Groundwater Hydraulics

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Darcy's law

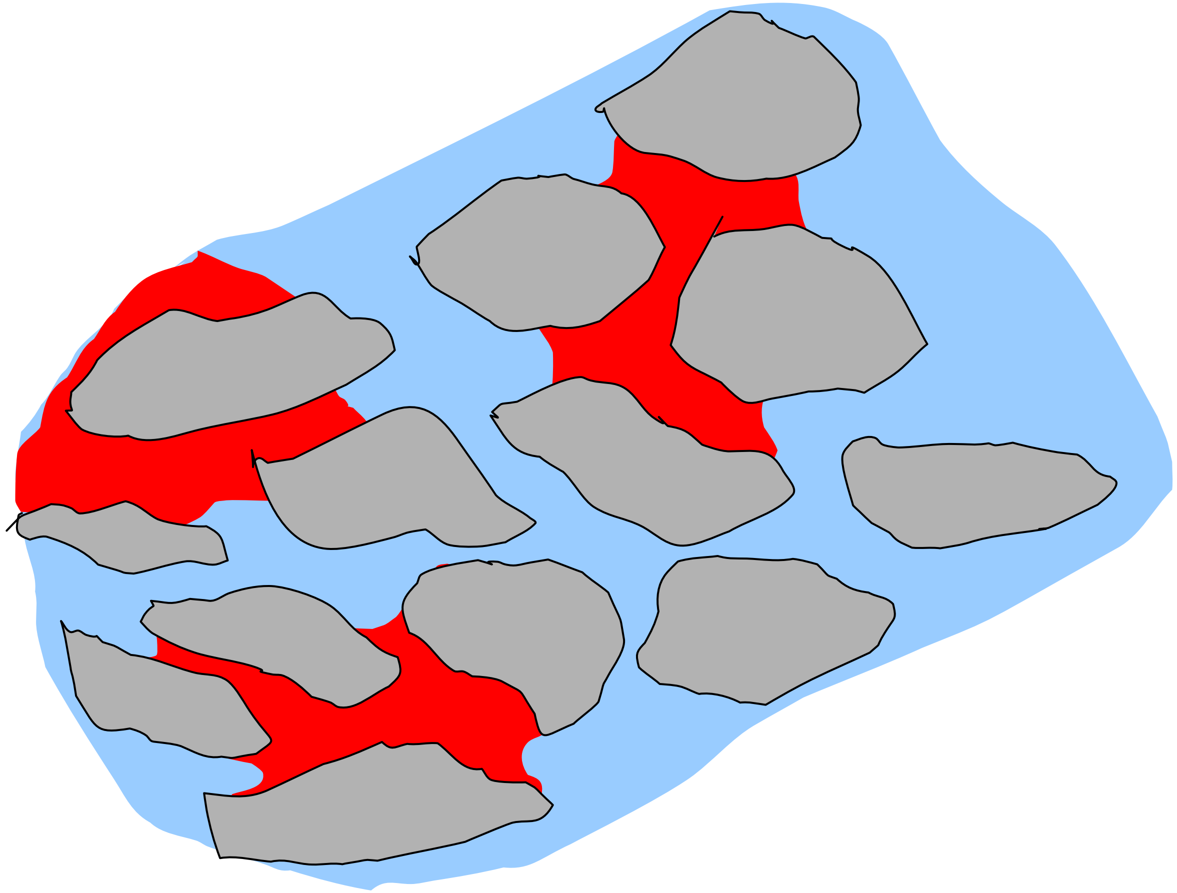
$$\vec{q} = -K_f \vec{\nabla} h$$

K_f: Hydraulic conductivity

$[K_f]$ = m/s

Darcy's law

Flow of two fluids



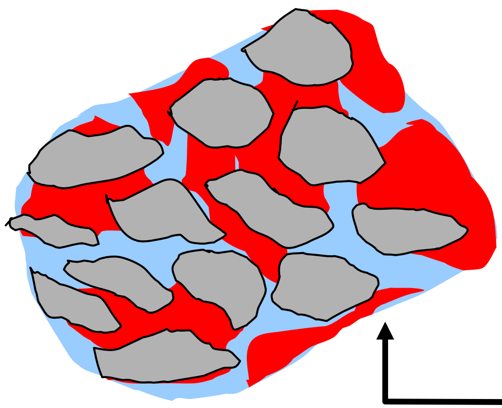
Darcy's law

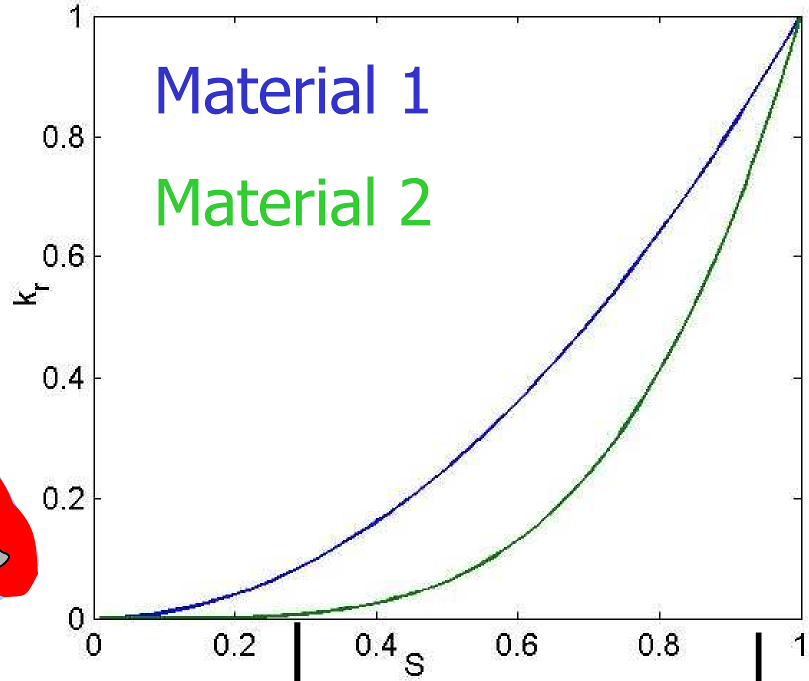
Buckingham Darcy's law for water in the unsaturated zone

Assumption: air is infinitely mobile → at atmospheric pressure

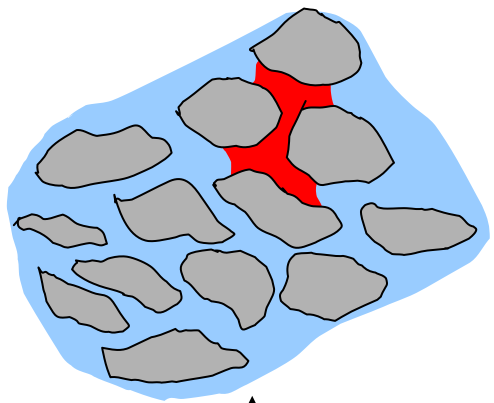
$$\vec{q}_w = -k_r(S_w)k_f\vec{\nabla}\left(\frac{P_w}{\rho g} + z\right)$$

Relative permeability





Relative permeability of the blue fluid



Relative Hydraulic Conductivity

- Presence of a second fluid (gas) acts as obstacle for water flow
- Van Genuchten (1980) parameterization:

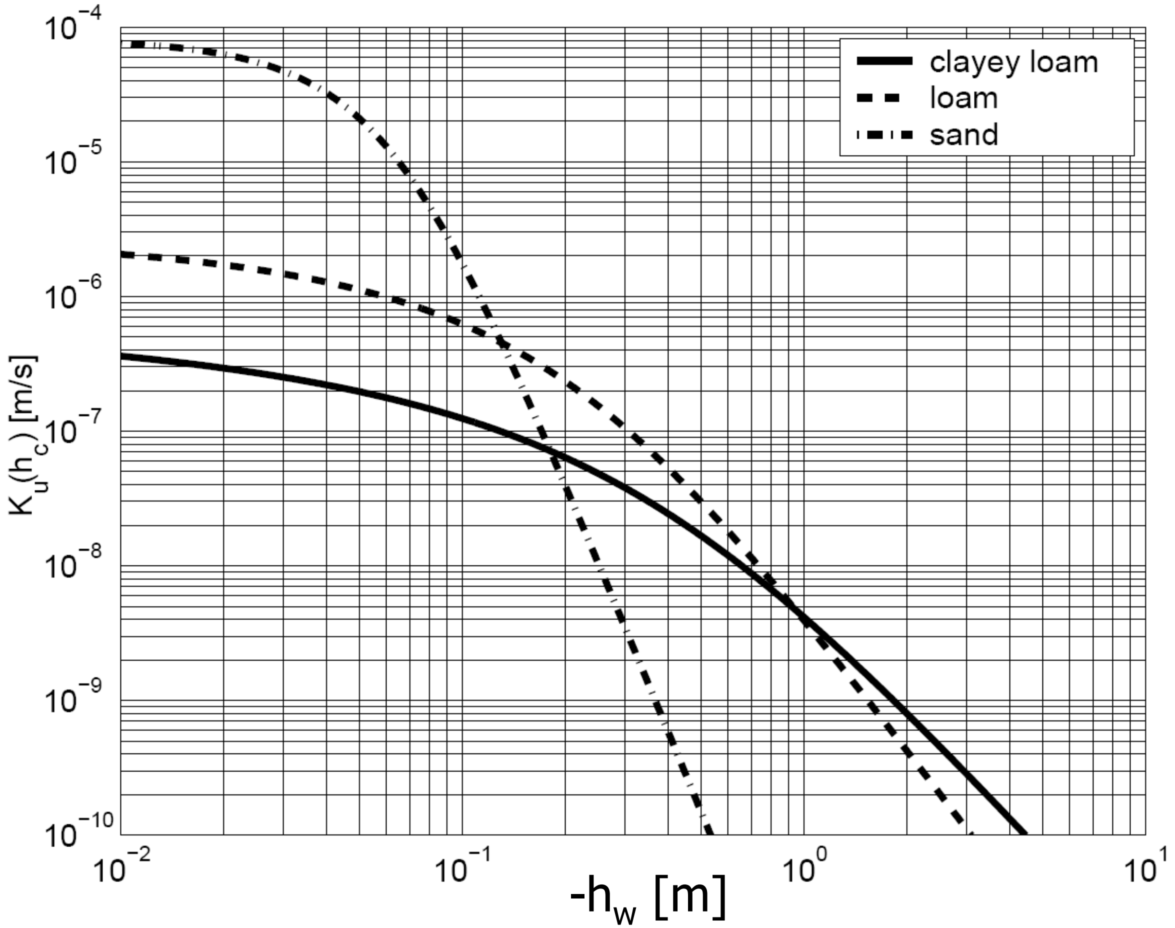
$$k_r(S_e) = \sqrt{S_e}\left(1 - \left(1 - S_e^{\frac{N}{N-1}}\right)^{\frac{N-1}{N}}\right)^2$$

- Other parameterizations exist

Darcy's law

Unsaturated Conductivity $K_u(h_c) = k_r(h_c(S_e)) \times K$

according to Carsel and Parrish (1988)



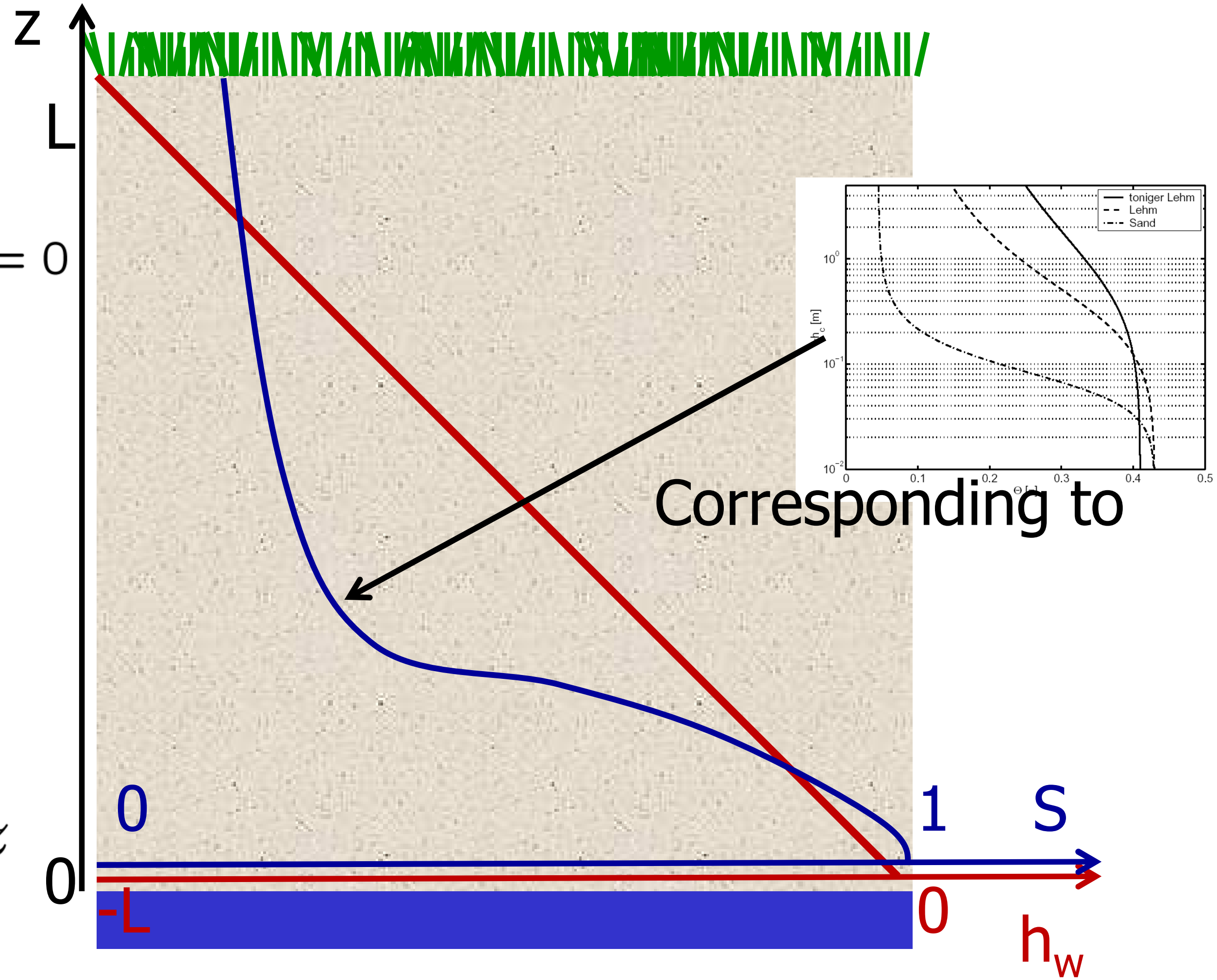
Darcy's law

Example: Vertical water profiles for steady state flow
No flow

$$-K_u(S)\left(\frac{\partial h_w}{\partial z}+1\right)=0$$

<0 >0

$$h_w = -z$$



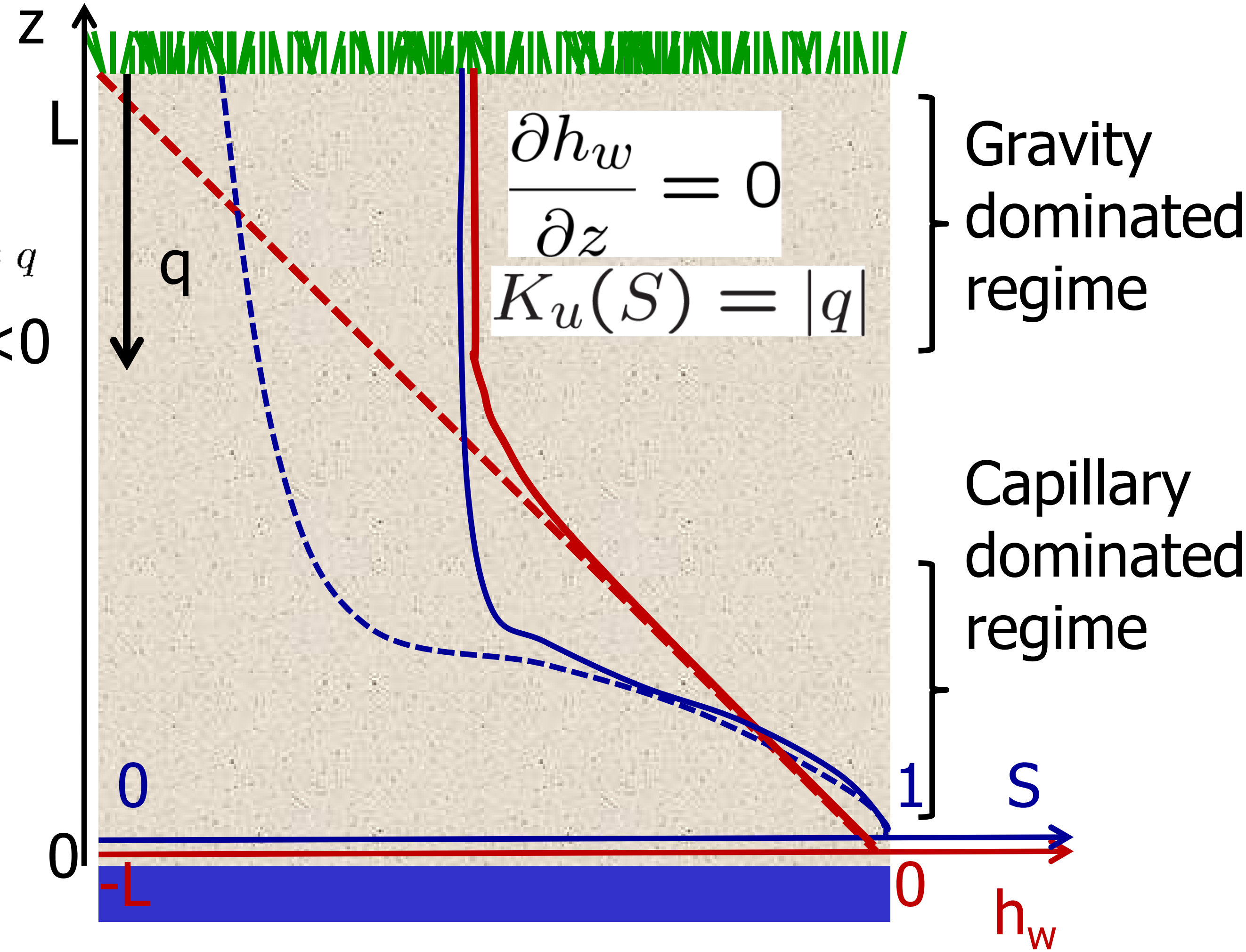
Darcy's law

Example: Vertical water profiles for steady state flow
Infiltration

$$-K_u(S)\left(\frac{\partial h_w}{\partial z}+1\right)=q$$

<0 >0 <0

>0



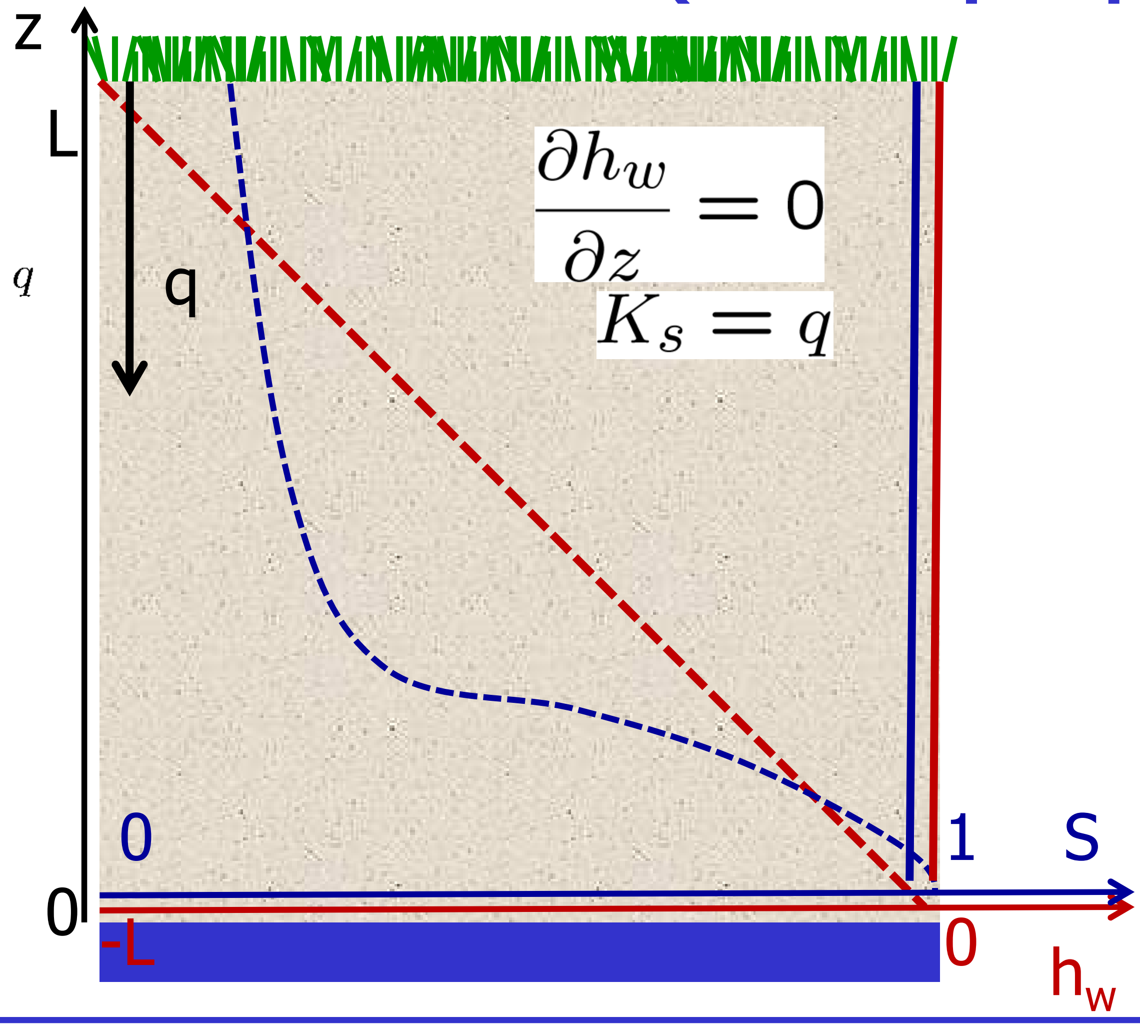
Darcy’s law

Example: Vertical water profiles for steady state flow Infiltration with maximum rate (without pumping)

$$-K_u(S)\left(\frac{\partial h_w}{\partial z}+1\right)=q$$

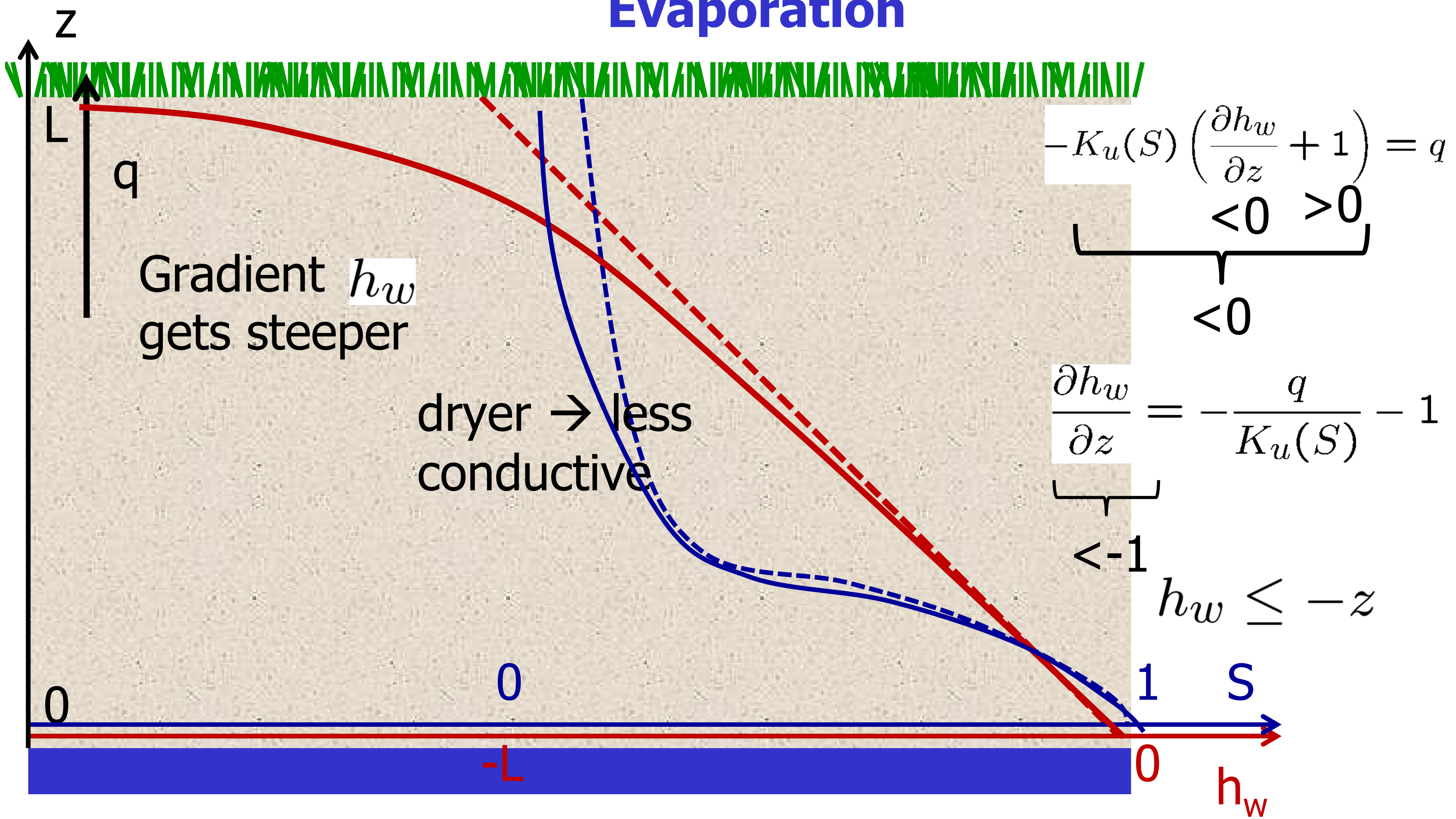
<0 >0

>0



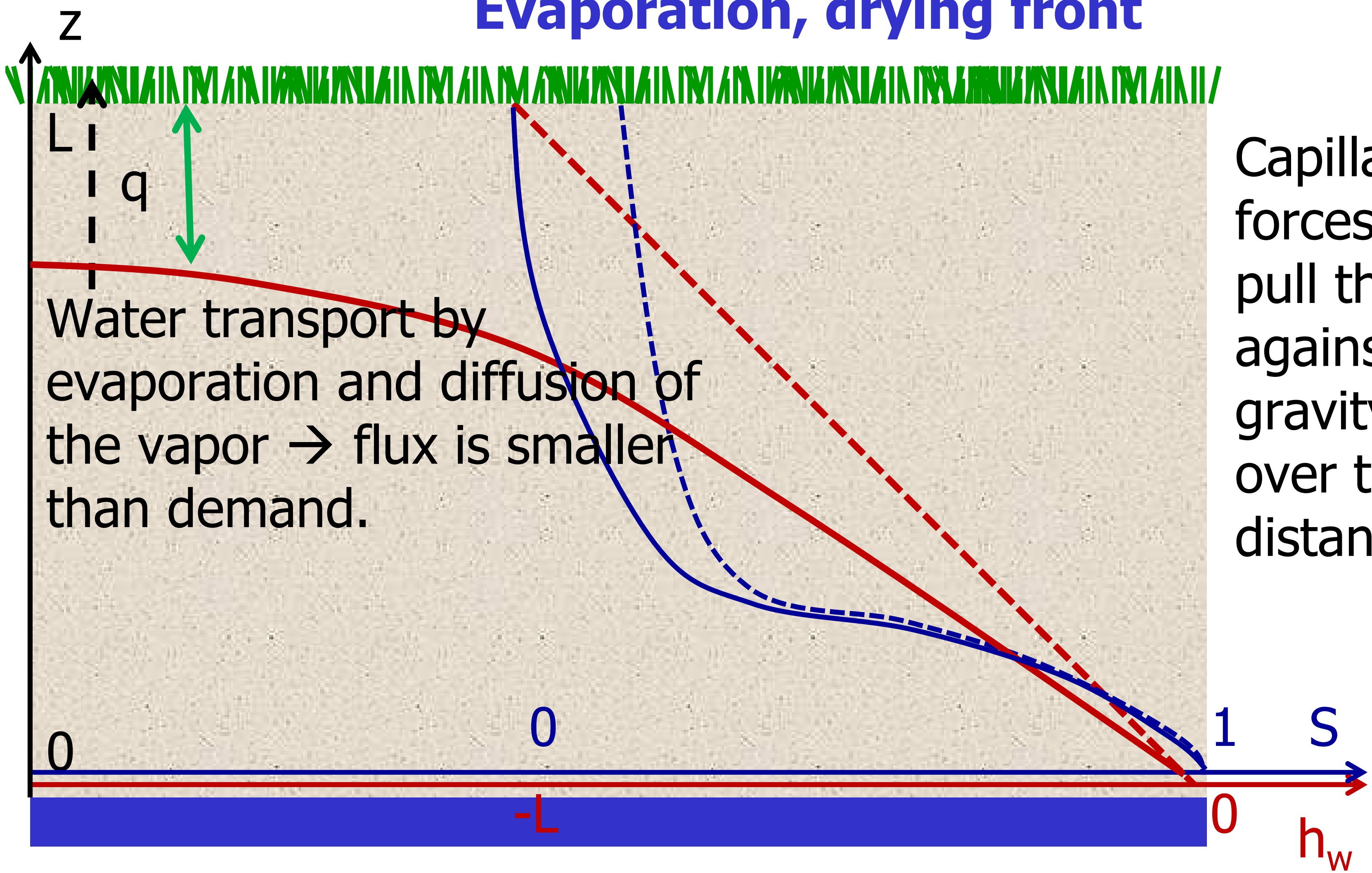
Darcy's law

Example: Vertical water profiles for steady state flow
Evaporation



Darcy's law

Example: Vertical water profiles for steady state flow
Evaporation, drying front



Capillary forces cannot pull the water against gravity forces over the distance L.

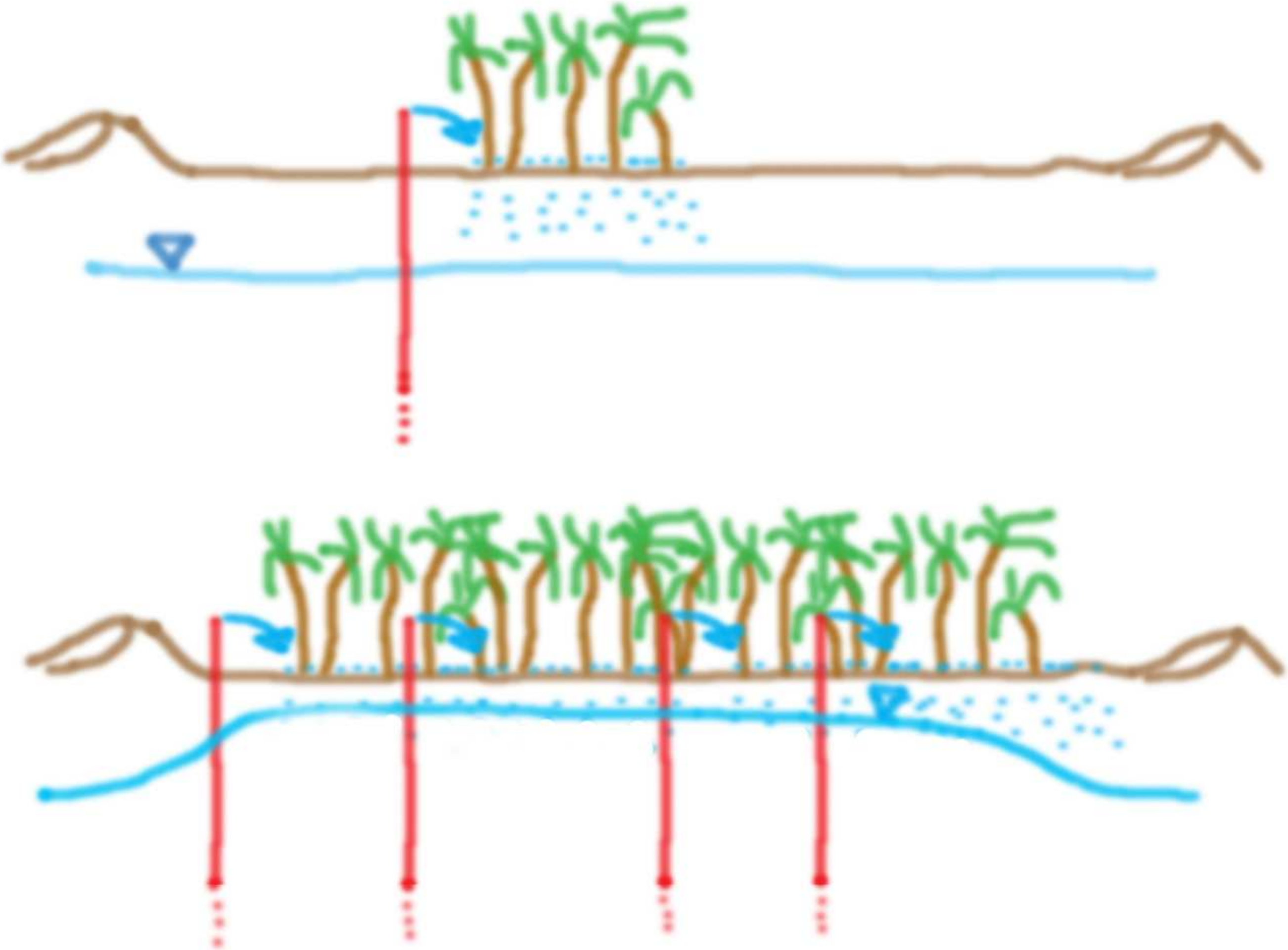
Darcy’s law

Evaporation

Salinization in irrigated areas

Darcy's law

Evaporation - Salinization



Continuity equation

How does the flow change if heads change?
How can head AND flow be determined?

Continuity equation

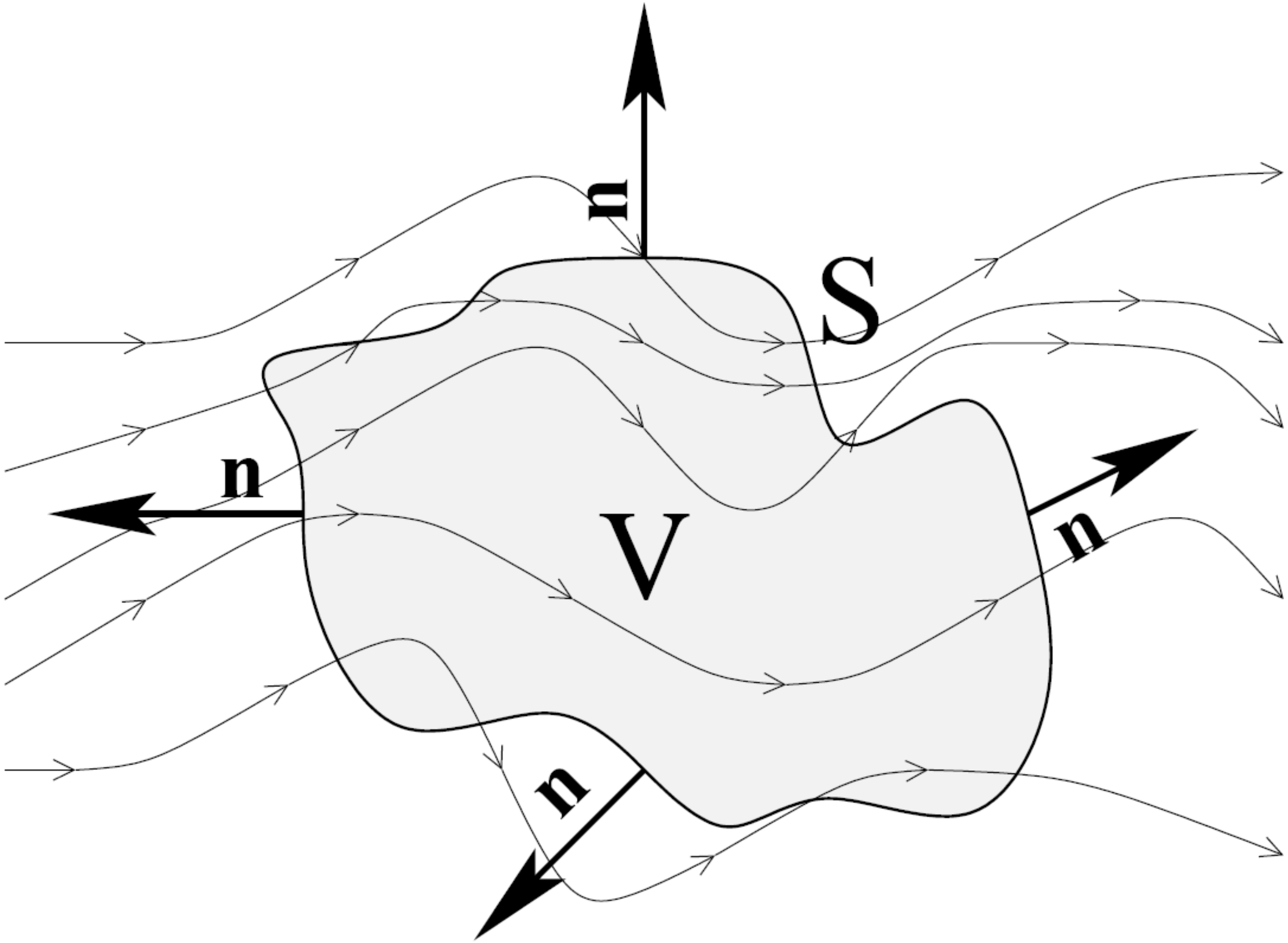
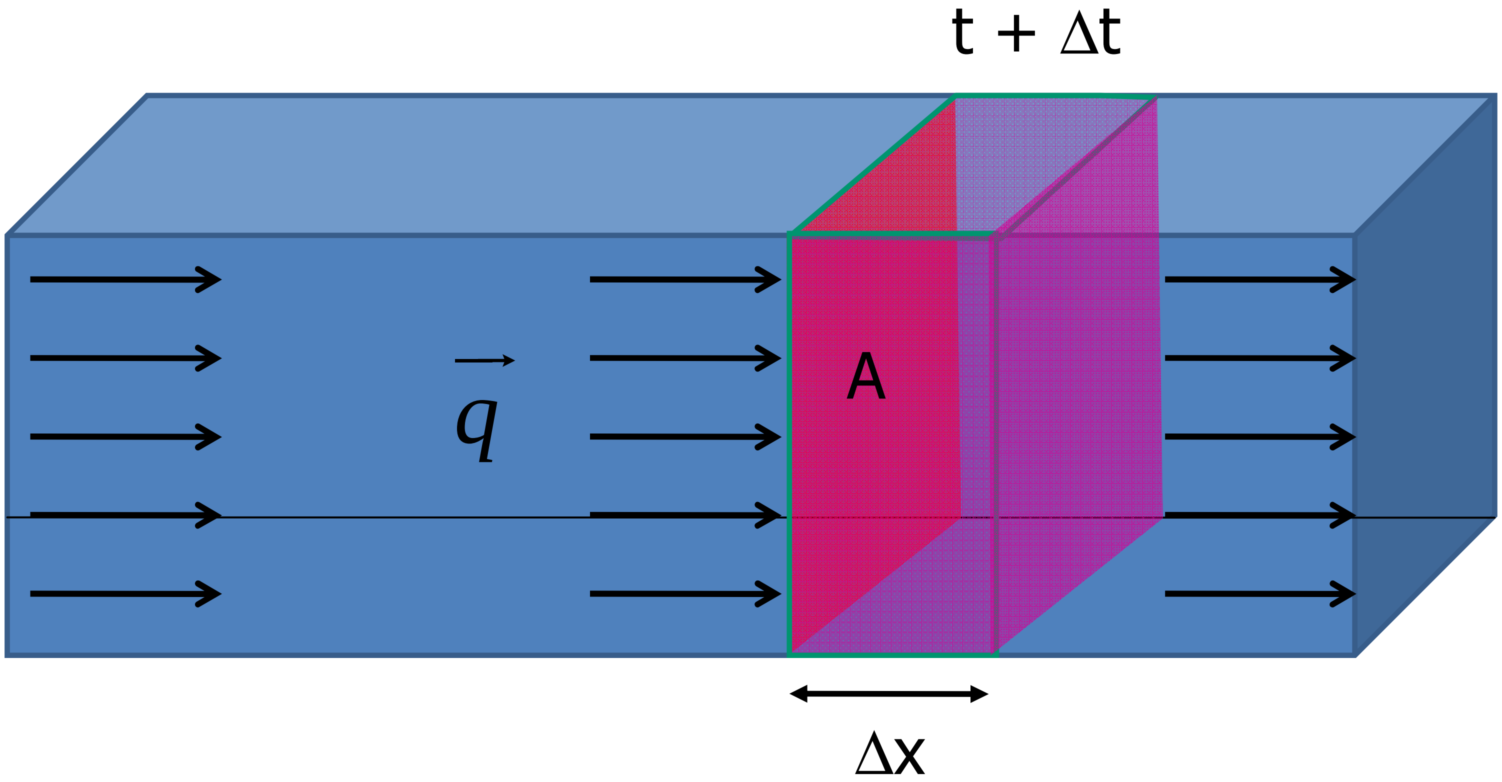


Figure 4.7: Control volume with volume V and surface S.

Continuity equation

Mass flux density: $\dot{m}/A = \rho q$



Continuity equation

Mass balance over a control volume

Mass of water:

$$m = \int_{CV} \rho \varphi_f S dV$$

Mass flux over the boundaries:

$$\dot{m} = -\int_{CA} \rho \vec{q} \cdot \vec{n} dA = -\int_{CV} \vec{\nabla} \cdot (\rho \vec{q}) dV$$

Sources and sinks:

$$\int_{CV} \rho W_0 dV$$

Continuity equation

Mass balance over an infinitesimal small control volume

$$\frac{\partial(\rho n_f)}{\partial t} + \vec{\nabla} \cdot (\rho \vec{q}) = \rho W_0$$