

Nonlinear dynamics of rotor of an auto-balancing ball device, considering eccentricity and changes in horizontal axes of rotation

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Abstract. This study is devoted to the dynamics of an auto-balancing device (ABD) when the rotor rotates at critical angular speeds. In this article, a model of an auto-balancing device (ABD) is proposed. The rotor is not only eccentric about the symmetry axis but also has a horizontal axis of rotation. A mathematical model is proposed based on the Lagrange equations; it describes the nonlinear motion of a balancing system (BS) with two running lines with a different number of balls, considering eccentricity, angular error, and changes in the horizontal axis of rotation. The nonlinear dynamics of the rotor motion with two running lines and two balls at a constant rotation speed and in the accelerating mode of the rotor motion is studied by a numerical method, taking into account the change in the horizontal axis of rotation.

1 Introduction

The problem of balancing rapidly rotating rotors is especially acute with the emergence of mechanisms such as gyroscopic inertial navigation devices and gas turbine engines. A sufficient number of scientific publications were devoted to the problem of rotor dynamics with balancing devices.

These publications can be divided into two parts. One part relates to an increase in the angular speed of the rotor in the subcritical zone; it is reduced to a design with high values of the rotor rigidity, and the other part relates to the issue of a smoother passage of the critical speed, which allows using the designs with thin rotors mounted on elastic supports. The use of balancing systems allows the rotor to more smoothly pass the critical angular speed and rotate at critical angular speeds while the loads on the supports remain within the allowable range.

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General issues of the dynamics of various types of rotors mounted on elastic supports (such as determining the critical speeds for various models of the rotor and supports, stability of partial motions, dynamic characteristics for elastic and combined supports, and the influence of rotor parameters on critical speed) are considered in [1-10].

Various balancing devices and methods to solve the problem of motion at critical speeds are considered in numerous studies. Researchers often use auto-balancing devices for dynamic balancing since, unlike complex electronic balancing systems, they are quite simple to manufacture.

Fundamental results were obtained in [11-18], where the dynamics of the motion of an unbalanced rotor mounted on an elastic shaft is considered, taking into account the eccentricity of the center of the auto-balancing device (ABD) during installation. The ABD presents a flat disk with a running line for balancing balls, and the rotor is modeled as a flat thin disk that performs a plane motion. The running line has a circular or elliptical path.

When the running line of balancing balls has a circular path, the equations of motion are obtained in the form of the Lagrange equations in generalized coordinates. In the coordinate system rotating with the rotor, the conditions for the existence of stationary motion with different arrangements of balls along the running line are obtained, and the issues of rotor acceleration when passing through the critical angular velocity and stability of partial motions are investigated.

In the case when the running path of the balancing balls is an ellipse, it turns out that the full balancing mode is not realizable, but a semi-balanced stable stationary motion exists. In these studies, the issue of the rotor dynamics in various modes, the stability of stationary motions using the A.M. Lyapunov methods, and a rather deep mechanical analysis of the model of a single-disk rotor with a multi-row balancing mechanism were considered. It was shown that the law of motion of balancing balls along the running line depends on external damping forces, friction forces, and the eccentricity of the balancing device (BD).

In [10], the issues of stable periodic motions of a vertically located rotor and the causes of the oscillatory motion were considered using classical methods of the theory of nonlinear oscillations. Numerical results were obtained for specific values of the system parameters.

References [22-23] consider the problem of balancing a horizontally located rigid rotor with two balancing devices, each containing two balls. The issues of stability of balanced stationary motions and the issues of numerical solution with various initial conditions were considered. In contrast to [13], the eccentricity of the balancing system is not taken into account.

Below we consider a model of an auto-balancing device, where the running lines can perform a rotational motion about a certain axis in a horizontal plane rotating together with the rotor. In a particular case, equations that consider the angular error of the balancing device can be easily obtained from the equations of motion.

2 Methods

2.1 Mechanical model

A rotor model is considered a rigid cylinder fixed along the vertical longitudinal axis with an elastic shaft on two supports. The model considered in [12] was taken as a mathematical model. The running lines for balls can rotate about a horizontal axis, and the rotor performs a plane motion. Below (Fig. 1), for convenience and comparison, we will keep the parameters' designations in [1].

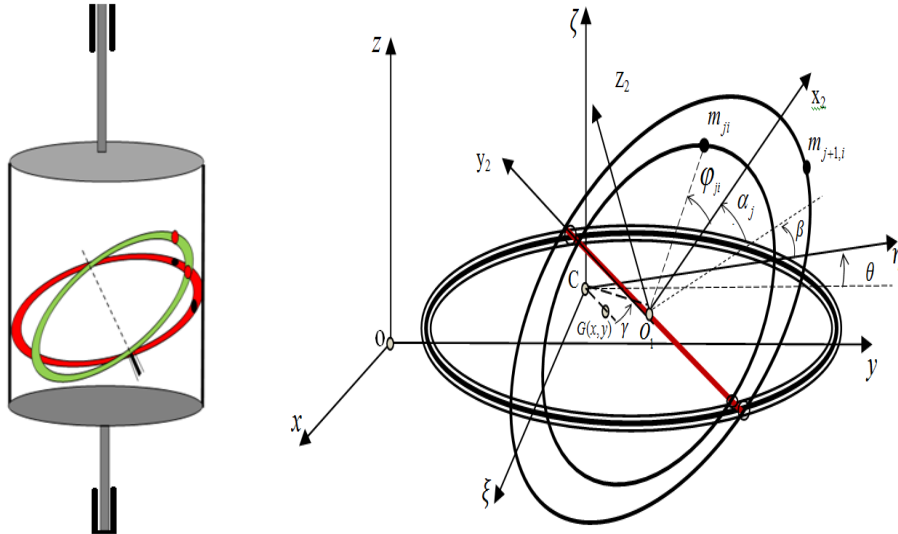


Fig 1. Calculation scheme of the auto-balancing device

2.2 Mathematical model

Since the constraints imposed on the system are geometric, the Lagrange equation in independent coordinates was used to build a mathematical model. In this model, the system has $s \times n + 3$ degrees of freedom. Coordinates x and y of point O , the angle of rotation of the rotor θ , angles α_j ($j = 1, \dots, s$) between the coordinate plane Oxy and the plane of the tubes, and angles φ_{ji} ($i = 1, \dots, n$; $j = 1, \dots, s$) for determining the positions of the balls inside the tubes (Fig. 1) are taken as the generalized coordinates. The kinetic energy of the system under consideration has the following form:

$$T = \frac{1}{2} \bar{m}_1 \bar{v}_G^2 + \frac{1}{2} J_G \dot{\theta}^2 + \frac{1}{2} \bar{m}_2 \bar{v}_{O_1}^2 + \frac{1}{2} \sum_{j=1}^s (J_{y_{j2}} \dot{\alpha}_j^2 + J_{x_{j2}} \dot{\theta}^2 \sin^2 \alpha_j + J_{z_{j2}} \dot{\theta}^2 \cos^2 \alpha_j) + \frac{1}{2} \sum_{j=1}^s \sum_{i=1}^n m_{ji} \bar{v}_{ji}^2$$

where $\bar{v}_G = (\dot{x} - s_1 \sin \theta \dot{\theta}) \vec{i} + (\dot{y} + s_1 \cos \theta \dot{\theta}) \vec{j}$ is the speed of the center of mass of the cylinder; $\bar{v}_{O_1} = (\dot{x} - s_2 \sin(\theta + \gamma) \dot{\theta}) \vec{i} + (\dot{y} + s_2 \cos(\theta + \gamma) \dot{\theta}) \vec{j}$ is the speed of the center of mass of the balancing mechanism;

$$\begin{aligned} \bar{v}_{ji} = & (\dot{x} - (s_2 \sin(\theta + \beta) + r_j \sin \varphi_{ji} \sin \delta - r_j \cos \varphi_{ji} \cos \delta \cos \alpha_j) \dot{\theta} - \cos \varphi_{ji} \sin \alpha_j \sin \delta r_j \dot{\alpha}_j + \\ & + (\cos \varphi_{ji} \cos \delta - \sin \varphi_{ji} \cos \alpha_j \sin \delta) r_j \dot{\varphi}_{ji}) \vec{i} + (\dot{y} + (s_2 \cos(\theta + \beta) + r_j \sin \varphi_{ji} \cos \delta + \\ & + r_j \cos \varphi_{ji} \cos \alpha_j \sin \delta) \dot{\theta} + r_j \dot{\alpha}_j \cos \varphi_{ji} \sin \alpha_j \cos \delta + r_j \dot{\varphi}_{ji} (\cos \varphi_{ji} \sin \delta + \sin \varphi_{ji} \cos \alpha_j \cos \delta)) \vec{j} + \\ & + (r_j \cos \varphi_{ji} \cos \alpha_j \dot{\alpha}_j - r_j \dot{\varphi}_{ji} \sin \varphi_{ji} \sin \alpha_j) \vec{k}, \delta = \theta + \gamma + \beta, (j = 1, \dots, s; i = 1, \dots, n) \end{aligned}$$

is the speed of the ball located in the tube with serial number j ; $\bar{m}_1, \bar{m}_2, m_i$ are the masses of the rotor, balancing device, and ball, respectively; J_G , is the moment of inertia relative to the principal axis of the rotor; $J_{xj_2}, J_{yj_2}, J_{zj_2}$ are the moments of inertia of the tube relative to the principal axes; $\vec{i}, \vec{j}, \vec{k}$ is the corresponding vector basis of the fixed coordinate system. Accordingly, the kinetic energy of the system under consideration in generalized coordinates has the following form:

$$\begin{aligned}
 T = & \frac{1}{2}(\bar{m}_1 + \bar{m}_2 + \sum_{j=1}^s \sum_{i=1}^n m_{ji})(\dot{x}^2 + \dot{y}^2) + \frac{1}{2} \sum_{j=1}^s (J_G + J_{zj_2} \cos^2 \alpha_j + J_{xj_2} \sin^2 \alpha_j) \dot{\theta}^2 + \\
 & + \frac{1}{2}(\bar{m}_1 s_1^2 + \bar{m}_2 s_2^2 + \sum_{j=1}^s \sum_{i=1}^n m_{ji}(s_2^2 + r_j^2 (\sin^2 \varphi_{ji} + \cos^2 \varphi_{ji} \cos^2 \alpha_j) + 2r_j s_2 (\sin \varphi_{ji} \cos \beta + \\
 & + \cos \varphi_{ji} \cos \alpha_j \sin \beta)) \dot{\theta}^2 + \frac{1}{2} (\sum_{j=1}^s \sum_{i=1}^n m_{ji} r_j^2 \cos^2 \varphi_{ji} + \sum_{j=1}^s J_{y_{j_2}}) \dot{\alpha}_j^2 + \frac{1}{2} \sum_{j=1}^s \sum_{i=1}^n m_{ji} r_j^2 \dot{\varphi}_{ji}^2 + \\
 & + [\bar{m}_1 s_1 (\dot{y} \cos \theta - \dot{x} \sin \theta) + \bar{m}_2 s_2 (\dot{y} \cos(\theta + \gamma) - \dot{x} \sin(\theta + \gamma)) + \sum_{j=1}^s \sum_{i=1}^n m_{ji} (s_2 (\dot{y} \cos(\theta + \gamma) - \\
 & - \dot{x} \sin(\theta + \gamma)) + r_j \sin \varphi_{ji} (\dot{y} \cos(\delta) - \dot{x} \sin(\delta)) + r_j \cos \varphi_{ji} \cos \alpha_j (\dot{x} \cos(\delta) + \dot{y} \sin(\delta))) \dot{\theta} + \\
 & + \sum_{j=1}^s \sum_{i=1}^n (m_{ji} (r_j \cos \varphi_{ji} (\dot{y} \sin \delta + \dot{x} \cos \delta) + \sin \varphi_{ji} \cos \alpha_j (\dot{y} \cos \delta - \dot{x} \sin \delta)) \dot{\varphi}_{ji} + \\
 & + \sum_{j=1}^s \sum_{i=1}^n m_{ji} r_j \cos \varphi_{ji} \sin \alpha_j (\dot{y} \cos \delta - \dot{x} \sin \delta) \dot{\alpha}_j + [\sum_{j=1}^s \sum_{i=1}^n m_{ji} (s_2 r_j (\cos \varphi_{ji} \sin \beta + \\
 & \sin \varphi_{ji} \cos \alpha_j \cos \beta) + r_j^2 \cos \alpha_j) \dot{\theta} \dot{\varphi}_{ji}] + \sum_{j=1}^s \sum_{i=1}^n m_{ji} (s_2 r_j \cos \varphi_{ji} \sin \alpha_j \cos \beta + \\
 & + r_j^2 \sin \varphi_{ji} \cos \varphi_{ji} \sin \alpha_j) \dot{\alpha}_j \dot{\theta}.
 \end{aligned} \tag{1}$$

Taking into account the potential energy $\Pi = \frac{1}{2} k(x^2 + y^2)$ of the elastic shaft and the dissipative function

$$F = \frac{1}{2} (c(\dot{x}^2 + \dot{y}^2) + c_\theta \dot{\theta}^2 + c_\varphi \sum_{j=1}^s \sum_{i=1}^n \dot{\varphi}_{ji}^2 + c_\alpha \sum_{j=1}^s \dot{\alpha}_j^2) \tag{2}$$

with (1), (2), the Lagrange equation has the following form:

$$\begin{aligned}
& (\bar{m}_1 + \bar{m}_2 + \sum_{j=1}^s \sum_{i=1}^n m_{ji}) \ddot{x} + c\dot{x} + kx = \bar{m}_1 s_1 (\dot{\theta}^2 \cos \theta + \ddot{\theta} \sin \theta) + (\bar{m}_2 s_2 + \sum_{j=1}^s \sum_{i=1}^n m_{ji} s_2) (\cos(\theta + \beta) \dot{\theta}^2 + \\
& + \sin(\theta + \gamma) \ddot{\theta}) + \sum_{j=1}^s \sum_{i=1}^n m_{ji} r_j [(\sin \varphi_{ji} \cos \delta + \cos \varphi_{ji} \cos \alpha_j \sin \delta) \dot{\theta}^2 + (\sin \varphi_{ji} \cos \delta + \\
& + \cos \varphi_{ji} \cos \alpha_j \sin \delta) \dot{\varphi}_{ji}^2 + \cos \varphi_{ji} \cos \alpha_j \sin \delta \dot{\alpha}_j^2 + (\sin \varphi_{ji} \cos \alpha_j \sin \delta - \cos \varphi_{ji} \cos \delta) \ddot{\varphi}_{ji} + \\
& + (\sin \varphi_{ji} \sin \delta - \cos \varphi_{ji} \cos \alpha_j \cos \delta) \ddot{\theta} + \cos \varphi_{ji} \sin \alpha_j \sin \delta \ddot{\alpha}_j + 2(\cos \varphi_{ji} \sin \delta + \\
& + \sin \varphi_{ji} \cos \alpha_j \cos \delta) \dot{\varphi}_{ji} \dot{\theta} + 2 \cos \varphi_{ji} \sin \alpha_j \cos \delta \dot{\alpha}_j \dot{\theta} - 2 \sin \varphi_{ji} \sin \alpha_j \sin \delta \dot{\varphi}_{ji} \dot{\alpha}_j], \\
& (\bar{m}_1 + \bar{m}_2 + \sum_{j=1}^s \sum_{i=1}^n m_{ji}) \ddot{y} + c\dot{y} + ky = \bar{m}_1 s_1 (\dot{\theta}^2 \sin \theta - \ddot{\theta} \cos \theta) + (\bar{m}_2 s_2 + \sum_{j=1}^s \sum_{i=1}^n m_{ji} s_2) (\sin(\theta + \beta) \dot{\theta}^2 - \\
& - \cos(\theta + \beta) \ddot{\theta}) + \sum_{j=1}^s \sum_{i=1}^n m_{ji} r_j [(\sin \varphi_{ji} \sin \delta - \cos \varphi_{ji} \cos \alpha_j \cos \delta) \dot{\theta}^2 + (\sin \varphi_{ji} \sin \delta - \\
& \cos \varphi_{ji} \cos \alpha_j \cos \delta) \dot{\varphi}_{ji}^2 - \cos \varphi_{ji} \cos \alpha_j \cos \delta \dot{\alpha}_j^2 - (\sin \varphi_{ji} \cos \alpha_j \cos \delta + \cos \varphi_{ji} \sin \delta) \ddot{\varphi}_{ji} - \\
& - (\sin \varphi_{ji} \cos \delta + \cos \varphi_{ji} \cos \alpha_j \sin \delta) \ddot{\theta} - \cos \varphi_{ji} \sin \alpha_j \cos \delta \ddot{\alpha}_j - 2(\cos \varphi_{ji} \cos \delta - \\
& \sin \varphi_{ji} \cos \alpha_j \sin \delta) \dot{\varphi}_{ji} \dot{\theta} + 2 \cos \varphi_{ji} \sin \alpha_j \sin \delta \dot{\alpha}_j \dot{\theta} + 2 \sin \varphi_{ji} \sin \alpha_j \cos \delta \dot{\varphi}_{ji} \dot{\alpha}_j], \\
& (\bar{m}_1 + \bar{m}_2 + \sum_{j=1}^s \sum_{i=1}^n m_{ji}) \ddot{z} + c\dot{z} + kz = \bar{m}_1 s_1 (\dot{\theta}^2 \sin \theta - \ddot{\theta} \cos \theta) + (\bar{m}_2 s_2 + \sum_{j=1}^s \sum_{i=1}^n m_{ji} s_2) (\sin(\theta + \beta) \dot{\theta}^2 - \\
& - \cos(\theta + \beta) \ddot{\theta}) + \sum_{j=1}^s \sum_{i=1}^n m_{ji} r_j [(\sin \varphi_{ji} \sin \delta - \cos \varphi_{ji} \cos \alpha_j \cos \delta) \dot{\theta}^2 + (\sin \varphi_{ji} \sin \delta - \\
& \cos \varphi_{ji} \cos \alpha_j \cos \delta) \dot{\varphi}_{ji}^2 - \cos \varphi_{ji} \cos \alpha_j \cos \delta \dot{\alpha}_j^2 - (\sin \varphi_{ji} \cos \alpha_j \cos \delta + \cos \varphi_{ji} \sin \delta) \ddot{\varphi}_{ji} - \\
& (\sin \varphi_{ji} \cos \delta + \cos \varphi_{ji} \cos \alpha_j \sin \delta) \ddot{\theta} - \cos \varphi_{ji} \sin \alpha_j \cos \delta \ddot{\alpha}_j - 2(\cos \varphi_{ji} \cos \delta - \\
& \sin \varphi_{ji} \cos \alpha_j \sin \delta) \dot{\varphi}_{ji} \dot{\theta} + 2 \cos \varphi_{ji} \sin \alpha_j \sin \delta \dot{\alpha}_j \dot{\theta} + 2 \sin \varphi_{ji} \sin \alpha_j \cos \delta \dot{\varphi}_{ji} \dot{\alpha}_j], \\
& m_{ji} r_j^2 \ddot{\varphi}_{ji} + c_{\varphi} \dot{\varphi}_{ji} = -m_{ji} r_j \cos \varphi_{ji} (\ddot{x} \cos \delta + \ddot{y} \sin \delta) + m_{ji} r_j \sin \varphi_{ji} \cos \alpha_j (\ddot{x} \sin \delta - \ddot{y} \cos \delta) + \\
& + 2m_{ji} r_j^2 \sin \alpha_j \cos^2 \varphi_{ji} \dot{\theta} \dot{\alpha}_j - m_{ji} r_j s_2 (\sin \beta \cos \varphi_{ji} + \sin \varphi_{ji} \cos \alpha_j \cos \beta + r_j^2 \cos \alpha_j) \ddot{\theta} + \\
& + m_{ji} r_j (r_j \sin \varphi_{ji} \cos \varphi_{ji} \sin^2 \alpha_j + s_2 (\cos \varphi_{ji} \cos \beta - \sin \varphi_{ji} \cos \alpha_j \sin \beta)) \dot{\theta}^2 - \\
& - m_{ji} r_j^2 \sin \varphi_{ji} \cos \varphi_{ji} \dot{\alpha}_j^2 + m_{ji} r_j g \sin \varphi_{ji} \sin \alpha_j, \\
& (J_{y_{j2}} + \sum_{i=1}^n m_{ji} r_j^2 \cos^2 \varphi_{ji}) \ddot{\alpha}_j + c_{\alpha} \dot{\alpha}_j = \sum_{i=1}^n m_{ji} r_j \cos \varphi_{ji} \sin \alpha_j (\ddot{x} \sin \delta - \ddot{y} \cos \delta) - \\
& - 2 \sum_{i=1}^n m_{ji} r_j^2 \sin \alpha_j \cos^2 \varphi_{ji} \dot{\theta} \dot{\varphi}_{ji} - \sum_{i=1}^n m_{ji} r_j \sin \alpha_j \cos \varphi_{ji} (s_2 \cos \beta + r_j \sin \varphi_{ji}) \ddot{\theta} + \\
& + \sum_{i=1}^n m_{ji} r_j^2 \sin 2\varphi_{ji} \dot{\varphi}_{ji} \dot{\alpha}_j + \sum_{i=1}^n (J_{x_{j2}} - J_{z_{j2}}) \cos \alpha_j \sin \alpha_j - \sum_{i=1}^n m_{ji} (r_j^2 \cos^2 \varphi_{ji} \cos \alpha_j \sin \alpha_j + \\
& + r_j s_2 \cos \varphi_{ji} \sin \beta \sin \alpha_j) \dot{\theta}^2 - \sum_{i=1}^n m_{ji} g \cos \varphi_{ji} \cos \alpha_j, \quad (j = 1, \dots, s)
\end{aligned}$$

$$\begin{aligned}
& (\bar{m}_1 s_1^2 + \bar{m}_2 s_2^2 + \sum_{j=1}^s \sum_{i=1}^n m_{ji} s_2^2 + J_G + \sum_{j=1}^s (J_{x_{j2}} \sin^2 \alpha_j + J_{z_{j2}} \cos^2 \alpha_j) + \sum_{j=1}^s \sum_{i=1}^n m_{ji} (r_j \sin^2 \varphi_{ji} + \\
& + r_j^2 \cos^2 \varphi_{ji} \cos^2 \alpha_j + 2r_j s_2 \sin \varphi_{ji} \cos \beta + 2r_j s_2 \cos \varphi_{ji} \cos \alpha_j \sin \beta) \ddot{\theta} + c_\theta \dot{\theta} = \\
& = s_1 \bar{m}_1 (\ddot{x} \sin \theta - \ddot{y} \cos \theta) + (\bar{m}_2 s_2 + \sum_{j=1}^s \sum_{i=1}^n m_{ji} s_2) (\ddot{x} \sin(\theta + \gamma) - \ddot{y} \cos(\theta + \gamma)) + \\
& + \sum_{j=1}^s \sum_{i=1}^n m_{ji} (r_j \sin \varphi_{ji} (\ddot{x} \sin \delta - \ddot{y} \cos \delta) - r_j \cos \varphi_{ji} \cos \alpha_j (\ddot{x} \cos \delta + \ddot{y} \sin \delta)) - \\
& - \sum_{j=1}^s \sum_{i=1}^n m_{ji} (r_j^2 \sin 2\varphi_{ji} \sin^2 \alpha_j + 2s_2 r_j (\cos \varphi_{ji} \cos \beta - \sin \varphi_{ji} \cos \alpha_j \sin \beta)) \dot{\theta} \dot{\varphi}_{ji} + \\
& + \sum_{j=1}^s \sum_{i=1}^n m_{ji} s_2 r_j (\sin \varphi_{ji} \sin \beta - \cos \varphi_{ji} \cos \alpha_j \cos \beta) \dot{\varphi}_{ji}^2 + \sum_{j=1}^s \sum_{i=1}^n m_{ji} (2s_2 r_j \sin \varphi_{ji} \sin \alpha_j \cos \gamma + \\
& + 2r_j^2 \cos^2 \varphi_{ji} \sin \alpha_j) \dot{\varphi}_{ji} \dot{\alpha}_j - (s_2 r_j \cos \varphi_{ji} \sin \alpha_j \cos \beta + r_j^2 \sin \varphi_{ji} \cos \varphi_{ji} \sin \alpha_j) \ddot{\alpha}_j - \\
& - \sum_{j=1}^s \sum_{i=1}^n m_{ji} (s_2 r_j \cos \varphi_{ji} \cos \alpha_j \cos \beta + r_j^2 \sin \varphi_{ji} \cos \varphi_{ji} \cos \alpha_j) \dot{\alpha}_j^2 - \sum_{j=1}^s \sum_{i=1}^n m_{ji} (s_2 r_j \cos \varphi_{ji} \sin \beta + \\
& + s_2 r_j \sin \varphi_{ji} \cos \alpha_j \cos \beta + r_j^2 \cos \alpha_j) \ddot{\varphi}_{ji} + M_\theta.
\end{aligned} \tag{3}$$

The generalized forces in variables α_j and φ_i of the nonlinear system of the equation of motion (3) in the rotating coordinate system with the rotor take the following form:

$$\begin{aligned}
& [M_0 (\ddot{\zeta} - \zeta \dot{\theta}^2 + i(2\dot{\zeta} \dot{\theta} + \zeta \ddot{\theta})) + c(\dot{\zeta} + i\zeta \dot{\theta}) + 2i\nu] \dot{\zeta} + k\zeta e^{i\theta} = -\frac{d}{dt} [[m_1 s_1 i + \\
& + \sum_{i=1}^n m_i (s_2 i e^{i\beta} + r i \sin \varphi_i e^{i(\gamma+\beta)} + r \cos \varphi_i \cos \alpha e^{i(\beta+\gamma)}) \dot{\theta} + \sum_{i=1}^n (\cos \varphi_i e^{i(\beta+\gamma)} + \\
& + i \sin \varphi_i \cos \alpha e^{i(\beta+\gamma)}) r \dot{\varphi}_i + i \sum_{i=1}^n r \sin \alpha \cos \alpha e^{i(\beta+\gamma)}] e^{i\theta}, \\
& (J_{x_2} + \sum_{i=1}^n m_i r^2 \cos^2 \varphi_i) \ddot{\alpha} - 2 \sum_{i=1}^n m_i r^2 \cos \varphi_i \sin \varphi_i \dot{\varphi}_i \dot{\alpha} + \\
& + \sum_{i=1}^n m_i r \sin \alpha \cos \varphi_i (s_2 \cos \gamma + r \sin \varphi_i) \ddot{\theta} - \sum_{i=1}^n m_i r \cos \varphi_i \sin \alpha (-\ddot{\zeta} - 2\dot{\eta} \dot{\theta} - \xi \dot{\theta}^2 - \\
& - \eta \ddot{\theta}) \sin(\beta + \gamma) + (\ddot{\eta} + 2\dot{\xi} \dot{\theta} - \eta \dot{\theta}^2 + \xi \ddot{\theta}) \cos(\beta + \gamma) - 2 \sum_{i=1}^n m_i r^2 \sin \alpha \sin^2 \varphi_i \dot{\theta} \dot{\varphi}_i - \\
& - [(-J_{z_2} + J_{y_2}) \cos \alpha \sin \alpha - \sum_{i=1}^n m_i (r^2 \cos^2 \varphi_i \cos \alpha \sin \alpha + r s_2 \cos \varphi_i \sin \gamma \sin \alpha)] \dot{\theta}^2 = \\
& = -\sum_{i=1}^n m_i g \cos \varphi_i \cos \alpha,
\end{aligned}$$

$$\begin{aligned}
 & m_i r^2 \ddot{\varphi}_i + c_{\varphi} \dot{\varphi} + m_i r (\cos \varphi_i ((\ddot{\xi} - 2\dot{\eta}\dot{\theta} - \xi\dot{\theta}^2 - \eta\ddot{\theta}) \cos(\beta + \gamma) + (\ddot{\eta} + 2\dot{\xi}\dot{\theta} - \eta\dot{\theta}^2 + \\
 & + \xi\ddot{\theta}) \sin(\gamma + \beta)) + \sin \varphi_i \cos \alpha (-(\ddot{\xi} - 2\dot{\eta}\dot{\theta} - \xi\dot{\theta}^2 - \eta\ddot{\theta}) \sin(\beta + \gamma) + (\ddot{\eta} + 2\dot{\xi}\dot{\theta} - \eta\dot{\theta}^2 + \\
 & + \xi\ddot{\theta}) \cos(\beta + \gamma)) - m_i r (s_2 \sin \varphi_i \cos \gamma \sin \alpha + r \sin \alpha) \dot{\alpha} \dot{\theta} + m_i (s_2 \cos \varphi_i \sin \gamma + \\
 & + \sin \varphi_i \cos \alpha \cos \gamma) \ddot{\theta} - m_i (r^2 \sin^2 \alpha \sin \varphi_i \cos \varphi_i + r s_2 (\cos \varphi_i \cos \gamma - \\
 & - \sin \varphi_i \cos \alpha \sin \gamma)) \dot{\theta}^2 + m_i r^2 \cos \varphi_i \sin \varphi_i \dot{\alpha}^2 + r s_2 \sin \alpha \cos \gamma \sin \varphi_i \dot{\alpha} \dot{\theta} - \\
 & - m_i r^2 \sin \alpha \cos 2\varphi_i \dot{\alpha} \dot{\theta} = m_i g \sin \varphi_i \sin \alpha. \quad (i = 1, 2, \dots, n)
 \end{aligned}
 \tag{4}$$

where the following relationship holds between the Cartesian coordinates of the center of mass and its coordinates in the rotating system [15]

$$z = x + iy = \zeta e^{i\theta}, \quad \dot{z} = \dot{x} + i\dot{y} = (\dot{\zeta} + \zeta\dot{\theta}i)e^{i\theta}, \quad \ddot{z} = \ddot{x} + i\ddot{y} = (\ddot{\zeta} + 2\dot{\zeta}\dot{\theta}i - \zeta\dot{\theta}^2 + \zeta\ddot{\theta}i)e^{i\theta}. \quad (5)$$

If, in the auto-balancing system, the running lines are located in the same plane, and the device is installed at a small angle α_j , then from the system of equations (1), it is easy to obtain the equation of motion for this case, eliminating the equations concerning variables α_i . For $\alpha = 0$, we get the equations of motion that completely coincide with the equations obtained in [15]

3 Results and Discussion

The issue of the existence of stationary motion for the model under consideration can be obtained by substituting a partial solution

$$\zeta = A e^{i\phi_0}, \quad \varphi_{ji} = \varphi_{ji}^0 = \text{const}, \quad A = \text{const}, \quad \phi_0 = \text{const}, \quad \alpha_j = \alpha_j^0 = \text{const}. \quad (6)$$

into the equation of motion (6), where A and ϕ_0 are the constant amplitude and phase of the rotational shear. Substituting the solution of the form (6) into the equation of motion (4), we obtain the conditions for the existence of stationary motion in the following form

$$(k - M_0 v^2)(\xi_0 + i\eta_0) = -(\widehat{m}_i s_1 i + \sum_{i=1}^n m_i (s_2 i e^{i\beta} + r i \sin \varphi_i e^{i(\gamma+\beta)} + r \cos \varphi_i \cos \alpha e^{i(\gamma+\beta)}) i v^2$$

$$\begin{aligned}
 & m_i r (\cos \varphi_i (-\xi_0 v^2 \cos(\beta + \gamma) - \eta_0 v^2 \sin(\beta + \gamma) + \sin \varphi_i \cos \alpha (\xi_0 v^2 \sin(\beta + \gamma) - \\
 & - \eta_0 v^2 \cos(\beta + \gamma))) - m_i (r^2 \sin^2 \alpha \sin \varphi_i \cos \varphi_i + r s_2 (\cos \varphi_i \cos \gamma - \sin \varphi_i \cos \alpha \sin \gamma)) v^2 = \\
 & = m_i g \sin \varphi_i \sin \alpha. \quad (i = 1, 2, \dots, n)
 \end{aligned}$$

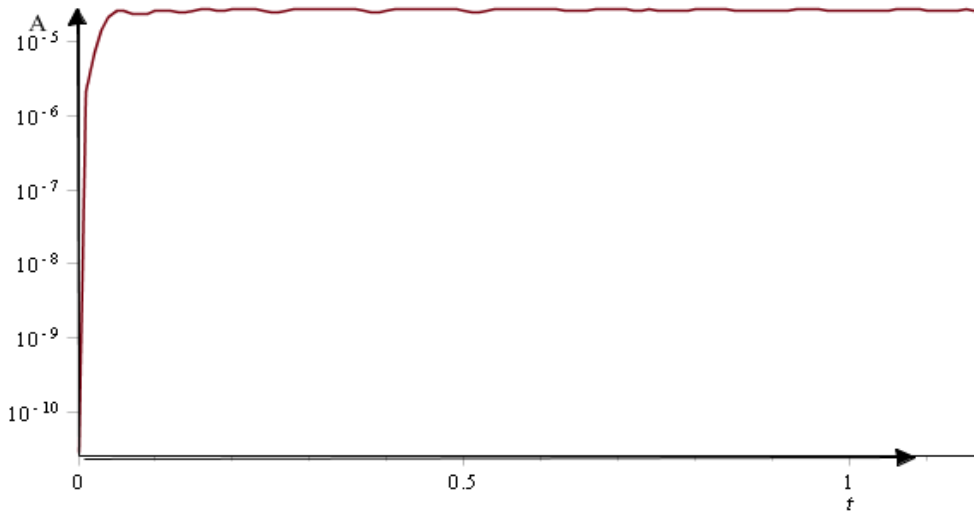
$$\sum_{i=1}^n m_i r \cos \varphi_i \sin \alpha (\xi_0 v^2 \sin(\beta + \gamma) - \eta_0 v^2 \cos(\beta + \gamma)) + \sum_{i=1}^n m_i (r^2 \cos^2 \varphi_i \cos \alpha \sin \alpha + r s_2 \cos \varphi_i \sin \gamma \sin \alpha) v^2 = \sum_{i=1}^n m_i g \cos \varphi_i \cos \alpha,$$

Since the system of equations (2) has a complex structure, integration is a difficult task. Therefore, we study the solution of the nonlinear system of equations (4) by the numerical method in various combinations of the ABD arrangement with two balancing balls. Using the MAPLE 18 software package, the following numerical results were obtained:

1. The balancing system is installed with eccentricity s_1 and small angular error α , with one running line. The rotor rotates at a constant angular speed. Numerical results were obtained for the following values of the system parameters (Fig. 1):

$m_1:=1.5; m_2:=0.7; m_3:=0.05; m_4:=0.05; r:=0.3; s_1:=0.001; s_2:=0.001; \alpha(t):=\pi/40; \beta:=\pi/6; c_1:=0.1; c:=0.1; c_0:=0.1; \theta(t):=600 \cdot t.$

The results obtained are presented in Fig. 2: Fig. 2a shows the change in the center of mass of the system over time; Fig. 2b shows the change in angle φ_1 that determines the position of the first ball in the tube over time; Fig. 2c shows the change in angle φ_2 that determines the position of the second ball in the tube over time.



a)

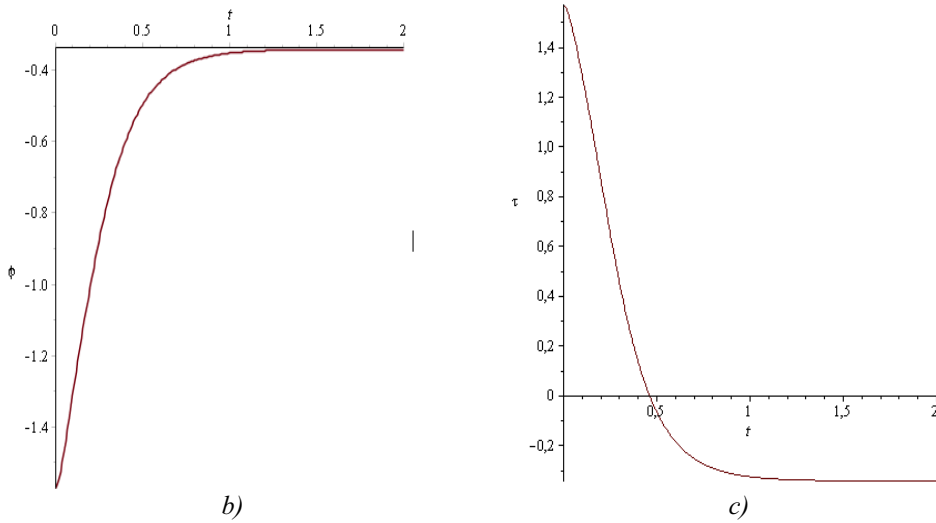


Fig. 2. Change in the center of mass of the rotor and position of the balls in the tube over time

The analysis of these results shows that the center of mass of the rotor after a certain time makes a precessional motion, and the balancing balls occupy a fixed position along the path; that is, a semi-balanced case takes place.

2. The balancing system is installed with eccentricity s_1 , and small angular error α , with two running lines and two balls while the rotor rotates at a constant speed. Numerical results were obtained for the following values of the system parameters (Fig.1): $m1:=2.5$; $m2:=1$; $m3:=0.05$; $m4:=0.08$; $r1:=0.3$; $r1:=0.4$; $s1:=0.001$; $s2:=0.001$; $\alpha(t):=\pi/40$; $\beta:=\pi/6$; $c1:=0.1$; $c:=0.1$; $co:=0.1$; $\theta(t):=300*t$.

The results obtained, i.e., the change in the system's center of mass under rotational motion at a constant angular speed, are shown in Fig. 3.

An analysis of these results shows that the change in the position of the center of mass over time occurs in terms of beating pattern since the masses of the balls installed in the lines and the radii of the tube are close to each other.

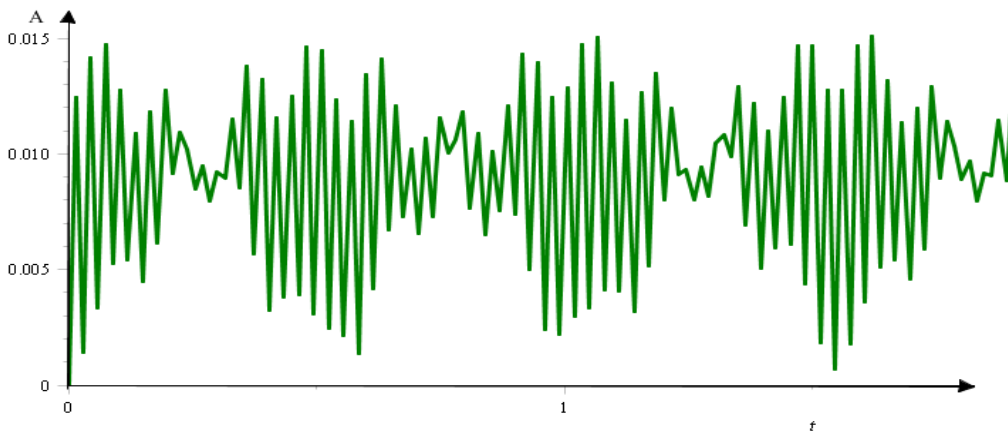


Fig. 3. Change in the center of mass of the system over time under rotational motion at a constant angular speed

3. Acceleration mode. In this case, the system's center of mass (Fig. 1) with two running lines and two balls rotates about a certain axis in a short time. Numerical results were obtained for the following values of the system parameters (Fig.1): $m_1:=2.5$; $m_2:=1$; $m_3:=0.05$; $m_4:=0.08$; $r_1:=0.3$; $r_1:=0.4$; $s_1:=0.001$; $s_2:=0.001$; $\alpha(t):=\pi/40$; $\beta:=\pi/6$; $c_1:=0.1$; $c:=0.1$; $c_0:=0.1$; $\theta(t):=100t^2$.

Figure 4 shows the results of calculating a system with two lines and two balls during acceleration mode: Fig. 4a shows the change in the center of mass of the system over time; Fig.4b shows the change in angle φ that determines the position of the first ball in the tube over time; Fig.4c shows the change in angle α between the coordinate plane Oxy and the plane of the tubes.

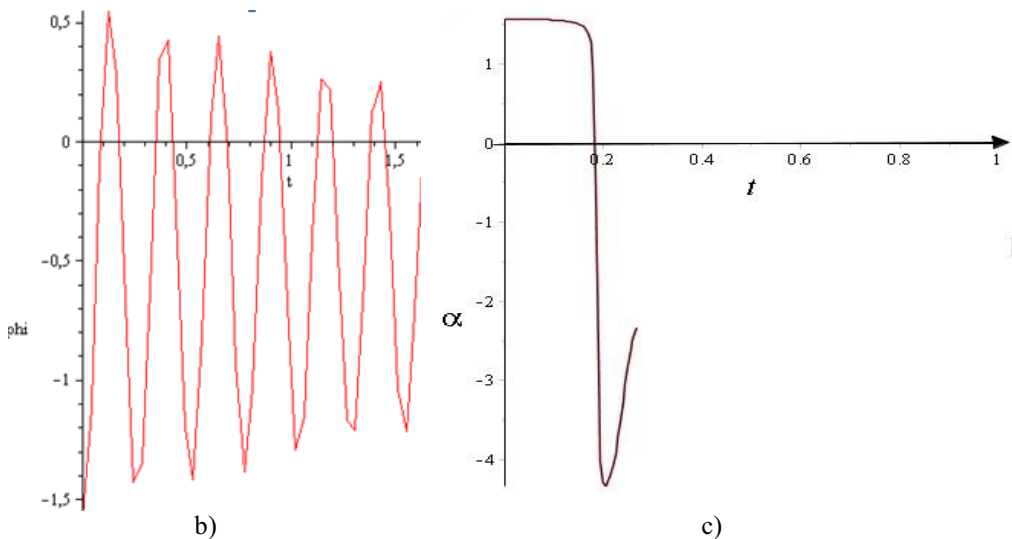


Fig. 4. Change in the center of mass and angle φ of the ball position, and angle α between the coordinate plane and the plane of the tube over time

4 Conclusions

1. A mathematical model was proposed that describes the motion of the balancing system with two running lines and various combinations of ball arrangement, taking into account eccentricity, angular error, and changes in the horizontal axis of rotation.

2. The motion of the balancing system with eccentricity and a small angular error was investigated at the rotor's constant angular speed of rotation with one running line.

3. The motion of the balancing system with eccentricity and a small angular error was investigated at the rotor's constant angular speed of rotation with two running lines and two balls.

4. The motion of the balancing system (Fig. 1) with two running lines and two balls was studied under the accelerating motion of the rotor, taking into account eccentricity, small angular error, and change in the horizontal axis of rotation.

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