



FAN:

NAZARIY MEXANIKA

MAVZU

06

**KINEMATIKA. NUQTA
KINEMATIKASI**



Husanov Q.

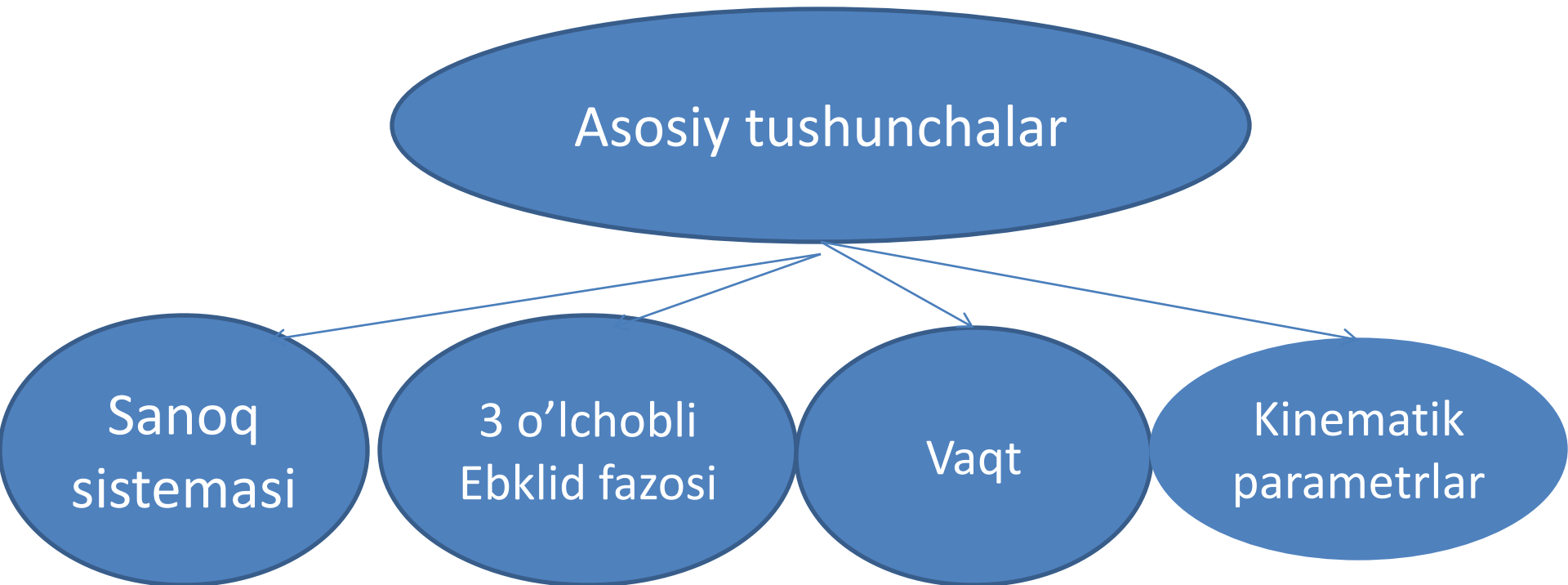


TAQDIMOT REJASI

1. Kinematikaning asosiy tushunchalari.
2. Nuqta harakatining vektor usulida berilishi.
Nuqtaning tezlik va tezlanishi.
3. Nuqta harakatini koordinatalar usulida berilishi.
Nuqtaning tezlik va tezlanishi.
4. Nuqta harakatining tabiiy usulda berilishi. Nuqtaning harakati tabiiy usulda berilganda, uning tezlanishini aniqlash

Kinematikaning asosiy tushunchalari

kinematika bo'limida *nuqta va jismning harakatlarini geometrik nuqtai nazardan, ya'ni ularga ta'sir etuvchi kuchlarni va ularning massalarini hisobga olmagan holda o'rganiladi.*



kinematikaning ikki asosiy masalasi

1. Nuqta harakatining berilishi usulini aniqlash.
2. Nuqta harakatining berilishi usuliga asosan, uning *kinematik xarakteristikasini* (harakat qonuni, trayektoriyasi, tezligi, tezlanishi) aniqlash.

Agar nuqtaning biror sanoq sistemasiga nisbatan istalgan vaqt oralig'ida fazodagi o'rnini aniqlash usuli ma'lum bo'lsa, nuqtaning harakati berilgan deyiladi.

Nuqtaning harakatini uch usulda, ya'ni *vektor*, *koordinatalar* va *tabiiy* usulda aniqlash mumkin.

Nuqta harakatining vektor usulida berilishi.

Nuqtaning tezlik va tezlanishi

$$\vec{r} = \vec{r}(t)$$

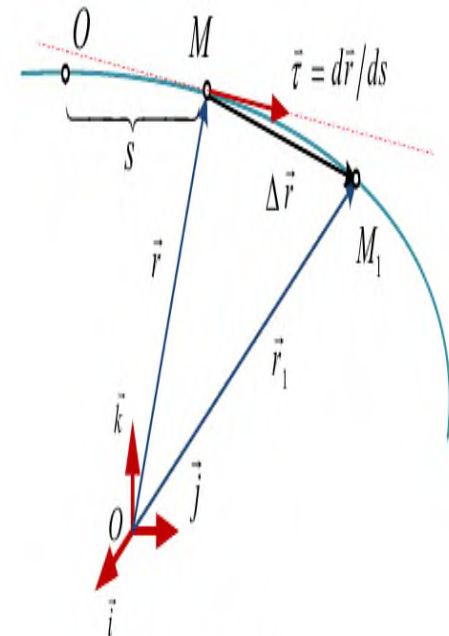
1. $\vec{r}(t)$ funksiya bir qiymatlii aniqlangan.
2. $\vec{r}(t)$ funksiya o'zining aniqlanish sohasida uzluksiz
3. $\vec{r}(t)$ funksiya o'zining aniqlanish sohasida vaqt bo'yicha hosilalarga ega bo'lishi.

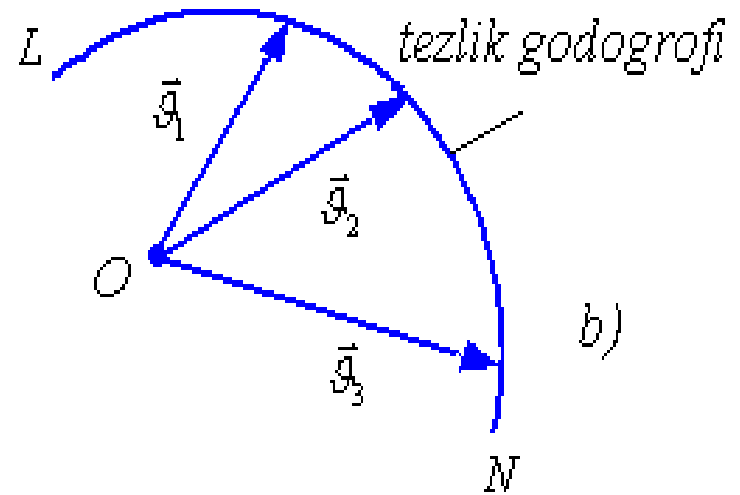
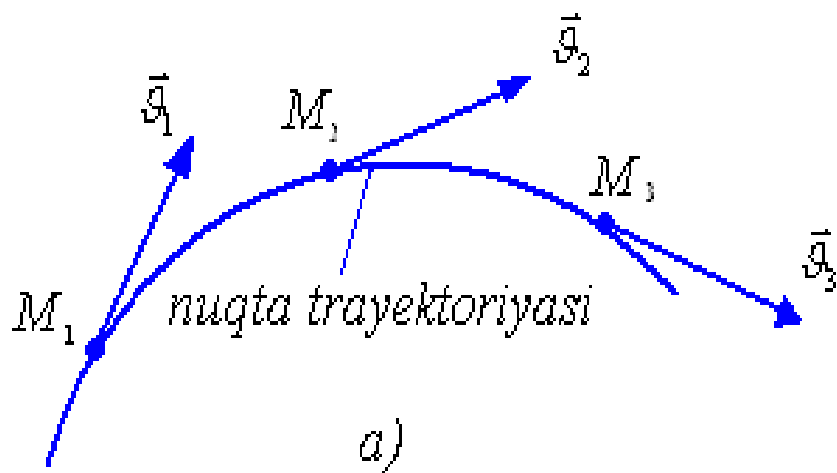
$$\vec{g}_{o'r} = \frac{\Delta \vec{r}}{\Delta t}$$

$$\vec{g} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{a} = \frac{d\vec{g}}{dt} = \frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}}$$

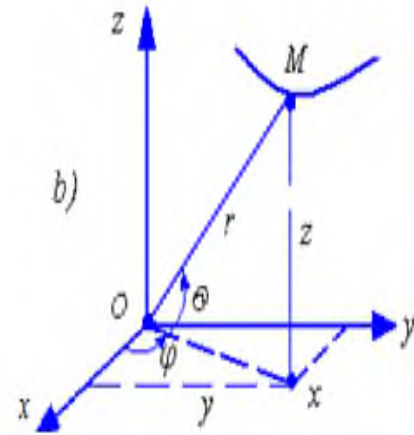
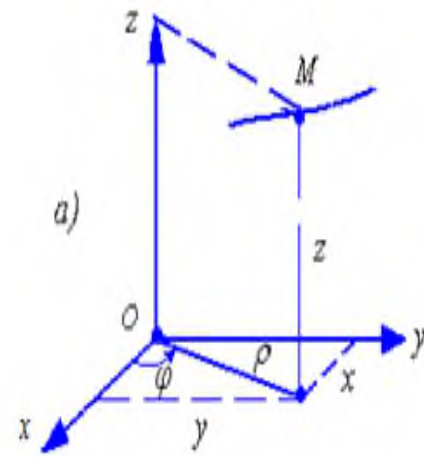
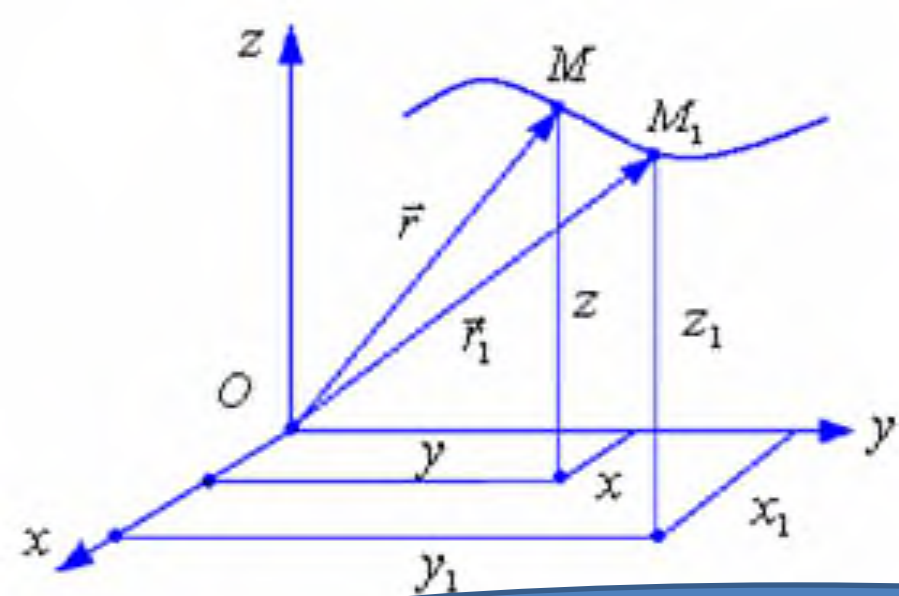
$$\vec{a}_{o'r} = \frac{\Delta \vec{g}}{\Delta t}$$





Nuqta harakatini koordinatalar usulida berilishi

Agarda vaqtning ixtiyoriy qiymatida nuqtaning fazodagi o'rnini aniqlovchi koordinatalar berilgan bo'lsa, u holda nuqtaning harakati koordinatalar usulida berilgan deyiladi.



$$x = x(t), \quad y = y(t), \quad z = z(t).$$

$$\vec{r} = x \cdot \vec{i} + y \cdot \vec{j} + z \cdot \vec{k}$$

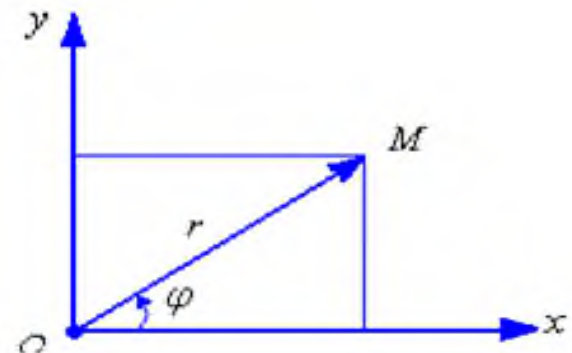
$$\rho = \rho(t), \quad \varphi = \varphi(t), \quad z = z(t),$$

$$r = r(t), \quad \varphi = \varphi(t), \quad \theta = \theta(t),$$

$$r = r(t), \quad \varphi = \varphi(t)$$

$$x = \rho \cdot \cos \varphi, \quad y = \rho \cdot \sin \varphi, \quad z = z,$$

$$x = r \cdot \cos \theta \cdot \cos \varphi, \quad y = r \cdot \cos \theta \cdot \sin \varphi, \quad z = r \cdot \sin \theta.$$



Nuqtaning harakati koordinatalar usulida berilgandagi tezligi va tezlanishini aniqlash

$$\vec{g} = \frac{d\vec{r}}{dt} = \frac{d(x \cdot \vec{i} + y \cdot \vec{j} + z \cdot \vec{k})}{dt} = \frac{dx}{dt} \cdot \vec{i} + \frac{dy}{dt} \cdot \vec{j} + \frac{dz}{dt} \cdot \vec{k}$$

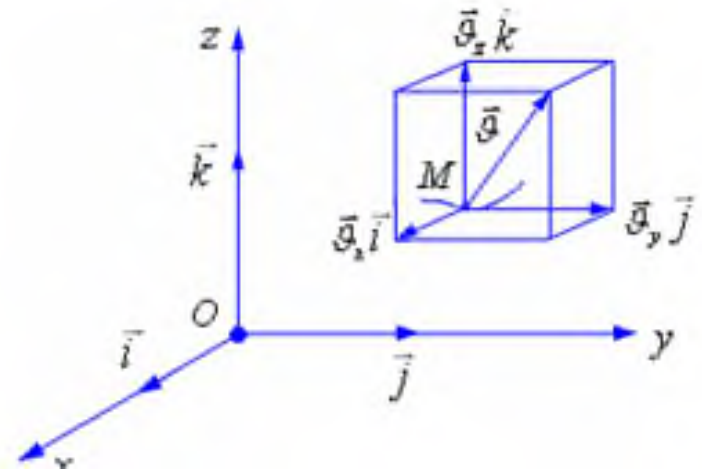
$$\vec{g} = g_x \cdot \vec{i} + g_y \cdot \vec{j} + g_z \cdot \vec{k}$$

$$g = \sqrt{g_x^2 + g_y^2 + g_z^2} = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

$$\cos(\vec{g}, \vec{i}) = \frac{g_x}{g} = \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}},$$

$$\vec{a} = \frac{dg_x}{dt} \cdot \vec{i} + \frac{dg_y}{dt} \cdot \vec{j} + \frac{dg_z}{dt} \cdot \vec{k}$$

$$\vec{g} = \frac{d}{dt}(r \cdot \vec{r}^0) = \frac{dr}{dt} \cdot \vec{r}^0 + r \cdot \frac{d\vec{r}^0}{dt}$$



$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}$$

$$\cos(\vec{a}, \vec{j}) = \frac{a_y}{a} = \frac{\ddot{y}}{\sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}},$$

$$\vec{a} = (\ddot{r} - r \cdot \dot{\varphi}^2) \cdot \vec{r}^0 + (r \cdot \ddot{\varphi} + 2\dot{r} \cdot \dot{\varphi}) \cdot \vec{p}^0$$

Nuqta harakatining tabiiy usulda berilishi

Nuqtaning trayektoriyasi va shu trayektoriya bo'ylab harakat qonuni aniqlangan bo'lsa, u holda nuqtaning harakati tabiiy usulda berilgan deyiladi.

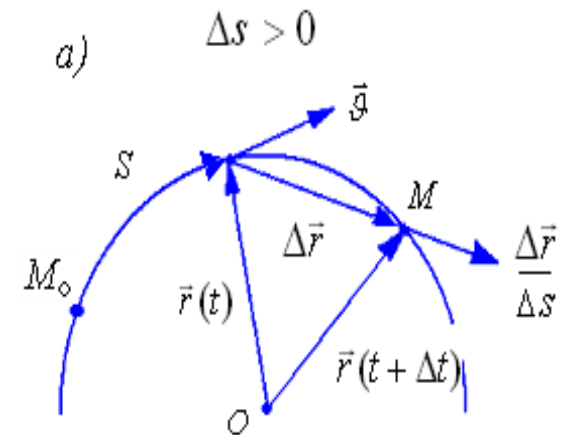
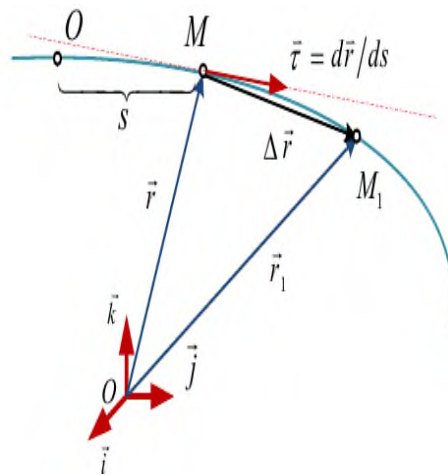
$$\vec{r} = \vec{r}(s(t)).$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}}{ds} \cdot \frac{ds}{dt} = \dot{s} \cdot \frac{d\vec{r}}{ds}$$

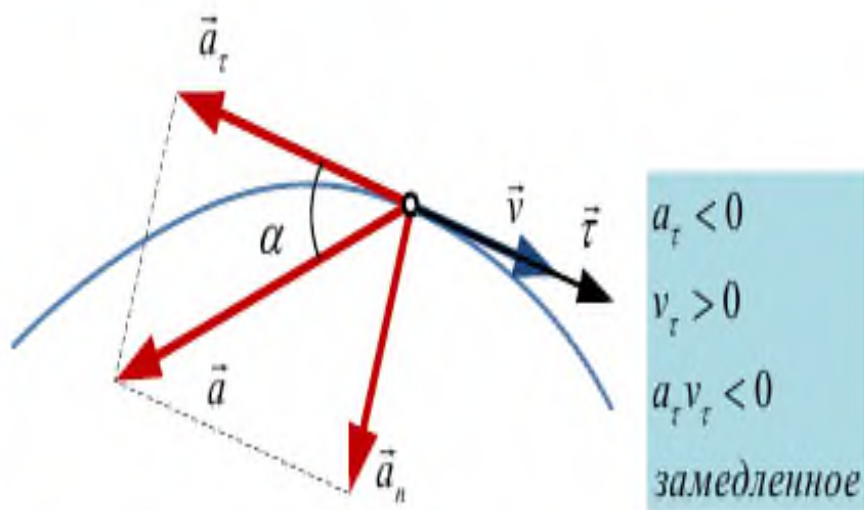
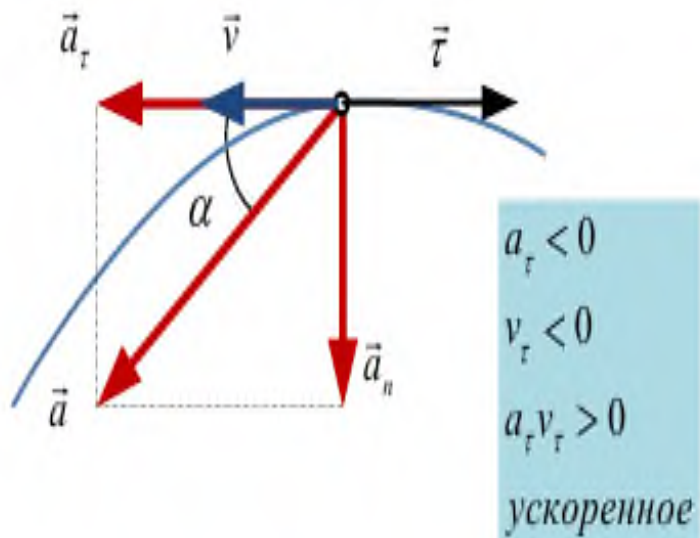
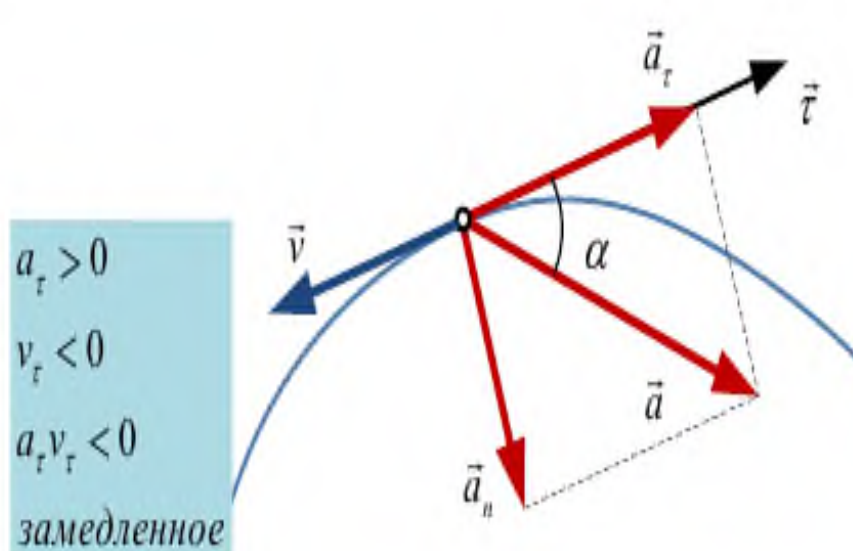
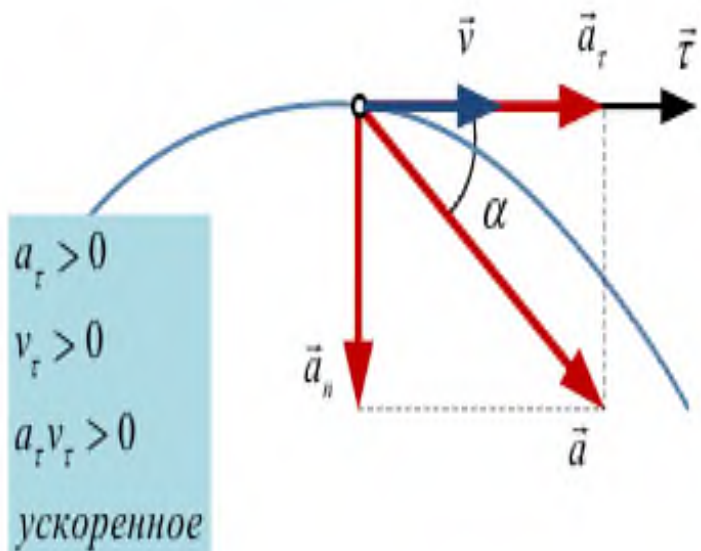
$$\left| \frac{d\vec{r}}{ds} \right| = \lim_{\Delta s \rightarrow 0} \left| \frac{\Delta \vec{r}}{\Delta s} \right| = 1,$$

$$\vec{g} = \dot{s} \cdot \vec{\tau}$$

$$s = \pm \int_0^t \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} dt$$



$$\vec{a} = \frac{d\vec{g}}{dt} = \frac{d}{dt} \cdot (\dot{s} \cdot \vec{\tau}) = \frac{d\dot{s}}{dt} \cdot \vec{\tau} + \dot{s} \cdot \frac{d\vec{\tau}}{dt}$$



K-1 V-05**Berilgan:**

$$t = 1 \text{ sek.}$$

$$x = 6 \cos\left(\frac{\pi}{6}t\right) - 3$$

$$y = 10 \sin\left(\frac{\pi}{6}t\right)$$

**Tezlik, tezlanish
egrilik radiusi**

Yechish: Masalaning shartiga asosan berilgan harakat qonuniga ko'ra harakatlanayotgan nuqtaning traektoriya tenglamasining tezligi va tezlanishini topish kerak. Buning uchun avvalo nuqtaning traektoriya tenglamasini topamiz va berilgan harakat tenglamalaridan parametr t ni yo'qotish kerak, ya'ni

$$\cos\left(\frac{\pi}{6}t\right) = \frac{x+3}{6} \quad \text{yoki} \quad \cos^2\left(\frac{\pi}{6}t\right) = \left(\frac{x+3}{6}\right)^2 ;$$

$$\text{Ma'lumki,} \quad \sin\left(\frac{\pi}{6}t\right) = \frac{y}{10} \quad \text{yoki} \quad \sin^2\left(\frac{\pi}{6}t\right) = \left(\frac{y}{10}\right)^2$$

$$\text{Demak,} \quad \left(\frac{y}{10}\right)^2 + \left(\frac{x+3}{6}\right)^2 = 1 \quad (\text{ellips})$$

$$t = 1 \text{ sek. da} \quad x = 2,2 \text{ sm} ; \quad y = 5,0 \text{ sm}$$

1. Tezlikni topish uchun harakat tenglamasidan t bo'yicha hosila olamiz;

$$\mathcal{V}_x = \frac{dx}{dt} = \dot{x} = -6 \cdot \frac{\pi}{6} \cdot \sin\left(\frac{\pi}{6}t\right) = -\pi \cdot \sin\left(\frac{\pi}{6}t\right) \text{ sm/s} ; \quad \mathcal{V}_y = \frac{dy}{dt} = \dot{y} = 10 \frac{\pi}{6} \cdot \cos\left(\frac{\pi}{6}t\right) \text{ sm/s}$$

$$t = 1 \text{ sek. da} \quad \mathcal{V}_x = -1,57 \text{ sm/s} ; \quad \mathcal{V}_y = 4,5 \text{ sm/s} ;$$

U holda tezlikning to'la qiymati

$$g = \sqrt{g_x^2 + g_y^2} = \sqrt{(-1,57)^2 + (4,5)^2} = 4,76 \text{ sm/s} ;$$

2. Tezlanishni topish uchun tezlikdan t bo'yicha hosila olamiz;

$$a_x = \frac{dg_x}{dt} = -\frac{\pi^2}{6} \cdot \cos\left(\frac{\pi}{6}t\right) \text{ sm/s}^2 ; \quad a_y = \frac{dg_y}{dt} = -10 \frac{\pi}{6} \cdot \frac{\pi}{6} \sin\left(\frac{\pi}{6}t\right) \text{ sm/s}^2 .$$

$$t = 1 \text{ sek. da} \quad a_x = -1,41 \text{ sm/s}^2 ; \quad a_y = -1,36 \text{ sm/s}^2 ;$$

U holda nuqta tezlanishining moduli

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-1,41)^2 + (-1,36)^2} = 1,95 \text{ sm/s}^2$$

3. Nuqtaning urinma va normal tezlanishlarining qiymatlarini aniqlaymiz:

$$\vec{a} = \vec{a}_\tau + \vec{a}_n$$

Urinma tezlanish

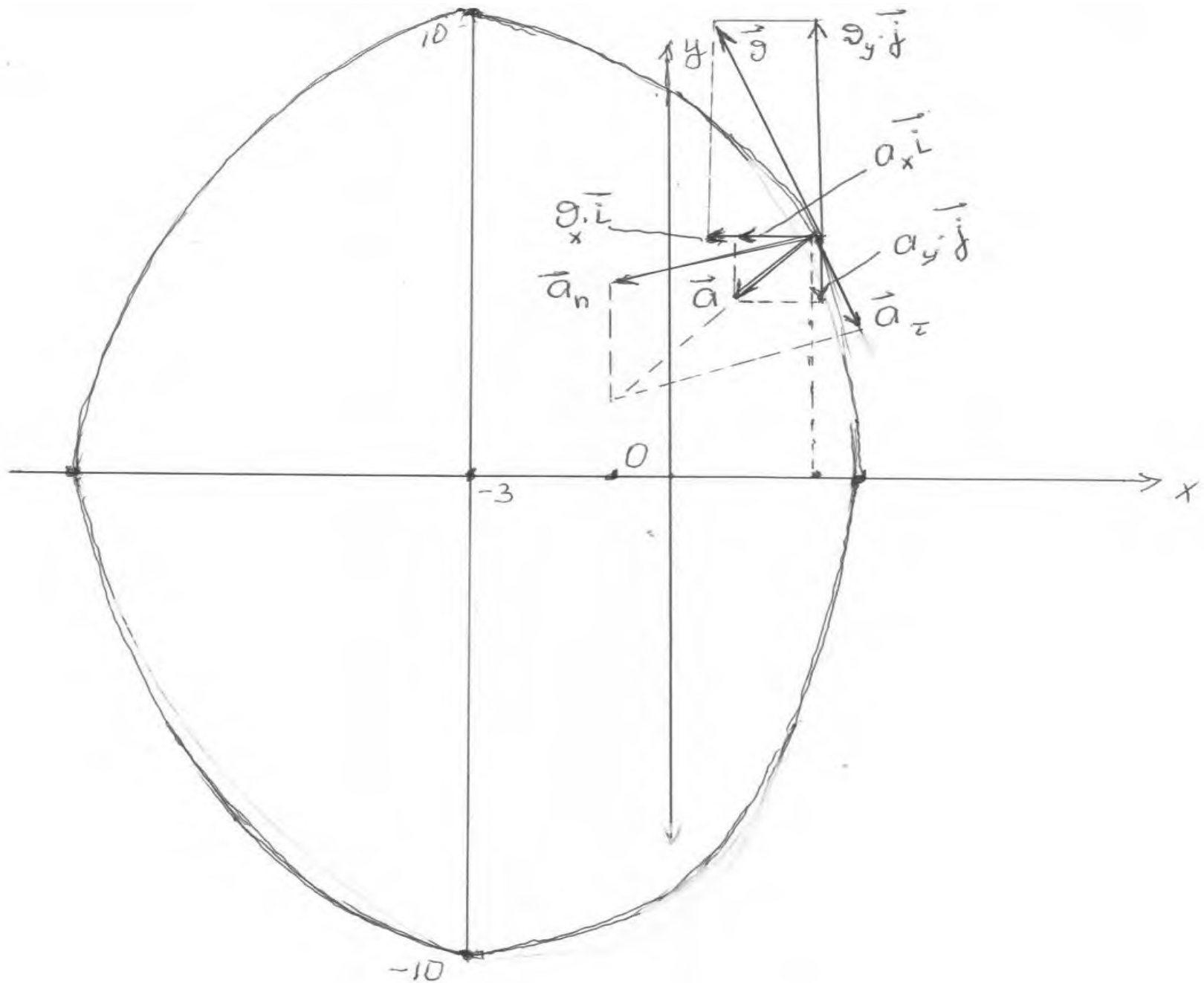
$$a_{\tau} = \frac{d\vartheta}{dt} = \frac{a_x \vartheta_x + a_y \vartheta_y}{\vartheta} = \frac{1,41 \cdot 1,57 - 1,36 \cdot 4,5}{4,76} = -0,82 \quad sm/s^2$$

Normal tezlanish

$$a_n = \sqrt{a^2 - a_{\tau}^2} = \sqrt{(1,95)^2 - (-0,82)^2} = 1,76 \quad sm/s^2$$

4. Traektoriyaning egrilik radiusi

$$\rho = \frac{\vartheta^2}{a_n} = \frac{(4,76)^2}{1,76} = 12,8 \quad sm.$$



К-1 В-04**Берилган:**

$$t = 1 \text{ сек.}$$

$$x = 6 \cos\left(\frac{\pi}{6}t\right) - 3$$

$$y = 3 \cos\left(\frac{\pi}{3}t\right)$$

**Тезлик, тезланиш
эгрилик радиуси**

Ечиш: Масаланинг шартига асосан берилган ҳаракат қонунига кўра ҳаракатланаётган нуқтанинг траектория тенгламасининг тезлиги ва тезланишини топиш керак. Бунинг учун аввало нуқтанинг траектория тенгламасини топамиз ва берилган ҳаракат тенгламаларидан параметр t ни йўқотиш керак, яъни

$$\cos\left(\frac{\pi}{6}t\right) = \frac{x+3}{6};$$

$$y = 3 \cos\left(\frac{\pi}{3}t\right) = 3 \cos\left(2 \cdot \frac{\pi}{6}t\right) = 3 \left[\cos^2\left(\frac{\pi}{6}t\right) - \sin^2\left(\frac{\pi}{6}t\right) \right] = 3 \left[\cos^2\left(\frac{\pi}{6}t\right) - 1 \right]$$

$$y = 6 \frac{(x+3)^2}{6} - 3 \quad \text{/парабола /}$$

$$t = 1 \text{ сек. да} \quad x = 2,19 \text{ см} \quad y = 1,5 \text{ см}$$

1. Тезликни топиш учун ҳаркат тенгламасидан t бўйича ҳосила оламиз;

$$g_x = \frac{dx}{dt} = \dot{x} = -3 \cdot \frac{\pi}{6} \sin\left(\frac{\pi}{6}t\right) = -\frac{\pi}{2} \cdot \sin\left(\frac{\pi}{6}t\right) = 6 \text{ см/с};$$

$$g_y = \frac{dy}{dt} = \dot{y} = -3 \cdot \frac{\pi}{3} \sin\left(\frac{\pi}{3}t\right) = -\pi \cdot \sin\left(\frac{\pi}{3}t\right) \text{ см/с}$$

$$t = 1 \text{ сек. да} \quad g_x = -\frac{\pi}{2} \text{ см/с}; \quad g_y = -\pi \cdot \frac{\sqrt{3}}{2} \text{ см/с};$$

У ҳолда тезликнинг тўла қиймати

$$g = \sqrt{g_x^2 + g_y^2} = \pi \sqrt{\left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \frac{\pi}{2} \cdot 2 = \pi \text{ см/с}$$

2. Тезланишни топиш учун тезликдан t бўйича ҳосила оламиз;

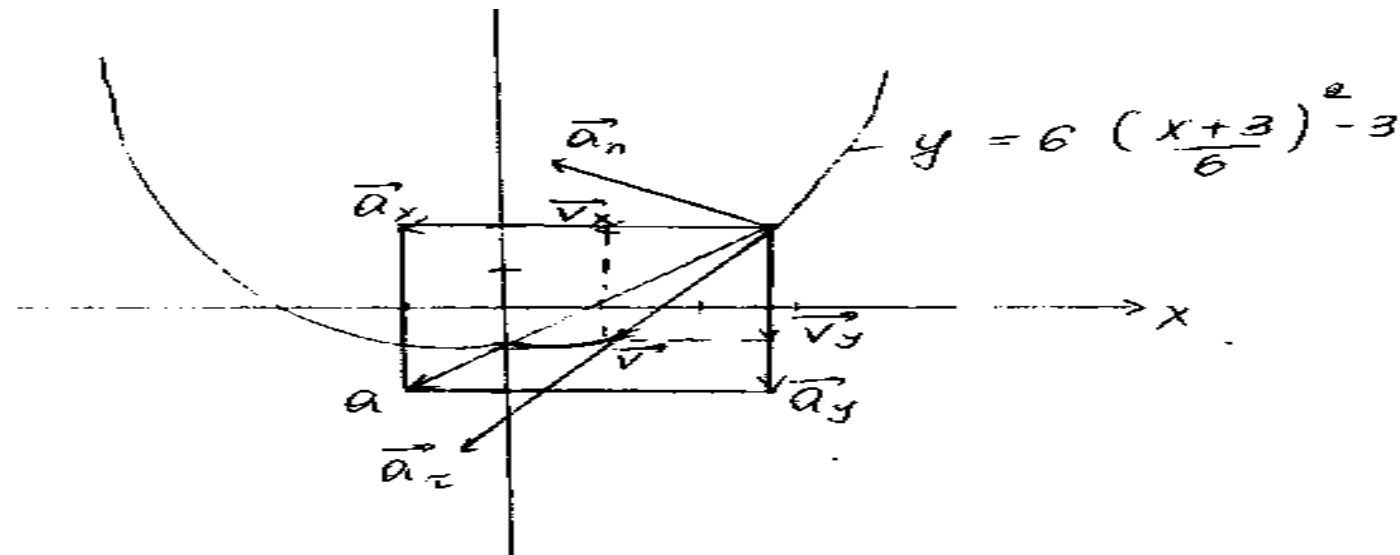
$$a_x = \frac{d\mathcal{G}_x}{dt} = \ddot{x} = -\frac{\pi^2}{12} \cdot \cos\left(\frac{\pi}{6}t\right) \text{ см/с}^2 ; \quad a_y = \frac{d\mathcal{G}_y}{dt} = \ddot{y} = -\frac{\pi^2}{3} \cos\left(\frac{\pi}{3}t\right) \text{ см/с}^2$$

$$t = 1 \text{ сек. да } a_x = -\pi^2 \cdot \frac{\sqrt{3}}{12} \text{ см/с}^2 ; \quad a_y = -\frac{\pi^2 \cdot 1}{6} \text{ см/с}^2 ;$$

У ҳолда нуқта тезланишининг модули

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(-\pi^2 \left|\frac{\sqrt{3}}{12}\right|\right)^2 + \left(-\left|\frac{\pi^2 \cdot 1}{6}\right|\right)^2} = \pi^2 \cdot 1,16 \text{ см/с}^2$$

3. Нуқтанинг уринма ва нормал тезланишларининг қийматларини аниқлаймиз:



$$a_t = \frac{d\vartheta}{dt} = \frac{a_x \vartheta_x + a_y \vartheta_y}{\vartheta} = \frac{\left(-\frac{\pi}{2}\right) \left(-\pi^2 \cdot \frac{\sqrt{3}}{12}\right) + \left(-\pi \frac{\sqrt{3}}{2}\right) \left(-\frac{\pi^2 \cdot 1}{6}\right)}{\pi} = \frac{\frac{\pi^3 \cdot \sqrt{3}}{24} + \frac{\pi^3 \cdot \sqrt{3}}{12}}{\pi} = 2,11 \text{ см/с}^2,$$

Нормал тезланиш

$$a_n = \sqrt{(\pi^2 \cdot 1,16)^2 - 2,11^2} = 11,17 \text{ см/с}^2,$$

4. Траекториянинг эгрилик радиуси

$$\rho = \frac{\vartheta^2}{a_n} = \frac{\pi^2}{11,17} = 0,882 \text{ см}$$



- 1. Kinematika bo'limida ko'riladigan asosiy masalalar nimadan iborat?**
- 2. Harakatning berilish usullarini qanday tushunasiz?**
- 3. Harakat qachon vektor usulda berildi deyiladi?**
- 4. Harakat qachon koordinatalar usulida berildi deyiladi?**
- 5. Harakat qachon tabiiy usulida berildi deyiladi?**
- 6. Nuqtaning tezligi qanday aniqlaniladi?**
- 7. Nuqtaning tezlanishi qanday aniqlaniladi?**
- 8. Birlik vektorlardan vaqt bo'yicha olingan xosilalarni tushuntiring?**
- 9. Qutb, silindrik va sferik koordinatalar sistemalaridan qanday harakatlarni uchun qo'llaniladi?**
- 10. Harakatning qanday xususiy hollarini bilasiz?**



TOSHKENT IRRIGATSIYA VA QISHLOQ
XO'JALIGINI MEXANIZATSIYALASH
MUHANDISLARI INSTITUTI



E'TIBORINGIZ UCHUN RAHMAT!



HUSANOV Q.



Nazariy va qurilish
mexanikasi kafedrasini
dotsenti

