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Mathematical description of water flow quantity for microhydroelectric station

D Kodirov[1*], O Tursunov[1,2,3], S Khushiev[1], O Bozarov[4], G Tashkhodjaeva[5], and S Mirzaev[6]

[1]Department of Power Supply and Renewable Energy Sources, Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, 100000 Tashkent, Uzbekistan
[2]School of Mechanical and Power Engineering, Shanghai Jiao Tong University, 200240 Shanghai, China
[3]Research Institute of Forestry, 111104 Tashkent, Uzbekistan
[4]Andijan branch of Tashkent State Agrarian University, Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, 170600 Andijan, Uzbekistan
[5]Department of Accounting and Audit, Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, 100000 Tashkent, Uzbekistan
[6]Department of Information Technologies, Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, 100000 Tashkent, Uzbekistan

*Email: d.kodirov@mail.ru

Abstract. The main methods of regulating water flow and power of a microhydroelectric power plant are considered**.** New technical solutions are proposed for screw jet hydraulic turbines adapted to low heads and water flow rates. Preliminary calculations carried out by the authors show that the power of a micro-hydroelectric power plant depends on the individual factors of the area.As the speed of the water flow increases, the speed of the water wheel also increases, and in turn, the electric power of the micro-hydroelectric power plant is increased.

1. Introduction

This has been the case for many years Q,V and M sizes are accepted as flow norms. These values were taken as the magnitude for the average water flow in practics. If the above magnitudes are constant in geographical conditions and there are changes in the river water flow cycles of not less than two of the same level of water flow into the river basin, then in practice it will not be possible to directly measure the flow data. Flow rates are calculated for the whole country and the archival sources of the flow module specified in the relevant literature are determined on the basis of maps (reference books). The flow modulus for Tashkent region varies from 20 l/sec $\cdot$ km^2 in mountainous areas to 0.5 l/sec $\cdot$ km^2 in flat areas [1-3].

The process of river flow formation is a multifactorial complex natural phenomenon. These factors include precipitation, melting intensity of snow and rain, freezing and moisture of soils, evaporation, and so on. In these cases, when an event or phenomenon occurs under the influence of a set or multiplication of unrelated or less related factors, according to the central boundary theorem of probability theory, the event or process is incidental and obeys a definite statistical law.

Therefore, mathematical statistical methods are used in the study of water flows [4-7]. In addition to the general purpose for the design of hydropower facilities, and to determining the flow rate in the implementation of hirological calculations, it is also necessary to calculate its potential oscillations during the operation of the device.

2. Methodology

The speed of flow in a river is the distance traveled by a stream of water in one second. In practice, the flow rate can be monitored by the movement of lightly floating objects (empty glass, matchbox, stick, etc.) in the water. But keep in mind that the object moves along the water at the speed at the top of the stream. The deeper the water layer, the lower its flow rate. The true average velocity for all layers of the flow is taken as 2/3 of the depth.

River flow is studied by systematic measurement of water consumption at time t (Q, m^3/sec) and levels on the river cross-sectional surface [2, 8].

$$Q = v \cdot \omega \tag{1}$$

where v is the average flow rate of the measured water in the river under study, m/s; ω is the cross-sectional area of the river, m^2.

Determined water consumption is recorded in the logbook of water resources surveyors. Along with the water consumption indicators of the river, the following indicators of water flow are given:

Average annual water consumption Q,

$$Q = \frac{\sum_{i=1}^{T} Q_i}{T}, \qquad m^3/\,sec \tag{2}$$

Where, $\sum_{i=1}^{T} Q_i$ - daily sum of average annual water consumption T – daily number of the year (T = 365 day or $31.54 \cdot 10^6$ s);

Annual water inflow V:

$$V = Q \cdot 31.54 \cdot 10^{6}, м^3 \tag{4}$$

There, $31.54 \cdot 10^{6}$ - the average number of seconds per year. V_0 is determined by the following formula:

$$V_0 = \frac{V_1 + V_2 + \ldots + V_n}{n}, \qquad m^3 \tag{5}$$

There $V_1, V_2, \ldots V_n$ - flow volumes for individual years; n is the number of years in the period;

Average annual water consumption. Q_0 is determined by the following formula:

$$Q_0 = \frac{\sum_1^n Q}{n} = \frac{V_0}{31{,}54 \cdot 10^6}, \qquad m^3/\,sec \tag{6}$$

Average annual flow modulus M_0:

$$M_0 = \frac{1000\,Q_0}{F} = \frac{1000\,V_0}{31{,}54 \cdot 10^6 F} \qquad l/s \cdot km^2 \tag{7}$$

where F is the catchment area of the river basin for the cross-sectional area of the river in question, km^2.

The coefficient of the average annual flow modulus k_i:

$$k_i = \frac{Q}{Q_0} = \frac{W_i}{W_0} \tag{8}$$

Where ***I*,** is the serial number of the year.
Based on the above, the amount of water in the area where the micro-hydroelectric power plant is planned to be installed during the year was measured and included in Table 3.3. [3].
Figure 1 highlights the average amount of water reported by month in the area.

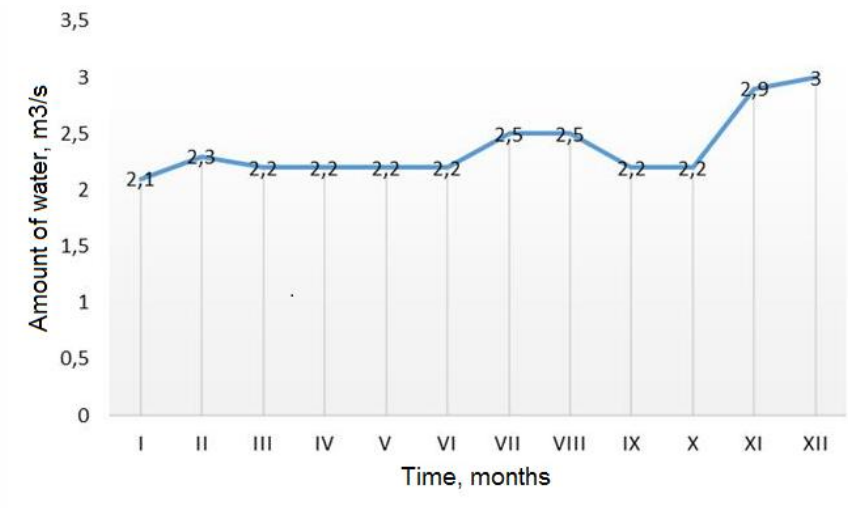

Figure 1. Water flow of the test area (2020)

The river flowing through the Tashkent region was seasonally variable throughout the year, with the lowest annual flow in November and December of the year, during which the canal flows at a rate of 2.9-3 m3 / sec. In the remaining months of the year, water flow was detected at an average rate of 2.2 - 2.5 m^3/sec (see Figure 1). The maximum velocity of water is 3 m^3/sec, and the minimum velocity is 2.1 m^3/sec. The average velocity of the observed water flow was 2.4 m^3/sec.

3. Results and Discussions

Based on the above data, we construct mathematical models that represent the degree of dependence of water consumption on flow rate and flow rate. We look for the appearance of the mathematical model in the form Z = a + blnx + clny [3]. To do this, we use the following information (Table 1) [5].

Table 1. Dependence of relative water consumption on flow rate and flow rate (low)

№	Z-water cost, *Q (m^3/sec)*	Y-flow rate, *v (м/sec)*	X-flow power, *N (kW)*
1	0,8	0,8	0,25
2	1,0	1,0	0,5
3	1,2	1,2	0,85
4	1,4	1,4	1,37
5	1,6	1,6	2,06
6	1,8	1,8	3,09
7	2,0	2,0	4,0
8	2,2	2,2	5,33
9	2,4	2,4	6,9
10	2,6	2,6	8,8
11	2,8	2,8	10,99
12	3,0	3,0	13,51
13	3,2	3,2	16,45

Based on Table 1 above, we construct the regression coefficients constructed for the relative water permeability of the device accordingly (Table 2).

Table 2. Line regression coefficients for the relative water permeability of the device

№	lnx	lny	lnx·lny	lnx^2	lny^2	Z·lnx	Z·lny
1	-1.38629	-0.22314	0.309343	1.921812	0.049793	-1.10904	-0.17851
2	-0.69315	0	0	0.480453	0	-0.69315	0
3	-0.16252	0.182322	-0.02963	0.026412	0.033241	-0.19502	0.0218786
4	0.314811	0.336472	0.105925	0.099106	0.113214	0.440735	0.471061
5	0.722706	0.470004	0.339674	0.522304	0.220903	1.15633	0.752006
6	1.128171	0.587787	0.663124	1.27277	0.345493	2.030708	1.058016
7	1.386294	0.693147	0.960906	1.921812	0.480453	2.772589	1.386294
8	1.673351	0.788457	1.319366	2.800104	0.621166	3.681373	1.734606
9	1.931521	0.875469	1.690987	3.730077	0.766446	4.63561	2.101125
10	2.174752	0.955511	2.078	4.729545	0.913002	5.654354	2.48433
11	2.396986	1.029619	2.467983	5.745541	1.060116	6.71156	2.882934
12	2.60343	1.098612	2.86016	6.777849	1.206949	7.81029	3.295837
13	2.800325	1.163151	3.257201	7.841823	1.35292	8.961042	3.722083
∑	14.89039	7.957408	16.02304	37.87031	7.164195	41.85743	19.92856

Based on the regression coefficients constructed for the relative water permeability of the device, we can obtain the following:

$z=-1.63221-3.6656\times \ln x+12.79505\times \ln y$,

F(The Fisher statistics)=174.5653

```
>> x=[0.25 0.5 0.85 1.37 2.06 3.09 4 5.33 6.9 8.8 10.99 13.51 16.45];
y=[0.8 1 1.2 1.4 1.6 1.8 2 2.2 2.4 2.6 2.8 3 3.2];
[x,y]=meshgrid(x,y);
z=-1.63221-3.6656×log(x)+12.79505×log(y);
surf(x,y,z)
>> xlabel('Flow power,'FontSize',14)
>> ylabel('Flow speed,'FontSize',14)
>> zlabel('Water cost,'FontSize',14)
```

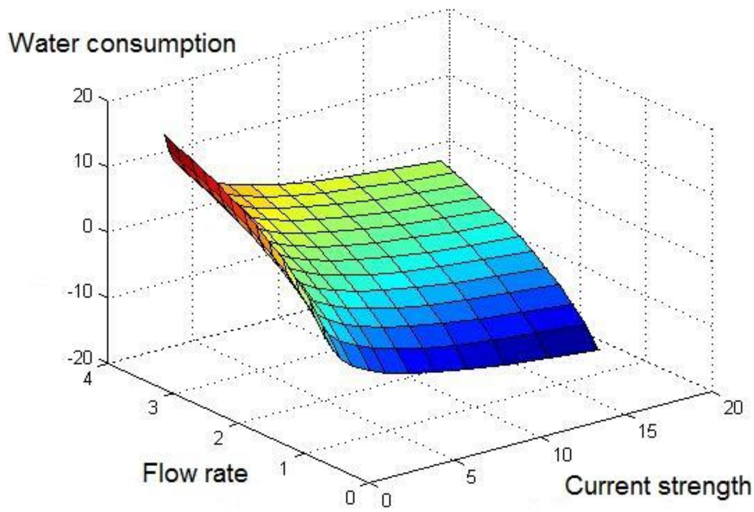

Figure 2. Graph of the results of regression coefficients in Matlab

From Figure 2, we can see that an increase in water consumption of 2 m3 leads to an increase of 4 kW of power obtained by the device, but we can observe that this increase in power is not significant. Therefore, by increasing the relative water consumption without changing the flow rate, we obtain Table 3 as follows:

Table 3. Dependence of relative water consumption on flow rate without changing the flow rate (medium)

№	Z-water cost, *Q* (m^3/sec)	Y-flow speed, *v (m/sec)*	X-flow power, *N (kW)*
1	4	0.8	1.25
2	5	1.0	2.5
3	6	1.2	4.3
4	7	1.4	6.87
5	8	1.6	10.28
6	9	1.8	15.45
7	10	2.0	20.0
8	11	2.2	26.65
9	12	2.4	34.5
10	13	2.6	44.0
11	14	2.8	54.9
12	15	3.0	67.5
13	16	3.2	82.2

Based on the dependence of the relative water consumption on the flow rate without changing the flow rate (Table 3), we construct the corresponding regression coefficients for the relative water permeability of the device (Table 4).

Table 4. Constructed regression coefficients for the relative water permeability of the device without changing the flow rate

№	lnx	lny	lnx·lny	lnx^2	lny^2	Z·lnx	Z·lny
1	0.223144	-0.22314	-0.04979	0.049793	0.049793	0.892574	-0.89257
2	0.916291	0	0	0.839589	0	4.581454	0
3	1.458615	0.182322	0.265937	2.127558	0.033241	8.75169	1.093929
4	1.927164	0.336472	0.648437	3.713961	0.113214	13.49015	2.355306
5	2.3302	0.470004	1.095520	5.429833	0.220903	18.6416	3.760029
6	2.737609	0.587787	1.60913	7.494503	0.345493	24.63848	5.29008
7	2.995732	0.693147	2.076483	8.974412	0.480453	29.95732	6.931472
8	3.282789	0.788457	2.588339	10.7767	0.621665	36.11068	8.673031
9	3.540959	0.875469	3.099999	12.53839	0.766446	42.49151	10.50562
10	3.78419	0.955511	3.615837	14.32009	0.913002	49.19447	12.42165
11	4.005513	1.029619	4.124154	16.04414	1.060116	56.07719	14.41467
12	4.212128	1.098612	4.627495	17.74202	1.206949	63.18191	16.47918
13	4.409155	1.163151	5.128513	19.44065	1.35292	70.54648	18.61041
∑	35.82349	7.957408	28.82973	119.4916	7.164195	418.555	99.64282

z=-7.09055+12.85657×lnx-29.9582×lny
F(The Fisher statistics)=99.3098

```
>> x=[1.25 2.5 4.3 6.87 10.28 15.45 20 26.65 34.5 44 54.9 67.5 82.2];
>> y=[0.8 1 1.2 1.4 1.6 1.8 2 2.2 2.4 2.6 2.8 3 3.2];
>> [x,y]=meshgrid(x,y);
>> z=-7.09055+12.85657×log(x)-29.9582×log(y);
>> surf(x,y,z)
>> xlabel(Flow power,'FontSize',14)
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>> ylabel('Flow speed,'FontSize',14)
>> zlabel('Water cost,'FontSize',14)

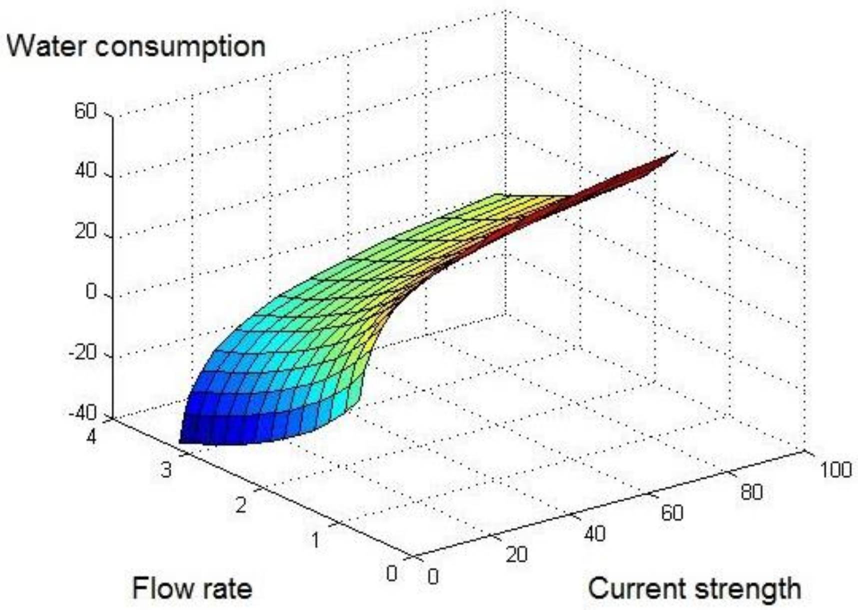

Figure 3. Graph of the results of regression coefficients without changing the water flow rate

From Figure 3, we can analyze that an increase in water consumption of 12 m3 led to an increase in the power received by the device by 34.5 kW. The fact that the water flow rate remains the same, taking into account that it has not changed, shows that this model is acceptable. However, we will continue the analysis again to determine if this conclusion is acceptable. In this case, we connect the flow rate by sharply increasing the relative water consumption without changing the flow rate (Table 5).

Table 5. Dependence of flow rate on the flow rate (high), which is sharply increased without changing the flow rate

№	Z-water cost, *Q (m^3/sec)*	Y-flow speed, *v (m/sec)*	X-flow power, *N (kW)*
1	16	0.8	5
2	20	1.0	10
3	24	1.2	17
4	28	1.4	28
5	32	1.6	41
6	36	1.8	62
7	40	2.0	80
8	44	2.2	107
9	48	2.4	138
10	52	2.6	176
11	56	2.8	219
12	60	3.0	270
13	64	3.2	329

Based on the dependence of the flow rate of the sharply increased relative water consumption without changing the flow rate (Table 5), we construct the corresponding regression coefficients for the relative water permeability of the device (Table 6).

Table 6. Regression coefficients for the relative water permeability of a device that is sharply increased without changing the flow rate

№	lnx	lny	lnx·lny	lnx^2	lny^2	Z·lnx	Z·lny
1	1.609438	-0.22314	-0.35913	2.590290	0.049791	25.75101	-3.57024
2	2.302585	0	0	5.301897	0	46.0517	0
3	2.833213	0.182322	0.516557	8.027095	0.033241	67.99711	4.375728
4	3.332205	0.336472	1.121194	11.10359	0.113213	93.30174	9.421216
5	3.713572	0.470004	1.745394	13.79061	0.220903	118.8343	15.040128
6	4.127134	0.587787	2.425876	17.03323	0.345493	148.5768	21.160332
7	4.382027	0.693147	3.037389	19.20216	0.480452	175.2811	27.72588
8	4.672829	0.788457	3.684325	21.83533	0.621664	205.6045	34.692108
9	4.927254	0.875469	4.313658	24.27783	0.766445	236.5082	42.022512
10	5.170484	0.955511	4.940454	26.73390	0.913001	268.8652	49.686572
11	5.389072	1.029619	5.548691	29.04209	1.060115	301.788	57.658664
12	5.598422	1.098612	6.150494	31.34232	1.206948	335.9053	65.91672
13	5.796058	1.163151	6.741691	33.59428	1.352920	370.9477	74.441664
∑	53.85429	7.957411	39.86659	243.8746	7.164191	2395.412	398.57128

z=-21.2005+17.30607×lnx-17.1412×lny
F(The Fisher statistics)=121.2742

```
>> x=[5 10 17 28 41 62 80 107 138 176 219 270 329];
y=[0.8 1 1.2 1.4 1.6 1.8 2 2.2 2.4 2.6 2.8 3 3.2];
[x.y]=meshgrid(x.y);
z=-21.2005+17.30607×log(x)-17.1412×log(y);
surf(x.y.z)
xlabel('Flow power.'FontSize'.14)
ylabel('Flow speed.'FontSize'.14)
zlabel('Water cost.'FontSize'.14)
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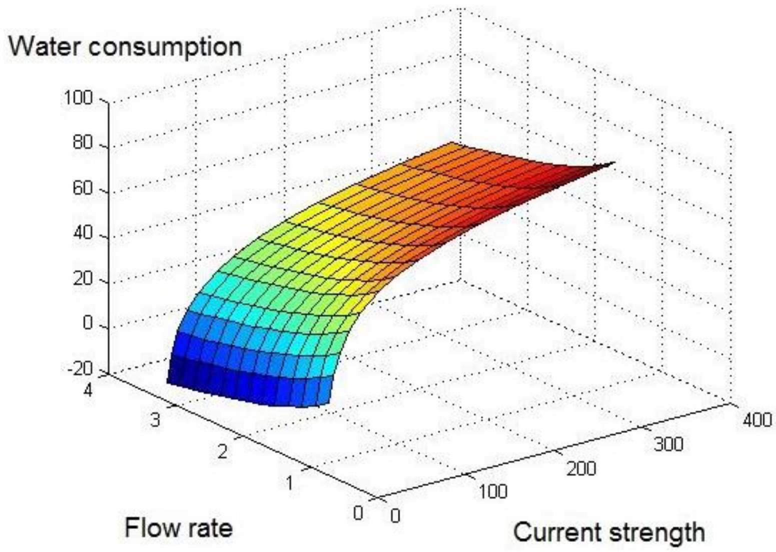

Figure 4. Graph of the results of regression coefficients without changing the water flow rate

From the Figure 4, we can analyze that the increase in water consumption by 60 m^3 led to an increase in the power received by the device by 270 kW. Of course. this is a good indicator for small hydropower plants. for the low-pressure micro hydropower plants we are looking at. such water flows are rare in our country. Even if available. small and medium hydropower plants are more efficient than micro hydropower plants.

For each of the above results. we apply the least squares method of mathematical statistics:

F(a.b.c)= $\sum_{k=1}^{n}(Z_k-(a+b\ln x_k+c\ln y_k))^2$.

$$\frac{\partial F}{\partial a}=2\sum_{k=1}^{n}(Z_k-a-b\ln x_k-c\ln y_k)\,(-1)$$

$$\frac{\partial F}{\partial b}=2\sum_{k=1}^{n}(Z_k-a-b\ln x_k-c\ln y_k)\,(-\ln x_k)$$

$$\frac{\partial F}{\partial c}=2\sum_{k=1}^{n}(Z_k-a-b\ln x_k-c\ln y_k)\,(-\ln y_k)$$

$\frac{\partial F}{\partial a}=0$

$\frac{\partial F}{\partial b}=0$

$\frac{\partial F}{\partial c}=0.$

By convention, we construct the following system of normal equations:

an+b $\sum_{k=1}^{n}\ln x_k+c\sum_{k=1}^{n}\ln y_k=\sum_{k=1}^{n}Z_k$

$$a\sum_{k=1}^{n}\ln x_k+b\sum_{k=1}^{n}(\ln x_k)^2+c\sum_{k=1}^{n}\ln x_k*\ln y_k\sum_{k=1}^{n}Zk\ln x_k$$

$$a\sum_{k=1}^{n}\ln y_k+b\sum_{k=1}^{n}\ln x_1*\ln y_k)^2+c\sum_{k=1}^{n}(\ln y_k)^2=\sum_{k=1}^{n}Zk\ln y_k$$

The resulting system was designed for water permeability based on Tables 1 and 2 and the following result was obtained:

13a+b14.89039+c7.9574=26

a14.89039+b37.87031+c16.02304=41.85743

a7.957408+b16.02304+c7.164195=19.92856

Answer.

a=-1.6321

b=-3.66656

c=12.79505. F(The Fisher statistics)=174.5653

Z=-1.6321-3.6665lnx+12.7905lny

The following result was obtained for the water permeability based on Tables 3 and 5.

13a+35.82349b+7.959408c=130

35.82349b+19.4916b+28.82973c=418.5555

7.957408a+28.8973b+7.164495c=99.64282

Answer.

a=-7.09055

b=12.95657

c=-29.9582 F(The Fisher statistics)=99.3098

z=-7.09055+12.85657*lnx-29.9582*lny

The following result was obtained for the water permeability based on Tables 5 and 6

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13a+53.85429b+7.957408c=520
53.85429a+243.8747b+39.86659c=2395.413
7.957408a+39.6659b+7.164195c=398.5713
Answer
a=-21.2005
b=17.30607
c=-17.1412 F(The Fisher statistics)=121.2742
z=-21.2005+17.30607×lnx-17.1412×lny

As a result of mathematical modeling of the degree of dependence of water consumption on flow rate and flow rate, we developed the following mathematical model z = a + b · lnx + c · lny. based on which the following squares of mathematical statistics were determined (Table 7):

Table 7. Squares of mathematical statistics

№	Water cost	Water cost mathematical modeling	The Fisher statistics
1	Low	z=-1.63221-3.6656×lnx+12.79505×lny	F=174.5653
2	Medium	z=-7.09055+12.85657×lnx-29.9582×lny	F=98.3098
3	High	z=-21.2005+17.30607×lnx-17.1412×lny	F=121.2772

4. Conclusions

A river flowing through Tashkent region was selected for the test area, and seasonal fluctuations were observed throughout the year. Observations revealed the presence of the required amount of water flow in the area for the experiment.
A mathematical description of the operation of the micro-hydroelectric power plant was developed, taking into account the amount of water flow. Analyzes have shown that the proposed microhydroelectric power model works effectively in low-pressure water flows.
The flow rate of the water body was calculated. It was observed that the amount of water flow varies seasonally throughout the year in the area where the test is planned. The average velocity of the observed water flow was found to be 2.4 m^3/sec. From this it can be concluded that a test sample of a micro-hydroelectric power plant has been installed in this area and research work is underway.

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