TOSHKENT IRRIGATSIYA VA QISHLOQ
XO'JALIGINI MEXANIZATSIYALASH
MUHANDISLARI INSTITUTI

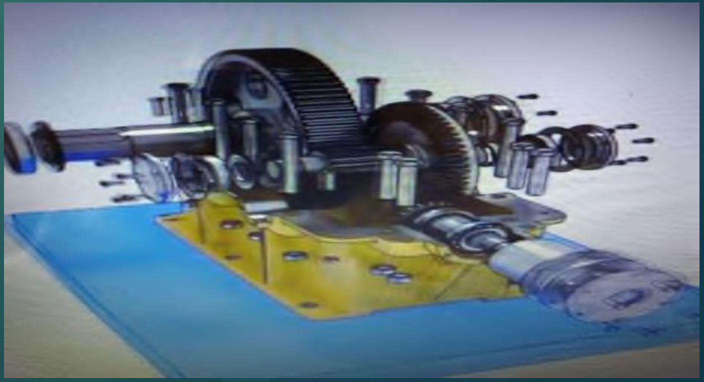
FAN: NAZARIY MEXANIKA

MAVZU 12

MODDIY NUQTA UCHUN DINAMIKANING UMUMIY TEOREMALARI

Husanov Q.

Nazariy va qurilish
mexanikasi
kafedrasi dotsenti



TAQDIMOT REJASI

1. NUQTA HARAKAT MIQDORINING O'ZGARISHI HAQIDAGI TEOREMA.

2. NUQTA HARAKAT MIQDORI MOMENTINING O'ZGARISHI HAQIDAGI TEOREMA.

3. NUQTA KINETIK ENERGIYASINING O'ZGARISHI HAQIDAGI TEOREMALARDAN IBORAT.

Moddiy nuqta harakat miqdori va moddiy nuqta harakat miqdorining o'zgarishi haqidagi teorema

NUQTANING MASSASI VA TEZLIGINING KO'PAYTMASIGA TENG BO'LIB, YO'NALISHI SHU NUQTA TEZLIGI BO'YLAB YO'NALGAN VEKTOR KATTALIKKA, NUQTANING HARAKAT MIQDORI DEYILADI.

$$m\ddot{\vec{r}} = \overline{F}$$

$$\frac{d(m\vec{\vartheta})}{dt} = \vec{F}$$

$$d(m\vec{\vartheta}) = \overline{F}dt$$

$$d(m\vec{\vartheta}) = d\vec{S}$$

$$d\vec{S} = \vec{F}dt$$

$$m\vec{\vartheta} - m\vec{\vartheta}_0 = \int_0^t \vec{F}dt$$

$$m\vec{\vartheta} - m\vec{\vartheta}_0 = \vec{S}$$

$$\overline{S} = \int_0^t \overline{F}dt$$

$$m\vartheta_x - m\vartheta_{0x} = S_x \ ,$$

$$m\vartheta_y - m\vartheta_{0y} = S_y \ ,$$

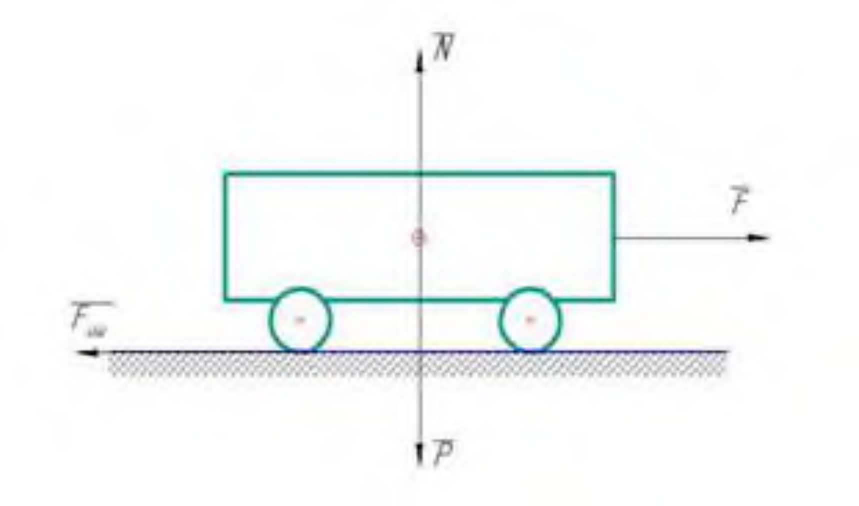
$$m\vartheta_z - m\vartheta_{0z} = S_z \ .$$

$$S_x = \int_o^t F_x dt; \quad S_y = \int_o^t F_y dt; \quad S_z = \int_o^t F_z dt.$$

$$\overline{F} = 0 \qquad d(m\vec{\vartheta}) = 0 \qquad m\vec{\vartheta} = const$$

$$F_x = 0 \qquad m\vartheta_x = const$$

Masala. Yuk ortilgan vagonlarning umumiy og'irligini o'lchash maqsadida vagon bilan teplovoz orasiga dinamometr o'rnatil-gan. Dinamometrning 2 minut oralig'idagi o'rtacha ko'rsatkich 100,8t. Shu vaqt oralig'ida vagonning tezligi 57,6 *km/soat* (boshlang'ich vaqtda poezd tinch holatda bo'lgan). Agar ishqalanish koeffitsienti $f = 0{,}02$ bo'lsa, vagonlarning umumiy og'irligi topilsin



$$\frac{P}{g}(\vartheta - \vartheta_0) = (F - F_{\text{ш}})t$$

$$m\vartheta - m\vartheta_0 = \int_0^t (F - F_{\text{ш}})dt$$

$$P = \frac{F \cdot t \cdot g}{\vartheta - \vartheta_0 + f \cdot t \cdot g} \qquad \vartheta_0 = 0$$

$$F_{\text{ш}} = f \cdot N = f \cdot P$$

$$P = \frac{F \cdot t \cdot g}{\vartheta + f \cdot t \cdot g} = \frac{100{,}8 \cdot 120 \cdot 9{,}81}{16 + 0{,}02 \cdot 9{,}81 \cdot 120} = 3000$$

Nuqta harakat miqdori momenti va uning o'zgarishi haqidagi teorema

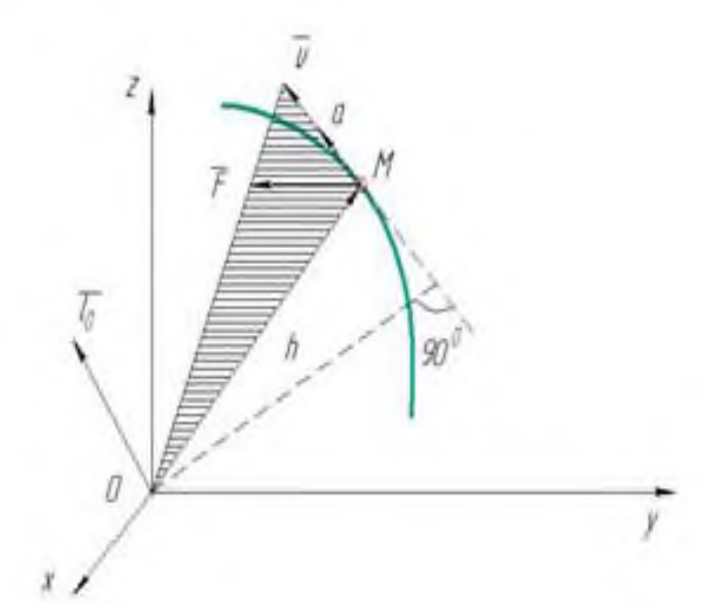
$$\vec{\ell}_0 = \vec{r} \times m\vec{\vartheta} \qquad m\vec{a} = \vec{F} \qquad m\ddot{\vec{r}} = \vec{F}$$

$$\vec{r} \times (m\vec{a}) = \vec{r} \times \left(m\frac{d\vec{\vartheta}}{dt} \right) = \frac{d}{dt}\left(\vec{r} \times m\vec{\vartheta}\right) - \frac{d\vec{r}}{dt} \times (m\vec{\vartheta}) = \frac{d}{dt}\left(\vec{r} \times m\vec{\vartheta}\right) - \vec{\vartheta} \times m\vec{\vartheta}$$

$$\frac{d}{dt}\left(\vec{r} \times m\vec{\vartheta}\right) = \vec{r} \times \vec{F}$$

$$\frac{d\vec{\ell}_0}{dt} = \vec{r} \times \vec{F} \qquad \frac{d\vec{\ell}}{dt} = \vec{M}_O$$

$$\ell_0 = m_0(m\vec{\vartheta}) = \pm h \cdot m\vec{\vartheta}$$



$$\frac{d}{dt}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ m\dot{x} & m\dot{y} & m\dot{z} \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\frac{d\ell_x}{dt} = M_x, \qquad m\frac{d}{dt}\left(y\dot{z} - z\dot{y}\right) = yF_z - zF_y;$$

$$\frac{d\ell_y}{dt} = M_y, \qquad m\frac{d}{dt}\left(z\dot{x} - x\dot{z}\right) = zF_x - xF_z;$$

$$\frac{d\ell_z}{dt} = M_z. \qquad m\frac{d}{dt}\left(x\dot{y} - y\dot{x}\right) = xF_y - yF_x.$$

Biror o'qqa nisbatan nuqta harakat miqdori momentidan vaqt bo'yicha olingan hosila nuqtaga ta'sir etuvchi kuchlardan shu o'qqa nisbatan olingan momentlarning algebraik yig'indisi teng.

Markaziy kuch, ya'ni nuqtaga ta'sir etuvchi kuchlarning ta'sir chiziqlari fazoning biror nuqtasidan o'tib, bu kuchning moduli, shu nuqtadan chiqqan $\vec{r}$ radiusga bog'liq bo'ladi.

$$\vec{m}_0(\vec{F}) = 0 \Rightarrow \vec{\ell}_0 = \vec{r} \times m\vec{\vartheta} = \overrightarrow{const}$$

Nuqta kinetik energiyasi va nuqta kinetik energiyasining o'zgarishi haqida teorema

$$\frac{m\vartheta^2}{2} \Rightarrow m \cdot \frac{d\vec{\vartheta}}{dt} \cdot d\vec{r} = \vec{F} \cdot d\vec{r} \quad m \cdot d\vartheta \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot d\vec{r}$$

$$m \cdot d\vec{\vartheta} \cdot \vec{\vartheta} = \overline{F} \cdot d\vec{r} \Rightarrow d\left(\frac{1}{2} m\vec{\vartheta}^2\right) = \vec{F} \cdot d\vec{r}$$

$$d\left(\frac{1}{2}m\vec{\vartheta}^2\right) = dA$$

$$\frac{d\left(\frac{1}{2}m\vartheta^2\right)}{dt} = \frac{dA}{dt}$$

$$\frac{d\left(\frac{1}{2}m\vartheta^2\right)}{dt} = N,$$

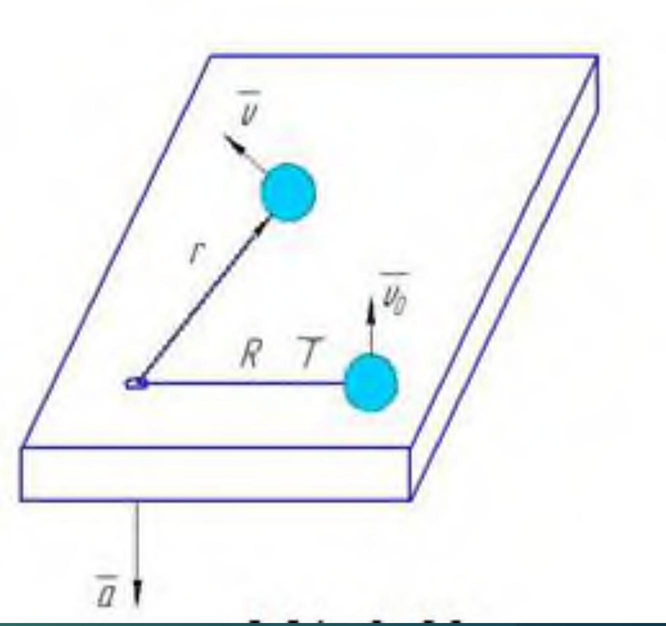
$$\frac{1}{2}m\vartheta^2 - \frac{1}{2}m\vartheta_0^2 = A$$

nuqtaning biror chekli ko'chishida nuqta kinetik energiyasining o'zgarishi unga ta'sir etuvchi kuchning mazkur ko'chishidagi bajargan ishiga teng.

ϑ_0 $\vec{T}$ $\vec{a}$

Masala.R

$$\frac{d\vec{\ell}}{dt} = \vec{M}_0 \quad M_z = 0$$



$$\vec{\ell}_0 = \vec{r} \times m\vec{\vartheta} = \overrightarrow{const}$$

$$m\vartheta_0 \cdot R = m\dot{\varphi}r^2 \qquad \dot{\varphi} = \frac{\vartheta_o R}{r^2}$$

$$\frac{dr}{dt} = -a \qquad r = c_1 \pm at \qquad c_1 = R$$

$$r = R - at \qquad \frac{d\varphi}{dt} = \frac{R\vartheta_o}{(R-at)^2}$$

$$\varphi = \frac{R\vartheta_0}{a(R-at)} + c_2 \Rightarrow c_2 = \frac{RV_0}{aR} \qquad r = R - at$$

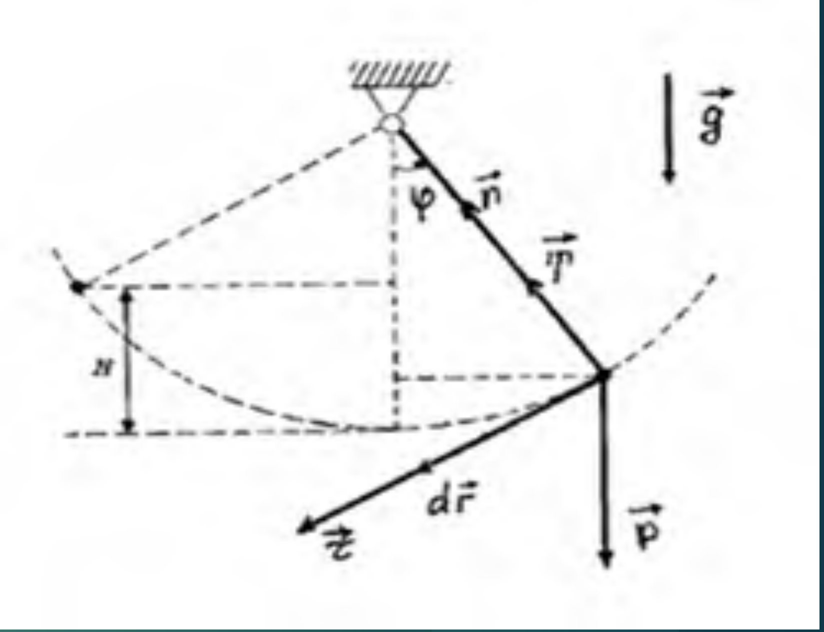
$$\varphi = \frac{\vartheta_0 R}{R - at} \qquad \varphi = \frac{\vartheta_0 R}{R - at} \qquad m\bar{a}_n = \bar{F}_n$$

$$m\frac{\vartheta^2}{r} = T$$

$$T = m.\frac{(\dot{\varphi}.r)^2}{r} = m.\frac{\frac{\vartheta_0^{\ 2}.R^2}{(R-a.t)^2}}{(R-at)}. = \frac{m.\vartheta_0^{\ 2}.R^2}{(R-at)^3}$$

Задача2. 0,5 m . $\vartheta_0 = 2,1\ m/s$

$$A(\vec{T}) = 0 \qquad \vec{T} \perp d\vec{r}$$



A(P)=mg(1-cos60)

$$\frac{m\vartheta^2}{2} - \frac{m\vartheta_0^2}{2} = mg(1-\cos 60^0)$$

$$\vartheta^2 = \vartheta_0^2 + 2g\ell(1-\cos 60^0)$$

$$ma_n = T - P\cos\varphi$$

$$T = P\cos\varphi + ma_n = P\cdot\cos\varphi + m\cdot a_n = P\cdot\cos\varphi + m\frac{\vartheta^2}{\ell} = mg + \frac{m\cdot\vartheta^2}{\ell}$$

$$= m\left[\frac{1}{\ell}(\vartheta_0^2 + 2g\ell(1-60^0)) + g\right] = m\cdot g(\frac{\vartheta_0^2}{g\cdot\ell} + 2) = 1\cdot 9,8(\frac{2,1^2}{9,8\cdot 0,5} + 2) = 28,4H$$

$$0 - \frac{m\vartheta_0^2}{2} = -mg\left[H - \ell(1-\cos 60^0)\right]$$

$$H = \frac{\vartheta_0^2}{2\cdot g} + \ell(1-\cos 60^0) = \frac{2,1^2}{2\cdot 9,8} + 0,5(1-0,5) = 0,475$$

$$T = P\cos\varphi + ma_n = P\cdot\cos\varphi + m\cdot a_n = P\cdot\cos\varphi + m\frac{\vartheta^2}{\ell} = mg + \frac{m\cdot\vartheta^2}{\ell}$$

$$= m\left[\frac{1}{\ell}(\vartheta_0^2 + 2g\ell(1-60^0)) + g\right] = m\cdot g(\frac{\vartheta_0^2}{g\cdot\ell} + 2) = 1\cdot 9{,}8(\frac{2{,}1^2}{9{,}8\cdot 0{,}5} + 2) = 28{,}4H$$

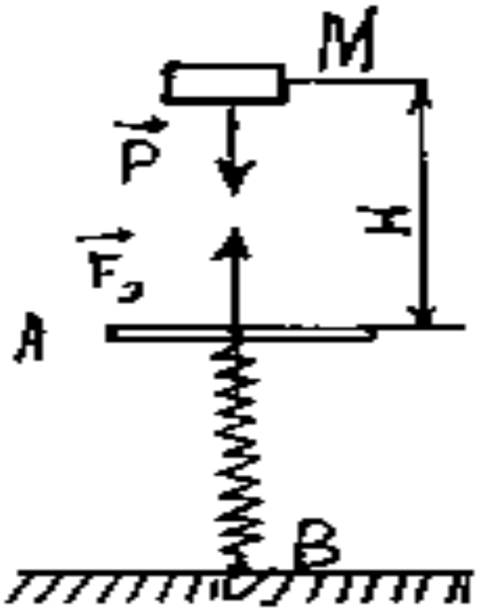
N balandlikni aniqlash uchun yana (3.104) tenglamadan foydalanamiz:

$$0 - \frac{m\vartheta_0^2}{2} = -mg\left[H - \ell(1-\cos 60^0)\right]$$

Bu tenglamadan

$$H = \frac{\vartheta_0^2}{2\cdot g} + \ell(1-\cos 60^0) = \frac{2{,}1^2}{2\cdot 9{,}8} + 0{,}5(1-0{,}5) = 0{,}475 \ m.$$

3- *masala* (31.12-[2]). Og'irligi P bo'lgan M yuk B spiral prujinada turgan A plitaga N balandlikdan tashlanadi. Yuk tashlanadigan nuqtada uning boshlang'ich tezligi nolga teng. Tushgan M yuk ta'sirida prujina h miqdopga qisiladi. A plita og'irligi va qarshiliklarni hisobga olmay, prujina h miqdorga qisilguncha o'tgan T vaqt va prujinaning shu vaqt ichidagi elastiklik kuch impulsi S hisoblansin



Yechish. Yukning A plitaga tushish vaqtidagi tezligini aniqlash uchun yuqoridaci tenglamadan foydalanamiz, ya'ni

$$\frac{m\vartheta_1^2}{2} = mg \cdot H \quad \text{yoki} \quad \vartheta_1 = \sqrt{2g \cdot H}$$

$$x = c_1 \cdot \cos\sqrt{\frac{c}{m}}t + c_2 \cdot \sin\sqrt{\frac{c}{m}}t + \frac{mg}{c}$$

$$t_1 = 0 \text{ da} \quad x_0 = 0, \quad \dot{x}_0 = \sqrt{2g \cdot H}.$$

Bu kattaliklarni yuqoridagi tenglamalarga qo'ysak,

$$c_1 = -\frac{mg}{c}\ ; \ c_2 = \sqrt{\frac{2g \cdot H \cdot m}{c}}\ .$$

$$x = \frac{mg}{c}\left(1 - \cos\sqrt{\frac{c}{m}}t\right) + \sqrt{\frac{2g \cdot H \cdot m}{c}} \cdot \sin\sqrt{\frac{c}{m}}t$$

$$\dot{x} = g\sqrt{\frac{m}{c}} \cdot \sin\sqrt{\frac{c}{m}}t + \sqrt{2g \cdot H} \cdot \cos\sqrt{\frac{c}{m}}t)$$

$$g\sqrt{\frac{m}{c}} \cdot \sin\sqrt{\frac{c}{m}}t + \sqrt{2g \cdot H} \cdot \cos\sqrt{\frac{c}{m}}T = 0$$

$$T = -\sqrt{\frac{m}{c}} \cdot arctg\sqrt{\frac{2H \cdot c}{m \cdot g}}t$$

$$-\frac{m\vartheta_0^2}{2} = mg \cdot h - \frac{c}{2}h^2$$

$$A(\vec{F}_{uu}) = -\frac{c}{2}h^2$$

$$c = \frac{2mg(H + h)}{k^2}$$

$$T = -\frac{h}{\sqrt{2g(H+h)}} \cdot arctg \frac{2\sqrt{H(H+h)}}{h} =$$

$$= \frac{h}{\sqrt{2g(H+h)}} \left[\frac{\pi}{2} + arctg \cdot \frac{2\sqrt{H(H+h)}}{h} \right] =$$

$$= \frac{h}{\sqrt{2g(H+h)}} \left[\frac{\pi}{2} + arctg \cdot \frac{h}{2\sqrt{H(H+h)}} \right]$$

$$S(\vec{F}_э) = \int_0^T F_э \cdot dt = \int_0^T cx \cdot dt = \int_0^T c \left[\frac{mg}{c} (1 - \cos\sqrt{\frac{c}{m}} \cdot t) + \sqrt{\frac{2g \cdot H \cdot m}{c}} \cdot \sin\sqrt{\frac{c}{m}} \cdot t) \right] \cdot dt =$$

$$mg(T - \frac{h}{\sqrt{2g(H+h)}} \cdot \sin\sqrt{\frac{c}{m}} \cdot T + m\sqrt{2g \cdot H}(1 - \cos\sqrt{\frac{c}{m}} \cdot T) = mg \cdot T + m\sqrt{2g \cdot H}$$

$$mg \cdot \frac{h}{\sqrt{2g(H+h)}} \cdot \sin\sqrt{\frac{c}{m}} \cdot T + m\sqrt{2g \cdot H} \cdot \cos\sqrt{\frac{c}{m}} \cdot T) = m \cdot \dot{x}(T) = 0$$

Moddiy nuqta uchun dinamikaning umumiy teoremalari nima uchun kiritiladi va qanday nomlanadi ?

Masalalar yechishda nuqta harakat miqdorining o'zgarishi haqidagi teoremani qanday hollarda qo'llaniladi ?

3. Nuqta harakat miqdorining saqlanish qonuni qanday shartlar asosida amalga oshiriladi ?
4. Nuqta harakat miqdori momentining o'zgarishi haqidagi teoremani qo'llanish sohasini tushuntiring.
5. Nuqta harakat miqdori momentining saqlanish qonunnini tushuntiring.
6. Markaziy kuchni misollar orqali tushuntiring.
7. Kuch impulsining mexanik ma'nosini tushuntiring.
8. Nuqta kinetik energiyasini o'zgarishi haqidagi teoremani qo'llanish sohasini tushuntiring.
9. Kuchning bajargan ishini mexanik ma'nosini tushuntiring.
10. Nuqtaning potensial energiyasi va kinetik energiyasi o'rtasidagi o'xshashlikni qanday tushunasiz ?

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E'TIBORINGIZ UCHUN RAHMAT!

HUSANOV Q.

Nazariy va qurilish mexanikasi kafedrasi dotsenti