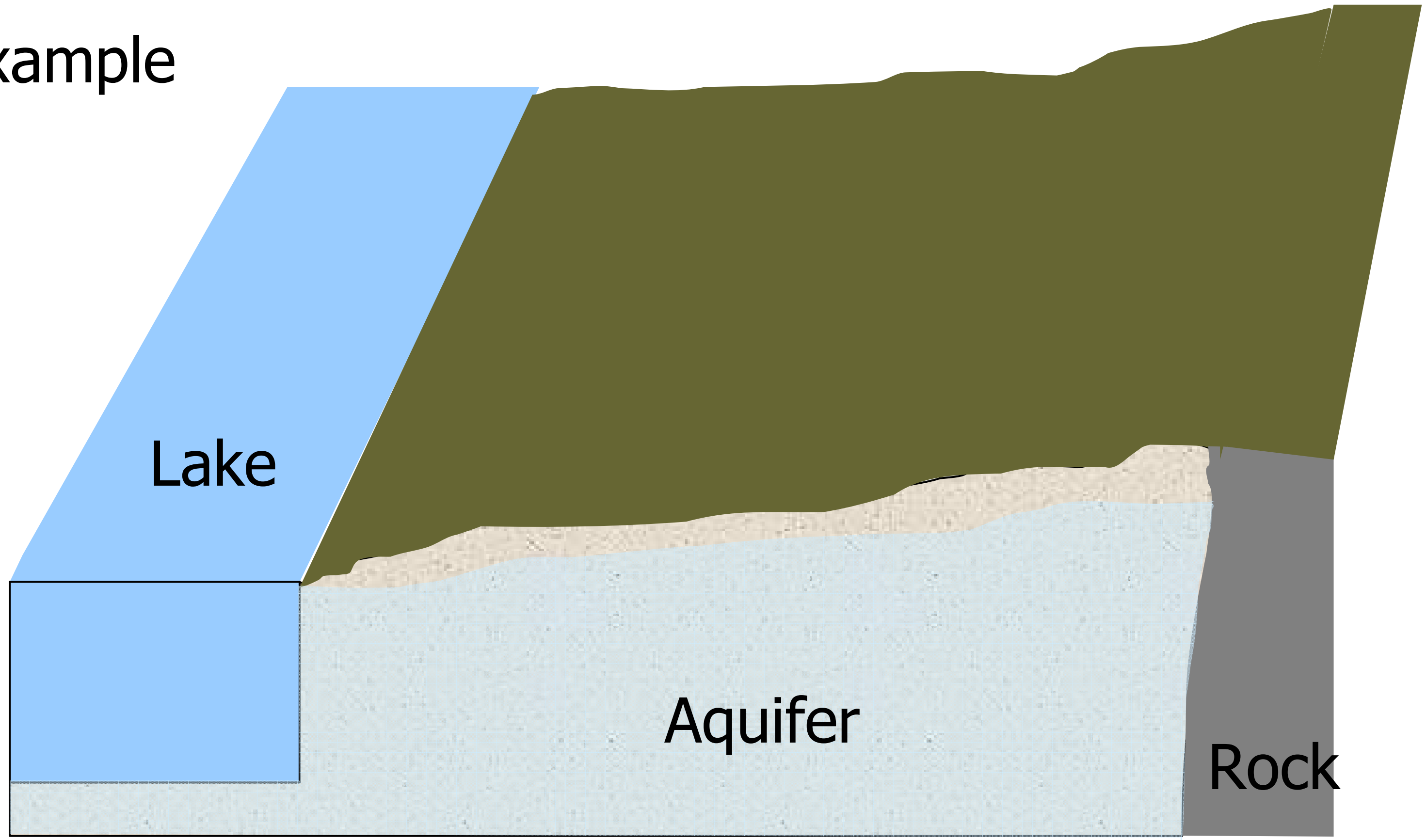
Procedure:

- Simplify as much as possible -> 2d auf 1d
- Classification of the aquifer
- Boundary conditions
- Parameters
- Solution of the problem

1d Solutions of groundwater flow

1d Systems, steady state flow

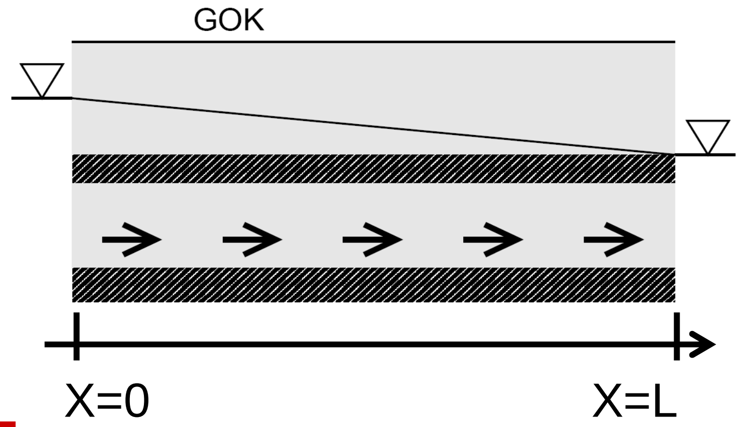
For example



1d Solutions of groundwater flow

Confined aquifer

$$S\frac{\partial h}{\partial t} - \vec{\nabla}_{(x,y)} \cdot \left(T\vec{\nabla}_{(x,y)}h\right) = 0$$



$$T\frac{\partial^2 h}{\partial x^2} = 0,$$

$$q = -\frac{T}{m}\frac{\partial h}{\partial x}$$

Boundary conditions: $h(x=0) = h_0,\ h(x=L) = h_L$

1d Solutions of groundwater flow

Sepcific solution: confined aquifer

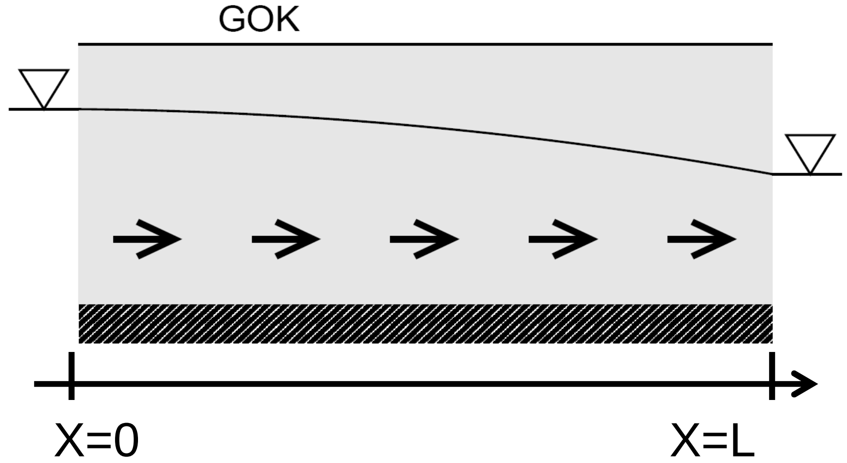
$$h(x) = h_0 + \frac{h_L - h_0}{L} x$$

$$Q(x) = -T \frac{h_L - h_0}{L}$$

1d Solutions of groundwater flow

Phreatic aquifer with recharge

$$n_f \frac{\partial h}{\partial t} - \frac{1}{2} \vec{\nabla}_{(x,y)} \cdot \left(\overline{k_f} \vec{\nabla}_{(x,y)} h^2 \right) = N$$



$$k_f \frac{\partial^2 h^2}{\partial x^2} = -2N,$$

$$q = -k_f \frac{\partial h}{\partial x}$$

Boundary conditions: $h(x = 0) = h_0, \; h(x = L) = h_L$

1d Solutions of groundwater flow

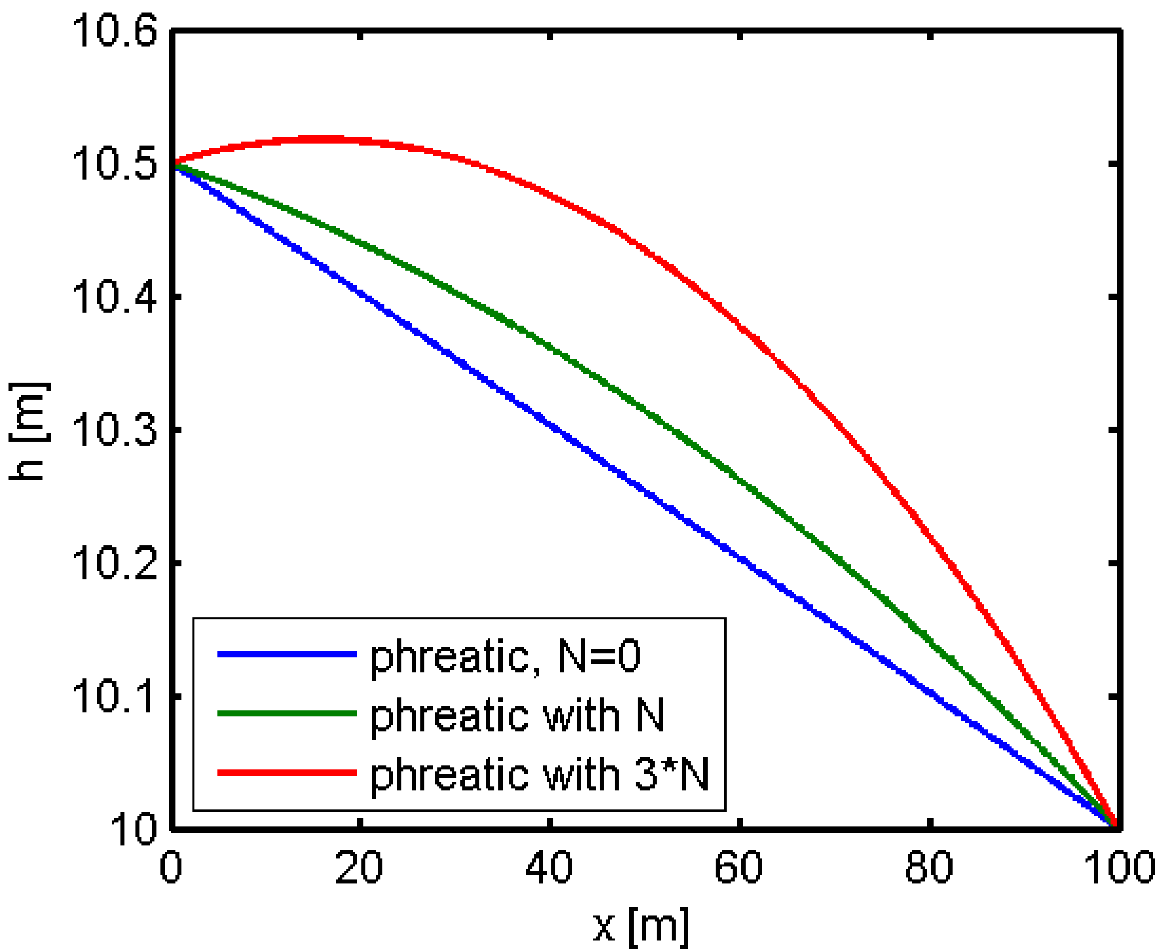
Specific solution: phreatic aquifer with recharge

$$h(x) = \sqrt{h_0^2 + \frac{N}{k_f} x (L - x) + (h_L^2 - h_0^2) \frac{x}{L}}$$

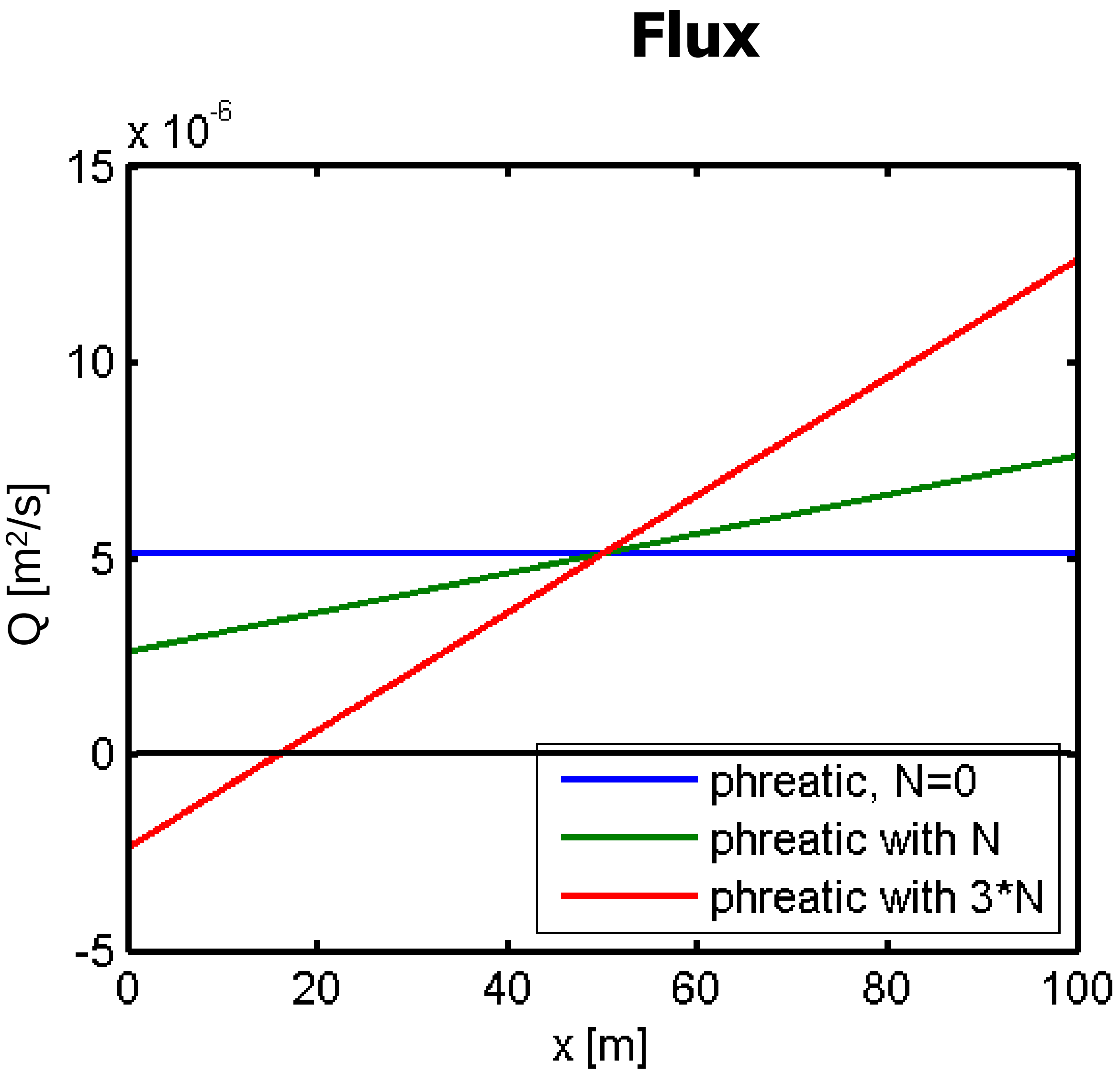
$$Q(x) = \frac{k_f (h_0^2 - h_L^2)}{2L} + N \left(x - \frac{L}{2} \right)$$

1d Solutions of groundwater flow

Piezometric head



1d Solutions of groundwater flow

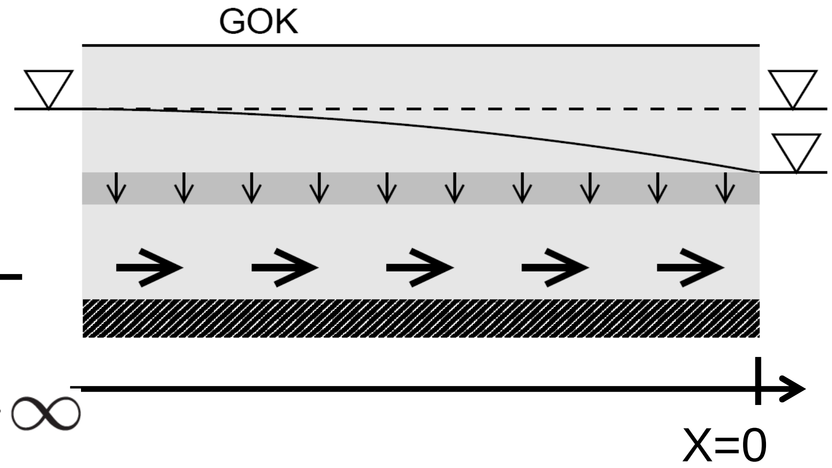


1d Solutions of groundwater flow

Leaky aquifer

$$\frac{S}{T}\frac{\partial h}{\partial t} - \vec{\nabla}^2_{(x,y)} h - \frac{h_1 - h}{\lambda_1^2} - \frac{h_2 - h}{\lambda_2^2} = \frac{W}{T}$$

Boundary at infinity



$$\frac{\partial^2 h}{\partial x^2} - \frac{h - h_{-\infty}}{\lambda^2} = 0, \qquad q = -k_f \frac{\partial h}{\partial x}$$

Boundary conditions: $h(x=0) = h_0,\ h(x \to -\infty) = h_{-\infty}$

1d Solutions of groundwater flow

Leaky aquifer

boundary at – infinity

$$h(x) = h_{-\infty} + (h_0 - h_{-\infty}) \exp\left(\frac{x}{\lambda}\right)$$

$$Q(x) = -T\frac{h_0 - h_{-\infty}}{\lambda} \exp\left(\frac{x}{\lambda}\right)$$

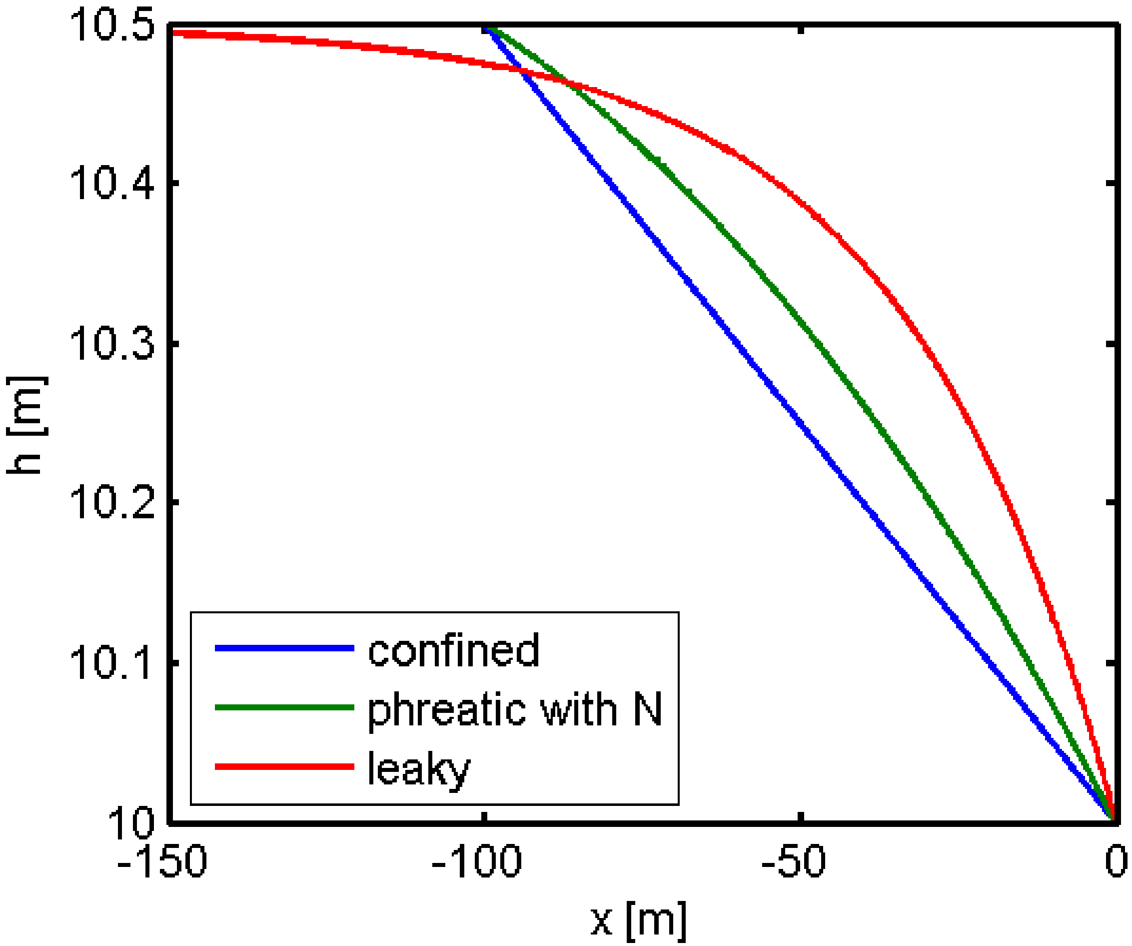
boundary at + infinity

$$h(x) = h_{\infty} + (h_0 - h_{\infty}) \exp\left(\frac{-x}{\lambda}\right)$$

$$Q(x) = -T\frac{h_{\infty} - h_0}{\lambda} \exp\left(\frac{-x}{\lambda}\right)$$

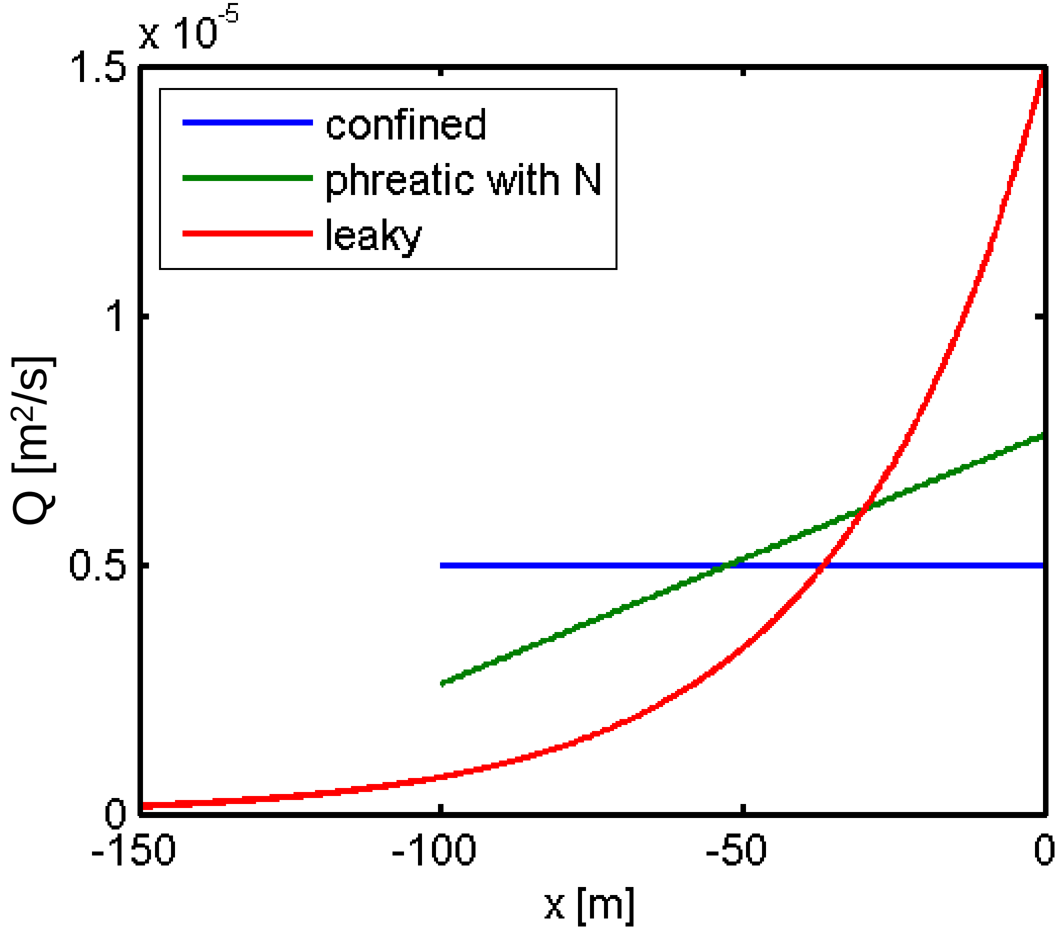
1d Solutions of groundwater flow

Piezometric head



1d Solutions of groundwater flow

Piezometric head



1d Solutions of groundwater flow

Piezometric heads

Confined $h(x) = h_0 + \frac{h_L - h_0}{L} x$

Phreatic $h(x) = \sqrt{h_0^2 + \frac{N}{k_f} x (L - x) + (h_L^2 - h_0^2) \frac{x}{L}}$

Leaky

$$h(x) = h_\infty + (h_0 - h_\infty) \exp\left(\frac{-x}{\lambda}\right)$$

Infinite boundary to the right

$$h(x) = h_{-\infty} + (h_0 - h_{-\infty}) \exp\left(\frac{x}{\lambda}\right)$$

Infinite boundary to the left

1d Solutions of groundwater flow

Discharges (in m²/s)

Confined

$$Q(x) = -T\frac{h_L - h_0}{L}$$

Phreatic

$$Q(x) = \frac{k_f(h_0^2 - h_L^2)}{2L} + N\left(x - \frac{L}{2}\right)$$

Leaky

$$Q(x) = -T\frac{h_\infty - h_0}{\lambda}\exp\left(\frac{-x}{\lambda}\right)$$

Infinite boundary to the right

$$Q(x) = -T\frac{h_0 - h_{-\infty}}{\lambda}\exp\left(\frac{x}{\lambda}\right)$$

Infinite boundary to the left

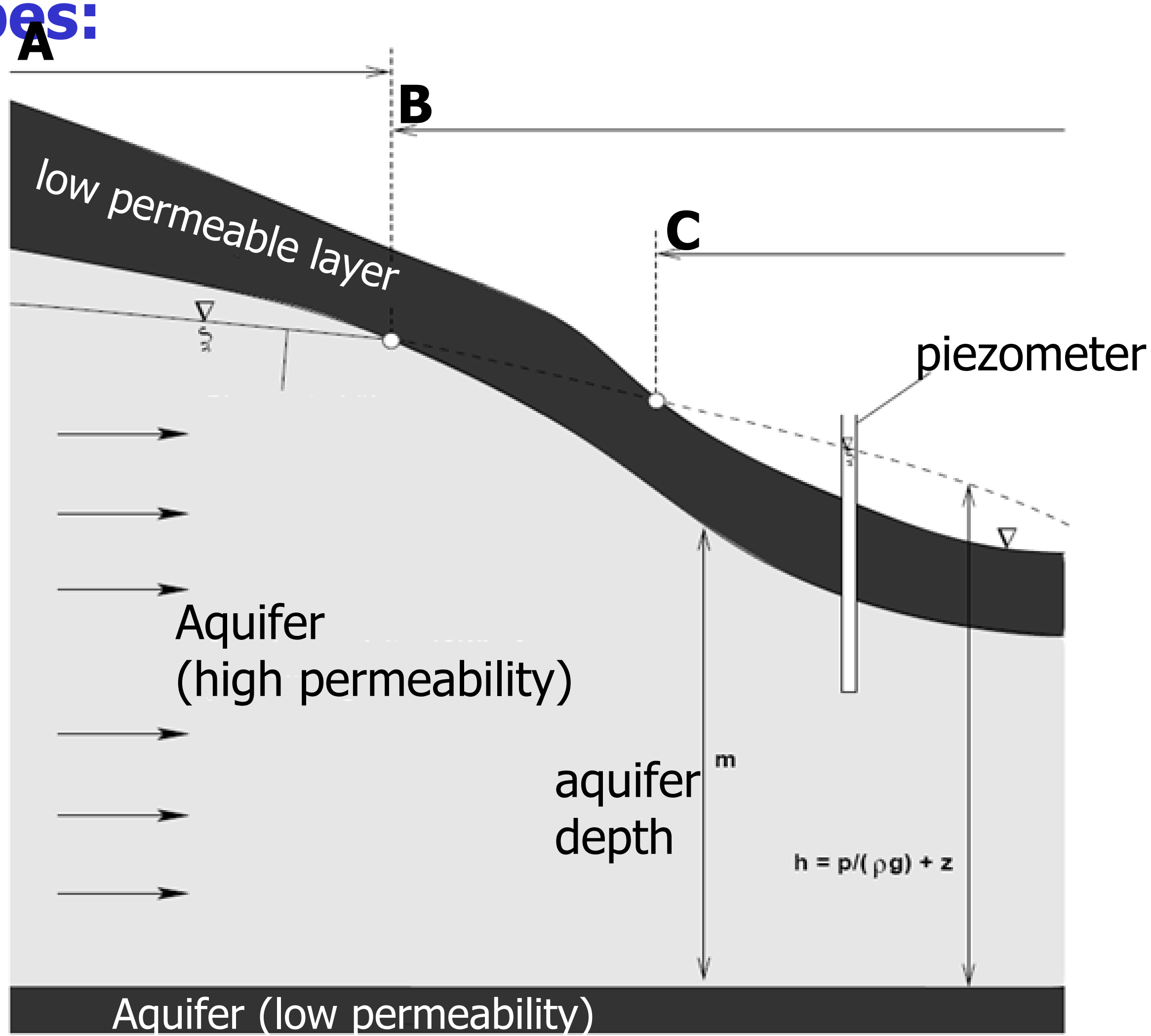
Continuity equation

Examples aquifer types:

A ???

B ???

C ???



Continuity equation

Examples aquifer types:

A ???

B ???

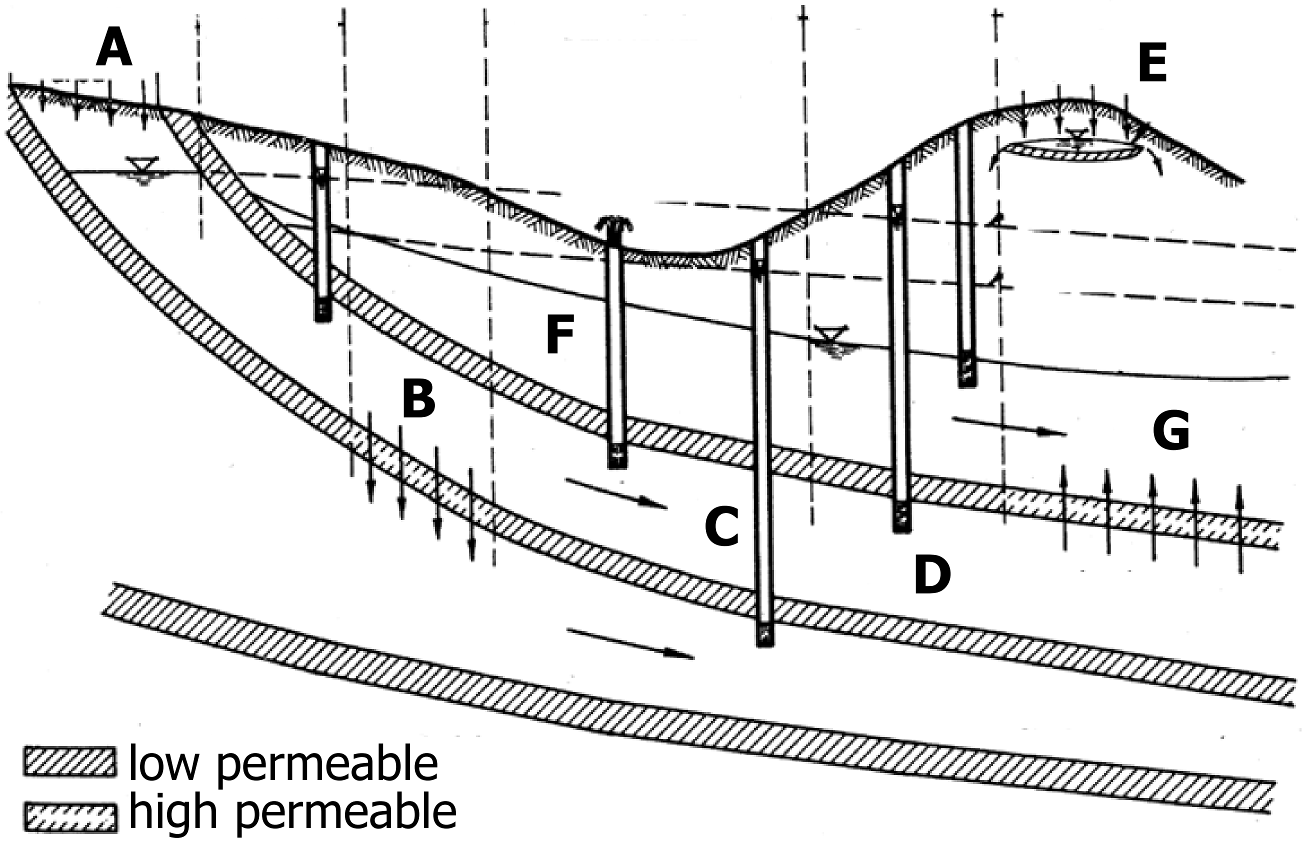
C ???

D ???

E ???

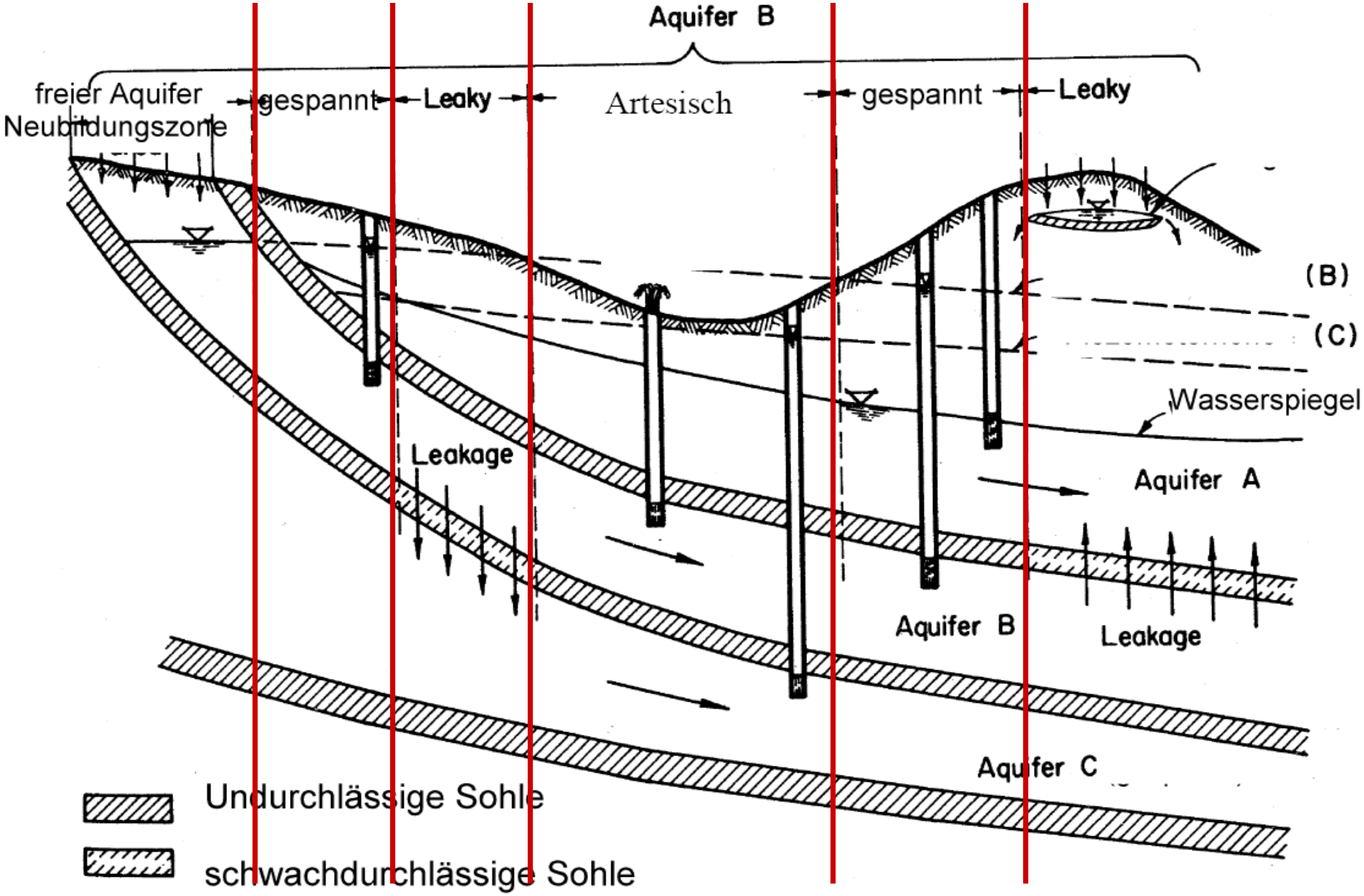
F ???

G ???



1d Solutions of groundwater flow

Method of fragmentation

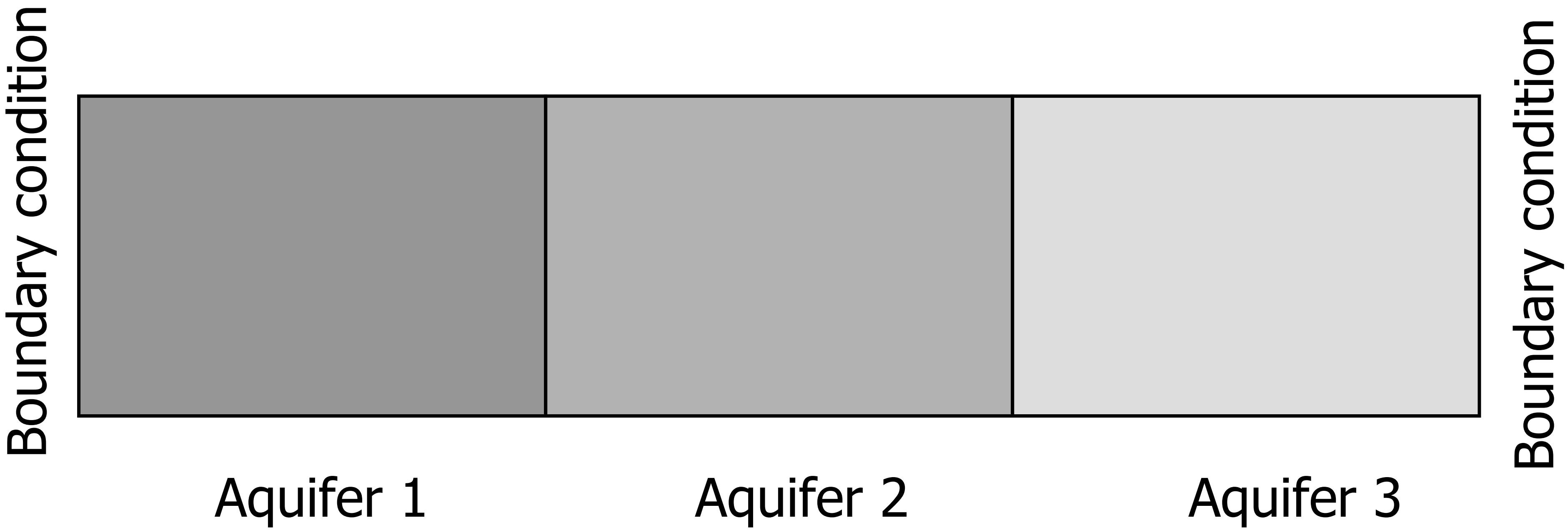


1d Solutions of groundwater flow

Method of fragmentation

• Continuous piezometric head,
• Continuous flux

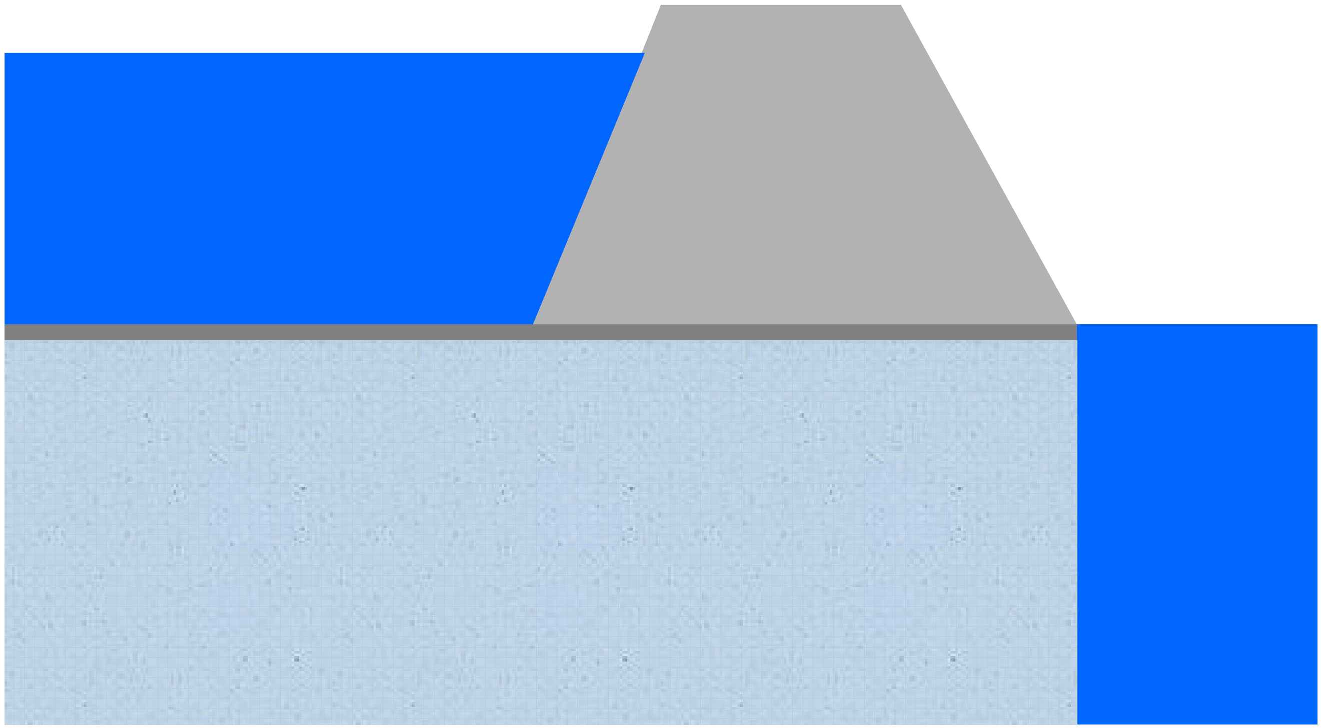
• Continuous piezometric head,
• Continuous flux



1d Solutions of groundwater flow

Method of fragmentation

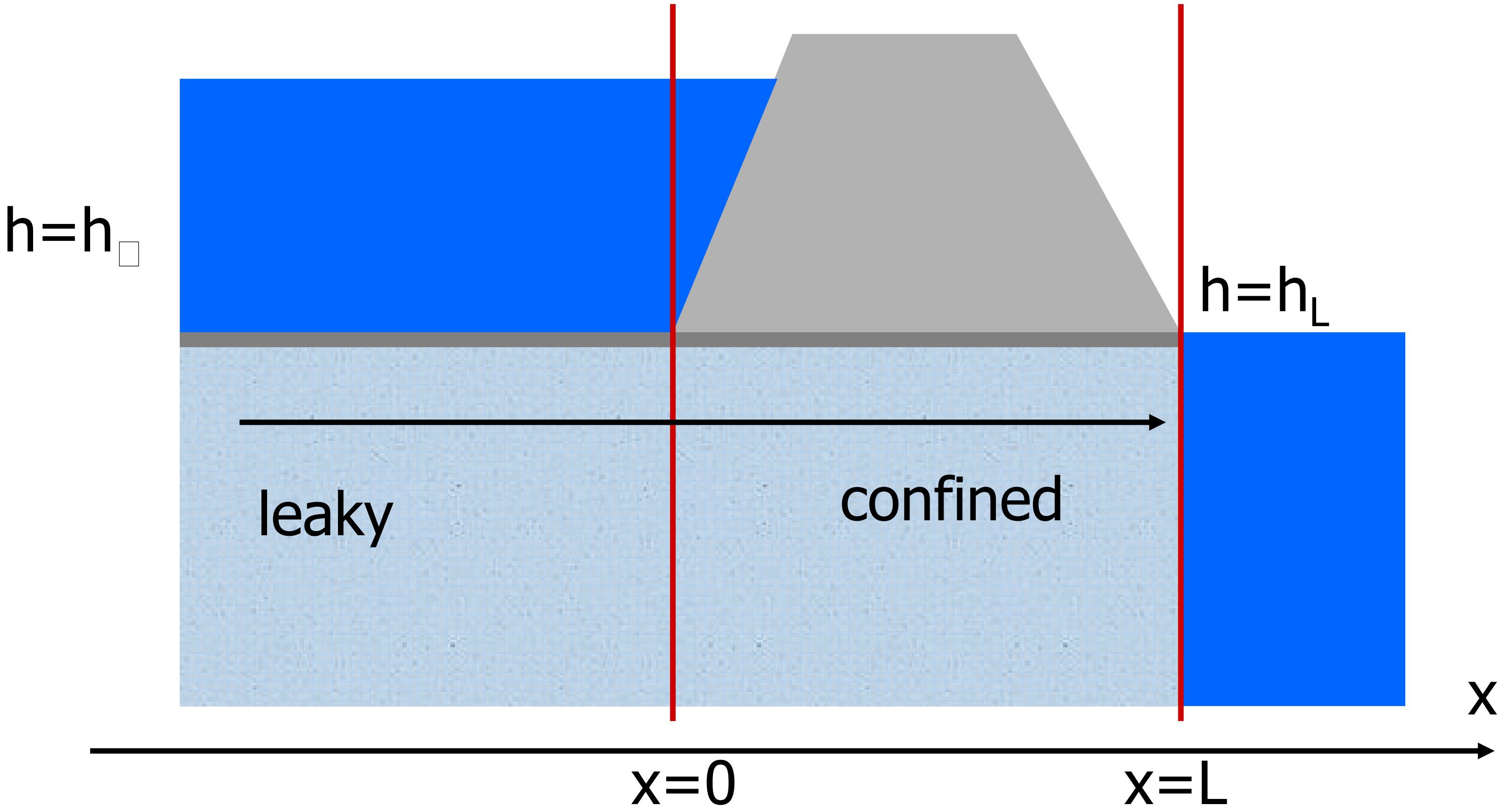
Example: Flow underneath a dam



1d Solutions of groundwater flow

Method of fragmentation

Example: Flow underneath a dam



1d Solutions of groundwater flow

Boundary conditions at the domain boundary

Boundary left: $h(x\text{->-Infinity})=h_{Inf}$

Boundary right: $h(x=L)=h_L$

Head at the interface

Boundary leaky right: $h(x=0) = h_{0l}$

Boundary confined left: $h(x=0) = h_{0r}$

Continuous piezometric head: $h_{0l} = h_{0r} = h_0$

1d Solutions of groundwater flow

1) leaky:

$$h_{\text{leaky}}(x) = h_\infty + (h_0 - h_\infty)\exp\left(\frac{x}{\lambda}\right)$$

$$Q_{\text{leaky}}(x) = -T\frac{h_0 - h_\infty}{\lambda}\exp\left(\frac{x}{\lambda}\right)$$

2) confined:

$$h_{\text{conf}}(x) = h_0 + \frac{h_L - h_0}{L}x$$

$$Q_{\text{conf}}(x) = -T\frac{h_L - h_0}{L}$$

Continuous flux: $Q_{\text{conf}}(x=0) = Q_{\text{leaky}}(x=0)$

1d Solutions of groundwater flow

Continuous flux:

$$-T\frac{h_0 - h_\infty}{\lambda}\exp\left(\frac{0}{\lambda}\right) = -T\frac{h_L - h_0}{L}$$

$$\frac{h_0 - h_\infty}{\lambda} = \frac{h_L - h_0}{L}$$

→ Head at the interface:

$$h_0 = \frac{\frac{h_L}{L} + \frac{h_\infty}{\lambda}}{\frac{1}{L} + \frac{1}{\lambda}} = \frac{h_L\lambda + h_\infty L}{\lambda + L}$$

1d Solutions of groundwater flow