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Criteria for the existence of established modes of power systems

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Criteria for the existence of established modes of power systems

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Abstract. This article considers the criteria for the existence of established modes of power systems. Nonlinear nodal equations of steady-state modes are presented, which have many solutions or do not have any physically realizable solutions. Criteria for the existence of solutions are given based on derivatives of power losses depending on the parameters of the power system modes. Equivalent circuit of the electrical system was performed using the South-Western MEN (MAIN ELECTRIC NETWORKS). The paper highlights the results of calculations of steady-state modes of the electrical system at weighted values of node 12. The maximum normal modes of electrical systems for the power of nodes was determined by the criteria are set $\frac{\partial \Delta P_c}{\partial \Pi_i} = \infty$.

1. Introduction

A system of equations of a steady-state electrical system due to non-linearity with respect to the desired variables can formally have a set of solutions or not have any physically realizable solutions. Since, in general, the solution of nonlinear nodal equations (NE) for a complex electrical system can only be obtained iteratively [1], it is necessary to solve the problem of the relationship between the convergence of the process and the existence of the solution. Indeed, if the iterative process does not converge when calculating a certain mode, the solution can sometimes be obtained by improving the initial approximation, correcting the course of the process (using accelerating coefficients, introducing an additional parameter, etc.), or using another method. There is also a problem of stability of the obtained solution [2-3]. Thus, it is necessary to solve the problem of the relationship between the properties of nonlinear equations and real modes, i.e., the adequacy of the properties of the steady-state mode of the real electric power system and its accepted mathematical model [4-6].
Determining the steady-state mode, the limit for any of its parameters (power or voltage modulus of individual nodes, power flow, etc.) is a common task in practice [7]. In general, this problem can be

solved through a series of calculations of steady-state modes with their "weights". In this case, the options and trajectories of weighting the mode from the source to the limit are determined by the nature of the problem being solved.

The limit on convergence of the iterative process under weighting corresponds to the limit mode only under the assumption of convergence of the applied algorithm to physically implemented solutions in a simply connected region containing the source and limit modes [8, 9].

Since the loss of convergence of the iterative process cannot serve as a criterion for the impracticability of the assumed mode without additional conditions, it is necessary to have another, physically justified criterion. The most suitable system function for the formulation of the mode limit criterion is the total power loss function [10-12], which depends on all parameters of the Electric power system mode.

2. Method

The research was carried out on the example of an electrical system, the scheme of which is shown in Figure 1. Weighting was carried out by the active power of the node 12.

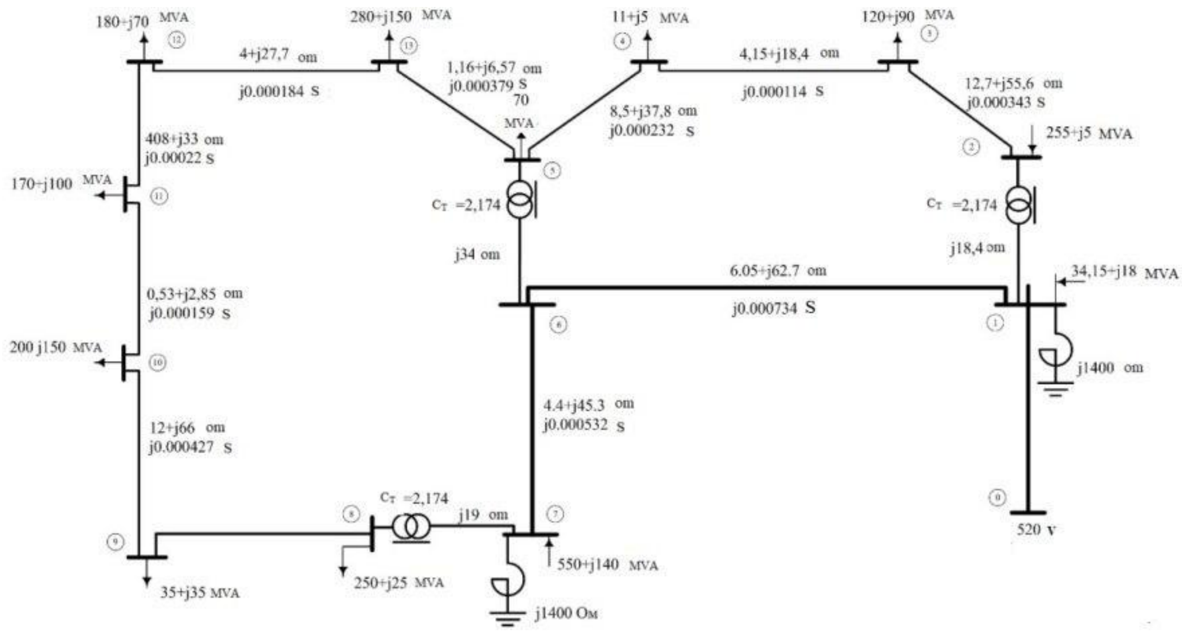

Figure 1. Equivalent diagram of the electrical system of the South-Western MEN (MAIN ELECTRIC NETWORKS)

Based on computational and experimental studies, the following position is established: the steady state mode is the limit for the deviation of any independent parameter $\Pi_i \in P_i, Q_i$, if small changes in this parameter cause infinitely large changes in the total loss of active power in the system.

$$\frac{\partial \Delta P_c}{\partial \Pi_i} \to \infty \Leftrightarrow \Pi_i \to \Pi_{i\square\square\square} \qquad (1)$$

Where,

Since the dependence of active power losses on the mode parameters is explicitly expressed in terms of the matrix of nodal resistances, we get the expression (1) in the form of the Jacobian NE. (nodal equations)

$$\Delta S_c = -\hat{I}_0 U_0 - \sum_{i=1}^{N} S_i = (\hat{Y}_{0э} U_0 - \hat{B}\hat{I}) U_0 - \sum_{i=1}^{N} S_i \qquad (2)$$

From (2) for the active loss component, we have

$$\Delta P_c = g_{0э}U_0^2 + U_0 \sum_{i=1}^{N}[B©_i(P_iU'_i + QU"_i) - B"_i(P_iU"_i - QU'_i)]/U_i^2 - \sum_{i=1}^{n} P_i \tag{3}$$

The expression of a partial derivative of a function over an independent variable $\Pi_i \in P_i, Q_i$ has the form

$$\frac{\partial \Delta P_c}{\partial \Pi_i} = \frac{\partial \Delta \bar{P}_c}{\partial \Pi_i} + \sum_{k=1}^{N}\left(\frac{\partial \Delta P_c}{\partial U'_k}\cdot\frac{\partial U'_k}{\partial \Pi_i} + \frac{\partial \Delta P_c}{\partial U"_k}\cdot\frac{\partial U"_k}{\partial \Pi_i}\right) \tag{4}$$

где

$$\frac{\partial \Delta P_c}{\partial U'_k} = \frac{U_0}{U_k^4}\left\{B'_k\left[P_k(U"^2_k - U'^2_k) - 2Q_kU'_kU"_k\right] + B"_k\left[Q_k(U"^2_k - U'^2_k) + 2P_kU'_kU"_k\right]\right\} \tag{5}$$

$$\frac{\partial \Delta P_c}{\partial U"_k} = \frac{U_0}{U_k^4}\left\{B'_k\left[Q_k(U'^2_k - U"^2_k) - 2P_kU'_kU"_k\right] - B"_k\left[P_k(U'^2_k - U"^2_k) + 2Q_kU'_kU"_k\right]\right\} \tag{6}$$

Expressions of partial derivatives $\frac{\partial U'_k}{\partial \Pi_i}$ и $\frac{\partial U"_k}{\partial \Pi_i}$ we get from the nodal equations system by writing them in the form of a stress balance in matrix form:

$$\begin{vmatrix} W'(U) \\ W"(U) \end{vmatrix} = \begin{vmatrix} U' \\ U" \end{vmatrix} - \begin{vmatrix} A' \\ A" \end{vmatrix} U_0 - \begin{vmatrix} \mathrm{Re}(Z\hat{S}) & -Jm(Z\hat{S}) \\ Jm(Z\hat{S}) & \mathrm{Re}(Z\hat{S}) \end{vmatrix} \cdot \begin{vmatrix} (U'_i/U_i^2) \\ (U"_i/U_i^2) \end{vmatrix} \tag{7}$$

Where, $\mathrm{Re}(Z\hat{S}),\ Jm(Z\hat{S})$ - square matrices of order $2N$, which elements are

$$\mathrm{Re}(Z\hat{S}) = R_{ik}P_k + X_{ik}Q_k,\ Jm(Z\hat{S}) = X_{ik}P_k - R_{ik}Q_k.$$

Differentiating the system of implicit functions of $2N$ dependent variables (7) by some independent variable $\Pi_i \in P_i, Q_i$, we have

$$\begin{vmatrix} \frac{\partial W'}{\partial U'} & \frac{\partial W'}{\partial U"} \\ \frac{\partial W"}{\partial U'} & \frac{\partial W"}{\partial U"} \end{vmatrix} \cdot \begin{vmatrix} \frac{\partial U'}{\partial \Pi_i} \\ \frac{\partial U"}{\partial \Pi_i} \end{vmatrix} = -\begin{vmatrix} \frac{\partial W'}{\partial \Pi_i} \\ \frac{\partial W"}{\partial \Pi_i} \end{vmatrix} \tag{8}$$

Then the matrix-column of the desired partial derivatives is defined as

$$\begin{vmatrix} \frac{\partial U'}{\partial \Pi_i} \\ \frac{\partial U"}{\partial \Pi_i} \end{vmatrix} = -[J(\mathrm{U})]^{-1} \cdot \begin{vmatrix} \frac{\partial W'}{\partial \Pi_i} \\ \frac{\partial W"}{\partial \Pi_i} \end{vmatrix} \tag{9}$$

The following equation was obtained by using the Jacobian NE (nodal equations) in the form of stress balance:

$$J(\mathrm{U}) = \begin{vmatrix} \frac{\partial W'}{\partial U'} & \frac{\partial W'}{\partial U"} \\ \frac{\partial W"}{\partial U'} & \frac{\partial W"}{\partial U"} \end{vmatrix} \tag{10}$$

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The following equations were obtained by differentiating the equations (7). We find them element by element ($k \neq i$):

$$\frac{\partial W'_i}{\partial U'_k} = [\mathrm{Re}(Z_{ik}\hat{S}_k)(U''^2_k - U'^2_k) + 2\,Jm(Z_{ik}\hat{S}_k)U'_k\,U''_k\,]\,/\,U^4_k\,,$$

$$\frac{\partial W'_i}{\partial U''_k} = -[\mathrm{Re}(Z_{ik}\hat{S}_k)2U'_k\,U''_k + Jm(Z_{ik}\hat{S}_k)(U'^2_k - U''^2_k)]\,/\,U^4_k\,,$$

$$\frac{\partial W''_i}{\partial U'_k} = [-\,\mathrm{Re}(Z_{ik}\hat{S}_k)2U'_k\,U''_k + Jm(Z_{ik}\hat{S}_k)(U''^2_k - U'^2_k)]\,/\,U^4_k\,,$$

$$\frac{\partial W''_i}{\partial U''_k} = [\mathrm{Re}(Z_{ik}\hat{S}_k)(U'^2_k - U''^2_k) - 2\,Jm(Z_{ik}\hat{S}_k)U'_k\,U''_k\,]\,/\,U^4_k\,, \qquad (11)$$

Besides:

$$\frac{\partial W'_i}{\partial U'_i} = 1 + [\mathrm{Re}(Z_{ii}\hat{S}_i)(U''^2_i - U'^2_i) + 2\,Jm(Z_{ii}\hat{S}_i)U'_i\,U''_i\,]\,/\,U^4_i \qquad (12)$$

From (4) and (9) we obtain an expression of the derivative of the active power loss function in the form of the NE Jacobian in the form of a voltage balance:

$$\frac{\partial \Delta P_c}{\partial \Pi_i} = \frac{\partial \Delta \bar{P}_c}{\partial \Pi_i} - \left[\frac{\partial \Delta P_c}{\partial U'},\, \frac{\partial \Delta P_c}{\partial U''}\right] \cdot [\mathrm{J(U)}]^{-1} \cdot \begin{vmatrix} \dfrac{\partial W'}{\partial \Pi_i} \\ \dfrac{\partial W''}{\partial \Pi_i} \end{vmatrix} \qquad (13)$$

Below are the derivatives included in this formula depending on the variable $\Pi_i \in P_i, Q_i$:

$$\frac{\partial \Delta P_c}{\partial P_i} = \frac{U_0}{U_i^2}(B'_i\,U'_i - B''_i\,U''_i) - 1,$$

$$\frac{\partial \Delta P_c}{\partial Q_i} = \frac{U_0}{U_i^2}(B'_i\,U''_i + B''_i\,U'_i) \qquad (14)$$

$$\frac{\partial W'_i}{\partial P_k} = \frac{1}{U_k^2}(R_{ik}U'_k - X_{ik}U''_k), \quad \frac{\partial W''_i}{\partial P_k} = \frac{1}{U_k^2}(X_{ik}U'_k + R_{ik}U''_k),$$

$$\frac{\partial W'_i}{\partial Q_k} = \frac{1}{U_k^2}(X_{ik}U'_k + R_{ik}U''_k), \quad \frac{\partial W''_i}{\partial Q_k} = \frac{1}{U_k^2}(-R_{ik}U'_k + X_{ik}U''_k). \qquad (15)$$

Condition for the existence of a derivative $\frac{\partial \Delta P_c}{\partial \Pi_i}$ in (13) will be:

$$\det[\mathrm{J(U)}] \neq 0 \qquad (16)$$

If the Jacobian NE (nodal equations) is zero at the solution point, then (13) follows $\frac{\partial \Delta P_c}{\partial \Pi_i} = \infty$.
Thus, the limit on the existence of solutions of nonlinear control systems corresponds to the limit mode defined by criterion (1). In other words, if the nodal equations system has a solution for certain values of independent parameters and the Jacobian is different from zero at the solution point, then even with a small change in the parameters, the system will have a well-defined solution. If the

Jacobian is equal to or close to zero at the point of solution, then a slight change in the parameters in the direction of weighting the mode will cause the absence of a real solution of the nodal equations. The formulated criteria for the existence and limit of the established regime are also valid for NE Jacobians written in the form of current balance or power balance.

3. Results and Discussions

The results of calculations of the steady-state modes of the electrical system with weighted values of the power of the node 12 are shown in Table 1.

Table 1. The steady-state modes of the electrical system with weighted values of the power of the node 12: ΔP_c *– active power loss;* ΔQ_c *– reactive power loss;* U_{12}*-node 12' voltage module and* P_{12} *– active power*

P_{12}	ΔP_c	ΔQ_c	U_{12}
180	15.706	147.268	215.141
230	20.362	187.478	208.188
280	27.139	245.826	198.534
330	38.705	345.698	182.500
340	42.631	379.795	177.082
341	43.100	383.873	176.435
342	43.588	388.125	175.760
343	44.098	392.571	175.054
344	44.634	397.236	174.314
345	45.198	402.152	173.534
346	45.795	407.361	172.708
347	46.431	412.914	171.827
348	47.114	418.883	170.881

Figure 2 shows a graph of the dependence of the total loss of active power in the load system of node 12 during the weighting.

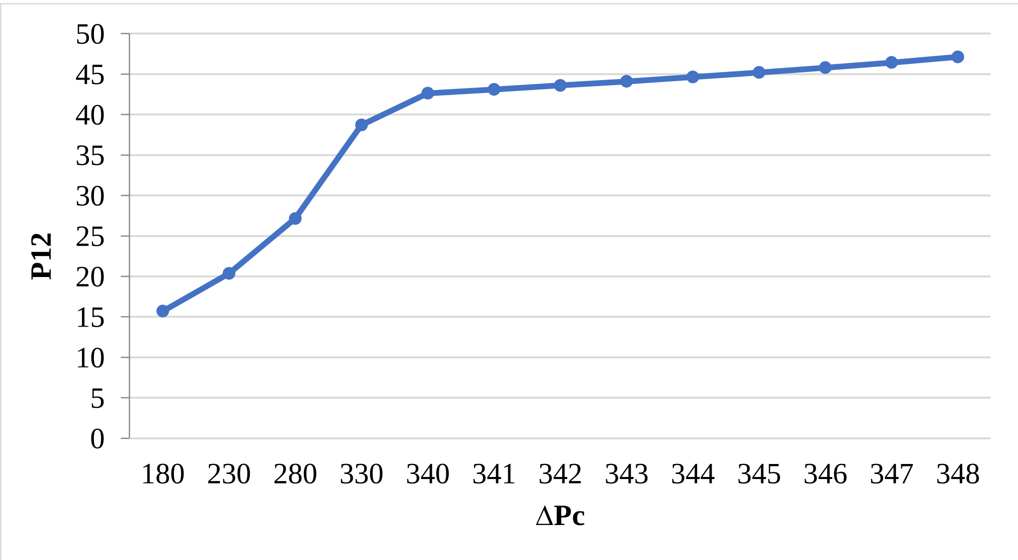

Figure 2. A graph of the dependence of the total losses of active power in the load system of node 12 with weighting in the form of row 1

As can be seen from the graph in Figure 2 the limit mode occurs when $P_{12} = 348 MBm$, after which the iterative process diverges. The solution obtained after $P_{12} > 350 M\text{в}m$, is physically unrealizable because it does not meet the criteria $\frac{\partial \pi}{\partial P_i} > 0$.

4. Conclusions

Thus, the limit normal modes of electrical systems for the power of nodes can be determined by the criterion $\frac{\partial \Delta P_c}{\partial \Pi_i} = \infty.$.

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