TOSHKENT IRRIGATSIYA VA QISHLOQ XO'JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI

FAN: NAZARIY MEXANIKA

MAVZU 11

Moddiy nuqtaning tebranma harakati

Husanov Q.

Nazariy va qurilish mexanikasi kafedrasi dotsenti

TAQDIMOT REJASI

1. Erkin tebranma harakat.
2. So'nuvchi tebranma harakat (muhit qarshiligidagi tebranma harakat).
3. Majburiy tebranma harakat (davriy ta'sir etuvchi, uyg'otuvchi nomli kuch ta'siridagi tebranma harakat).
4. Majburiy tebranma harakat (muhit qarshiligidagi majburiy tebranma harakat).

Moddiy nuqtaning tebranma harakati

Moddiy nuqta harakatining texnikada alohida ahamiyatga ega bo'lgan turlaridan biri tebranma harakat hisoblanadi. Masalan, inshootlar poydevorini tebranishi; mashina va mexanizm qismlarini tebranishi; mayatnik, prujinaga osilgan yuk va vagon kuzovlarining tebranishi.

moddiy nuqtaning davriy ravishda takrorlanadigan harakatiga tebranma harakat deyiladi.

moddiy nuqtaning tebranma harakati texnikada asosan besh xilga bo'linadi.

1. Erkin tebranma harakat.
2. So'nuvchi tebranma harakat (muhit qarshiligidagi tebranma harakat).
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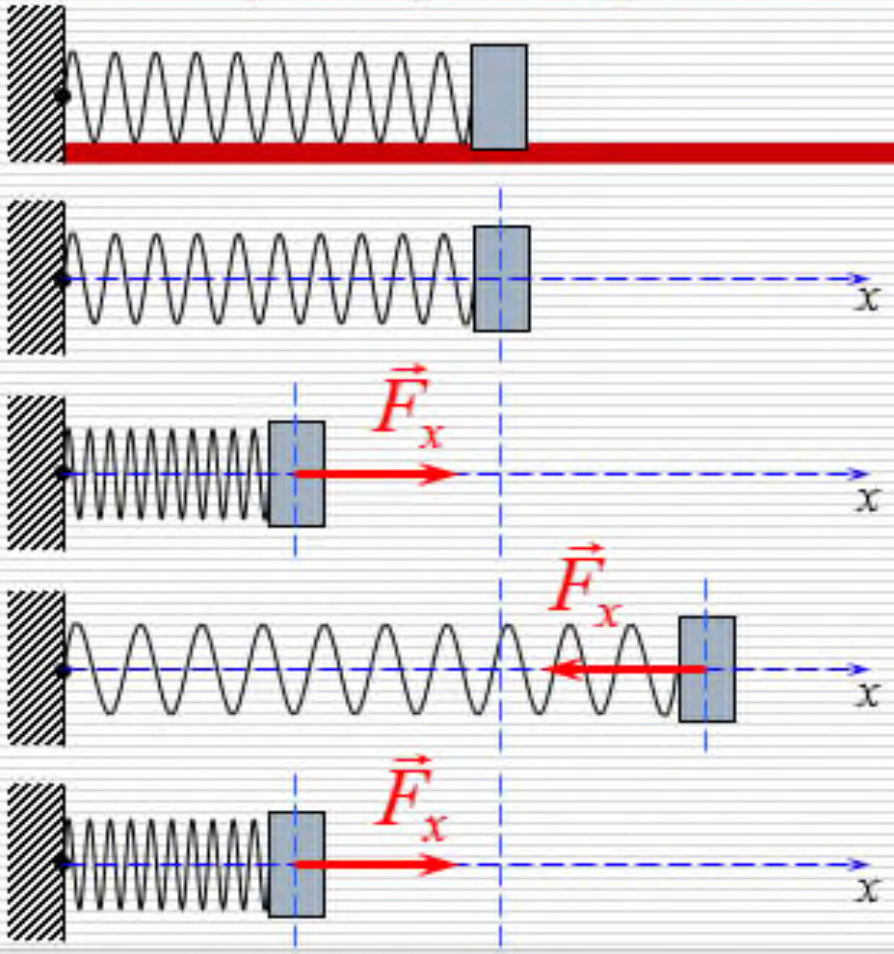
4. Majburiy tebranma harakat (muhit qarshiligidagi majburiy tebranma harakat).
5. Kichik tebranma harakat nazaryasi

Moddiy nuqtaning tebranma harakati

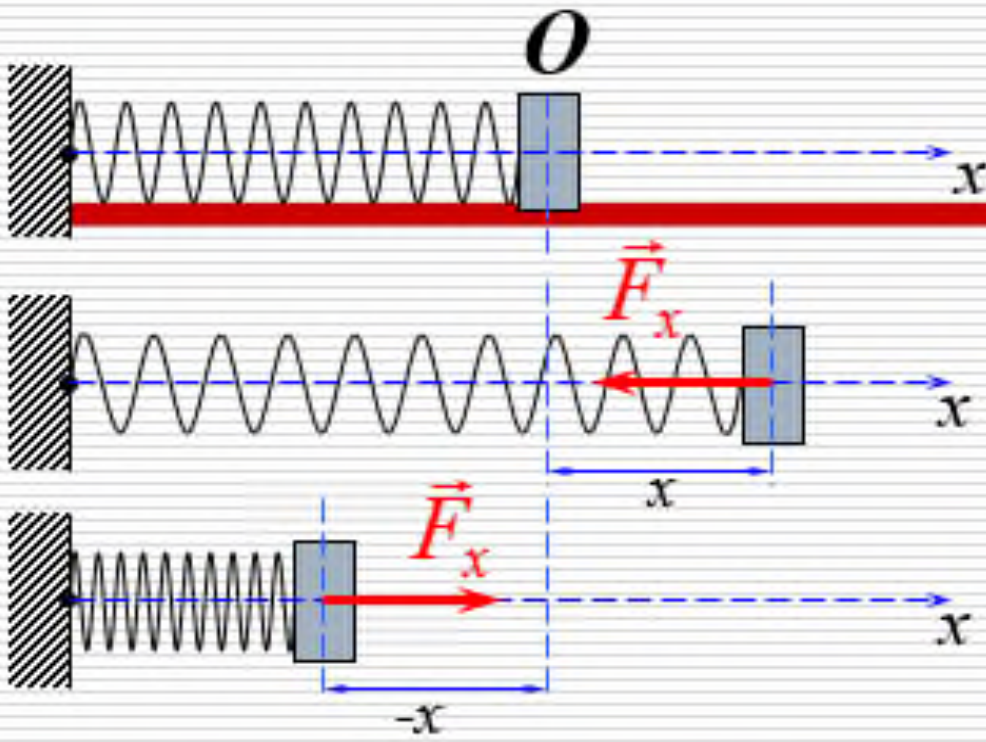
Moddiy nuqtaning erkin tebranma harakati



moddiy nuqta tinch muvozanat holatidan chetlatilganda moddiy nuqtaning tebranma harakati vujudga keladi.

moddiy nuqtaning muvozanat holatiga qaytarishga intiluvchi kuchga qaytaruvchi kuch deyiladi.

Moddiy nuqtaning tebranma harakati



nuqta muvozanat holatidan x masofaga og'dirilsa, u holda unga x o'qi bo'ylab hamisha O nuqtaga yo'nalgan qaytaruvchi kuch ta'sir etadi.

bu kuchning Ox o'qdagi proektsiyasi quyidagicha aniqlanadi:

$$X = -cx$$

bunda s proporsionallik koeffitsienti (prujinaning bikrligi).

moddiy nuqtaning qaytaruvchi kuch ta'siridagi harakat differentsial tenglamasi:

$$m\ddot{x} = -cx$$

$$\frac{c}{m} = k^2$$ desak,

$$\ddot{x} + k^2 x = 0$$

Moddiy nuqtaning tebranma harakati

$\ddot{x}+k^2x=0$ → **moddiy nuqtaning erkin tebranma harakat differensial tenglamasi deyiladi.**

$t=0$ **da** $x=x_0,\ \upsilon=\dot{x}=\upsilon_0$

$$x=C_1\cos kt+C_2\sin kt$$

$$\dot{x}=-C_1k\sin kt+C_2k\cos kt$$

$$C_1=x_0,\ C_2=\upsilon_0/k$$

$$x=x_0\cos kt+\frac{\upsilon_0}{k}\sin kt$$

$$x=a\sin(kt+\alpha)$$

$$\dot{x}=ak\cos(kt+\alpha)$$

$$x_0=a\sin\alpha,\ \upsilon_0=ak\cos\alpha$$

$a=\sqrt{x_0^2+\upsilon_0^2/k^2}$ tebranish amplitudasi – nuqtaning tebranish markazidan (O dan) eng katta og'ishi.

$kt+\alpha$ tebranish fazasi.

α tebranishlarning boshlang'ich fazasi.

Moddiy nuqtaning tebranma harakati

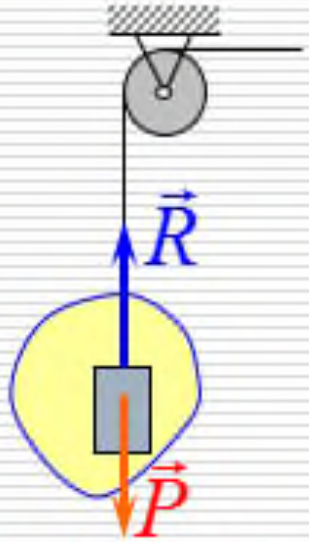
$x_0 = a\sin\alpha$ - nuqtaning boshlang'ich paytdagi og'ishini ifodalaydi

$T = 2\pi/k$ - tebranishlar davri: nuqta bir marta to'liq tebranishi uchun ketgan vaqt.

$\nu = 1/T = k/2\pi$ - tebranishlar chastotasi: bir sekunddagi tebranishlar sonini ifodalaydi.

Moddiy nuqtaning tebranma harakati

misol. Troga osilgan 2 *t* yuk blok orqali 5 *m/s* tezlik bilan tekis harakat bilan pastga tushirilayotganda kutilmaganda trosning blokdagi qismi to'xtab qoldi. Trosning og'irligini hisobga olmay, yukning tebranma harakatidagi trosning maksimal zo'riqishini toping. Trosning bikrligi $4 \cdot 10^6$ *n/m*.

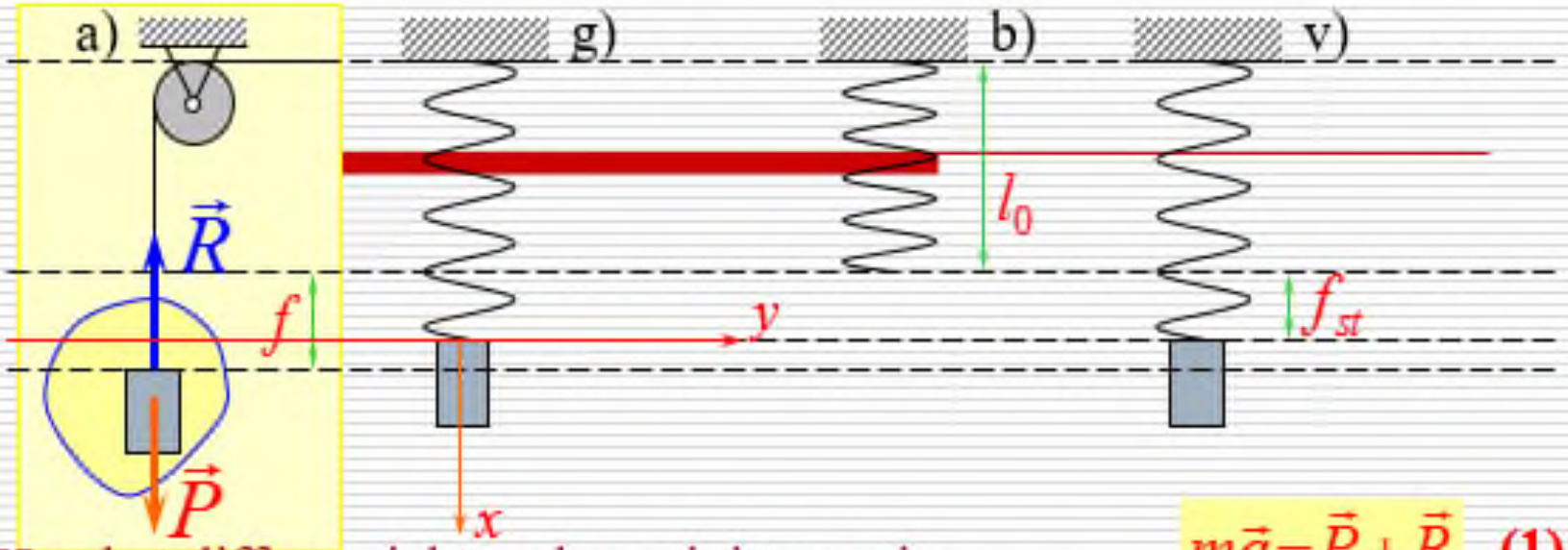


1. harakatlanayotgan ob'ektni (yukni) moddiy nuqta deb qaraymiz.
2. yukni bog'lanishdan (trosdan) ozod etamiz va bog'lanish reaksiyasini qo'yamiz.
3. yukka aktiv kuchni (og'irlik kuchini) qo'yamiz.

Moddiy nuqtaning tebranma harakati

berilgan: $m = 2000\ кг,\quad \upsilon_0 = 5\ м/с,\quad c = 4\cdot 10^6\ Н/м.$

topish kerak: T_{max}



$$m\vec{a} = \vec{P} + \vec{R} \quad (1)$$

4. Harakat differensial tenglamasini tuzamiz:

4.1 masalani yechishda asosiy sxemadan tashqari yordamchi sxemadan foydalaniladi. Elastik tros elastik prujina deb qaraladi.

b-da prujinaning cho'zilmagan qismi l_0 ga teng;

v-da yukning muvozanat holati (prujina f_{st} ga cho'zilgan);

g-da yukning boshlang'ich holati (t_0=0);

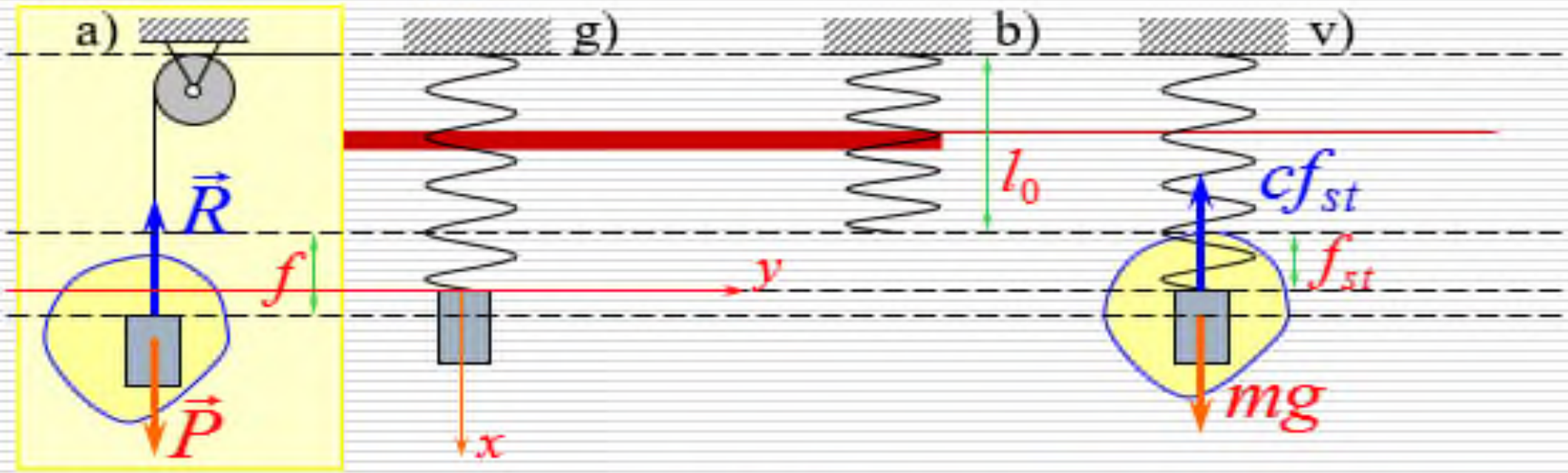
Asosiy sxema (a) uchun harakat tenglamasini tuzamiz (ixtiyoriy t vaqtda prujinaning cho'zilishi f ga teng).

koordinata boshini muvozanat holatiga mos qo'yamiz.

Moddiy nuqtaning tebranma harakati

berilgan: $m = 2000\ кг,\quad \upsilon_0 = 5\ м/с,\quad c = 4\cdot 10^6\ Н/м.$

topish kerak: T_{max}



4.2 (1) tenglamani x o'qiga proeksiyalaymiz: $m\ddot{x} = P - R$ **(2)**

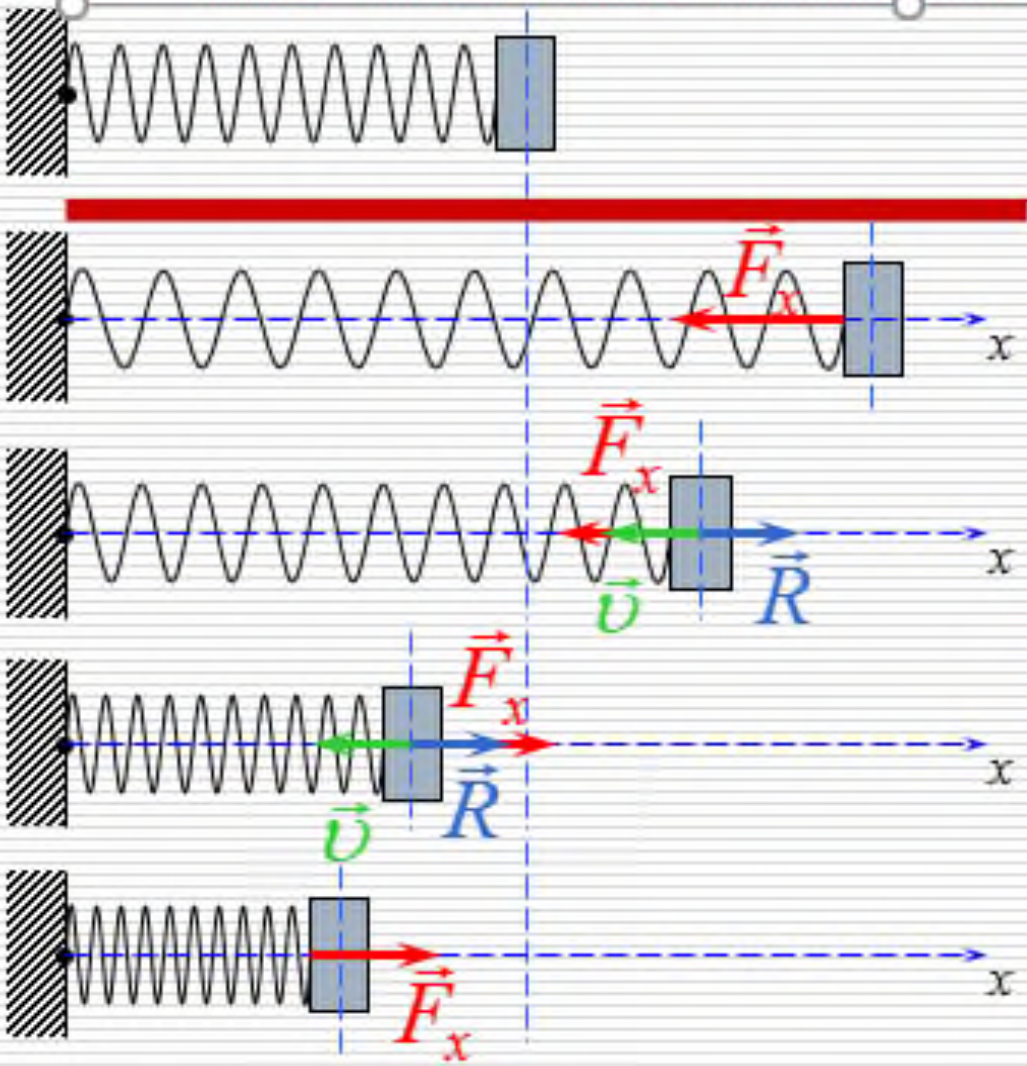
$P = mg,\ R = cf = c(f_{st} + x)$ ekanidan $m\ddot{x} = mg - c(f_{st} + x)$ **(3)**

statik muvozanat holatidan (v-sxemadan) f_{st} ni topamiz: $f_{st} = mg/c$

$m\ddot{x} = mg - c(mg/c + x) = -cx \Rightarrow \ddot{x} + k^2 x = 0$ $\quad k^2 = c/m$

Moddiy nuqtaning tebranma harakati

Moddiy nuqtaning so'nuvchi tebranma harakati



moddiy nuqtaning erkin tebranishida havoning qarshiligini hisobga olaylik.

Ox o'q bo'yicha harakatlanuvchi moddiy nuqtaga qaytaruvchi kuchdan tashqari nuqtaning tezligiga qarama-qarshi yo'nalgan qarshilik kuchi ta'sir etadi.

$$\vec{R} = -\mu\vec{\upsilon}$$

μ - qarshilik koeffitsienti.

harakat differensial tenglamasi: $m\vec{a} = \vec{F}_x + \vec{R}$

qaytaruvchi va qarshilik kuchlarining Ox o'qdagi proektsiyalari:

$$F_x = -cx;\ R = -\mu\upsilon = -\mu\dot{x}$$

Moddiy nuqtaning tebranma harakati

$m\vec{a}=\vec{F}_x+\vec{R}$ **harakat differensial tenglamasi:**

$m\ddot{x}=-cx-\mu\dot{x}$ yoki $\ddot{x}+2n\dot{x}+k^2x=0$

$k^2=c/m$
$2n=\mu/m$

qaytaruvchi kuch va nuqta tezligiga proporsional bo'lgan qarshilik kuchi ta'siridagi moddiy nuqtaning harakat differensial tenglamasini ifodalaydi.

$t=0$ **da** $x=x_0,\ \upsilon=\dot{x}=\upsilon_0$

Xarakteristik tenglamasi: $\lambda^2+2n\lambda+k^2=0$ bo'lib, u ildizlarga ega.

$\lambda_{1,2}=-n\pm\sqrt{n^2-k^2}$

a. muhitning qarshiligi uncha katta bo'lmagan hol.

$n<k$ - xarakteristik tenglamaning ildizlari qo'shma kompleks sonlardan iborat bo'ladi.

b. muhitning qarshiligi katta bo'lgan hol.

$n>k$ - xarakteristik tenglamaning ildizlari haqiqiy va turlicha bo'ladi.

$n=k$ - xarakteristik tenglamaning ildizlari haqiqiy va o'zaro teng bo'ladi.

Moddiy nuqtaning tebranma harakati

A. muhitning qarshiligi uncha katta bo'lmagan hol.

$n<k$ - xarakteristik tenglamaning ildizlari qo'shma kompleks sonlardan iborat bo'ladi.

$k^2-n^2=k_1^2$ belgilash kiritilsa, $\lambda_{1,2}=-n\pm ik_1$ bo'ladi va

$$x=e^{-nt}(C_1\cos k_1t+C_2\sin k_1t) \quad \text{yoki} \quad x=ae^{-nt}\sin(k_1t+\alpha)$$

tebranish dekrementi: $\frac{a_{i+1}}{a_i}=e^{-n\frac{T_1}{2}}$

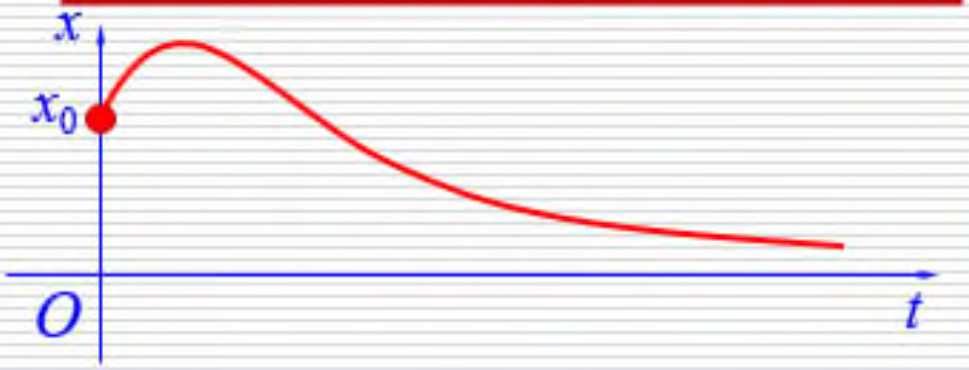
$$T_1=\frac{2\pi}{k_1}=\frac{2\pi}{k\sqrt{1-(n/k)^2}}=\frac{T}{\sqrt{1-(n/k)^2}}$$

Moddiy nuqtaning tebranma harakati

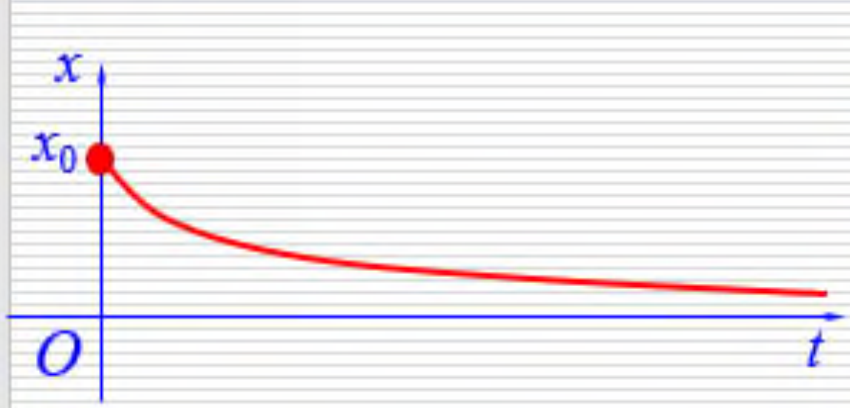
b. muhitning qarshiligi katta bo'lgan hol.

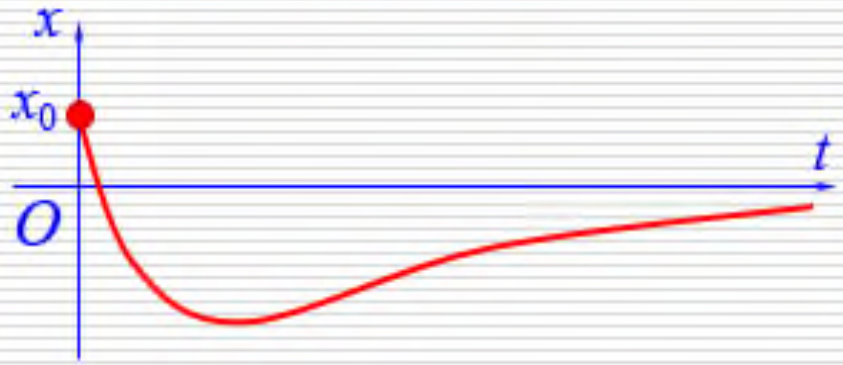
nuqtaning so'nuvchi tebranma harakat grafigi

a) Agar nuqta boshlang'ich vaqtda *Ox* o'qning musbat yo'nalishi bo'yicha yo'nalgan boshlang'ich tezlikka ega bo'lsa



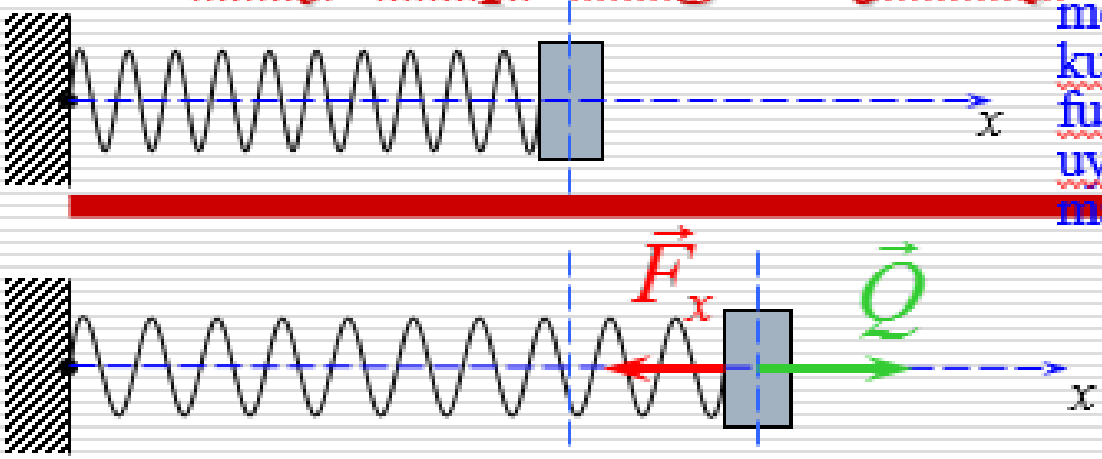
a) Agar nuqta boshlang'ich vaqtda *Ox* o'qqa qarama-qarshi yo'nalgan boshlang'ich tezlikka ega bo'lsa





Moddiy nuqtaning tebranma harakati

Moddiy nuqtaning majburiy tebranma harakati



moddiy nuqtaga qaytaruvchi kuchdan tashqari vaqtning davriy funksiyasidan iborat bo'lgan uyg'otuvchi kuch ham ta'sir etsa, u majburiy harakatda bo'ladi.

moddiy nuqtaga ta'sir etuvchi uyg'otuvchi kuch: $\vec{Q}$ bo'lsa,

harakat differensial tenglamasi: $m\vec{a}=\vec{F}_x+\vec{Q}$

qaytaruvchi va uyg'otuvchi kuchlarining Ox o'qdagi proeksiyalari:

$F_x=-cx$; $Q_x=H\sin(pt+\delta)$,

n – uyg'otuvchi kuchning amplitudasi;
r – uning doiraviy chastotasi;
δ – boshlang'ich faza.

qaytaruvchi va uyg'otuvchi kuchlar ta'siridagi moddiy nuqtaning harakat differensial tenglamasi:

$m\ddot{x}=-cx+H\sin(pt+\delta)$ yoki $\ddot{x}+k^2x=H_0\sin(pt+\delta)$

$k^2=c/m$

$H_0=H/m$

Moddiy nuqtaning tebranma harakati

$$\ddot{x}+k^2x=H_0\sin(pt+\delta) \longrightarrow x=x_1+x_2$$

$m\ddot{x}+k^2x=0$
ning umumiy yechimi

$$x_1=a\sin(kt+\alpha)$$

$\ddot{x}+k^2x=H_0\sin(pt+\delta)$
ning hususiy yechimi

$$x_2=A\sin(pt+\delta)$$
$$\ddot{x}_2=-Ap^2\sin(pt+\delta)$$
$$A(k^2-p^2)\sin(pt+\delta)=H_0\sin(pt+\delta)$$
$$A(k^2-p^2)=H_0 \longrightarrow A=H_0/(k^2-p^2)$$
$$x_2=\frac{H_0}{k^2-p^2}\sin(pt+\delta)$$

$$\ddot{x}+k^2x=H_0\sin(pt+\delta)$$

$$x=a\sin(kt+\alpha)+\frac{H_0}{k^2-p^2}\sin(pt+\delta)$$

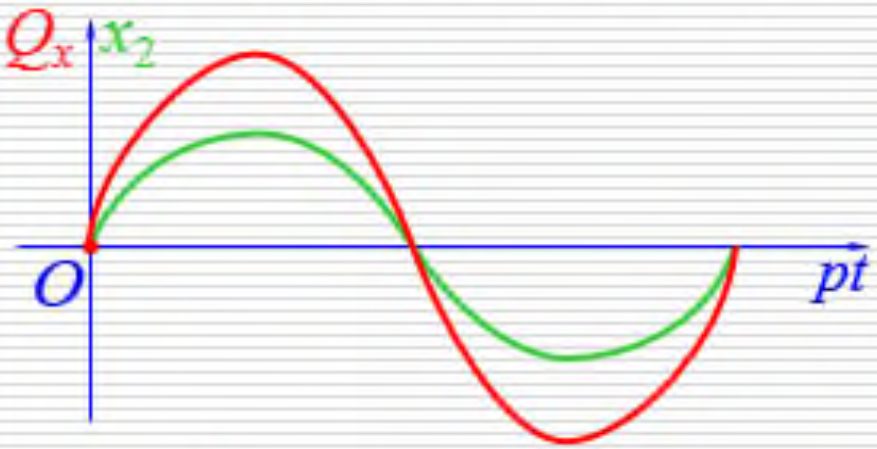
moddiy nuqtaga bir vaqtning o'zida qaytaruvchi va uyg'otuvchi kuchlar ta'sir etsa, mazkur nuqta ***k*** chastota bilan sodir bo'ladigan *erkin tebranma harakat*, hamda uyg'otuvchi kuch chastotasi *r* bilan sodir bo'ladigan *majburiy tebranma harakatlaridan* tashkil topgan murakkab harakatda ishtirok etadi.

Moddiy nuqtaning tebranma harakati

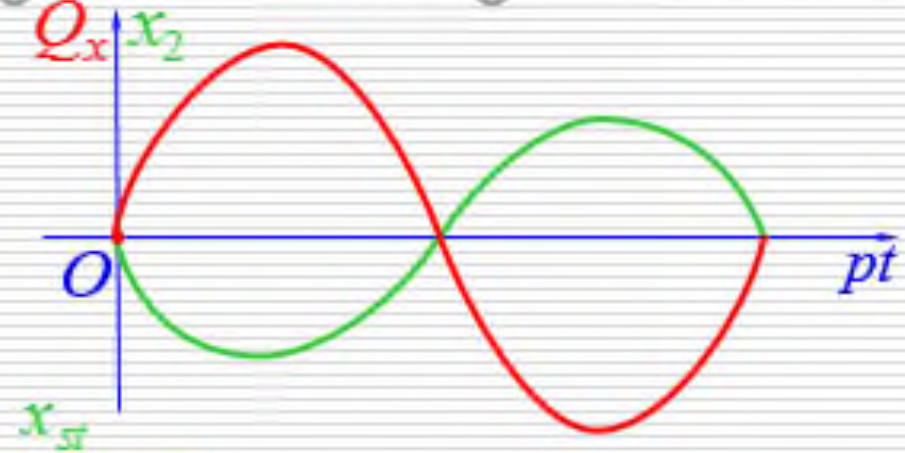
$$x_2 = \frac{H_0}{k^2 - p^2}\sin(pt+\delta)$$ ⟹ majburiy tebranma harakat, nuqta harakatining boshlang'ich shartlariga bog'liq bo'lmaydi.

majburiy tebranma harakat amplitudasi $A_1 = \frac{H_0}{|k^2 - p^2|}$ desak,

$k > p$ bўлса, $x_2 = A_1 \sin(pt+\delta)$



$k < p$ bўлса, $x_2 = -A_1 \sin(pt+\delta)$



$$A_1 = \frac{H_0}{|k^2 - p^2|} = \frac{H_0}{k^2|1-(p/k)^2|} = \frac{x_{st}}{|1-(p/k)^2|}$$

η – dinamiklik koeffisienti:

$$\eta = \frac{A_1}{x_{st}} = \frac{1}{|1-(p/k)^2|}$$

Moddiy nuqtaning tebranma harakati

$p \approx k$ bo'lganda, $\ddot{x} + k^2 x = H_0 \sin(pt + \delta)$

ning hususiy yechimini quyidagicha tanlab olamiz:

$$x_2 = \frac{H_0}{k^2 - p^2}(\sin pt - \sin kt)$$

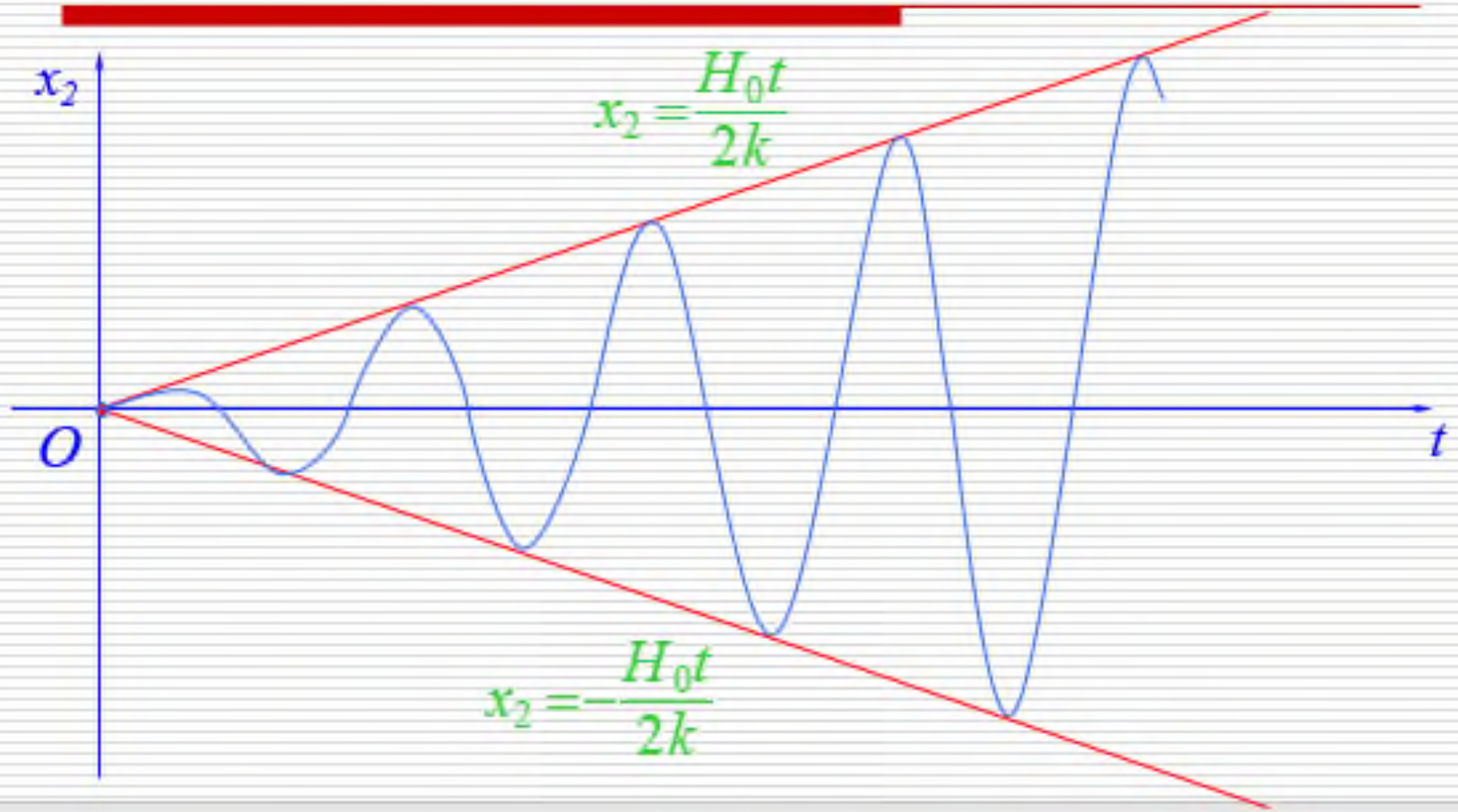
$p = k$ bo'lsa, Lopital' qoidasiga asosan

$$x_2 = H_0 \left[\frac{\frac{d}{dp}(\sin pt - \sin kt)}{\frac{d}{dp}(k^2 - p^2)} \right]_{p=k} = -\frac{H_0 t}{2k} \cos kt$$

$$\ddot{x} + k^2 x = H_0 \sin(pt + \delta) \longrightarrow x = a\sin(kt + \alpha) + \frac{H_0 t}{2k} \cos kt$$

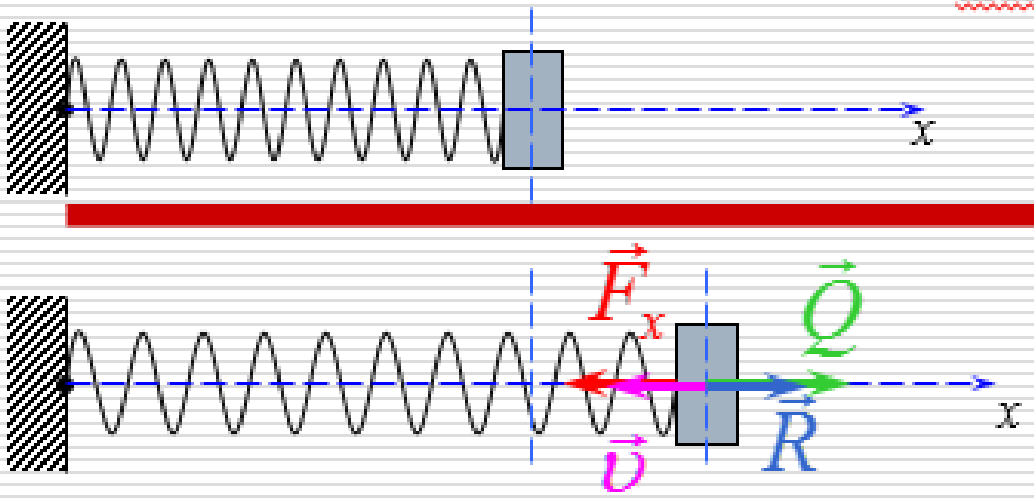
Moddiy nuqtaning tebranma harakati

$p=k$ bo'lganda, x_2 funksiyaning grafigi vaqtning o'tishi bilan tebranish amplitudasi vaqtning chiziqli funksiyasi sifatida cheksiz orta boradi. Bu hodisaga **rezonans** deyiladi.



Moddiy nuqtaning tebranma harakati

nuqtaning majburiy tebranishiga muhit qarshiligining ta'siri



harakat differensial tenglamasi:

$$m\vec{a}=\vec{F}_x+\vec{Q}+\vec{R}$$

qaytaruvchi, uyg'otuvchi va qarshilik kuchlarining Ox o'qdagi proeksiyalari:

$$F_x=-cx;\quad Q_x=H\sin(pt+\delta),\quad R=-\mu\upsilon=-\mu\dot{x}$$

qaytaruvchi, uyg'otuvchi va qarshilik kuchlari ta'siridagi moddiy nuqtaning harakat differensial tenglamasi:

$$m\ddot{x}=-cx+H\sin(pt+\delta)-\mu\dot{x}$$ yoki

$$\ddot{x}+2n\dot{x}+k^2x=H_0\sin(pt+\delta)$$

$$k^2=c/m$$
$$2n=\mu/m$$
$$H_0=H/m$$

Moddiy nuqtaning tebranma harakati

$$\ddot{x}+2n\dot{x}+k^2x=H_0\sin(pt+\delta)$$

$x=x_1+x_2$

$$m\ddot{x}+2n\dot{x}+k^2x=0$$

ning umumiy yechimi

$$\ddot{x}+2n\dot{x}+k^2x=H_0\sin(pt+\delta)$$

ning hususiy yechimi

$$x_2=A\sin(pt+\delta-\varepsilon) \Rightarrow x_2=\frac{H_0}{\sqrt{(k^2-p^2)^2+4n^2p^2}}\sin(pt+\delta-\varepsilon)$$

$n<k$ bo'lsa, $x=e^{-nt}(C_1\cos k_1t+C_2\sin k_1t)+A\sin(pt+\delta-\varepsilon)$

$k^2-n^2=k_1^2$

$n>k$ bo'lsa, $x=e^{-nt}(C_1e^{ht}+C_2e^{-ht})+A\sin(pt+\delta-\varepsilon)$

$n^2-k^2=h^2$

$n=k$ bo'lsa, $x=e^{-nt}(C_1t+C_2)+A\sin(pt+\delta-\varepsilon)$

TOSHKENT IRRIGATSIYA VA QISHLOQ XO'JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI

E'TIBORINGIZ UCHUN RAHMAT!

HUSANOV Q.

Nazariy va qurilish mexanikasi kafedrasi dotsenti

Masala. Prujina vositasida qo'zg'almas A nuqtaga biriktirilgan massasi 2 kg. ga teng yuk gorizont bilan α burchak hosil qiluvchi silliq qiya tekislik ustida $S = 180 \cdot \sin 10t$ N uyg'otuvchi kuch va tezlikka proporsional $R = -29{,}4\vartheta$ (R-H hisobida) qarshilik kuchi ta'sirida harakat qiladi. Prujinaning bikirlik koeffitsienti $c = 5кн/м$. Boshlang'ich vaqtda jism statik muvozanat holatida tinch turgan. Jismning harakat tennglamasi, erkin va majburiy tebranishlarning davrlari T va T_1, majburiy tebranish va uyg'otuvchi kuchning faza silgishi topilsin.

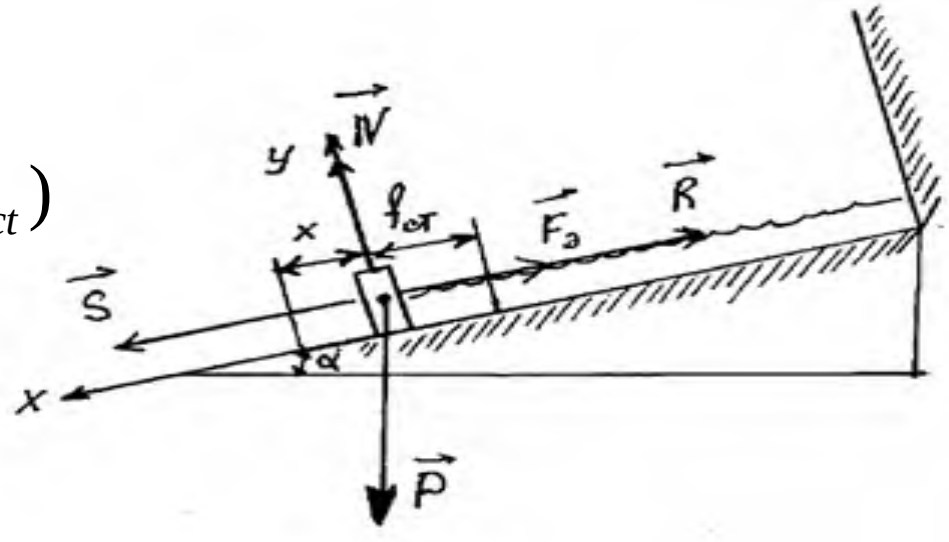
Yechish: Yukka ta'sir etuvchi kuchlarni shaklda ko'rsatamiz: R-yukning og'irlik kuchi; R- qarshilik kuchi; F - prujinaning elastiklik kuchi; N- tekislikning normal reaksiya kuchi.

$$m\ddot{x} = P\sin\alpha + S_x + R_x - c(x + f_{ct})$$

$$m\ddot{x} = mg\sin\alpha + 180\sin 10t - 29{,}4\dot{x} - cx - c\cdot f_{ct})$$

$$\sum F_{kx} = 0 \qquad P\sin\alpha - F_э' = 0$$

$$mg\sin\alpha = c\cdot f_{ct}$$



$t = 0$ da $x = x_0 = 0;\ \dot{x}_0 = 0\ m/s$

$$\ddot{x} + 14{,}7\dot{x} + 2500\,x = 90\sin 10t \qquad x = x_1 + x_2$$

$$\ddot{x} + 2n\dot{x} + k^2 x = 0$$

$$\lambda^2 + 2n\cdot\lambda + k^2 = 0 \qquad \lambda^2 + 14{,}7\cdot\lambda + 2500 = 0 \qquad \lambda_{1,2} = -7{,}35 \pm 49{,}46i$$

$$x_1 = e^{-7{,}35t}\left(c_1^* \cos 49{,}46t + c_2^* \sin 49{,}46t\right) \qquad x_2 = A\cdot\sin(10t - \varepsilon)$$

$$\dot{x}_2 = A\cdot 10\cos(10t - \varepsilon) \qquad \ddot{x}_2 = -A\cdot 100\sin(10t - \varepsilon)$$

$$-100A\cdot\sin(10t-\varepsilon) + 14{,}7\cdot 10A\cdot\cos(10t-\varepsilon) + 2500\cdot A\cdot\sin(10t-\varepsilon) =$$

$$= 90\cdot\sin(10t - \varepsilon + \varepsilon)$$

$$-100A\cdot\sin(10t-\varepsilon) + 147A\cdot\cos(10t-\varepsilon) + 2500\cdot A\cdot\sin(10t-\varepsilon) =$$

$$= 90\cdot\sin(10t-\varepsilon)\cdot\cos\varepsilon + 90\cdot\cos(10t-\varepsilon)\cdot\sin\varepsilon$$

$$\begin{cases} -100 \cdot A + 2500 = 90 \cdot \cos\varepsilon \\ 147 \cdot A = 90 \cdot \sin\varepsilon \end{cases}$$

$$tg\varepsilon = \frac{147}{2400} = 0{,}06 \; ; \quad \varepsilon = 3^0 30' \; ; \quad A = \sqrt{\frac{90}{(2500)^2 + (147)^2}} = 3{,}74 \; sm.$$

$$x_1 = e^{-7{,}35t}\left(c_1^* \cos 49{,}46t + c_2^* \sin 49{,}46t\right) + A \cdot \sin(10t - 3^0 30')$$

$$\dot{x} = -7{,}35e^{-7{,}35t}(c_1^* \cos 49{,}46t + c_2^* \cdot \sin 49{,}46t) + e^{-7{,}35t}(49{,}46 \cdot c_1^* \sin 49{,}46t +$$

$$+ 49{,}46 \cdot c_2^* \cdot \cos 49{,}46t) + A \cdot 10 \cos(10t - 3^0 30')$$

$$\begin{cases} 0 = c_1 + A \cdot \sin 3^0 30' \\ 0 = -7{,}35 \cdot c_1 + 49{,}46 \cdot c_2 + 10 \cdot A \cdot \cos 3^0 30' \end{cases}$$

$$T = \frac{2\pi}{k_1} = \frac{2\pi}{49{,}46} = 0{,}127$$

$$c_1 = 0{,}028 \quad ; \quad c_2 = 0{,}7$$

$$\left| x = e^{-7{,}35t}(0{,}028 \cos 49{,}46t + 0{,}7 \cdot \sin 49{,}46t) + 3{,}74 \sin(10t - 3^0 30') \; sm. \right.$$

$$T_1 = \frac{2\pi}{P} = 10 = 0{,}628$$