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Construction of mathematical modelling of a population of microalgae

To cite this article: Sh Rakhmanov *et al* 2021 *IOP Conf. Ser.: Earth Environ. Sci.* **939** 012054

Construction of mathematical modelling of a population of microalgae

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Abstract. This article devoted to the development of the mathematic model of the technological process of the chlorella cultivation process, its features and solving of this mathematic model. The Exponential growth of microalgae population under conditions of unlimited nutrient resources and population space proceeds at a rate proportional to the number of species of predominant cells and is described by the differential equation. In the presence of several inhibitors, specific velocity equations with the number of inhibitors can be used, but, as a rule, there are practically no elements acting as inhibitors in the cultivation of Chlorella microalgae. The modeling of this particular class of objects did not take into account the effect of inhibitors on the growth of microalgae. The consumption of nutrients to support the life of microalgae is described by the differential equation. In the course of this work, the processes of cultivation of microalgae were brought together into a system of equations. As a result, a system of differential equations of the technological process of Chlorella cultivation was obtained. Thus, the obtained system of equations describes the process of cultivation of microalgae and its technological process, implemented in a periodic mode.

1. Introduction

A critical analysis of the current state of the theory and practice of modeling and control of microbiological processes based on the application of modern methods, principles and algorithms of the theory of optimal control using computer technology indicates the need to revise a number of issues related to the algorithmization of the complex of problems being solved. This approach should contribute to a scientifically grounded choice of information presentation methods for predicting the course of technological processes, optimizing operating parameters and optimal control of the course of the technological process [1].

IOP Conf. Series: Earth and Environmental Science **939** (2021) 012054 doi:10.1088/1755-1315/939/1/012054

The analysis of the peculiarities of the production of microalgae as an object of modeling and control showed that the mathematical models have not been sufficiently developed to take into account the physicochemical, biotechnological and hydrodynamic laws of the phenomena under study [2-5]. This is due to the lack of knowledge of the specific features of production, the variety of types of cultivators, a significant number of factors affecting the process, the lack of information about enzymatic kinetics, the real hydrodynamic situation in the apparatus and about the laws of heat and mass exchange, as well as about biotechnological laws due to the rate of reproduction of microorganisms.
The exponential growth of the population of microalgae in conditions of unlimited resources of nutrients and space of the population proceeds at a rate proportional to the number of species of prevailing cells and is described by the following differential equation [6, 7, 8]

$$\frac{dx}{dt} = \mu x. \quad (1)$$

where μ is the proportionality coefficient characterizing the growth rate of the population; x is the concentration of microalgae.

2. Methods

The specific growth rate characterizes the physiological properties of microalgae and depends on the concentration of the substrate, the presence of inhibitors and activators in the medium, the quantity and quality of the inoculation of the nutrient medium, illumination, temperature, pH of the medium, etc. [9, 10, 11].
Figure 1 shows the dependences of the change in μ on the concentration of the hydrogen ion, temperature and illumination.

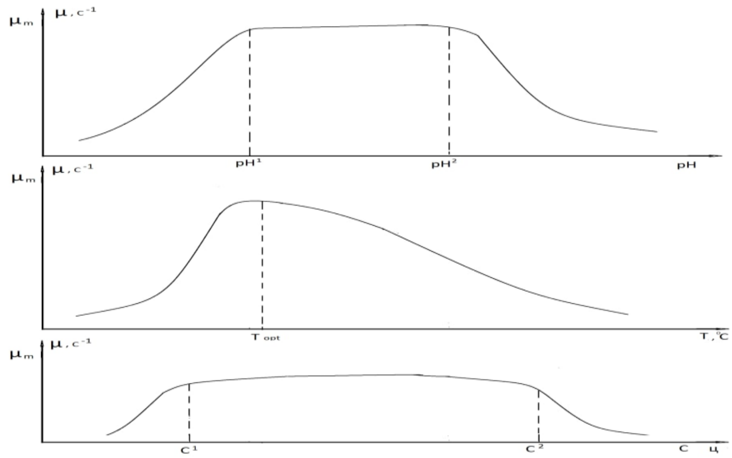

Figure 1 Dependences of the change in μ on the concentration of pH - hydrogen ion, T - temperature and C - illumination

Most attempts to trace the law of population growth from chemical or physiological factors were unsuccessful, although an attempt was made to link the growth with the concentration of resources [12, 13, 14, 15]. The analogy with rectangular hyperbola and the laws of enzymatic kinetics and active mass has led to widespread use in microbiology of the following equation:

$$\mu = \frac{\mu_m S}{K_s + S}, \quad (2)$$

where: μ_m is the maximum specific growth rate that can appear in the absence of inhibitors and activators, as well as with constant physicochemical factors, K_s is the inhibitor concentration constant, S is the concentration of the limiting substrate.

Depending on one inhibitor, equation (2) is calculated by the formula:

$$\mu = \mu_m \frac{S_i}{K_{s_i} + P_i} \quad (3)$$

where: S_i is the concentration of the i-limiting substrate, K_{s_i}- is the concentration constant of the *i*-inhibitor, p_i is the concentration of i-nutrients.

Here, instead of the numerical value of si, the concentration of nutrients N, P, Mg, K, CO_2, O_2 and others can be used, as a substrate - an element necessary for the vital activity of microalgae [16, 17, 18].

It is accepted that some of the many nutrients can act as activators or inhibitors. those. μ depends on the following factors

$$\mu = f(N, P, Mg, K, CO_2, O_2, T, pH, I, C, ...) \quad (4)$$

Then equation (2) has the following form: $\mu = f(\mu_m, z)$,

$$z = z^{(1)}, z^{(2)} \dots z^{(n)}$$

z is the number of considered groups of parameters.

The algorithms described for the selection of the most essential elements of the set Z are used. As a result, three minimum necessary groups of parameters are obtained. The first group of parameters ($Z^{(1)}$) includes activators, the second ($Z^{(2)}$) inhibitors, and the third ($Z^{(3)}$) physicochemical variables.

So, the set Z has the following more suitable form for modeling purposes:

$$Z^{(1)} = [M, P, CO^2, K]$$

$$Z^{(2)} = [O^2, I]$$

$$Z^{(3)} = [pH, T, C]$$

In the presence of an inhibitor in the medium, the specific growth rate decreases by the value $\frac{M_i}{K_I + I}$, which is obtained from the equation of the enzymatic reaction in the presence of an inhibitor:

$$\mu_1 = \mu_0 - \frac{\mu_0 I_1}{K_{I_1} + I_1} = \frac{\mu_0 K_{I_1}}{K_{I_1} + I_1} \quad (5)$$

where: K1 is a constant numerically equal to the inhibitor concentration at which the specific growth rate reaches half of its maximum possible value

$$\mu = \frac{\mu_m}{2} \quad (6)$$

When calculating the specific growth rate taking into account L (the number of inhibitors), equation (5) takes the following form:

$$\mu_1 = \mu_0 \frac{K_{I_1} * K_{I_2} \dots K_{I_n}}{(K_{I_1}+I_1)(K_{I_2}+I_2)\dots(K_{I_n}+I_n)} \quad (7)$$

After transformations, we have:

$$\mu_1 = \mu_0 \prod_{i_1}^{n} \frac{K_{I_1}}{K_{I_i}+I_i}, \quad \text{where, i = 1,2… n} \quad (8)$$

In the presence of several inhibitors, equation (6) can be used, but, as a rule, during the cultivation of microalgae chlorella, there are practically no elements acting as inhibitors. In this regard, in the future, when modeling the specific class of objects under consideration, the effect of inhibitors on the growth of microalgae is not taken into account [19, 20, 21]. The consumption of nutrients for maintaining the vital activity of microalgae is described by the equation

$$\frac{dS_m^i}{dt} \quad (9)$$

where: S^i_m - the amount of the i-th nutrient consumed to maintain the vital activity of microalgae cells; m_i is the rate of nutrient consumption to maintain the vital activity of a unit of microalgae; X is the number of microalgae cells.

The total amount of the i-th nutrient consumed for the growth and maintenance of the vital activity of microalgae can be determined by the equation:

$$\frac{dS_m^i}{dt} = (d_i\mu + m_i X), \quad (10)$$

where: d_i- coefficient taking into account the consumption of the i-th nutrient for the growth of a unit of microalgae.

In accordance with (4), for each parameter, you can write

$$\mu_{z_1} = F_i(Z_i)$$

For an analytical description of the degree of influence of the parameters of the medium on the specific growth rate of microorganisms, some simplifications are adopted [22]. Let the specific growth rate of microalgae μ in the studied intervals$\{Z_i \in Z^i\}$ range from *0* to *i*.

We take the following notation:

$$min\{\mu(Z_i)\} = \mu Z_i^{(0)}$$
$$max\{\mu(Z_i)\} = \mu Z_i^{(m)}$$

3 Results

Research results Based on the experimental data, the dependence of the specific growth rate of microalgae μ on the elements of the set can be represented in the following form

$$r_{z_i} = \exp\left(\frac{(z_i-\varphi_i)^2}{2\delta_i}\right) \quad (11)$$

where φ_i- is the optimal value of the i-th root-mean-square deviation of the distribution of the physicochemical factor; δ_i - the difference between the limiting value of the parameter and its optimal value.
The formula for calculating the specific growth rate of microalgae can be written as follows:

$$\mu = \mu_m \frac{Z_i^{(1)}}{K_{Z_i}+Z_i^{(1)}} \prod_{i=1}^{N} r_{z_i}. \quad (12)$$

All the above equations (8, 9, 10, 11, 12) are the basic kinetic equations under the conditions of the periodic regime of the process of growing microalgae.
The unified system of equations describing the process of cultivation of microalgae is as follows:

$$\begin{cases} \frac{dx}{dt} = \mu x; \\ \frac{dS_{P_i}^i}{dt} = d_\varepsilon \mu x; \\ \frac{dS_{m_i}^i}{dt} = m_\varepsilon x; \\ \mu = \mu_m \frac{S_i}{K_{S_i}} exp\left[\frac{(pH-\varphi_1)^2}{\delta_1{}^2} - \frac{(T-\varphi_2)^2}{\delta_2{}^2} - \frac{(C-\varphi_3)^2}{\delta_3{}^2}\right] \\ \frac{ds}{dt} = (\alpha_1 \mu + m_i)x. \end{cases} \quad (13)$$

Thus, the resulting system of equations (13) describes the process of cultivation of microalgae, which is carried out in a periodic mode. When the set value x is reached, the process is transferred to a continuous mode in order to increase the productivity of cultivators and the stable use of sown crops and nutrients.

4. Conclusions

Based on the theoretical and experimental studies carried out within the framework of this work, the following main results were obtained:
When modeling the process of cultivation of microalgae, it is advisable to build an analytical model based on physicochemical, biological, hydrodynamic; mass transfer patterns, which is designed to contribute to a more accurate prediction of the growth and reproduction of microalgae and the solution of problems associated with the design of new cultivators, optimization of existing processes and their automatic control.
The results of computer simulation made it possible to quantify the consumption of nutrients required for the growth and reproduction of microalgae, to establish the required rate of culture selection from the cultivator, to determine the growth rate of microorganisms with the required accuracy, taking into account the factors that directly affect the cultivation process.
In the process of carrying out this work, the equations of the process of cultivation of microalgae were brought together into a system. As a result, a system of

IOP Conf. Series: Earth and Environmental Science **939** (2021) 012054 doi:10.1088/1755-1315/939/1/012054

differential equations for the technological process of chlorella cultivation was obtained, which describes the process of cultivation of microalgae and its technological process, which is implemented in a periodic mode.

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