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Stabilization of mechanical systems with nonholonomic servoconstraints

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Abstract. The proposed work is devoted to the study of mechanical systems with releasing nonholonomic servoconstraints on controllability and stabilization. In this case, the reaction forces of the servoconstraints are decomposed into the normal and tangent components. The factors of tangent components are taken as control parameters. A technique is proposed for determining the reaction forces of servoconstraints providing stabilization of motion relative to the variety determined by nonholonomic servoconstraints. As an example, the problem of a gyroscope in a cardan suspension with nonholonomic releasing servoconstraints is considered.

1. Introduction

The aim of the paper is to control the movement of the system under consideration with tangential components of the reactions of servoconstraints, carried out by servomotors. A servomotor is an unpretentious working element that is part of industrial equipment [1]. This paper lists all types of servomotors and stepper motors.

In the studies by A.G. Azizov [2] a constructive method for determining the reaction forces of servo-links, containing mismatches and allowing to restore the broken constraints was developed. To study the systems with servoconstraints, he applied the theory of parametric release, the development for a class of mechanical systems given in the works of V.V. Rumyantsev [3] and V.I. Kirgetov [4]. In [5], the dynamics of systems with servo-constraints was discussed, when the constraints are realized by controlling the inertial properties of the system. It was shown that the presence of symmetries allows one to reduce dynamic equations to a closed system of differential equations with quadratic right-hand sides. In [6], methods were shown for determining the reaction force of servoconstraints at given A-displacements, at which the work of the reaction force of servoconstraints is zero, i.e. servoconstraints are ideal for these displacements.

In [8-12], mechanical systems under the influence of various forces are considered; nonlinear positional, potential, dissipative, gyroscopic forces, and radial correction forces. Gyroscopic forces are considered dominant, which is expressed by the presence of a large parameter - the multiplier for them in the equations of motion. The conditions for the parameter growth rate are established, which guarantee the asymptotic stability of the equilibrium position both for the linear system and for substantially nonlinear dissipative forces specified by the homogeneous Rayleigh function. It is shown that for any values of the system parameters (mass of loads, spring stiffness), it is possible to provide asymptotic stability of the equilibrium position due to the addition of an additional load with a damper via a non-linear spring. An approach to the study of stability and stabilization of mechanical systems with nonlinear positional forces based on the decomposition of the equations of perturbed motion into two isolated subsystems is proposed. The stabilization conditions for systems with nonlinear non-stationary potential forces due to forces of a different structure are found.

However, in these works, the explicit forms of the tangent and normal components of servoconstraints, which determine the number of control parameters, were not distinguished.



2. Methods

In this paper, in contrast to the above-mentioned studies, we consider the problem of composing the equations of the disturbed motion of mechanical systems with nonholonomic servoconstraints, taking into account the release from servoconstraints. It is accepted here that the work of the reactions of servoconstraints at virtual displacements is nonzero, for the systems of non-released and parametrically released from servoconstraints.

Let the position of the mechanical system be determined by Lagrange coordinates q_1, q_2, \dots, q_n . a of non-ideal linear nonholonomic servoconstraints are imposed on the movement of the system

$$\sum_i a_{\alpha i} \dot{q}_i = 0; \quad (\alpha = 1, 2, \dots, a; \quad i = 1, 2, \dots, n) \quad (1)$$

We introduce kinematic characteristics independent with respect to \dot{q}_i

$$e_\nu = \sum_i \alpha_{\nu i} \dot{q}_i; \quad (\nu = 1, 2, \dots, n) \quad (2)$$

from here

$$\dot{q}_i = \sum_\nu \beta_{i\nu} e_\nu \quad (3)$$

Suppose that the forces of servoconstraints reactions are of the form

$$R_\nu = \sum_\alpha \lambda_\alpha a_{\alpha i} + \sum_\nu \mu_\nu \frac{\partial \dot{q}_i}{\partial e_\nu}; \quad (\nu = 1, 2, \dots, k; \quad k = n - a)$$

where λ_α are the servoconstraints factors, μ_ν some proportionality coefficients.

Denoting by ξ_α the left-hand sides of the equations of servoconstraints computed under the actual movement of the system, instead of (3) we obtain

$$\dot{q}_i = \sum_\gamma \beta_{i\gamma} e_\gamma + \sum_\alpha \beta_{i\alpha} \xi_\alpha; \quad (\alpha = 1, 2, \dots, a; \quad \gamma = a + 1, a + 2, \dots, n)$$

which allows us to write down the equations of motion and kinetic energy obtained in [7], in the form of Voronets-Hamel equation:

$$\frac{d}{dt} \frac{\partial \tilde{T}}{\partial e_\nu} - \frac{\partial \tilde{T}}{\partial \pi_\nu} = \sum_r \sum_s \frac{\partial \tilde{T}}{\partial e_r} W_{\nu s}^r e_s = \tilde{Q}_\nu + \sum_s \mu_s A_s \quad (4)$$

$$\frac{d}{dt} \frac{\partial \tilde{T}}{\partial \xi_\rho} - \frac{\partial \tilde{T}}{\partial \omega_\rho} + \sum_r \sum_s \frac{\partial \tilde{T}}{\partial e_r} W_{\rho s}^r e_s = Q_\rho^* + \lambda_\rho + \sum_\nu \mu_\nu B_{\rho \nu}$$

$$(\nu, s = 1, 2, \dots, k; \quad k = n - a; \quad \rho = k + 1, \dots, n)$$

where

$$W_{vs}^r = \sum_i \sum_j \left(\frac{\partial \alpha_{ri}}{\partial q_j} - \frac{\partial \alpha_{rj}}{\partial q_i} \right) \cdot \beta_{iv} \beta_{js}$$

$$W_{\rho s}^r = \sum_i \sum_j \left(\frac{\partial \alpha_{ri}}{\partial q_j} - \frac{\partial \alpha_{rj}}{\partial q_i} \right) \cdot \beta_{ip} \beta_{js}$$

$$A_{vs} = \sum_i \alpha_{vi} \beta_{is}, \quad B_{\rho v} = \sum_i \alpha_{vi} \frac{\partial \dot{q}_i}{\partial \xi_\rho}$$

Now we compose the equations of the disturbed motion of the system. To do this, we first write equation (4) in a more compact form:

$$\frac{d}{dt} \frac{\partial \tilde{T}}{\partial e_s} - \frac{\partial \tilde{T}}{\partial \pi_s} + \sum_r \sum_t \frac{\partial \tilde{T}}{\partial e_r} W_{ts}^r e_t = \tilde{Q}_s + \lambda_\tau + \sum_v \mu_v A_{sv} \tag{5}$$

$$(e_{k+\alpha} = \xi_\alpha; \quad \alpha = 1, 2, \dots, \alpha; \quad s = 1, 2, \dots, n; \quad \tau = k + 1, \dots, n)$$

In the case under consideration, the expression of kinetic energy has the form

$$\tilde{T} = \frac{1}{2} \sum_s \sum_k \tilde{A}_{sk} e_s e_k \quad (s, k = 1, 2, \dots, n) \tag{6}$$

where

$$\tilde{A}_{sk} = \tilde{A}_{sk}(q_1, q_2, \dots, q_n)$$

Suppose further that at $\lambda_s = \lambda_s^0$ and $\mu_v = \mu_v^0$, equations (5) with (2) allow a particular solution in the form

$$q_s = q_s(t), \quad e_k = e_k(t)$$

Introducing the excitations

$$q_s = q_s(t) + x_s, \quad e_k = e_k(t) + p_k, \quad \lambda_s = \lambda_s^0 + u_s, \quad \mu_v = \mu_v^0 + u_v. \tag{7}$$

$$(\lambda_s = 0, u_s = 0,) \quad \text{if} \quad (s = 1, 2, \dots, n - a)$$

As is known, in the monographs [8, 9], the sequence of composing the equation of disturbed motion is shown. Performing this sequence, we obtain the equations of disturbed motion:

$$A\dot{p} + Bp + Cx = Du + \Phi \tag{8}$$

$$\dot{x} = B_1 p + C_1 x + \Phi_1$$

where

$$A = \tilde{A}_{sk}, \quad B = 2 \sum_t \sum_r W_{ts}^r \tilde{A}_{rk} e_t + 2 \sum_r [k, r, s]_q e_r - \frac{\partial P_s}{\partial e_k}$$

$$C = \sum_r \sum_\beta \left\{ \sum_t \left(W_{ts}^r \frac{\partial \tilde{A}_{r\beta}}{\partial q_k} + \tilde{A}_{r\beta} \frac{\partial W_{ts}^r}{\partial q_k} \right) e_\beta e_t + \frac{1}{2} \sum_\nu \left(\beta_{\sigma\beta} \frac{\partial^2 \tilde{A}_{sr}}{\partial q_\sigma \partial q_k} + \beta_{\sigma\tau} \frac{\partial^2 \tilde{A}_{s\beta}}{\partial q_\sigma \partial q_k} - \beta_{\sigma\tau} \frac{\partial^2 \tilde{A}_{\beta r}}{\partial q_\sigma \partial q_k} \right) + \left(\frac{\partial \tilde{A}_{sr}}{\partial q_\sigma} \frac{\partial \beta_{\sigma\beta}}{\partial q_k} + \frac{\partial \tilde{A}_{s\beta}}{\partial q_\sigma} \frac{\partial \beta_{\sigma\tau}}{\partial q_k} - \frac{\partial \tilde{A}_{\beta r}}{\partial q_\sigma} \frac{\partial \beta_{\sigma\tau}}{\partial q_k} \right) e_r e_\beta \right\} - \frac{\partial P_s}{\partial q_k} - \sum_\nu \mu_\nu^0 \frac{\partial A_{s\nu}}{\partial e_k}$$

$$d_{s\nu} = A_{s\nu}, \quad B_1 = \beta_{is}, \quad C_1 = \sum_\beta \frac{\partial \beta_{i\beta}}{\partial q_s}$$

$$\left\| \begin{array}{cccc|cccc} d_{11} & \dots & d_{1k} & & 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ d_{n-a,1} & \dots & d_{n-a,k} & & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ d_{n1} & \dots & d_{nk} & & 0 & \dots & \dots & 1 \end{array} \right\|$$

Φ and Φ_1 are the terms above the first order of smallness are relative to x , \dot{x} and p .

Next, we propose a technique for determining the forces of servoconstraints that ensure motion stabilization relative to the manifold determined by servoconstraints. It is based on the structural features of equations (5), for which the following theorem is true.

Theorem: The controlled system (5) and (3) can be stabilized with respect to the manifold determined by the servoconstraints (1).

For the proof, along with the equations of motion, we consider the system

$$\dot{e}_s = \varphi_s(t, e_\nu); \quad (s, \nu = 1, 2, \dots, a) \tag{9}$$

with asymptotically stable zero solution. Substituting the kinetic energy (6) into equations (5), we obtain

$$\sum_k \tilde{A}_{sk} \dot{e}_k + /**/ = P_s + \lambda_s = \sum_\nu \mu_\nu A_{s\nu}$$

where $/**/$ denotes the sum of terms that do not contain derivatives of quasi-coordinates in time. Since $\det(\tilde{A}_{sk})_{s,k=1}^n \neq 0$, then the equation can be represented as

$$\dot{e}_k = G_k(t, q_j, e_s, \mu_\nu) + \sum_s A_{ks}^* \lambda_s \tag{10}$$

$$(j, s, \tau = 1, 2, \dots, n)$$

where $A_{\tau s}^*$ are the elements of the determinant reciprocal with the determinant. $\|\tilde{A}_{sk}\|$. Substituting system (9), we obtain a system of equations with respect to

$$\sum_s A_{\tau s}^* \lambda_s = \varphi_\tau - G_\tau$$

The latter system is solvable relative to λ_s and allows explicit determination of the factors

$$\lambda_s = \lambda_s(t, q_i, e_s, \mu_\nu)$$

In the particular case of systems with ideal constraints in the obtained expressions, $\mu_\nu = 0$ is set and equations (5) and (3) determine the motion of the system

Now consider a gyroscope in a gimbal mount with nonholonomic servoconstraints.

$$\omega_z = \dot{\gamma} + \dot{\alpha} \sin \beta = 0 \quad (11)$$

This constraint means that the projection of the angular velocity of the gyro rotor on the axis O_{z1} is zero at each moment of time (O_{x_1}, y_1, z_1) connected with the inner ring [10], i.e. angular velocity always remains in the plane (O_{x_1}, y_1) . The equations of motion of the system are taken in the form:

$$\begin{aligned} [A_2 + (A_1 + A) \cos^2 \beta + C_1 \sin^2 \beta] \cdot \dot{e}_1 + C \cdot e_2 \cdot e_3 \cdot \cos \beta + \\ + (C - A - A_1) \cdot \sin 2\beta \cdot e_1 \cdot e_2 = (1 + \sin^2 \beta) \cdot \mu_1, \end{aligned} \quad (12)$$

$$(B_1 + A) \cdot \dot{e}_2 - (C_1 - A - A_1) \cos \beta \cdot \sin \beta \cdot e_1^2 - C \cdot e_1 \cdot e_3 \cdot \cos \beta = \mu_2,$$

$$C \cdot \dot{e}_3 = \lambda - \mu_1 \cdot \sin \beta.$$

Let the equations of motion of system (12) allow a particular solution

$$e_1 = 0, \quad \alpha = \alpha_0, \quad \mu_1 = 0 \quad (13)$$

$$e_2 = 0, \quad \beta = \beta_0, \quad \mu_2 = 0$$

$$e_3 = k, \quad \gamma = kt + \gamma_0, \quad \lambda = 0$$

For disturbed motion

$$e_1 = p_1, \quad \alpha = \alpha_0 + x_1, \quad \mu_1 = u_1 \quad (14)$$

$$e_2 = p_2, \quad \beta = \beta_0 + x_2, \quad \mu_1 = u_2$$

$$e_3 = k, \quad \gamma = kt + \gamma_0 + x_3, \quad \lambda = u_3$$

Substituting values from (14) into (12), we obtain

$$\left[A_2 + (A + A_1) \cos^2(\beta_0 + x_2) + C_1 \sin^2(\beta_0 + x_2) \right] \dot{p}_1 + (C_1 - A - A_1) \sin^2(\beta_0 + x_2) p_1 p_2 + C p_2 (k + p_3) \cos(\beta_0 + x_2) = \mu_1 [1 + \sin^2(\beta_0 + x_2)]$$

$$(A + B_1) \dot{p}_2 - (C_1 - A - A_1) \cos(\beta_0 + x_2) \sin(\beta_0 + x_2) p_1^2 - C(p_3 + k) \cos(\beta_0 + x_2) p_1 = u_2$$

$$C \dot{p}_3 = u_3 - u_1 \sin(\beta_0 + x_2)$$

$$\dot{x}_1 = p_1, \quad \dot{x}_2 = p_2, \quad \dot{x}_3 = p_3 - p_1 \sin(\beta_0 - x_2)$$

or

$$L \dot{p}_1 + N p_2 = D u_1 \quad (15)$$

$$G \dot{p}_2 - N p_1 = u$$

$$C \dot{p}_3 = u_3 - M u_1$$

$$\dot{x}_1 = p_1, \quad \dot{x}_2 = p_2, \quad \dot{x}_3 = p_3 - M p_1$$

where

$$L = A_2 + (A + A_1) \cos^2 \beta_0 + C_1 \sin^2 \beta_0$$

$$N = C \cos \beta_0 k, \quad G = B_1 + A, \quad D = 1 + \sin^2 \beta_0, \quad M = \sin \beta_0$$

For the convenience the equation (15) is written in the form

$$L \ddot{x}_1 + N \dot{x}_2 = D u_1 \quad (16)$$

$$G \ddot{x}_2 - N \dot{x}_1 = u_2$$

$$C(\ddot{x}_3 + M \ddot{x}_1) = u_3 - M u_1$$

Assuming that $y_{2i-1} = x_i$, $y_{2i} = \dot{x}_i$ a normal system of the six order is obtained

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = -\frac{N}{L} y_1 + \frac{D}{L} u_1$$

$$\dot{y}_3 = y_4$$

$$\dot{y}_4 = \frac{N}{G} y_2 + \frac{1}{G} u_2$$

$$\dot{y}_5 = y_6$$

$$\dot{y}_6 = \frac{MN}{L} y_2 - \frac{M(L+CD)}{CL} u_1 + \frac{1}{C} u_3$$

Composing matrices K_3 in the form (17), make a matrix [11-14],

$$K_3 = \left\| Q, PQ, P^2Q, P^3Q, P^4Q, P^5Q \right\| \tag{17}$$

It is easy to verify that the rank of the matrix K_3 is six. Indeed, the determinant of a matrix composed of Q and (PQ) is

$$K_3 = \begin{vmatrix} 0 & 0 & 0 & \frac{D}{L} & 0 & 0 \\ \frac{L}{C} & 0 & 0 & 0 & -\frac{N}{LG} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & \frac{1}{G} & 0 & \frac{DN}{GL} & 0 & 0 \\ 0 & 0 & 0 & -\frac{M(L+CD)}{CL} & 0 & \frac{1}{C} \\ \frac{M(L+CD)}{CL} & 0 & \frac{1}{C} & \frac{MND}{L^2} & 0 & 0 \end{vmatrix} = \frac{D}{C^3G^2}$$

So, the system is completely controllable. Similarly, it can be shown that at $u_1 = u_2 = 0$ a gyroscope, as a control object with nonholonomic constraints (11), becomes uncontrollable.

To solve the stabilization problem, we use the theorem.

Writing down the equation

$$\dot{e}_3 = \varphi(e_3, t)$$

with asymptotically stable zero solution and making up the system (10)

$$\dot{e}_1 = \frac{\left[-(A + A_1 - C) \cdot e_1 e_2 \cdot \sin 2\beta - C \cdot e_1 e_3 \cdot \cos \beta + (1 + \sin^2 \beta) \cdot \mu_1 \right]}{A_2 + (A + A_1) \cdot \cos^2 \beta + C_1 \sin^2 \beta}$$

$$\dot{e}_2 = \frac{C_1 - A - A_1}{A + B_1} \cdot \cos \beta \cdot \sin \beta \cdot e_1^2 + \frac{C}{A + B_1} \cdot e_1 e_3 \cdot \cos \beta + \frac{1}{A + B_1} \cdot \mu_2$$

$$\dot{e}_3 = \frac{1}{C} \lambda - \frac{\sin \beta}{C} \mu_1$$

we obtain

$$\lambda = \mu_1 \cdot \sin \beta + C \cdot \varphi \cdot (e_3, t) \tag{18}$$

3. Results and Discussion

This system, which describes the motion of the gyroscope, is asymptotically stable with respect to the variable characterizing the release of the gyroscope from condition (11).

Taking relation (13) as a particular solution, we obtain the following equations in a first approximation

$$\dot{p}_1 = \frac{[Ckp_2 \cos \beta + (1 + \sin^2 \beta_0) \mu_1]}{A_2 + (A + A_1) \cos^2 \beta + C_1 \sin^2 \beta_0}$$

$$\dot{p}_2 = \frac{C}{A + B_1} kp_1 \cos \beta + \frac{1}{A + B_1} \mu_1$$

$$\dot{p}_3 = \frac{1}{C} (\lambda - \mu_1 \sin \beta_0)$$

Then, relation (16) of the law of formation of the normal components of the servo-constraints reaction has the form

$$\lambda = C \frac{\partial \varphi}{\partial e_3} p_3 + \mu_1 \sin \beta_0$$

4. Conclusion

Equations of disturbed motion are derived containing the normal and tangent components of nonholonomic servoconstraints taken as the control parameters. A technique is proposed for determining the reaction forces of servoconstraints, which ensures motion stabilization relative to the manifold determined by nonholonomic servoconstraints. A theorem is proved that gives a method for determining the reaction force of a nonholonomic servoconstraints. A gyroscope in a gimbal mount with nonholonomic servoconstraints is considered. It is shown that when examining a gyroscope in a gimbal mount with non-holonomic servoconstraints on controllability, it is advisable to take servoconstraints as non-ideal ones. Otherwise, to achieve the system under consideration, it is necessary to impose new constraints or various (dissipative or gyroscopic) forces on controllability.

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