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Stabilization of mechanical system with holonomic servo constraints

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Abstract. The proposed study is devoted to the derivation of the equation of disturbed motion of mechanical systems with releasing servoconstraints, in which the multipliers of the normal and tangential components of the servoconstraints reaction forces are taken as control parameters. A technique is proposed for determining the reaction forces of servoconstraints providing motion stabilization relative to the manifold determined by holonomic servoconstraints. As an example, the problem of a gyroscope in a gimbal mount with holonomic releasing servoconstraints is considered.

1. Introduction

The aim of the study is to determine the reaction forces of servoconstraints that provide motion stabilization with respect to the manifold determined by holonomic servoconstraints, carried out by servomotors. A servomotor is an unpretentious working element that is part of industrial equipment.

Modern servomotors have combined all the achievements of scientific and technological innovative progress, therefore they are able to develop tremendous speeds of rotation at very high power. A large range of rotation adjustment of the servomotor shaft using software with significant acceleration or braking makes this equipment simply indispensable for use in machines or production lines and many other designs [1].

The concept of servoconstraints was first introduced by A. Begen [2]. A feature of A. Begen's systems with servoconstraints is that for such systems it is impossible to escape from the method of constraints realization, and the elementary work of the servoconstraints reactions at virtual displacements allowed by the constraints, generally speaking, is not zero. As is known [3], servoconstraints are invariant relations of the obtained differential equations of motion. Therefore, in the presence of disturbances that violate the servoconstraints conditions, the question arises of taking into account the release and solving problems of motion stabilization with respect to the manifold determined by servoconstraints [4]. In [5], the methods for determining the reaction force of servoconstraints at given displacements are shown, on which the work of the reaction force of servoconstraints is zero, i.e. servoconstraints are ideal for these displacements. However, in this work, explicit forms of the tangent and normal components of the servoconstraints are not distinguished.

2. Methods

Let the system, the position of which is determined by generalized coordinates $\tilde{q}_1,\ldots,\tilde{q}_n$, impose restrictions in the form of servoconstraints

$$f_\alpha(\tilde{q}_1, \tilde{q}_2, ..., \tilde{q}_n, t) = 0 \; ; \qquad (\alpha = 1,2,..., \alpha) \qquad (1)$$

The resulting system, while remaining holonomic, has $n-a$ degrees of freedom, and from equations (1) it is possible to express the parameter a in terms of the other parameters $n-a$ and these latter can be taken as the generalized coordinates of the new system, i.e.

$$\tilde{q}_i = \tilde{q}_i(q_1, q_2, ..., q_k, t); \; (i = 1,2,..., n \; ; \; k = n-a) \qquad (2)$$

and the variation of coordinates is interconnected by conditions

$$\delta\tilde{q}_i = \sum_s \frac{\partial \tilde{q}_i}{\partial q_s} \delta q_s \; ; \quad (s = 1,2,..., k)$$

Bearing in mind the continuous parametric release, we introduce additional independent quantities $\eta_p (p = k+1,..., n)$ characterizing the release of the system from servoconstraints. Subsequently, the left-hand sides of the servoconstraint equations (1) calculated on the real motion [4] are taken as quantities η_p. The possibility to provide continuous release in the dynamics of systems with servoconstraints by changing the parameters is of interest, first of all, from the point of view of determining the reactions of servoconstraints which provide the required change in parameters, in particular, stabilization of the system's motion with respect to servoconstraints.

It is assumed that the equations of motion released from servoconstraints obtained in [6]

$$\frac{d}{dt}\frac{\partial \tilde{T}}{\partial \dot{q}_s} - \frac{\partial \tilde{T}}{\partial q_s} = \tilde{Q}_s + \sum_\tau \mu_\tau a_{s\tau} \quad , \qquad (3)$$

$$\frac{d}{dt}\frac{\partial \tilde{T}}{\partial \dot{\eta}_p} - \frac{\partial \tilde{T}}{\partial \eta_p} = \tilde{Q}_p + \lambda_p + \sum_\tau \mu_\tau b_{p\tau} \quad , \qquad (4)$$

where $$\tilde{T} = \tilde{T}(q_s, \eta_p, \dot{\eta}_p, q_s, t)$$

$$a_{s\tau} = \sum_i \frac{\partial \tilde{q}_i}{\partial q_s} \cdot \frac{\partial \tilde{q}_i}{\partial q_\tau} \, , \qquad b_{p\tau} = \sum_i \frac{\partial \tilde{q}_i}{\partial \eta_p} \cdot \frac{\partial \tilde{q}_i}{\partial q_\tau}$$

allow some partial solutions that are compatible with servoconstraints. It is noted that the arbitrariness in the choice of excitations of the servoconstraints factors can be used to stabilize the undisturbed motion. To determine the reactions of servoconstraints providing motion stabilization relative to the manifold determined by holonomic servoconstraints, we make up the equation of disturbed motion. For convenience, equations (5) and (4) are written more compactly:

$$\frac{d}{dt}\frac{\partial \tilde{T}}{\partial \dot{q}_i} - \frac{\partial \tilde{T}}{\partial q_i} = \tilde{Q}_i + \lambda_i + \sum_\tau \mu_\tau a_{i\tau} \qquad (5)$$

where $a_{i\tau}$ is the known functions of coordinates and time

$$\widetilde{T} = \frac{1}{2}\sum_{i}\sum_{\nu} A_{i\nu}\dot{q}_i\dot{q}_\nu + \sum_{\nu} B_\nu \dot{q}_\nu + T_0 \qquad (6)$$

where T_0 and coefficients $A_{i\tau}$, B_ν are functions of generalized coordinates and time.

Bearing in mind the stabilization problem, consider some particular solution

$$q_i = q_i(t),\ q_{k+\alpha} = 0,\ \mu_\tau = \mu_\tau^0,\ \lambda_i = \lambda_i^0. \qquad (7)$$

$$q_{k+\alpha} = \eta_\alpha \quad (\alpha = 1,2,...,a),\quad \lambda_i = 0 \qquad (i = 1,2,...,k)$$

Coordinate values q_i, factors q_i and arbitrary coefficients μ_τ for disturbed motion are

$$q_i = q_i(t) + x_i,\ q_{k+\alpha} = 0,\ \mu_\tau = \mu_\tau^0 + u_\tau,\ \lambda_s = \lambda_s + u_s \qquad (8)$$

$$(s = k+1, k+2,...,n)$$

where u_τ $(\tau = 1,2,...,k)$, u_s $(s = k+1,...,n)$ are the control parameters.

Following the sequence in composing the equation of disturbed motion as shown in [7,8], we obtain the equations of disturbed motion:

$$A\ddot{x} + B\dot{x} + \Gamma\dot{x} + C\dot{x} = \mathrm{D}u + \Phi \qquad (9)$$

$$A = \|a_{i\nu}\|,\ a_{i\nu} = A_{i\nu} \qquad (10)$$

$$B = \|b_{i\nu}\|,\quad b_{i\nu} = \sum_j \frac{\partial A_{ij}}{\partial q_\nu}\dot{q}_j + \frac{\partial A_{i\nu}}{\partial t} - \frac{\partial \widetilde{Q}_i}{\partial \dot{q}_\nu}$$

$$\Gamma = \|g_{i\nu}\|,\quad g_{i\nu} = \sum_j \left(\frac{\partial A_{ij}}{\partial q_\nu} - \frac{\partial A_{\nu j}}{\partial q_i}\right)\dot{q}_j + \left(\frac{\partial B_i}{\partial q_\nu} - \frac{\partial B_\nu}{\partial q_i}\right)$$

$$C = \|c_{i\nu}\|,\quad C_{i\nu} = \sum_r \frac{\partial A_{ir}}{\partial q_\nu}\ddot{q}_r + \sum_j\sum_r \left(\frac{\partial^2 A_{ir}}{\partial q_j \partial q_\nu} - \frac{1}{2}\frac{\partial^2 A_{jr}}{\partial q_i \partial q_\nu}\right)\dot{q}_r\dot{q}_j +$$

$$+\sum_r\left(\frac{\partial^2 B_i}{\partial q_r\partial q_\nu}-\frac{\partial^2 B_r}{\partial q_i\partial q_\nu}\right)\cdot\dot{q}_r-\sum_r\frac{\partial^2 A_{ir}}{\partial q_\nu\partial t}\dot{q}_r-\frac{\partial\tilde{Q}_i}{\partial q_\nu}-\sum_\tau\mu_\tau^0\frac{\partial a_{i\tau}}{\partial q_\nu}+\frac{\partial^2 B_r}{\partial t\partial q_\nu}-\frac{\partial^2 T_0}{\partial q_i\partial q_\nu}$$

$$D=\|d_{i\nu}\|,\quad d_{i\nu}=a_{i\nu}$$

$$d=\begin{Vmatrix} d_{11} & \ldots & d_{1k} & 0 & \ldots & 0 \\ . & \ldots & . & . & \ldots & . \\ d_{n-a,1} & \ldots & d_{n-a,k} & 1 & \ldots & 0 \\ . & \ldots & . & . & \ldots & . \\ d_{n1} & \ldots & d_{nk} & 0 & \ldots & 1 \end{Vmatrix}$$

Φ are the terms of the higher than the first order of smallness with respect to $x, \dot{x}$ and u.

The solution to the problem of motion stabilization (5) is reduced to studying the stabilization problem in terms of the variable equations of disturbed motion (9). In these equations $a_{i\nu}, b_{i\nu}, q_{i\nu}$ and $c_{i\nu}$ for a given undisturbed motion are the known functions of time. Obviously, the terms $\sum_\nu \mathrm{b}_{i\nu}\dot{x}_\nu, \sum_\nu q_{i\nu}\dot{x}_\nu$ and $\sum_\nu c_{i\nu}x_\nu$ can be interpreted as forces, and the forces $\sum_\nu q_{i\nu}\dot{x}_\nu$ are gyroscopic ones. To verify this, it suffices to note that according to (10), a matrix composed of coefficients $q_{i\nu}$ is skew-symmetric one, i.e. $q_{i\nu}=-q_{\nu i}$. Thus, in deriving the equations of disturbed motion, as a rule, the terms appear that can be interpreted as gyroscopic forces.

Below we propose a technique for determining the forces of servoconstraints, which provides the motion stabilization relative to the manifold determined by servoconstraints. The technique is based on the structural features of equations (5), for which the following theorem is true.

3. Results and Discussion

Theorem: A controlled system (5) can always be stabilized with respect to the manifold defined by constraints (1).

For the proof, along with the equations of motion, we consider the system

$$\ddot{q}_s=\varphi_s(q_r,\dot{q}_r,\mathrm{t})\,;\ (s,r=k+1,k+2,\ldots,\mathrm{n}) \qquad (11)$$

with asymptotically stable zero solution. Substituting the kinetic energy into equations (5), we obtain

$$\sum_\nu A_{i\nu}\ddot{q}_\nu+/**/=\mathrm{Q_i}+\lambda_i+\sum_j\mu_j a_{ij}$$

where /**/ is the sum of terms not containing the second derivatives of the coordinates with respect to time. Since $\det(A_{i\nu})_{i,j=1}^n\neq 0$, then the equations can be represented as

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$$\ddot{q}_i = G_r(t, q_\lambda, \dot{q}_\lambda, \mu_{\mathrm{j}}) + \sum_{\mathrm{s}} \widetilde{A}_{\mathrm{si}} \lambda_s \qquad (12)$$

where $\widetilde{A}_{si}$ are the elements of the determinant, similar to the determinant $A_{i\nu}$. Substituting into the system (5) instead of $\ddot{q}_r (r = k+1,..., \mathrm{n})$ the value from (6), a system of equations is obtained with respect to

$$\sum_{\mathrm{s}} \widetilde{A}_{\mathrm{si}} \lambda_s = \varphi_r - G_r\,; \qquad (s = k+1, k+2,..., \mathrm{n})$$

The latter system is solvable relative to λ_s and allows explicit determining the factors

$$\lambda_s = \lambda_s(t, q_i, \dot{q}_i, \mu_{\mathrm{j}})$$

In a particular case of the systems with ideal constraints in the obtained expressions assume that $\mu_j = 0$; equations (5) and (1) determine the motion of the system.

As an example, consider a gyroscope in a gimbal mount with a fixed base. Let the equations of gyroscope motion with a holonomic release servoconstraints

$$\dot{\gamma} = \omega\,; \quad \gamma = \omega \cdot t + \gamma_o \qquad (13)$$

have the form

$$\left[A_2 + (A_1 + A)\cos^2\beta + (C_1 + C)\sin^2\beta\right] \cdot \ddot{a} + (C_1 - A_1 - A)\sin 2\beta\dot{\alpha}\dot{\beta} +$$
$$+ (\dot{\eta} + \omega + \sin\beta\dot{\alpha})C\cos\beta\dot{\beta} + C\sin\beta\ddot{\eta} = \mu_1\,, \qquad (14)$$

$$(A + B_1)\ddot{\beta} + \frac{1}{2}(A + A_1 - C - C_1)C\cos\beta\dot{\beta} + C\sin\beta\ddot{\eta} = \mu_1\,,$$
$$-C\omega\cos\beta\dot{\alpha} = \mu_2\,,$$

$$C\ddot{\eta} + C\sin\beta\ddot{\alpha} + C\cos\beta\dot{\alpha}\dot{\beta} = \lambda\,.$$

As a particular solution, we take the relations

$$\alpha = \alpha_0, \quad \beta = \beta_0, \quad \eta = 0, \quad \lambda = 0, \quad \mu_1 = \mu_2 = 0. \qquad (15)$$

Introducing disturbances

$$\alpha = \alpha_0 + x_1,\ \beta = \beta_0 + x_2,\ \eta = x_3,\ \lambda = u_3,\ \mu_1 = u_1,\ \ \mu_2 = u_2 \qquad (16)$$

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substitute them into equations (14) and obtain

$$\left[A_2+(A+A_1)\cos^2(\beta_0+x_2)+(C+C_1)\sin^2(\beta_0+x_2)\right]\ddot{x}_1+(C_1-A-A_1)\sin[2(\beta_0+x_2)]\dot{x}_1\dot{x}_2+$$

$$+C\cdot\cos(\beta_0+x_2)\cdot(\dot{x}_3+\omega)+\sin(\beta_0+x_2))\cdot\dot{x}_1\dot{x}_2+C\cdot\sin(\beta_0+x_2)\cdot\ddot{x}_3=u_1$$

$$(B_1+A)\cdot\ddot{x}_2+\frac{1}{2}(A+A_1-C-C_1)\cdot\sin[2(\beta_0+x_2)]\cdot\dot{x}_1^2-C\cos(\beta_0+x_2)\cdot\dot{x}_1\dot{x}_3-$$

$$-C\omega\cos(\beta_0+x_2)\dot{x}_1=u_2$$

$$C\cdot\ddot{x}_3+C\cdot\sin(\beta_0+x_2)\cdot\ddot{x}_1+C\cdot\cos(\beta_0+x_2)\cdot\dot{x}_1\dot{x}_2=u_3$$

Expanding the input functions in series in powers x_2, we have

$$\sin(\beta_0+x_2)=\sin\beta_0+\cos\beta_0\cdot x_2+\ \ldots\ ,$$

$$\cos(\beta_0+x_2)=\cos\beta_0-\sin\beta_0\cdot x_2+\ \ldots\ ,$$

$$\sin^2(\beta_0+x_2)=\sin^2\beta_0+\sin 2\beta_0\cdot x_2+\ \ldots\ ,$$

$$\cos^2(\beta_0+x_2)=\cos^2\beta_0-\sin 2\beta_0\cdot x_2+\ \ldots\ ,$$

where the dots denote terms containing x_2 in powers higher than the first one.

The equations of disturbed motion of the gyroscope in the first approximation take the form

$$L\cdot\ddot{x}_1+N\cdot\dot{x}_2+M\cdot\ddot{x}_3=u_1\ , \tag{17}$$

$$G\cdot\ddot{x}_2-N\cdot\dot{x}_1=u_2\ ,$$

$$C\cdot\ddot{x}_3+M\cdot\ddot{x}_1=u_3\ ,$$

where

$$L=A_2+(A+A_1)\cos^2\beta+(C+C_1)\sin^2\beta_0 \tag{18}$$

$$N=C\cdot\omega\cdot\cos\beta_0,\quad M=C\cdot\sin\beta_0,\quad G=B_1+A$$

Under conditions $u_i = 0$ ($i = 1,2,3$) the zero solution of system (17) is not stable or stable, but not asymptotically stable. Indeed, the characteristic equation of the system has the form

$$\begin{vmatrix} L\lambda^2 & N\lambda & M\lambda^2 \\ -N\lambda & G\lambda^2 & 0 \\ M\lambda^2 & 0 & C\lambda^2 \end{vmatrix} = 0$$

or

$$\lambda^6 (GLC - MG) + CN^2 \lambda^4 = 0 \tag{19}$$

The system under consideration is stable in the first approximation if all the roots relative to λ^2 are real negative numbers. Solving this equation relative to λ^2, we obtain

$$\lambda^4 = 0, \quad \lambda^2 = -\frac{CN^2}{G(LC - M)} CN^2$$

Since $M^2 < LC$, then equation (19) has a pair of purely imaginary roots $\pm \sqrt{CN^2 / G(LC - M^2)} \cdot i$. In this case, stability (not asymptotic one) and instability can take place.

To achieve asymptotic stability, we use controls u_i. By entering the notation

$$x_i = y_{2i-1}, \quad \dot{x}_i = y_{2i} \quad (i = 1,2,3)$$

with necessary calculations, we get the equations in normal form

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = -\frac{NC}{LC - M^2} y_4 + \frac{C}{LC - M^2} u_1 - \frac{M}{LC - M^2} u_3$$

$$\dot{y}_3 = y_4$$

$$\dot{y}_4 = \frac{N}{G} y_2 + \frac{1}{G} u_2$$

$$\dot{y}_5 = y_6$$

$$\dot{y}_6 = -\frac{NM}{LC - M^2} y_4 - \frac{M}{LC - M^2} u_1 + \frac{L}{LC - M^2} u_3$$

The controllability of the gyroscope as a control object with three controls is studied.
Let's compose a matrix [9 Krasovsky].

$$K = \left\| Q, PQ, P^2Q, P^3Q, P^4Q, P^5Q \right\| \tag{20}$$

where

$$P = \left\| \begin{matrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{NC}{M^2 - LC} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \dfrac{N}{G} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \dfrac{NM}{CL - M^2} & 0 & 0 \end{matrix} \right\| \qquad Q = \left\| \begin{matrix} 0 & 0 & 0 \\ \dfrac{C}{LC - M^2} & 0 & \dfrac{M}{M^2 - LC} \\ 0 & 0 & 0 \\ 0 & \dfrac{1}{G} & 0 \\ 0 & 0 & 0 \\ \dfrac{M}{M^2 - LC} & 0 & \dfrac{L}{LC - M^2} \end{matrix} \right\|,$$

$$PQ = \left\| \begin{matrix} \dfrac{C}{LC - M} & 0 & \dfrac{M}{M^2 - LC} \\ 0 & \dfrac{NC}{G(M^2 - LC)} & 0 \\ 0 & \dfrac{1}{G} & 0 \\ \dfrac{CN}{G(LC - M^2)} & 0 & \dfrac{MN}{G(M^2 - LC)} \\ \dfrac{M}{M^2 - LC} & 0 & \dfrac{L}{LC - M^2} \\ 0 & \dfrac{MN}{G(LC - M^2)} & 0 \end{matrix} \right\|$$

The determinant of a matrix composed of matrix columns Q and (PQ), is equal to $C(GL + MN)/G^3(LC - M^2)$ and, therefore, the rank of the matrix (20) is six. The latter means that the gyroscope in the gimbal mounts as a control object with non-ideal releasing servoconstraints (13) is completely controllable.

To study the gyroscope controllability with normal components of the servoconstraints reactions (13), we compose a matrix of the form (20) in which

$$
\mathrm{Q} = \begin{Vmatrix} 0 \\ \dfrac{M}{M^2 - LC} \\ 0 \\ 0 \\ 0 \\ \dfrac{L}{LC - M^2} \end{Vmatrix}, \qquad \mathrm{PQ} = \begin{Vmatrix} \dfrac{M}{M^2 - LC} \\ 0 \\ 0 \\ \dfrac{MN}{G(M^2 - LC)} \\ \dfrac{L}{LC - M^2} \\ 0 \end{Vmatrix}
$$

$$
\mathrm{P}^2\mathrm{Q} = \begin{Vmatrix} 0 \\ \dfrac{MCN^2}{G(M^2 - LC)^2} \\ \dfrac{MN}{G(M^2 - LC)} \\ 0 \\ 0 \\ -\dfrac{M^2N^2}{G(M^2 - LC)^2} \end{Vmatrix}, \qquad \mathrm{P}^3\mathrm{Q} = \begin{Vmatrix} \dfrac{MCN^2}{G(M^2 - LC)^2} \\ 0 \\ 0 \\ \dfrac{MCN^3}{G^2(M^2 - LC)^2} \\ -\dfrac{N^2M^2}{G(M^2 - LC)^2} \\ 0 \end{Vmatrix}
$$

$$
\mathrm{P}^4\mathrm{Q} = \begin{Vmatrix} 0 \\ \dfrac{N^4MC^2}{G^2(M^2 - LC)^3} \\ \dfrac{MCN^3}{G^2(M^2 - LC)^2} \\ 0 \\ 0 \\ -\dfrac{M^2CN^4}{G^2(M^2 - LC)^3} \end{Vmatrix}, \qquad \mathrm{P}^5\mathrm{Q} = \begin{Vmatrix} \dfrac{MC^2N^4}{G^2(M^2 - LC)^3} \\ 0 \\ 0 \\ \dfrac{N^5MC^2}{G^3(M^2 - LC)^3} \\ -\dfrac{N^4M^2C}{G^2(M^2 - LC)^3} \\ 0 \end{Vmatrix}
$$

The matrix K_1 is square, containing 6 rows and 6 columns. The determinant of the matrix K_1 made up of these columns is zero, and therefore, the rank of this matrix is less than six. The latter means that the gyroscope in gimbal mount as a control object with normal components of servoconstraints reactions is not completely controllable.

Thus, in the study of mechanical systems with servoconstraints on controllability, it is advisable to accept servoconstraints as non-ideal ones, otherwise, to achieve the system controllability, it is necessary to impose new constraints or various (dissipative or gyroscopic) forces [10, 11].

If to use Theorem 1 to solve the stabilization problems, then we should restrict ourselves to the case when the constraint (13) can be considered as ideal, so, equation (9) is written in the form

$$\ddot{\eta} = \varphi(\eta, \dot{\eta}) \tag{21}$$

Composing equations (14) and taking relation (15) as a partial solution, we obtain the following equations

$$L\ddot{x}_1 + N\dot{x}_2 + M\ddot{x}_3 = 0 \tag{22}$$

$$G\ddot{x}_2 + N\dot{x}_1 = 0$$

$$\ddot{\eta} = \left(\frac{\partial\varphi}{\partial\dot{\eta}}\right)_0 \dot{\eta} + \left(\frac{\partial\varphi}{\partial\eta}\right)\eta$$

where the coefficients L, M, N and G are determined by relations (19).

System (22), describing the equations of disturbed motion, is asymptotically stable with respect to the variables characterizing the release of the gyroscope from condition (13).

To determine the reaction forces of servoconstraints, the following equation is used

$$C(\sin\beta_0\ddot{\alpha} + \ddot{\eta}) = \lambda$$

which, together with the first equation of system (22), allows finding the following law of formation of the servoconstraint forces:

$$\lambda = \left[\left(\frac{\partial\varphi}{\partial\dot{\eta}}\right)_0 \dot{\eta} + \left(\frac{\partial\varphi}{\partial\eta}\right)_0 \eta\right]\left(C - \frac{C^2\sin^2\beta_0}{L}\right) - \\ - \frac{C^2\omega\cos\beta_0\sin\beta_0}{L}\dot{x}_2 \tag{23}$$

Note that during stabilization with respect to the angular velocity of rotation, function (21) can be taken in the form

$$\ddot{\eta} = \varphi(\dot{\eta})$$

and instead of (23) we have

$$\lambda = \left(\frac{\partial\varphi}{\partial\dot{\eta}}\right)_0 \dot{\eta}\left(C - \frac{C^2\sin^2\beta_0}{L}\right) - \frac{C^2\omega\cos\beta_0\sin\beta_0}{L}\dot{x}_2$$

4. Conclusion

The equations of disturbed motion are compiled containing the normal and tangent components, which are accepted as control parameters. A technique is proposed for determining the strength of servoconstraints, providing motion stabilization relative to the manifold determined by servoconstraints. In the study of mechanical systems with servoconstraints on controllability, it is advisable to accept servoconstraints as non-ideal ones, otherwise, in order to achieve the

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controllability of the system under consideration, it is necessary to impose new relationships or various (dissipative or gyroscopic) forces.

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