Dynamics of cutter rolls of setting out machines

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Dynamics of cutter rolls of setting out machines

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Abstract. The proposed research is devoted to studying the motion of the cutter roll of leather-processing setting out machines and, of the resonance, when the number of roll revolutions coincides with the frequency of its natural vibrations, violating its dynamic stability. Composing differential equations of forced vibrations of the cutter roll, taking into account the resistance force of the medium, the frequencies of natural vibrations and the critical speeds are determined for different diameters of the cutter roll. It is assumed that a servo motor is installed to the cutter roll, which ensures constant roll rotation and reduces the number of degrees of freedom. It is shown that the dynamic coefficient and the maximum amplitude of forced oscillations occur not at resonance but extreme values of the detuning coefficient less than one. In addition, it was shown that the phase shift of forced oscillations at resonance does not depend on the linear resistance. Methods for determining the vibration frequency for different diameters of the cutter roll are shown.

INTRODUCTION

During the mechanical processing of a semi-finished product, one of the main links of the leather processing machines is the pressure and cutter rolls. The cutter rolls rest on an elastic base and rotate at high speed [1-9].

In several publications in the study of a body with an electric drive, it is assumed that the angular velocity of the expansion roll relative to the x-axis of rotation is strictly constant. The condition for the constancy of the angular velocity of its own rotation can be considered as a servo constraint imposed on the roll rotating around the x-axis of the roll. Servo-drive with control through negative feedback allows precise control of motion parameters [10-14], i.e.

$$\dot{\varphi} = \omega = const \qquad (\varphi = \omega t + \varphi_o) \tag{1}$$

where ω is the constant angular velocity of the own rotation of the cutter roll relative to the x-axis of rotation, φ is the angle of rotation of the cutter roll concerning the x-axis. In this case, the cutter roll moves on a vertical plane since only centrifugal forces of inertia act on it, which press the roll against the semi-finished product.

If condition (1) is not met, then tangential forces of inertia appear. Then the force of inertia, directed along the diagonal of the rectangle, is built to the centrifugal and tangential forces of inertia that do not lie on the vertical plane and cause the cutter roll to deviate from the vertical line. Consequently, vibrations of the cutter roll appear. To improve the processing of the semi-finished product, the cutter rolls must rotate at a constant angular velocity.

To dampen the vibration of the cutter roll on the supports O and O_1 of the cutter roll, we will install four vertical springs. To determine the spring stiffness coefficient, let us consider the cutter roll in equilibrium position (Figure 1a). In the equilibrium position, the following forces act on the roll: $\vec{F}_y$ is the elastic force of the upper spring, $\vec{F}_y'$ is the elastic force of the lower spring (the springs are fixed so that the cutter roll always remains on the vertical plane), $Q = Mg$ is the roll gravity forces distributed along the length of the cutter roll.

The x-axis is directed along the axis of rotation of the cutter roll. Let us assume that the movements of the cutter roll occur on a vertical plane. This condition is fulfilled when the cutter roll rotates at a constant angular velocity, i.e. $\omega = const.$

Then the condition for the balance of the roll has the form:

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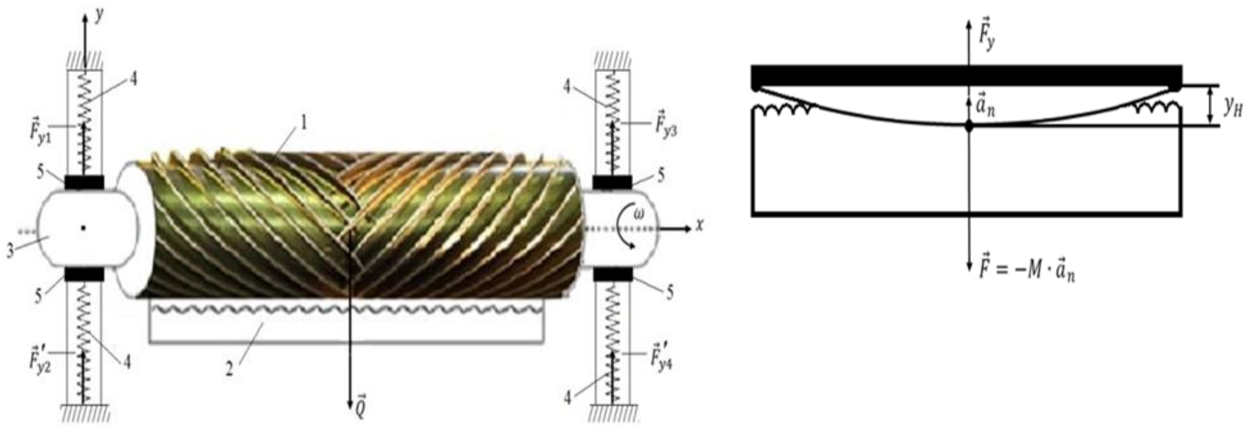
$$\sum F_{ky} = 0\ ;\ \ 2F_{yn} + 2F'_{yn} - Q = 0 \tag{2}$$



(a) *(b)*

FIGURE 1. To the calculation of the cutter roll on an elastic base a) 1 is cutter roll, 2 is hide, 3 are pulleys, 4 are springs, 5 are bearings b) roll in a strained position.

where $F_{yn} = c_1 f_{cm}$, $F'_{yn} = c_2 f'_{cm}$, $f'_{cm} = f_{cm} = a$, $c_1 = c_2$; a is the constant number set in advance, and the choice of number a depends on the diameter of the roll.

Equation (2) implies that $4cf_{cm} = Q$ or $c = \frac{Mg}{4a}$.

METHODS

Let us consider the cutter roll in an arbitrary position, i.e. when the cutter roll is straightened in the dynamic stability mode.

One of the factors of the loss of dynamic stability of the expanding rolls occurs due to resonance when the number of roll revolutions coincides with the frequency of its natural vibrations. Therefore, we determine the main parameters of the oscillatory motion of the cutter roll, which lead to the loss of dynamic stability.

To determine the frequency of natural vibrations of the roll, let us assume that the roll with one load Q was bent in the *xy* plane by the value of y_H under the point of load application, and then the force that caused the roll deflection was eliminated. Obviously, under the action of elastic forces, the roll will tend to a position of static equilibrium, reach it. Under the influence of inertia, it will pass through this position and begin to bend in the opposite direction. The elastic forces will resist the deflection, and the deflection of the roll will stop. However, being stressed without load, under the action of elastic forces, it will begin to return to the position of static equilibrium, and inertia will pass through it, etc. In other words, the roll will oscillate relative to its equilibrium (statically) position (Figure 1b).

Thus, the roll moves in a potential force field, i.e. for any position of the roll, we have the equation of constancy of the sum of potential (P) and kinetic (T) energy:

$$T + \Pi = H = const$$

The strain during the reverse deflection of the roll has the same magnitude as at the initial time. Since the phenomenon will be repeated in the future, we obtain transverse vibrations around the equilibrium (statically) position with the deflection value of $\pm y_H$. In addition, at any time point, the inertia forces $-M \cdot \ddot{y}$ and the elastic force equal to the limits of an elastic strain of the roll - *ky* acts on the roll, where *k* is the elastic constant of the roll, i.e. the force producing a static deflection equal to one (in *N/cm*).

During the straightening of the leather material, there are always forces that cause damping of the roll vibrations (friction of the medium and other forces). Suppose in the simplest case the resistance to motion (the damping force) is considered proportional to its speed (with the proportionality coefficient η) and periodic disturbing force $F=H\cdot\sin(pt+\delta)$ is applied. In that case, the equation of motion of the roll takes the following form:

$$\ddot{y} + 2n\dot{y} + k^2y = h \cdot \sin(pt + \delta), \tag{3}$$

where $k^2 = \frac{c}{m}$; $2n = \frac{\eta}{M}$; $h = \frac{H}{M}$ $\frac{cm}{c^2}$; k is the angular frequency of natural vibrations, n is the damping coefficient; h is the relative amplitude of the disturbing force.

A linear differential equation with constant coefficients of forced oscillations is obtained, taking into account linear resistance.

In [15-20], the general solution to the equation of forced oscillation $y = y_1 + y_2$ is shown, where y_1 is the general solution of the homogeneous equation of damped oscillations and y_2 is a particular solution of the inhomogeneous equation (3).

The general solution y_1 of the homogeneous differential equation $\ddot{y}_1 + 2n\dot{y}_1 + k^2y_1 = 0$ depending on the ratio between the quantities n and k, is expressed in one of three forms:

$$n < k\ ;\ \ y_1 = A_1e^{-nt} \cdot \sin(\sqrt{k^2 - n^2} \cdot t + \alpha)\ ;$$

$$n = k\ ;\ \ y_1 = e^{-nt}(c_1t + c_2)\ ;$$

$$n > k\ ;\ \ y_1 = e^{-nt}(c_1 \cdot e^{\sqrt{k^2-n^2}\cdot t} + c_2 \cdot e^{-\sqrt{k^2-n^2}\cdot t}\ :$$

It is known that in any of these cases, due to the presence of the factor e^{-nt}, y_1 tends to zero with time, i.e., attenuates. At small values of the damping coefficient ($n < k$), the damped motion y_1 is of an oscillatory nature, and at large values ($n \geq k$), the damping is so great that the motion is not oscillatory. Consequently, in the presence of linear resistance, after a certain time, the total forced motion y differs insignificantly from the forced oscillations, and we can assume that $y = y_2$.

A particular solution y_2 of equation (3) is sought in the following form

$$y_2 = A \cdot sin(pt + \delta - \varepsilon)$$

To determine A and ε, we calculate the first and second derivatives of y_2 and substitute their values into equation (3).

Let us calculate $\dot{y}_2 = A \cdot pcos(pt + \delta - \varepsilon)$; $\ddot{y}_2 = A \cdot p^2sin(pt + \delta - \varepsilon)$ and transform the right-hand side of equation (3):

$$h \cdot \sin(pt + \delta) = h \cdot sin[(pt + \delta - \varepsilon) + \varepsilon] == h \cdot sin\varepsilon \cdot \cos(pt + \delta - \varepsilon) + h \cdot cos\varepsilon \cdot \sin(pt + \delta - \varepsilon)$$

With this in mind, we substitute the value of y_2 and its derivatives into equation (3), and after simple transformations, we get

$$A(k^2 - p^2) = h \cdot cos\varepsilon\ ,\ \ An \cdot p = h \cdot sin\varepsilon$$

From these equations, we determine the amplitude of forced oscillations A and the phase shift ε:

$$A = \frac{h}{\sqrt{(k^2-p^2)^2+4n^2\cdot p^2}} \quad tg\varepsilon = \frac{2np}{k^2-p^2}\ .$$

Thus, the law of forced vibrations of the cutter roll is

$$y = A \cdot \sin(pt + \delta - \varepsilon) \tag{4}$$

where

$$A = \frac{h}{\sqrt{(k^2-p^2)^2+4n^2\cdot p^2}}\ ; tg\varepsilon = \frac{2np}{k^2-p^2}\ ;\ 0 \leq \varepsilon \leq \pi. \tag{5}$$

The amplitude A and the phase shift ε following (5) do not depend on the initial phase δ of the disturbing force. When calculating the amplitude and the phase shift, we can take $\delta = \pi/2$. Therefore, $A_o = h/k^2$ can be considered the "amplitude" of forced oscillations under the action of a constant disturbing force, which coincides in magnitude with the largest value of harmonic disturbing force. As is known, the value of A/A_o, called the dynamic coefficient, characterizes the relative magnitude of the amplitude of forced oscillations, i.e. shows how much the amplitude of forced oscillations under the action of a harmonic disturbing force differs from the static displacement caused by a constant disturbing force equal in magnitude to the largest value of harmonic force.

Taking into account A and A_0, after the transformation, we obtain

$$\mu = \frac{A}{A_o} = \frac{1}{\sqrt{(1-z^2)^2 + 4b^2 \cdot z^2}} \tag{6}$$

where $z = \frac{p}{k}$ is the detuning factor and $b = \frac{n}{k}$ is the relative damping coefficient.

DISCUSSION AND RESULTS

Thus, the dynamic coefficient μ depends on the parameters of the detuning factor and attenuation coefficient.

For a flexible cutter roll $(k < p)$, the action of the disturbing force of high frequency is not perceived by the oscillating cutter rolls. It does not violate the mode of natural vibrations, which damp out under the influence of resistance for linear systems.

Denoting the denominator in relation (6) by $(z) = (1 - z^2)^2 + 4b^2 z^2$, we get

$$\mu = \frac{1}{\sqrt{f(z)}}$$

Obviously, when *f(z)* reaches a maximum, then μ has a minimum, and vice versa. Let us define the extreme values of $f(z)$; $z_1 = 0$; $z_2 = \sqrt{1 - 2b^2}$.

Since the relative frequency can only be positive and equal to zero for a constant disturbing force, then $1 - 2b^2 > 0$ or $b < 0.7$. For $f''(z_1) < 0$, $f(z)$ reaches the maximum, and μ reaches the minimum.

For $f''(z_1) > 0$, *f (z)* reaches the minimum, and μ reaches the maximum.

If $1 - 2b^2 = 0$, then $z_1 = z_2 = 0$ and $f'(z) = 0$.

Using the third and fourth derivatives, we can show that *f(z)* at z = 0 reaches the minimum, and μ reaches the maximum.

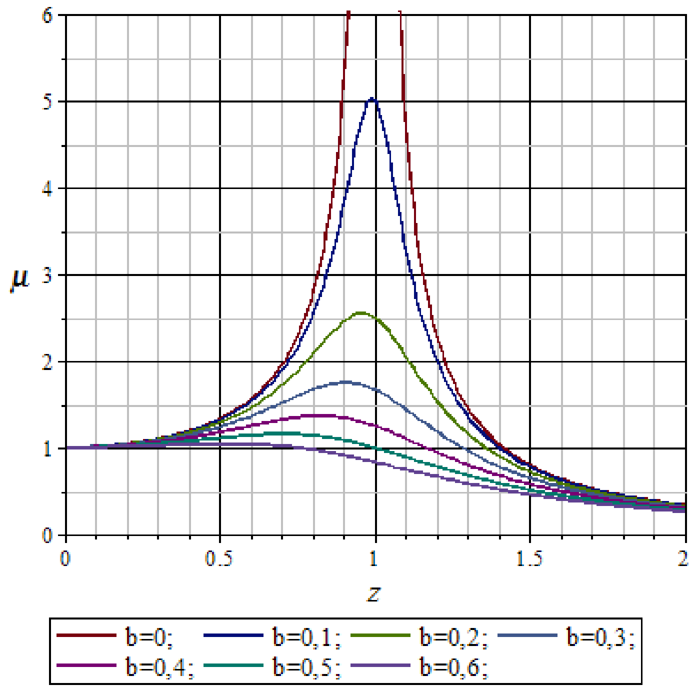


FIGURE 2. Frequency response of the cutter roll

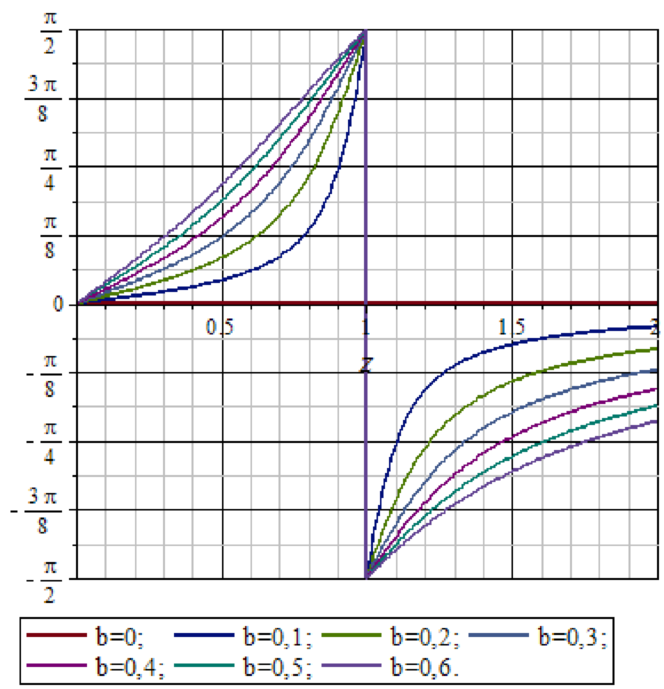


FIGURE 3. Changes in the phase shift of the cutter roll

If $1-2b^2<0$, then z_2 becomes purely imaginary, i.e. apart from $z=0$, the function $f(z)$ has no extremum point. At $z = 0$, $f(z)$ reaches the minimum, and μ reaches the maximum.

With increasing z, i.e. with an increase in the angular frequency of the disturbing force, dynamic coefficient μ decreases monotonically for $1-2b^2 \leq 0$.

Thus, the maximum μ, and hence the amplitude of forced oscillations, occurs not at resonance, at z = 1 (p = k) but at the value of $z = z_2 = \sqrt{1-2b^2}$ less than one (Figure 2). To obtain the value of the maximum amplitude A_{max}, we should substitute z by $z_2 = \sqrt{1-2b^2}$ in (6), which corresponds to the critical angular frequency of the disturbing force

$$p_{cr} = k\sqrt{1-2\frac{n^2}{k^2}} = \sqrt{k^2-2n^2} = \sqrt{k^2-\frac{n^2}{M^2}} \quad (7)$$

Therefore

$$A_{max} = \frac{A_o}{\sqrt{(1-z_2^2)^2+4b^2\cdot z_2^2}} = \frac{h}{2n\sqrt{k^2-n^2}} = \frac{A}{2b\sqrt{1-b^2}}$$

For *z=1* and from equation (6), we obtain:

$$A_{res} = \frac{h}{2nk} = \frac{A_o}{2b} < A_{max}$$

i.e., the amplitude of the forced vibrations at resonance is less than the maximum amplitude, reached at $p_{cr} = \sqrt{k^2-2n^2}$. The critical angular frequency at which the amplitude of the forced oscillations reaches its maximum decreases with an increase in the damping coefficient. The values of A_{max} and A_{res} also decrease in this case.

Equation (5) implies

$$tg\varepsilon = \frac{2bz}{1-z^2}; \quad 0 \le \varepsilon \le \pi \quad (8)$$

The tangent of the phase shift ε is expressed by a simple dependence on *z*. Using the monotonicity of the change in the tangent depending on the change in the argument, we plot the dependence of ε on *z* for various fixed values of *b* (Figure 3). It follows from (8) that for rigid rolls *(z<1)*; $\varepsilon = \frac{\pi}{2}$ and for flexible rolls *(z>1)*; *z=1*, $\varepsilon = \pi$.

Thus, at $z = 0$ and $\varepsilon = 0$, the range of variation of the detuning coefficient and phase shift is $0 < z < 1$, $0 < \varepsilon < \frac{\pi}{2}$. At *z=1*, $= \frac{\pi}{2}$, the range of variation of the detuning coefficient and phase shift is $0 < z < 1$, $0 < \varepsilon < \frac{\pi}{2}$. $(z \to \infty, \varepsilon = \pi)$.

Thus, at low values of the damping force and, consequently, its magnitude, the approach of the frequency of forced vibrations (for example, the number of roll revolutions) to the frequency of natural vibrations of the roll becomes dangerous from the point of view of the growth of strains and stresses.

If the frequency of the exciting force is less or greater than the frequency of free vibrations of the roll, then the strain directions caused by their action do not coincide in time.

A roll with the exciting force frequency lower than the frequency of free vibrations is called a rigid roll; for $p > \omega$, it is called flexible. For a roll with several concentrated transverse loads, deflections of several shapes are possible and, therefore, the same number of angular frequencies of natural vibrations are possible. The smallest of them is called the main one; the rest are called the higher ones. Naturally, the coincidence of the number of revolutions of such a roll with any of the mentioned frequencies causes resonance and is dangerous. This number of revolutions is called critical (the first critical n_{1cr} for the main frequency, the second critical n_{2cr} for the second frequency, etc.).

For a rigid roll, the number of revolutions *n* should not approach more than 75% to the first critical roll (i.e. $n \le 0.75 n_{1cr}$).

The recommended speed range for the flexible roll is:

$$1.4 n_{1cr} < n < 0.7 n_{2cr} \quad (9)$$

Determining the critical speed n_{1cr} of a roll having one transverse load does not cause any difficulties:

$$n_{cr} = 60v = 60\frac{\omega}{2\pi} = \frac{60}{2\pi}\sqrt{\frac{g}{f_{cm}}} \approx 300\sqrt{\frac{1}{f_{cm}}}\left|\frac{1}{min.}\right|$$

In [2], ignoring the mass of the pulleys, which is relatively small and located behind the bearings, it is assumed that the cutter roll carries only one distributed lateral load of its own weight. For this case, using the equation of the elastic line, the frequencies of natural vibrations and the critical speeds of the cutter roll for the oscillatory motion of the cutter roll are determined:

$$\omega_1 = k^2\sqrt{\beta} = \frac{\pi^2 n^2}{l^2}\sqrt{\frac{gEI}{q}} \quad (n = 1,2,3, \dots .)$$

i.e., the roll has an infinite number of vibration frequencies, of which the main (the smallest) one is

$$\omega_1 = k^2\sqrt{\beta} = \frac{\pi^2}{l^2}\sqrt{\frac{gEI}{q}}$$

Using these formulas, we determine the frequencies of natural vibrations and the critical speeds of the cutter roll with diameters *d* = 10; 15; 20; 25; 30 cm and lengths *l* = 180; 300 cm of the cutter roll.

TABLE 1. Values of natural vibration frequencies and critical speeds of the cutter roll

d *cm*	***l*** *cm*	***E*** *N/cm²*	***J*** *cm⁴*	***γ*** *N/cm³*	***q*** *N/cm*	$\boldsymbol{\omega_1}$ s^{-1}	$\boldsymbol{\omega_2}$ s^{-1}	$\boldsymbol{\omega_3}$ s^{-1}	$\boldsymbol{n_{1k}}$ min^{-1}	$\boldsymbol{n_{2k}}$ min^{-1}	$\boldsymbol{n_{3k}}$ min^{-1}
10	180	$2.1\cdot10^6$	500	$7.8\cdot10^{-3}$	0.61	395.4	1581.8	3558.9	3778.1	15112.3	34002.6
15	180	$2.1\cdot10^6$	2531	$7.8\cdot10^{-3}$	1.378	591.9	2367.8	5327	5655.5	22622.0	50899.6
20	300	$2.1\cdot10^6$	$8\cdot10^3$	$7.8\cdot10^{-3}$	2.5	281.3	1125	2531.5	2687.4	10749.5	24186.3
25	300	$2.1\cdot10^6$	19531.25	$7.8\cdot10^{-3}$	3.83	355.1	1420.3	3195.7	3392.5	13569.9	30532.4
30	300	$2.1\cdot10^6$	40500	$7.8\cdot10^{-3}$	5.51	426.3	1705.3	3836.7	4072.9	16291.6	36656.1

Critical speeds:

$\boldsymbol{n \le 0.75n_{1k}}$	$\boldsymbol{1.4n_{1k} < n < 0.7n_{2k}}$
a) $n \le 2833.58$ (d=10 cm)	*a*) $5289 < n < 10578.61$
b) $n \le 4241.6$ (d=15 cm)	*b*) $7917.7 < n < 16966.5$
c) $n \le 2015.55$ (d=20 cm)	*c*) $3762.36 < n < 8062.1$
d) $n \le 2544.4$ (d=25 cm)	*d*) $4746.5 < n < 10177.4$
e) $n \le 5054.8$ (d=30 cm)	*e*) $5702 < n < 11404.12$

Thus, the range of determination of the critical speeds of the flexible roll is wider than that of the rigid roll. Analyzing the values of the critical speeds given in the table, in the future, it is recommended to choose a spreading roll with a diameter of 20 cm.

CONCLUSIONS

Based on the previous, it can be concluded that the cutter roll with servo-constraint reduces vibration and eliminates defects on the face of the leather material.

The forced vibrations of the cutter roll were investigated, taking into account the force of resistance of the medium to the leather processing machines during flattening. It is assumed that a servo motor is installed on the cutter roll, which ensures constant rotation of the roll.

It is shown that the roll moves in a potential force field, i.e. for any roll position, we have the equation of constancy of the sum of potential (*P)* and kinetic (*T*) energy. The dynamic coefficient and the maximum amplitude of forced oscillations do not occur at resonance but at extreme values of the detuning coefficient less than one. In addition, it was shown that the phase shift of forced oscillations at resonance does not depend on the linear resistance. The state of the cutter roll is graphically determined near the resonance values. It is shown that the range of determination of the critical speeds of the flexible roll is wider than that of the rigid roll. In the future, it is recommended to choose a spreading roller with a diameter of 20 cm.

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