TOSHKENT IRRIGATSIYA VA QISHLOQ XO'JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI

FAN: NAZARIY MEXANIKA

MAVZU 9

Nuqtaning murakkab harakati

Husanov Q

Nazariy va qurilish mexanikasi kafedrasi dotsenti

TAQDIMOT REJASI

1. Nuqtaning nisbiy, ko’chirma va absalyut harakati.
2. Murakkab harakatdagi nuqtaning tezligi .
3. Murakkab harakatdagi nuqtaning tezlanishi. Koriolis teoremasi

Nuqtaning murakkab harakati

Ba'zi hollarda M nuqta bir vaqtning o'zida ham qo'zg'aluvchan, ham qo'zg'almas koordinatalar sistemalariga nisbatan harakatlanadi.

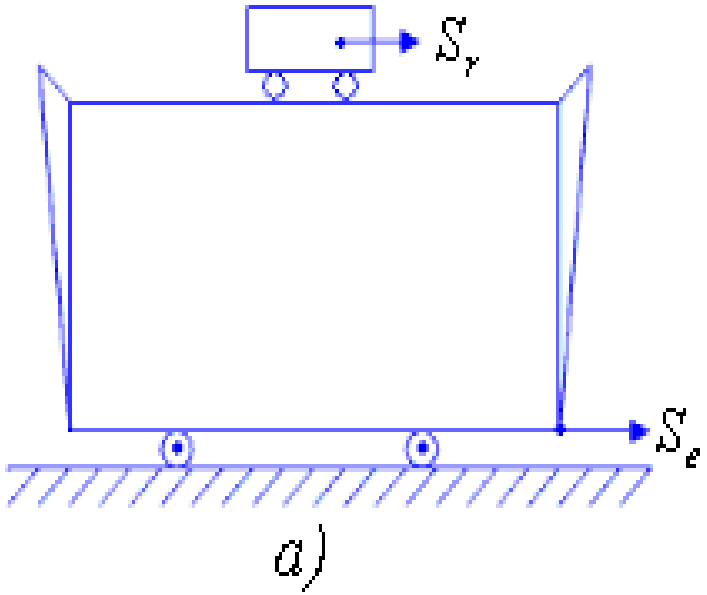
Aytaylik, M nuqta qo'zg'aluvchan $Axyz$ koordinatalar sistemasiga nisbatan va u o'z navbatida asosiy qo'zg'almas $Ox_1y_1z_1$ koordinatalar sistemasiga nisbatan harakatlansin.

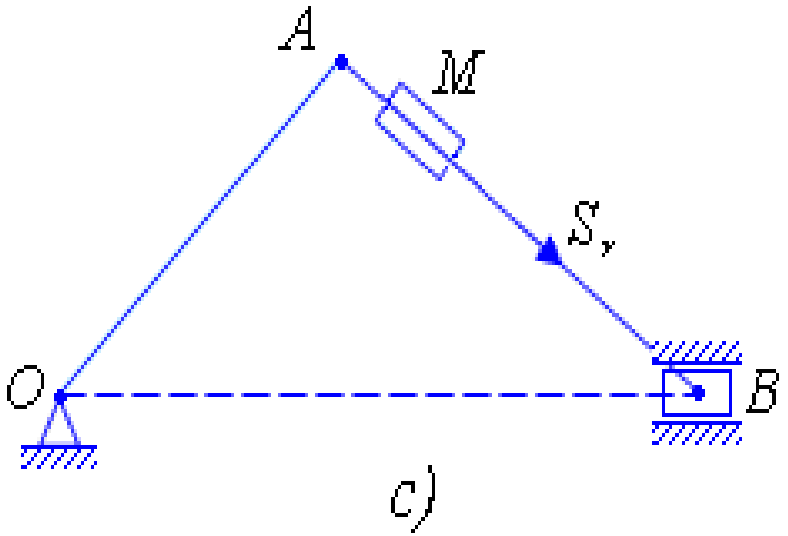
U holda quyidagi ta'riflar o'rinli bo'ladi:

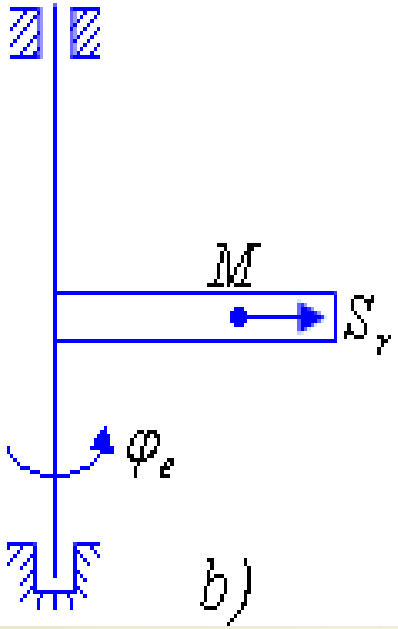
M nuqtaning qo'zg'almas $O_1x_1y_1z_1$ koordinatalar sistemasiga nisbatan harakati murakkab yoki absolyut harakat deyiladi. Murakkab harakatni S_a bilan belgilaymiz.

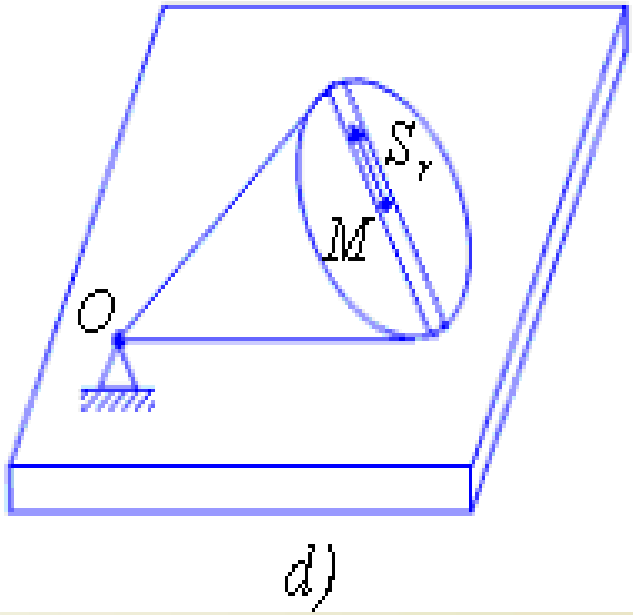
M nuqtaning qo'zg'aluvchan Axyz koordinatalar sistemasiga nisbatan harakati nisbiy harakat deyiladi. Nisbiy harakatni S_r bilan belgilaymiz.

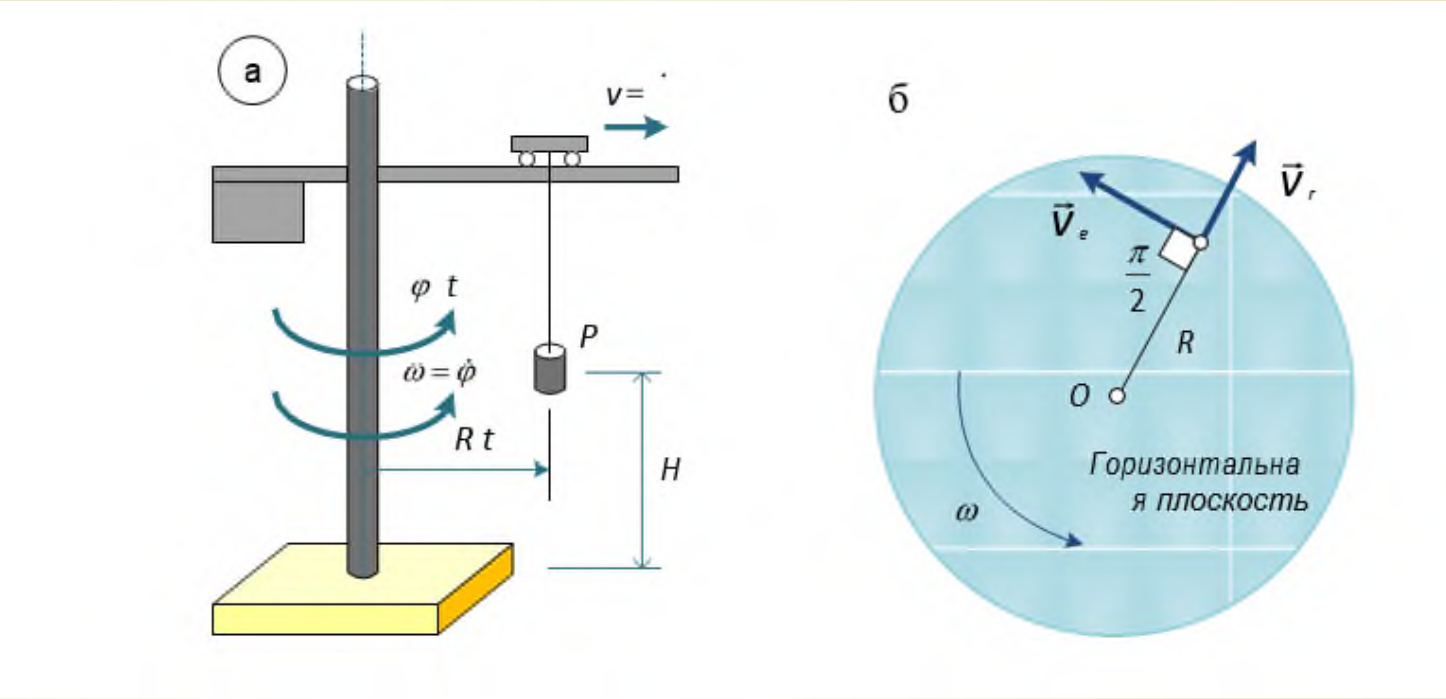
M nuqtaning qo'zg'aluvchan Axyz koordinatalar sistemasi bilan birgalikda (ya'ni M nuqtaning Axyz koordinatalar sistemasiga nisbatan qo'zg'almas holati) qo'zg'almas $Ox_1y_1z_1$ koordinatalar sistemasiga nisbatan harakati ko'chirma harakat deyiladi.

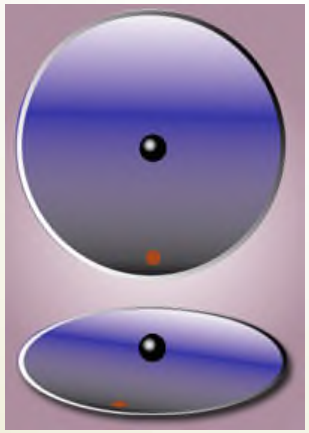

S_r
S_e
a)

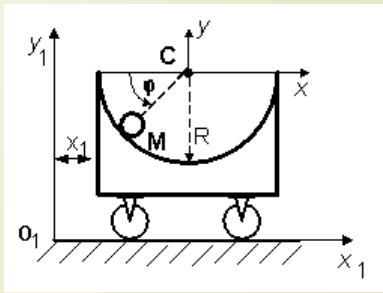

A
M
S_r
O
B
c)


M
S_r
φ_e
b)


S_r
M
O
d)


а
v=
φ t
ω = φ̇
P
R t
H
б
$\vec{V}_r$
$\vec{V}_e$
π/2
R
O
Горизонтальна
я плоскость
ω




y
y1
C
φ
x
x1
M
R
o1
x1

$$\vec{b} = \vec{b}(t)$$

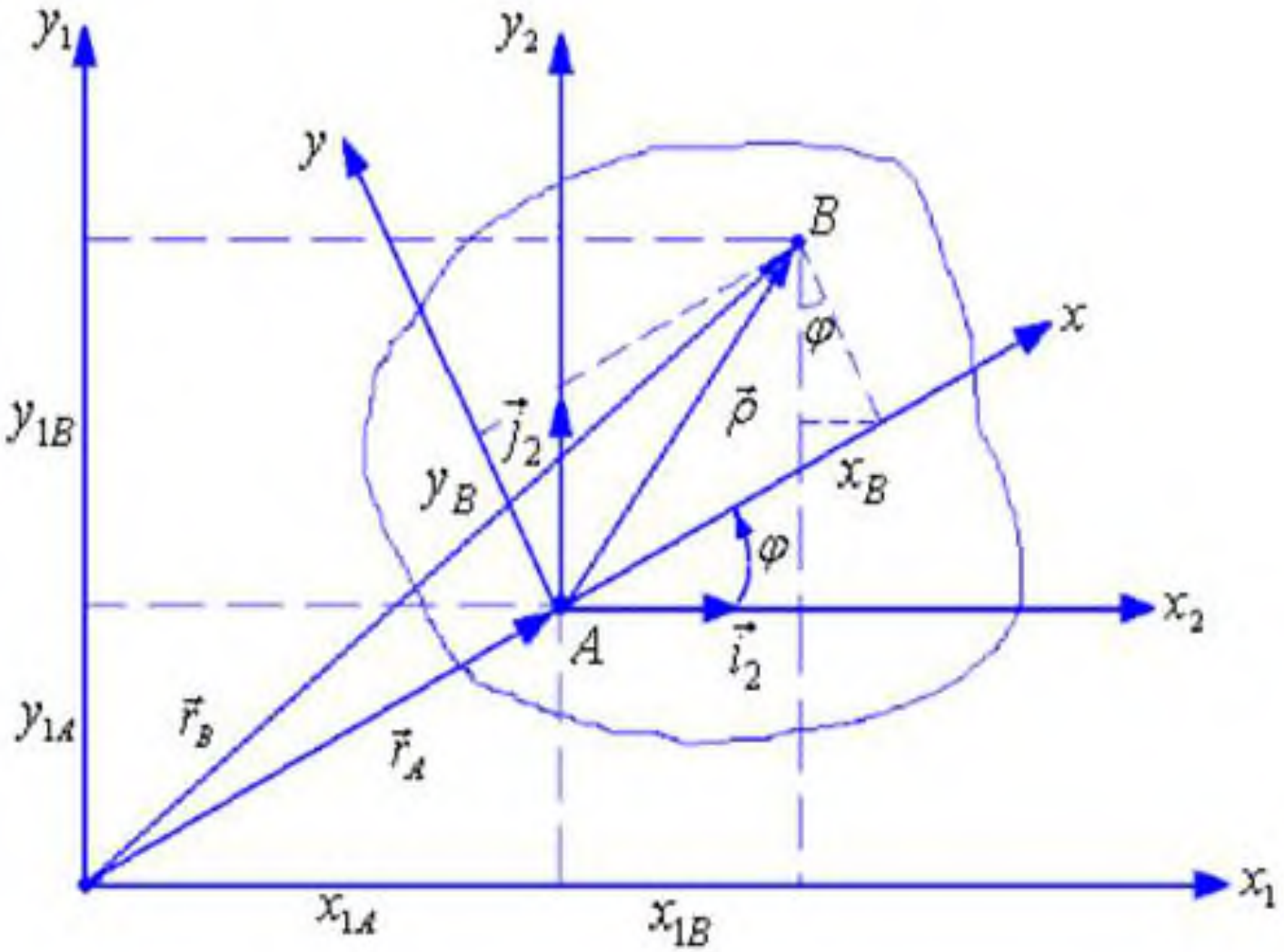
$$\vec{b} = b_x \cdot \vec{i} + b_y \cdot \vec{j} + b_z \cdot \vec{k}$$

yoki

$$\frac{d\vec{b}}{dt} = \frac{\tilde{d}\vec{b}}{dt} + \vec{\omega} \times \vec{b}$$

Bur formulasi

Yani , berilgan vektorning absolyut hosilasi shu vektorning nisbiy hosilasi bilan qo'zg'aluvchan koordinatalar sistemasi burchak tezligi va berilgan vektorning vektor ko'paytmasining geometrik yig'indisiga teng.


y_1
y_2
y
B
x
φ
y_{1B}
$\vec{j}_2$
$\vec{\rho}$
y_B
x_B
φ
x_2
A
$\vec{i}_2$
y_{1A}
$\vec{r}_B$
$\vec{r}_A$
x_1
x_{1A}
x_{1B}

$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\vec{\rho}}{dt} \qquad \frac{d\vec{\rho}}{dt} = \frac{\tilde{d}\vec{\rho}}{dt} + \vec{\omega} \times \vec{\rho}$$

$$\vec{\vartheta}_r = \frac{\tilde{d}\vec{\rho}}{dt} = \dot{x} \cdot \vec{i} + \dot{y} \cdot \vec{j} + \dot{z} \cdot \vec{k} \qquad \vec{\vartheta}_a = \vec{\vartheta}_A + \vec{\omega} \times \vec{\rho} + \vec{\vartheta}_r$$

bunda $\vec{\vartheta}_a = \vec{\vartheta}_e + \vec{\vartheta}_r$

ya'ni *nuqtaning absolyut tezligi uning ko'chirma tezligi bilan nisbiy tezliklarining geometrik yig'indisiga teng.*

Murakkab harakatdagi nuqtaning tezlanishi. Koriolis teoremasi

Nuqtaning absolyut tezlanishini aniqlash uchun tezlik vektorining har ikki tomonidan vaqt bo'yicha hosila olamiz:

$$\vec{a} = \frac{d\vec{\vartheta}_a}{dt} = \frac{d\vec{\vartheta}_A}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{\rho} + \vec{\omega} \times \frac{d\vec{\rho}}{dt} + \frac{d\vec{\vartheta}_r}{dt}$$

$\vec{\vartheta}_r$ vektorning absolyut hosilasini Bur formulasiga asosan aniqlaymiz:

$$\frac{d\vec{\vartheta}_r}{dt} = \frac{\tilde{d}\vec{\vartheta}_r}{dt} + \vec{\omega} \times \vec{\vartheta}_r$$

Nisbiy tezlik vektoridan olingan nisbiy hosila

$$\vec{a}_r = \frac{\tilde{d}\vec{\vartheta}_r}{dt} = \ddot{x}\cdot\vec{i} + \ddot{y}\cdot\vec{j} + \ddot{z}\cdot\vec{k}$$

$$\vec{a} = \vec{a}_A + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times\left[\vec{\vartheta}_r + (\vec{\omega}\times\vec{\rho})\right] + \vec{a}_r + \vec{\omega}\times\vec{\vartheta}_r$$

$$= \vec{a}_A + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho}) + \vec{a}_r + 2(\vec{\omega}\times\vec{\vartheta}_r)$$

Nuqtaning ko'chirma tezlanishini aniqlash uchun uni qo'zg'aluvchan koordinatalar sistemasiga nisbatan harakatlanmaydi deb hisoblaymiz, ya'ni $\vec{\vartheta}_r = 0$ va $\vec{a}_r = 0$ deb qabul qilamiz:

$$\vec{a}_e = \vec{a}_A + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})$$

ya'ni ko'chirma tezlanish vektori, erkin qattiq jism nuqtasining (qo'zg'aluvchan koordinatalar sistemasi jismga qattiq qilib mahkamlangan) tezlanish vektori bilan bir xil ko'rinishga ega bo'ladi.

Bu tenglikning oxirgi xadi *qo'shimcha* yoki *koriolis tezlanishi* deb ataladi va bilan belgilanadi:

$$\vec{a}_K = 2(\vec{\omega} \times \vec{\vartheta}_r)$$

yoki bu kattaliknung moduli.

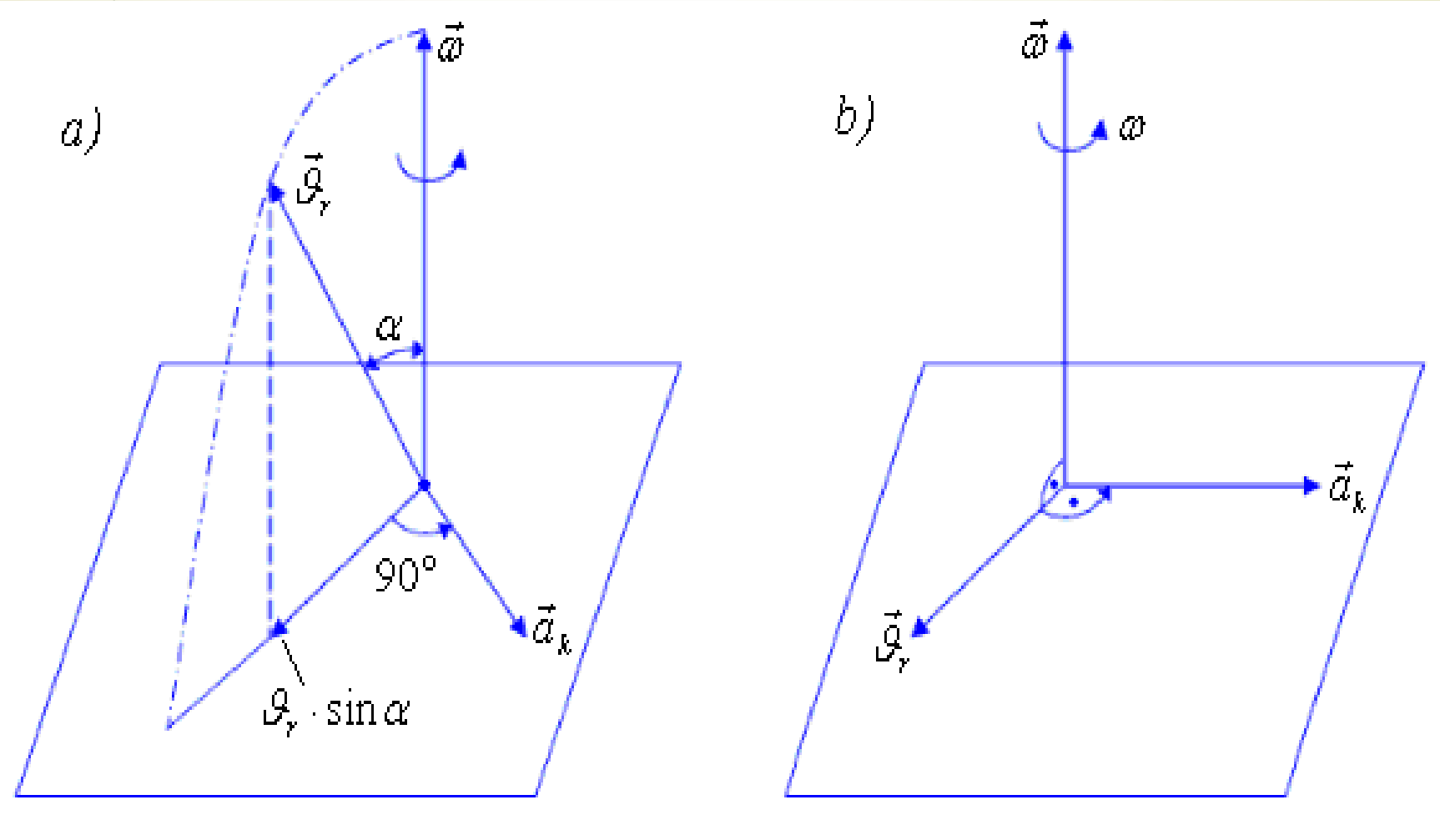
$$a_K = 2 \cdot \omega \cdot \vartheta_r \cdot \sin(\hat{\vec{\omega}, \vec{\vartheta}_r})$$

Shunday qilib, nuqtaning absolyut tezlanishi uchun quyidagi vektor tenglikni yozish mumkin:

$$\vec{a} = \vec{a}_e + \vec{a}_r + \vec{a}_K$$

ya'ni, murakkab harakatdagi nuqtaning absolyut tezlanishi uning ko'chirma, nisbiy va koriolis tezlanishlarining geometrik yig'indisiga teng.

Koriolis tezlanishining yo'nalishini aniqlash uchun N.E.Jukovskiy qoidasidan foydalanamiz.

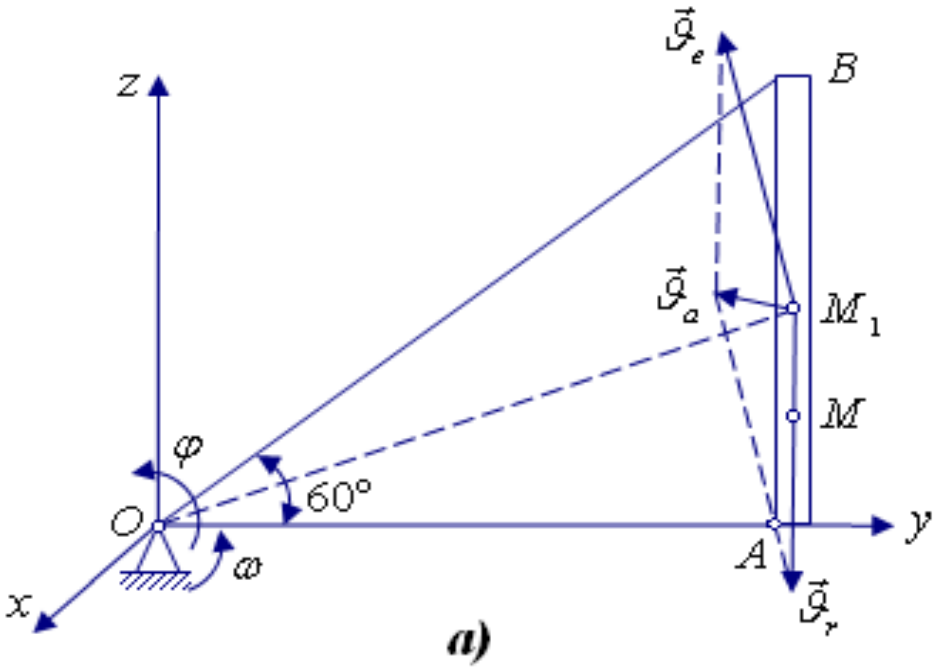


Koriolis tezlanish quyidagi hollarda nolga teng bo'ladi:

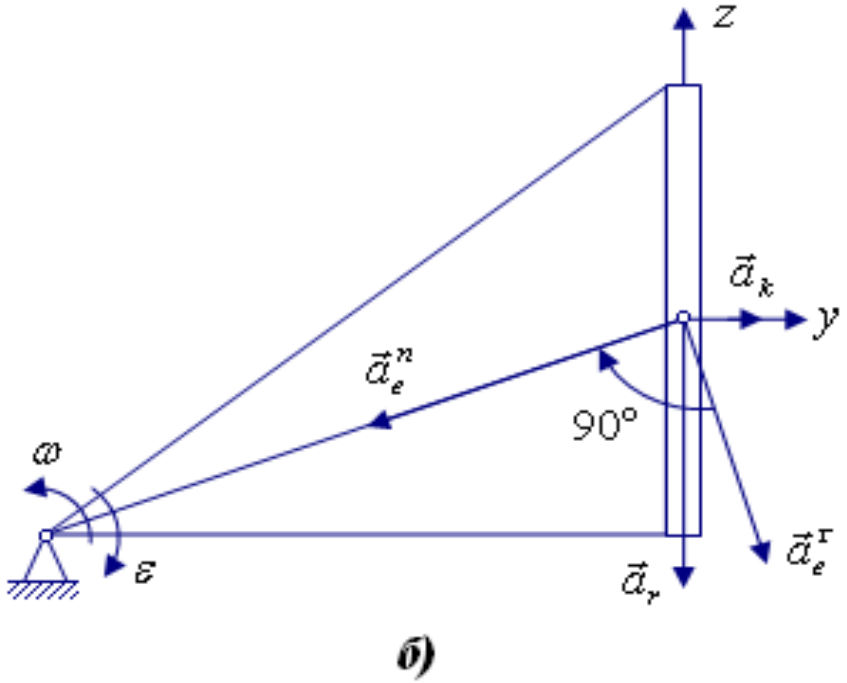
1. $\omega = 0$ bo'lsa, ya'ni qo'zg'aluvchan koordinatalar sistemasi ilgarilanma harakatda bo'lgan holda.
2. $\vec{\vartheta}_r$ va $\vec{\omega}$ vektorlar o'zaro parallel bo'lgan holda.
3. Nisbiy tezlik vektori nolga teng, ya'ni nuqta qo'zg'aluvchan koordinatalar sistemasiga nisbatan tinch holatda bo'lgan holda.

Murakkab harakatga doir masalalar yechishda "**to'xtatish**" usulidan foydalanish ancha qulaylik tug'diradi.

Масала. Berilgan: Vertikal tekislikda yotuvchi OAB uchburchakli plastinka AB tomonida naychadan iborat qurilma payvandlangan. Bu naycha bo‘ylab, M yuk $s = AM = 70(3t - 2t^2)$ sm bo‘yicha harakatlanadi. OAB plastina esa $\varphi = 6t^2 - 3t^3$ $rad.$ qonun bo‘yicha harakatlanadi. Agar $OA = 80$ sm bo‘lsa, M yukning $t = 1$ $sek.$ dagi absolyut tezligi va absolyut tezlanishi aniqlansin..



а)



б)

Yechish: Avvalo M yukning $t=1$ *sek*.da AB naychadagi o‘rnini aniqlaymiz:

$$s = AM_1 = 70(3\cdot 1 - 2\cdot 1) = 70\ sm\ , \qquad AB = AO\cdot tg60^\circ = 138{,}56\ \ sm$$

AB naychadan 70 *sm* o‘lchab, M yukning o‘rnini aniqlaymiz. M yukning AB naycha ichidagi harakati nisbiy harakatini M yukning M_1 nuqtadagi o‘rnida mahkamlangan deb, qarasak, u holda M yukning O nuqtadan o‘tuvchi x o‘qi atrofidagi aylanma harakati ko‘chirma harakatni beradi. M yukning qo‘zg‘almas koordinatalar sistemasiga nisbatan harakati absolyut harakatini bildiradi.

M yukning absolyut tezligini (2.37) tenglikka asosan aniqlaymiz. Buning uchun «to‘xtatish usuli»dan foydalanamiz, ya’ni yukning nisbiy tezligini aniqlash uchun OAB plastinkaning aylanishini to‘xtatamiz. U holda M yukning nisbiy harakati oddiy ilgarilanma harakatdan iborat

$$\vartheta_r = \frac{ds}{dt} = 70(3-4t)\ sm/s$$

M yukning ko‘chirma tezligini aniqlash uchun bu yukni M_1 nuqtada mahkamlangan deb qarasak, u holda M yukni ko‘chirma harakati oddiy Ox o‘qi atrofidagi aylanma harakatdan iborat bo‘ladi:

$$\vartheta_e = \left|\frac{d\varphi}{dt}\right| \cdot OM_1 = \omega \cdot OM_1, \quad \text{bunda} \quad \omega = \left|\frac{d\varphi}{dt}\right| = 12t - 9t^2 \quad rad/s,$$

$$OM_1 = \sqrt{OA^2 + AM_1^2} = \sqrt{80^2 + 70^2} = 106{,}3\,.$$

Bu kattaliklarning yo‘nalishini chizmada ko‘rsatish uchun, ularning $t = 1$ *sek.* dagi qiymatlarini aniqlaymiz:

$$\omega = \left|\frac{d\varphi}{dt}\right| = 3 \ rad/s, \quad OM_1 = 106{,}3 \ sm, \quad \vartheta_r = -70 \ sm/s, \quad \vartheta_e = 318{,}9 \ sm/$$

$/\vartheta_r$ - nisbiy tezlikning minus ishorasi shuni bildiradiki, $\vec{\vartheta}_r$ nisbiy tezlik vektori M_1 nuqtadan A nuqta tomon yo‘nalgan bo‘ladi (29, *a*-chizma).

$$\vartheta_a = \sqrt{\vartheta_r^2 + \vartheta_e^2 + 2\vartheta_r\vartheta_e \cdot \cos\left(\vec{\vartheta}_r, \hat{\vec{\vartheta}}_e\right)} =$$

$$= \sqrt{(70)^2 + (318{,}9)^2 + 2 \cdot 70 \cdot 318{,}9 \cdot \cos 139^\circ} = 270$$

Endi M yukning absolyut tezlanishini aniqlaymiz.

$$a_\tau = \left|\frac{d^2 s}{dt^2}\right| = |70 \cdot (-4)| = 280 \text{ sm} / \text{s}^2,$$

$$\vec{a}_e = \vec{a}_e^{\,\tau} + \vec{a}_e^{\,n}$$

bunda $a_e^\tau = \varepsilon \cdot OM_1;$

$$\varepsilon = \left|\frac{d^2\varphi}{dt^2}\right| = |12 - 8t|.$$

$t = 1$ *sek.* da $\dfrac{d^2\varphi}{dt^2} = -6 \text{ sm} / \text{s}^2,$ $a_e^\tau = 640{,}8 \ \dfrac{\text{sm}}{\text{s}^2},$

$$a_e^n = \omega^2 \cdot OM_1 = 9 \cdot 106{,}8 = 961{,}2 \ \text{ñì} / \tilde{n}^2 .$$

Ma'lumki a_e^n – ko'chirma normal tezlanish $\grave{I}_1$ nuqtadan O nuqta tomon yo'nalgan bo'ladi.

M yukning $\vec{a}_k$ – kariolis tezlanishi

$$\vec{a}_k = 2 \cdot \left| \vec{\omega} \times \vec{\vartheta}_r \right|,$$

yoki

$$a_k = 2 \cdot \omega \vartheta_r \cdot \sin 90^\circ = 2 \cdot 3 \cdot 70 = 420 \ \tilde{n}\grave{i} / \tilde{n}^2 .$$

$$a_x = 0$$

$$a_y = a_k - a_e^n \cdot \cos 31^\circ + a_e^\tau \cdot \cos 49^\circ = 16{,}49 \qquad \tilde{n}\grave{i} / \tilde{n}^2$$

$$a_z = -a_r - a_e^n \cdot \cos 49^\circ - a_e^\tau \cdot \cos 31^\circ = -1459{,}87 \qquad \tilde{n}\grave{i} / \tilde{n}^2$$

M yukning absolyut tezlanishining qiymati

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{(16{,}49)^2 + (-1459{,}87)^2} = 1459{,}97 \ \tilde{n}\grave{i} / \tilde{n}^2$$

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E'TIBORINGIZ UCHUN RAHMAT!

HUSANOV Q.

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