Stability of Nonlinear Vibrations of Elastic Plate and Dynamic Absorber in Random Excitations

Mirziyod Mirsaidov[1,2], *Olimjon* Dusmatov[3], and *Muradjon* Khodjabekov[4*]

[1]National Research University - Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, 39 Kori Niyoziy str., 100000, Tashkent, Uzbekistan
[2]Institute of Mechanics and Seismic Stability of Structures of the Academy of Sciences of the Republic of Uzbekistan, 33 Durmon str., 100125, Tashkent, Uzbekistan
[3]Samarkand State University, 15 University blv., 140104, Samarkand, Uzbekistan
[4]Samarkand State Architectural and Civil Engineering University, 70 Lolazor str., 140147, Samarkand, Uzbekistan

Abstract. In this work, the problem of exploring the stability of vibrations of a hysteresis-type elastic dissipative characteristic plate with a liquid section dynamic absorber under the influence of random excitations is considered. Expressions of mean square deviations of the generalized coordinates are presented, and the expression of the spectral density of the base acceleration is obtained in the form of a wide band. The integral expressions of mean square deviations were calculated, and the conditions for the existence of vertical tangents transferred to the graph of the function representing the mean square values of displacements of plate points were determined. It is shown that these conditions are not fulfilled, and the stability condition is determined depending on the structural parameters of the system.
Keywords: plate, nonlinear vibration, dynamic absorber, stability condition, hysteresis, resonance.

1 Introduction

Nowadays, the problems of reducing harmful vibrations in mechanical systems and identifying and eliminating the factors preventing their long-term perfect operation are important tasks that require solving. In this regard, mathematical modeling of the motion of the elastic vibration protected plate, taking into account the nonlinear deformation, exploring its dynamics and stability are considered urgent problems.

There are several works devoted to nonlinear vibrations of plates of various shapes, their stability and vibration damping. Among them:

The problem of finding and analyzing the solution of the Ito differential equation for Weiner random excitations is solved in the article [1]. The condition that the solution of the Ito differential equation does not change sharply is defined for non-autonomous system parameters.

* Corresponding author: uzedu@inbox.ru

In the article [2], the parametric stability of the motion of systems in real nois excitations is studied. The Lyapunov exponent was determined and analyzed numerically.

The article [3] examines the problem of exponential stability of systems with hysteresis-type connections under the influence of random excitations. In this case, the Ito differential equation was constructed by the stochastic averaging method, and the Lyapunov method was used to explore the stability of motion. In order to demonstrate the reliability of the obtained results, several problems have been resolved.

In the article [4], a thin plate is studied as a wall of a building, and its hysteresis-type elastic dissipative characteristic is numerically analyzed using the finite element method depending on the change in the ratio of plate thickness to height.

In the study [5], the energy losses in the plate material with elastic dissipative characteristics of the hysteresis type, with steel bars attached to all four ends, were determined and analyzed based on analytical methods and experiments.

In the article [6], the nonlinear vibrations of a thin plate with a composite coating under the influence of external forces and the stability of stationary motion were studied, in which the effectiveness of the damping coefficient of the plate, the frequency and the influence of the external force on the stationary motion of the plate and the stability of stationary motion were analyzed. Numerical solutions for the amplitude-frequency characteristic were obtained, compared with analytical solutions, and numerically analyzed.

In the study [7], the vibrations of the plate with elastic dissipative characteristics under the influence of wide-band random excitations was considered on an experimental basis. The analytical expression of the amplitudes was obtained as a function of the system parameters and numerically analyzed for the aluminum material.

In the article [8], the hysteresis-type elastic dissipative characteristic of the composite plate was analyzed and experiments were conducted for several types of materials. The resonance frequency is expressed analytically.

The vibrations of a plate with elastic dissipative characteristics were studied in [9] under the influence of various random excitations. Root mean square deviation and spectral densities were analytically expressed and numerically analyzed.

In the work [10], the natural frequency and mode shapes of a plate of different shapes were studied using finite differences, experiments and R-function methods for different boundary conditions. It is shown that the use of the R-function method gives more convenience in the determination and analysis of the characteristic frequency and characteristic vibration forms. The influence of the geometrical dimensions of the plate on the change of the natural frequency was numerically analyzed and the results are given in the form of a table.

The transverse vibrations of a plate passing between two fixed axes rotating rollers were studied in [11] using asymptotic methods. The relations between the forces representing the influence of the materials on the surface of the rollers and the deformations of the plate are expressed in the hysteresis type. The dynamic model of nonlinear forces is derived using the Duffing equation. The damping and uniformity coefficients of the rollers were analyzed and their optimal values for damping plate vibrations were determined.

The finite element analysis of energy to solve the problem of plate vibrations is presented in the work [12]. It is shown that the energy equations obtained in this analysis, averaged with respect to time and coordinate, represent the nature of vibrations.

The work [13-14] is devoted to determining the limits of failure of plate motion. In this case, the equations of motion and boundary conditions are determined using Hamilton's principle. Analytical expressions of uncertainty limits were derived from the Hurwitz criterion, and the effect of damping coefficients on uncertainty areas was investigated.

Mathematically modeled transverse vibrations of a hysteresis-type plate with elastic dissipative characteristics under the effects of harmonic and random excitations, studied its dynamics, and explored its stability in [15-19]. In this case, the hysteresis-type elastic characteristics of the plate material were obtained based on the Pisarenko-Boginich hypothesis. Based on the obtained results, numerical calculations were carried out and analyzed, conclusions were drawn and recommendations were developed.

Solving the mathematical stability of the motion of elastic plates protected against vibrations in random excitations, taking into account nonlinear deformation, is considered one of the urgent problems.

2 Materials and methods

The main relationships of the theory of random processes are used when solving problems related to the vibrations of mechanical systems under the influence of random excitations [15]. Based on them, the mean square deviations of the generalized coordinates represent the random vibrations of the considered systems.

$$\sigma_{ik}^2 = \int_{-\infty}^{\infty} |H_1(\omega)|^2 S_{W_0}(\omega) d\omega\,;$$
$$\sigma_{3*}^2 = \int_{-\infty}^{\infty} |H_2(\omega)|^2 S_{W_0}(\omega) d\omega\,; \tag{1}$$
$$\sigma_{4*}^2 = \int_{-\infty}^{\infty} |H_3(\omega)|^2 S_{W_0}(\omega) d\omega,$$

where σ_{ik} is the mean square value of displacements of plate points; σ_{3*} and $\sigma_{4*}$are mean square values of the displacement of the outer body of the liquid section dynamic absorber and the solid body inside the liquid, respectively; $S_{W_0}(\omega)$ is spectral density of base acceleration; $H_1(\omega), H_2(\omega), H_3(\omega)$ are system amplitude-frequency characteristics and are defined as follows:

$$H_1(\omega) = |u_{ik*}|\varepsilon p_0 \sqrt{\frac{\Psi_1^2 + \Psi_2^2}{\Upsilon_1^2 + \Upsilon_2^2}}\,; H_2(\omega) = |u_{ik*}|\varepsilon p_0 \sqrt{\frac{\Psi_3^2 + \Psi_4^2}{\Upsilon_1^2 + \Upsilon_2^2}}\,;$$
$$H_3(\omega) = |u_{ik*}|\varepsilon p_0 \sqrt{\frac{\Psi_5^2 + \Psi_6^2}{\Upsilon_1^2 + \Upsilon_2^2}}, \tag{2}$$

where $\Psi_1(\omega) = m_{ik}(c_1 - M_1\omega^2)(2c_2 - M_4\omega^2) - m_{ik}\omega^2(b_F b_S + M_2 M_3 \omega^2)$;
$\Psi_2(\omega) = ((c_1 - M_1\omega^2)b_S + (2c_2 - M_4\omega^2)b_F)m_{ik}\omega$;
$\Psi_3(\omega) = (2c_2M_1 - \Delta\omega^2)m_{ik}u_{ik1}\omega^2$; $\Psi_4(\omega) = b_S M_1 m_{ik} u_{ik1}\omega^3$;
$\Psi_5(\omega) = c_{1*}M_3 m_{ik} u_{ik1}\omega^2$; $\Psi_6(\omega) = b_F M_3 m_{ik} u_{ik1}\omega^3$;
$\Upsilon_1(\omega) = -\Delta m_{ik}\omega^6 + \Big((2c_2M_1 + c_1M_4)m_{ik} + (m_{ik} + u_{ik1}^2 M_1)b_F b_S + \Delta(c_{1ik} + c_1 u_{ik1}^2)\Big)\omega^4 + (b_S M_1 + b_F M_4)c_{2ik}\omega^3 - (2c_1c_2(m_{ik} + u_{ik1}^2 M_1) + (2c_2M_1 + c_1M_4 + b_F b_S)c_{1ik})\omega^2 - (b_S c_1 + 2c_2 b_F)c_{2ik}\omega + 2c_1c_2c_{1ik}$;
$\Upsilon_2(\omega) = (\Delta b_F u_{ik1}^2 + (b_S M_1 + b_F M_4)m_{ik})\omega^5 + \Delta c_{2ik}\omega^4 - \Big((b_S M_1 + b_F M_4)c_{1ik} + (m_{ik} + u_{ik1}^2 M_1)(b_S c_1 + 2c_2 b_F)\Big)\omega^3 - (2c_2M_1 + c_1M_4 + b_F b_S)c_{2ik}\omega^2 + (b_S c_1 + 2c_2 b_F)c_{1ik}\omega + 2c_1c_2c_{2ik}$; $M_1 = m_{13*} + m_2$; $M_2 = m_2 + m_v$; $M_3 = m_2 - m_v$; $M_4 = m_2 + m_4$;
$\Delta = M_1M_4 - M_2M_3$; m_1 is the mass of the outer body of the dynamic absorber surrounding the liquid; m_2 is the mass of the solid body of the dynamic absorber; m_3 is mass of liquid; m_4 is mass of the liquid attached to the object with mass m_2; b_F is damping coefficient; c_1

and c_2 are stiffnesses; $m_{13*} = m_1 + m_3$; m_v is the mass of liquid displaced by a solid body with mass m_2; b_S is the viscosity coefficient of the liquid; m_{ik} and c_{ik} are modal masses and stiffnesses, expressed as follows $(i, k = 1 \dots n)$:

$$m_{ik} = \iint_{0\,0}^{a\,b} \rho h u_{ik}^2 dxdy\,; c_{ik} = \left[\left(1 + D_0(-\eta_1 + J\eta_2)\right) \iint_{0\,0}^{a\,b} \rho h u_{ik}^2 dxdy\right] +$$

$$+\frac{3D}{\omega_{ik}^2}(-\eta_1 + J\eta_2) \sum_{R=1}^{s_1} D_R \sigma_{ika}^R \frac{h^R}{2^R(R+3)} \iint_{0\,0}^{a\,b} u_{ik}\, [\frac{\partial^2}{\partial x^2}((\frac{\partial^2 u_{ik}}{\partial x^2} +$$

$$+\mu_n \frac{\partial^2 u_{ik}}{\partial y^2}) \left|\frac{\partial^2 u_{ik}}{\partial x^2} + \mu_n \frac{\partial^2 u_{ik}}{\partial y^2}\right|^R) + \frac{\partial^2}{\partial y^2}((\frac{\partial^2 u_{ik}}{\partial y^2} + \mu_n \frac{\partial^2 u_{ik}}{\partial x^2}) \times$$

$$\times \left|\frac{\partial^2 u_{ik}}{\partial y^2} + \mu_n \frac{\partial^2 u_{ik}}{\partial x^2}\right|^R)]dxdy + \frac{6D}{\omega_{ik}^2}(1 - \mu_n)(\nu_1 - J\nu_2) \sum_{N=1}^{s_2} K_N \sigma_{ika}^N \times$$

$$\times \frac{h^N}{2^N(N+3)} \iint_{0\,0}^{a\,b} u_{ik} \frac{\partial^2}{\partial x \partial y}\left(\left(\frac{\partial^2 u_{ik}}{\partial x \partial y}\right)\left|\frac{\partial^2 u_{ik}}{\partial x \partial y}\right|^N\right) dxdy + \frac{2D}{\omega_{ik}^2}(1 - \mu_n) \times$$

$$\times (\nu_1 - J\nu_2) K_0 \iint_{0\,0}^{a\,b} u_{ik} \frac{\partial^4 u_{ik}}{\partial x^2 \partial y^2} dxdy] \omega_{ik}^2;$$

a, b and h are plate sides and thickness, respectively; ρ is the material density of the plate; $u_{ik} = u_{ik}(x, y)$ are mode shapes of plate; $D_0, D_1, \dots, D_{S_1}, K_0, K_1, \dots, K_{S_2}$ are experimentally determined parameters of plate material [20]; $D = \frac{Eh^3}{12(1-\mu_n^2)}$ are cylindrical stiffness; E is Young's module; μ_n is Poisson's coefficient; σ_{ika} are amplitude values of plate vibrations; ω_{ik} are natural frequencies of the plate; $\eta_1, \eta_{22}, \nu_1, \nu_{22}$ are statistical linearization coefficients [15]; $\eta_{22} sign(\omega), \nu_2 = \nu_{22} sign(\omega)$; ω is vibration frequency; $J^2 = -1$; $u_{ik0} = u_{ik}\left(\frac{x}{2}, 0\right), u_{ika} = u_{ik}\left(\frac{x}{2}, b\right)$ and $u_{ik1} = u_{ik}(x_1, y_1)$ are values of the mode shapes at the points where the forces are applied and at the point where the liquid joint dynamic damper is installed; εp_0 is the amplitude value of the base acceleration; $u_{ik*} = u_{ik0} + u_{ik1}$.

In order to analyze the stability of the system under consideration, the spectral density of the base acceleration in the expressions of mean square deviations (1) is obtained as follows [15]:

$$S_{W_0}(\omega) = \frac{D_{W_0} \varkappa \upsilon^3}{\pi(\upsilon^2 - \omega^2 + J\varkappa\upsilon\omega)(\upsilon^2 - \omega^2 - J\varkappa\upsilon\omega)}, \tag{3}$$

where D_{W_0} is the dispersion of base acceleration; $\varkappa$ is a parameter characterizing the spectrum width of vibrations; υ is a dominant frequency of vibration.

After putting the expression of the spectral density of the base acceleration (3) into the system of equations (1).

$$\sigma_{ik}^2 = \frac{D_{W_0} \varkappa \upsilon^3 (u_{ik*} \varepsilon p_0)^2}{\pi} \int_{-\infty}^{\infty} \frac{\Psi_1^2 + \Psi_2^2}{(\upsilon^2 - \omega^2 + J\varkappa\upsilon\omega)(\upsilon^2 - \omega^2 - J\varkappa\upsilon\omega)(\Upsilon_1^2 + \Upsilon_2^2)} d\omega\,; \tag{4}$$

$$\sigma_{3*}^2 = \frac{D_{W_0} \varkappa \upsilon^3 (u_{ik*} \varepsilon p_0)^2}{\pi} \int_{-\infty}^{\infty} \frac{\Psi_3^2 + \Psi_4^2}{(\upsilon^2 - \omega^2 + J\varkappa\upsilon\omega)(\upsilon^2 - \omega^2 - J\varkappa\upsilon\omega)(\Upsilon_1^2 + \Upsilon_2^2)} d\omega\,; \tag{5}$$

$$\sigma_{4*}^2 = \frac{D_{W_0} \varkappa \upsilon^3 (u_{ik*} \varepsilon p_0)^2}{\pi} \int_{-\infty}^{\infty} \frac{\Psi_5^2 + \Psi_6^2}{(\upsilon^2 - \omega^2 + J\varkappa\upsilon\omega)(\upsilon^2 - \omega^2 - J\varkappa\upsilon\omega)(\Upsilon_1^2 + \Upsilon_2^2)} d\omega. \tag{6}$$

In order to find the values of integrals in expressions (4) - (6) using the calculation method given in [21]. Calculations are stated for expression (4). To do this, it is necessary to transform this integral expression into the following form:

$$I_1 = \int_{-\infty}^{\infty} \frac{P(\omega)}{\mathbb{Z}(J\omega)\,\mathbb{Z}(-J\omega)} d\omega, \tag{7}$$

where $P(\omega) = c_{n-1}^{*}\omega^{2n-2} + c_{n-2}^{*}\omega^{2n-4} + \cdots + c_0^{*}; \mathbb{Z}(J\omega) = d_n^{*}(J\omega)^n + d_{n-1}^{*}(J\omega)^{n-1} + \cdots + d_0^{*}$.

In order to explore the stability of the transverse vibrations of the vibration protected plate under influence random excitations, the integrals will be made in the expressions (4) - (6) to form (7) and determine the mean square deviations.

3 Results and discussion

In order to explore the stability of the transverse vibrations of the vibration protected plate in random excitations, first the integral will be made in the expression (4) into the form (7). For this, the expression $\Upsilon_1^2(\omega) + \Upsilon_2^2(\omega)$ writes in the form

$$\Upsilon_1^2(J\omega) + \Upsilon_2^2(J\omega) = \big(\Upsilon_1(J\omega) + J\Upsilon_2(J\omega)\big)\big(\Upsilon_1(J\omega) - J\Upsilon_2(J\omega)\big) \tag{8}$$

The expressions will be analyzed $\Upsilon_1(J\omega) + J\Upsilon_2(J\omega)$ and $\Upsilon_1(J\omega) - J\Upsilon_2(J\omega)$.

$$\begin{gathered}\Upsilon_1(J\omega) = \tau_6(J\omega)^6 + \tau_5(J\omega)^5 + \tau_4(J\omega)^4 + J\tau_3(J\omega)^3 + \tau_2(J\omega)^2 + J\tau_1(J\omega) + \tau_0; \\ \Upsilon_2(J\omega) = J\phi_5(J\omega)^5 + \phi_4(J\omega)^4 + J\phi_3(J\omega)^3 + \phi_2(J\omega)^2 + J\phi_1(J\omega) + \phi_0,\end{gathered} \tag{9}$$

where $\tau_0 = 2c_1c_2c_{1ik}; \tau_1 = (b_Sc_1 + 2c_2b_F)c_{2ik};$

$\tau_2 = 2c_1c_2(m_{ik} + u_{ik1}^2M_1) + (2c_2M_1 + c_1M_4 + b_Fb_S)c_{1ik}; \tau_3 = (b_SM_1 + b_FM_4)c_{2ik};$

$\tau_4 = (2c_2M_1 + c_1M_4)m_{ik} + (m_{ik} + u_{ik1}^2M_1)b_Fb_S + \Delta(c_{1ik} + c_1u_{ik1}^2);$

$\tau_5 = 0;\ \tau_6 = \Delta m_{ik}; \phi_0 = 2c_1c_2\theta_{2ik};\ \phi_1 = -(b_Sc_1 + 2c_2b_F)c_{1ik};$

$\phi_2 = (2c_2M_1 + c_1M_4 + b_Fb_S)c_{2ik};$

$\phi_3 = -\Big((b_SM_1 + b_FM_4)c_{1ik} + (m_{ik} + u_{ik1}^2M_1)(b_Sc_1 + 2c_2b_F)\Big);$

$\phi_4 = \Delta c_{2ik}; \phi_5 = -\Delta b_Fu_{ik1}^2 - (b_SM_1 + b_FM_4)m_{ik};$

According to the method of calculating the integral mentioned above, expressions (9) should have real coefficients, but they contain complex coefficients. In order to eliminate these complex coefficients, expressions (9) will be put into expression (8) and, replacing it in the integral (4), multiply the numerator and denominator of the fraction by the following multiplier:

$$P_1(J\omega) = \big(\Upsilon_{1a}(J\omega) - J\Upsilon_{2a}(J\omega)\big)\big(\Upsilon_{1b}(J\omega) - J\Upsilon_{2b}(J\omega)\big), \tag{10}$$

where

$\Upsilon_{1a}(J\omega) = \tau_6(J\omega)^6 + \phi_5(J\omega)^5 + \tau_4(J\omega)^4 + \phi_3(J\omega)^3 + \tau_2(J\omega)^2 + \phi_1(J\omega) + \tau_0;$

$\Upsilon_{2a}(J\omega) = -\phi_4(J\omega)^4 + \tau_3(J\omega)^3 - \phi_2(J\omega)^2 + \tau_1(J\omega) - \phi_0;$

$\Upsilon_{1b}(J\omega) = \tau_6(J\omega)^6 - \phi_5(J\omega)^5 + \tau_4(J\omega)^4 - \phi_3(J\omega)^3 + \tau_2(J\omega)^2 - \phi_1(J\omega) + \tau_0;$

$\Upsilon_{2b}(J\omega) = \phi_4(J\omega)^4 + \tau_3(J\omega)^3 + \phi_2(J\omega)^2 + \tau_1(J\omega) + \phi_0.$

Then, the integral expression (7) will be

$$P(\omega) = (\Psi_1^2 + \Psi_2^2)Re\big(P_1(J\omega)\big);$$

$\mathbb{Z}(J\omega) = \big(\Upsilon_1(J\omega) + J\Upsilon_2(J\omega)\big)\big(\Upsilon_{1a}(J\omega) - J\Upsilon_{2a}(J\omega)\big))(\upsilon^2 + (J\omega)^2 + \varkappa\upsilon(J\omega));$

$\mathbb{Z}(-J\omega) = \big(\Upsilon_1(J\omega) - J\Upsilon_2(J\omega)\big)\big(\Upsilon_{1b}(J\omega) - J\Upsilon_{2b}(J\omega)\big))(\upsilon^2 + (J\omega)^2 - \varkappa\upsilon(J\omega)),$

After simplifications, the result is:

$$n = 14; P(\omega) = m_{ik}^2(c_{13}^{*}\omega^{26} + c_{12}^{*}\omega^{24} + \cdots + c_0^{*});$$

$$\mathbb{Z}(J\omega) = d_{14}^*(J\omega)^{14} + d_{13}^*(J\omega)^{13} + \cdots + d_0^*, \tag{11}$$

where $c_0^* = \Delta_1\Delta_f$; $c_1^* = \Delta_1\Delta_a + \Delta_4\Delta_f$; $c_2^* = \Delta_1\Delta_b + \Delta_4\Delta_a + \Delta_2\Delta_f$;

$c_3^* = \Delta_1\Delta_c + \Delta_4\Delta_b + \Delta_2\Delta_a + \Delta_3\Delta_f$; $c_4^* = \Delta_1\Delta_d + \Delta_4\Delta_c + \Delta_2\Delta_b + \Delta_3\Delta_a + \Delta^2\Delta_f$;

$c_5^* = \Delta_1\Delta_e + \Delta_4\Delta_d + \Delta_2\Delta_c + \Delta_3\Delta_b + \Delta^2\Delta_a$; $c_6^* = \Delta_1\tau_6^2 + \Delta_4\Delta_e + \Delta_2\Delta_d + \Delta_3\Delta_c + \Delta^2\Delta_b$;

$c_7^* = \Delta_4\tau_6^2 + \Delta_2\Delta_e + \Delta_3\Delta_d + \Delta^2\Delta_c$; $c_8^* = \Delta_2\tau_6^2 + \Delta_3\Delta_e + \Delta^2\Delta_d$;

$c_9^* = \Delta_3\tau_6^2 + \Delta^2\Delta_e$; $c_{10}^* = \Delta^2\tau_6^2$; $c_{11}^* = c_{12}^* = c_{13}^* = 0$; $\Delta_1 = (2c_{1*}c_{2*})^2$;

$\Delta_2 = 4c_1c_2\Delta + (2c_2M_1 + c_1M_4 + b_Fb_S)^2 - 2(b_Sc_1 + 2c_2b_F)(b_SM_1 + b_FM_4)$;

$\Delta_3 = -2\Delta(2c_2M_1 + c_1M_4 + b_Fb_S) + (b_SM_1 + b_FM_4)^2$;

$\Delta_4 = -4c_1c_2(2c_2M_1 + c_1M_4 + b_Fb_S) + (b_Sc_1 + 2c_2b_F)^2$; $\Delta_f = \tau_0^2 + \phi_0^2$;

$\Delta_a = -\tau_1^2 + 2\tau_0\tau_2 - \phi_1^2 + 2\phi_0\phi_2$; $\Delta_b = \tau_2^2 + 2\tau_0\tau_4 - 2\tau_1\tau_3 + \phi_2^2 + 2\phi_0\phi_4 - 2\phi_1\phi_3$;

$\Delta_c = -\tau_3^2 + 2\tau_0\tau_6 + 2\tau_2\tau_4 - \phi_3^2 - 2\phi_1\phi_5 + 2\phi_2\phi_4$; $\Delta_d = \tau_4^2 + 2\tau_2\tau_6 + \phi_4^2 - 2\phi_3\phi_5$;

$\Delta_e = 2\tau_4\tau_6 - \phi_5^2$; $d_0^* = \upsilon^2\Delta_f$; $d_1^* = \varkappa\upsilon\Delta_f - 2\phi_a\upsilon^2$; $d_2^* = \Delta_f - 2\varkappa\upsilon\phi_a + \phi_b\upsilon^2$;

$d_3^* = -2\phi_a + \varkappa\upsilon\phi_b + 2\phi_c\upsilon^2$; $d_4^* = \phi_b + 2\varkappa\upsilon\phi_c + \phi_d\upsilon^2$; $d_5^* = 2\phi_c + \varkappa\upsilon\phi_d - 2\phi_e\upsilon^2$;

$d_6^* = \phi_d - 2\varkappa\upsilon\phi_e + \phi_f\upsilon^2$; $d_7^* = -2\phi_e + \varkappa\upsilon\phi_f + 2\phi_g\upsilon^2$; $d_8^* = \phi_f + 2\varkappa\upsilon\phi_g + \phi_h\upsilon^2$;

$d_9^* = 2\phi_g + \varkappa\upsilon\phi_h - 2(\tau_4\phi_5 + \tau_6\phi_3)\upsilon^2$;

$d_{10}^* = \phi_h - 2\varkappa\upsilon(\tau_4\phi_5 + \tau_6\phi_3) + (\phi_5^2 + 2\tau_4\tau_6)\upsilon^2$;

$d_{11}^* = -2(\tau_4\phi_5 + \tau_6\phi_3) + \varkappa\upsilon(\phi_5^2 + 2\tau_4\tau_6) - 2\tau_6\phi_5\upsilon^2$;

$d_{12}^* = \upsilon^2\tau_6^2 - 2\varkappa\upsilon\tau_6\phi_5 + \phi_5^2 + 2\tau_4\tau_6$; $d_{13}^* = \varkappa\upsilon\tau_6^2 - 2\tau_6\phi_5$;

$d_{14}^* = \tau_6^2$; $\phi_a = \tau_0\phi_1 - \tau_1\phi_0$; $\phi_b = \tau_1^2 + 2\tau_0\tau_2 + \phi_1^2 + 2\phi_0\phi_2$;

$\phi_c = \tau_1\phi_2 - \tau_0\phi_3 + \tau_3\phi_0 - \tau_2\phi_1$; $\phi_d = \tau_2^2 + 2\tau_0\tau_4 + 2\tau_1\tau_3 + \phi_2^2 + 2\phi_0\phi_4 + 2\phi_1\phi_3$;

$\phi_e = \tau_0\phi_5 - \tau_1\phi_4 + \tau_2\phi_3 - \tau_3\phi_2 + \tau_4\phi_1$;

$\phi_f = \tau_3^2 + 2\tau_0\tau_6 + 2\tau_2\tau_4 + \phi_3^2 + 2\phi_1\phi_5 + 2\phi_2\phi_4$;

$\phi_g = \tau_3\phi_4 - \tau_2\phi_5 - \tau_4\phi_3 - \tau_6\phi_1$; $\phi_h = \tau_4^2 + 2\tau_2\tau_6 + \phi_4^2 + 2\phi_3\phi_5$.

If the value of the integral (7) is calculated based on the determined coefficients of the expressions (11) according to the method presented in [21], it will be as follows:

$$I_{2*} = \frac{\pi m_{ik}^2}{d_{14}^*} \frac{\begin{vmatrix} c_{13}^* & c_{12}^* & c_{11}^* & \dots & c_1^* & c_0^* \\ -d_{14}^* & d_{12}^* & -d_{10}^* & d_8^* & \dots & 0 \\ 0 & -d_{13}^* & d_{11}^* & -d_9^* & \dots & 0 \\ 0 & d_{14}^* & -d_{12}^* & d_{10}^* & \dots & 0 \\ 0 & \dots & & & \dots & 0 \\ 0 & \dots & & \dots & -d_2^* & d_0^* \end{vmatrix}}{\begin{vmatrix} d_{13}^* & -d_{11}^* & d_9^* & \dots & & 0 \\ -d_{14}^* & d_{12}^* & -d_{10}^* & d_8^* & \dots & 0 \\ 0 & -d_{13}^* & d_{11}^* & -d_9^* & \dots & 0 \\ 0 & d_{14}^* & -d_{12}^* & d_{10}^* & \dots & 0 \\ 0 & \dots & & & \dots & 0 \\ 0 & \dots & & \dots & -d_2^* & d_0^* \end{vmatrix}}. \tag{12}$$

So, based on the value of the integral (12), the mean square value of the vibrations of the hysteresis-type plate with elastic dissipative characteristics under the influence of random excitations is as follows:

$$\sigma_{ik}^2 = \frac{D_{W_0}\varkappa\upsilon^3(u_{ik*}\varepsilon p_0)^2}{\pi} I_{2*}. \tag{13}$$

The determined mean square values (13) allow to analyze the dynamics and stability of transverse vibrations of a hysteresis-type plate with elastic dissipative characteristics under the influence of random excitations, depending on the system parameters. For this purpose, using the expression of mean square values (13) and the method of vertical tangents, the stability of the transverse vibrations of the hysteresis-type plate with elastic dissipative characteristics under the influence of random excitations will be explored.

For this, the condition of existence of vertical tangents transferred to the graph of the function σ_{ik} are analyzed. In this case, the condition for the existence of vertical tangents transferred to the graph of the function σ_{ik} is as follows:

$$2\sigma_{ik} - \frac{D_{W_0}\varkappa\upsilon^3(u_{ik*}\varepsilon p_0)^2}{\pi}\frac{\partial I_{2*}}{\partial\sigma_{ik}} = 0. \quad (14)$$

Equation (14) has a solution when the condition $\frac{\partial I_{2*}}{\partial\sigma_{ik}} \geq 0$ is fulfilled. Values of the dominant frequency υ that satisfy the equation (14) represent the border of stability of the considered system.

$$\frac{\partial I_{2*}}{\partial\sigma_{ik}} = \frac{2\pi\sigma_{ik}}{D_{W_0}\varkappa\upsilon^3(u_{ik*}\varepsilon p_0)^2}. \quad (15)$$

By the nature of the vertical tangents method, if there are no vertical tangents transferred to the graph in question, this motion takes stability. Therefore, it is enough that $\frac{\partial I_{2*}}{\partial\sigma_{ik}}$ is negative definite so that the condition of existence of vertical tangents (14) is not fulfilled.

$$\frac{\partial I_{2*}}{\partial\sigma_{ik}} < 0. \quad (16)$$

The obtained condition (16) is considered as the stability condition of transverse vibrations of the hysteresis-type plate with elastic dissipative characteristics under the influence of random excitations, and it allows to determine different values of system parameters corresponding to stable and instable vibrations.

4 Conclusion

1. The stability of nonlinear transverse vibrations of a hysteresis-type elastic dissipative characteristic plate combined with a liquid section dynamic absorber under the influence of random excitations was investigated.
2. Based on the method of vertical tangents, the stability condition was determined, and the condition for its solution was determined.
3. The stability of the transverse vibrations of the hysteresis-type plate with elastic dissipative characteristics under the influence of random excitations can be analyzed depending on the system parameters according to determined stability condition.

References

1. J. Villarroel, J. Comput. Appl. Math. **158**, 225 (2003)
2. W.C. Xie, J. Sound Vib **239**, 139 (2001)
3. H. Zhang, Zh. Wu, Y. Xi, Journal of Automatica **50**, 599 (2014)
4. L. Guo, M. Jia, Li R., S. Zhang, Int. J. Steel Struct. **13**, 163 (2013)
5. P. Nogueiro, L.S.da Silva, R. Bento, *Numerical Implementation and calibration of a hysteretic model for cyclic response of end-plate beam-to-column steel joints under arbitrary cyclic loading*, in proceedings of the world congress on computational

methods in engineering and science EPMESC X, 21-23 August. 2006, Hainan, China (2006)
6. M. Sayed, A. Mousa, Math. Probl. Eng **2013**, 26 (2013). http://dx.doi.org/10.1155/2013/418374
7. M. Skeen, N. Kessissoglou, *Extraction of wavenumbers and amplitudes from plate vibration measurements,* in proceedings of the 14th international congress on sound and vibration, 9-12 July 2007, Cairns, Australia (2007)
8. Y. Baccouche, M. Bentahar, Ch. Mechri, R.El. Guerjouma, J. Acoust. Soc. Am. **133**, 153 (2013)
9. V. Dogan, Y. Erkal, *Vibration analysis of mindlin's sandwich plate (FSDT) under random excitation*, in proceedings of the 14th international congress on sound and vibration, 9-12 July 2007, Cairns, Australia (2007)
10. Z. Antonio, I. Giovanni, P. Francesco, Sh. Tetyana, Shock Vib. **2020**, 23 (2020) https://doi.org/10.1155/2020/8882867
11. L. Wang, Y. Zhao, Q. Zhu, Y. Liu, Q. Han, ISIJ int. **60**, 1237 (2020)
12. L. Zhili, Ch. Xiliang, Zh. Bo, L. Zhili, Ch. Xiliang, *Energy finite element analysis of vibrating thin plates at high frequency,* in IEEE International conference on cybernetics and intelligent systems and IEEE conference on robotics, automation and mechatronics, Ningbo, China (2017), https://doi.org/10.1109/ICCIS.2017.827478
13. D. Zhang, Y. Tang, L. Chen, Eur J Mech A Solids **75**, 142 (2019) https://doi.org/10.1016/j.euromechsol.2019.01.02
14. M.M. Mirsaidov, M.N. Sidikov, K.M. Turajonov, E3S Web Conf. **365**, (2023) https://doi.org/10.1051/e3sconf/202336504017
15. M.A. Pavlovsky, L.M. Ryzhkov, V.B. Yakovenko, O.M. Dusmatov, Nonlinear problems of the dynamics of vibration protection systems (Technique, 1997)
16. M.M. Mirsaidov, O.M. Dusmatov, M.U. Khodjabekov, J. Phys. Conf. Ser. **1921**, (2021) https://doi:10.1088/1742-6596/1921/1/012097
17. M.M. Mirsaidov, O.M. Dusmatov, M.U. Khodjabekov, AIP Conf. Proc. **2637**, (2021) https://doi.org/10.1063/5.0118289
18. M.U. Khodjabekov, Kh.M. Buranov, A.E. Qudratov, AIP Conf. Proc. **2637**, (2021) https://doi.org/10.1063/5.0118292
19. M.M. Mirsaidov, O.M. Dusmatov, M.U. Khodjabekov, PNRPU Mech. Bull. **3**, 51 (2022) https://doi.org/10.15593/perm.mech/2022.3.06
20. G.S. Pisarenko, O.E. Boginich, Oscillations of kinematically excited mechanical systems taking into account energy dissipation, (Nauk, Dumka, 1981)
21. J.B. Roberts, P.D. Spanos, Random vibrations and statistical linearization, (Dover publications press, New York, 2003)