TOSHKENT IRRIGATSIYA VA QISHLOQ XO'JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI

FAN: NAZARIY MEXANIKA

MAVZU 13

MODDIY NUQTALAR SISTEMASI

Husanov Q.

Nazariy va qurilish mexanikasi kafedrasi dotsenti

TAQDIMOT REJASI

1. Sistema massalar markazi
2. Jismning inersiya momenti
3. Oddiy bir jinsli jismlarning inersiya momentlarini Hisoblash
4. Jismning berilgan nuqtadan o'tuvchi ixtiyoriy o'qqa nisbatan inersiya momenti
5. Inersiya ellipsoidi
6. Masala

Sistema massalar markazi

- *Harakatlari o'zaro bir-biriga bog'liq bo'lgan moddiy nuqtalar to'plamiga moddiy nuqtalar sistemasi yoki sistema deyiladi.*
- Sistema tarkibiga kiruvchi nuqtalarga ta'sir etuvchi kuchlarni ichki va tashqi kuchlarga ajratamiz.

$$\overline{F} = \overline{F}^e + \overline{F}^i \qquad \vec{F}^e = \sum^N \vec{F}_k^a + \sum^N \vec{N}_k$$

N'yutonning uchinchi qonunidan kelib chiqqan holda, ichki kuchlarning quyidagi hossalarini qabul qilamiz.

1^0. Sistema nuqtalariga ta'sir etuvchi barcha ichki kuchlarning bosh vektori nolga teng, ya'ni $\vec{F}^i = \sum_{k=1}^{N} \vec{F}_k^i = 0$.

2^0. Sistema nuqtalariga ta'sir etuvchi ichki kuchlarning biror markazga nisbatan olingan momentlarining bosh momenti nolga teng, ya'ni:

$$\vec{M}_O^i = \sum_{k=1}^{N} \vec{m}_O(\vec{F}_k^i) = 0.$$

Mexanik sistemani tashkil etuvchi moddiy nuqtalar massalarining yig'indisi ***sistema massasi*** *deyiladi*. Mexanik sistema massasini M bilan belgilasak, ta'rifga binoan: $M = \sum_{i=1}^{n} m_i$.

$$\vec{r}_c = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{M}$$

$$x_c = \frac{\sum_{i=1}^{n} m_i x_i}{M}, \quad y_c = \frac{\sum_{i=1}^{n} m_i y_i}{M}, \quad z_c = \frac{\sum_{i=1}^{n} m_i z_i}{M}.$$

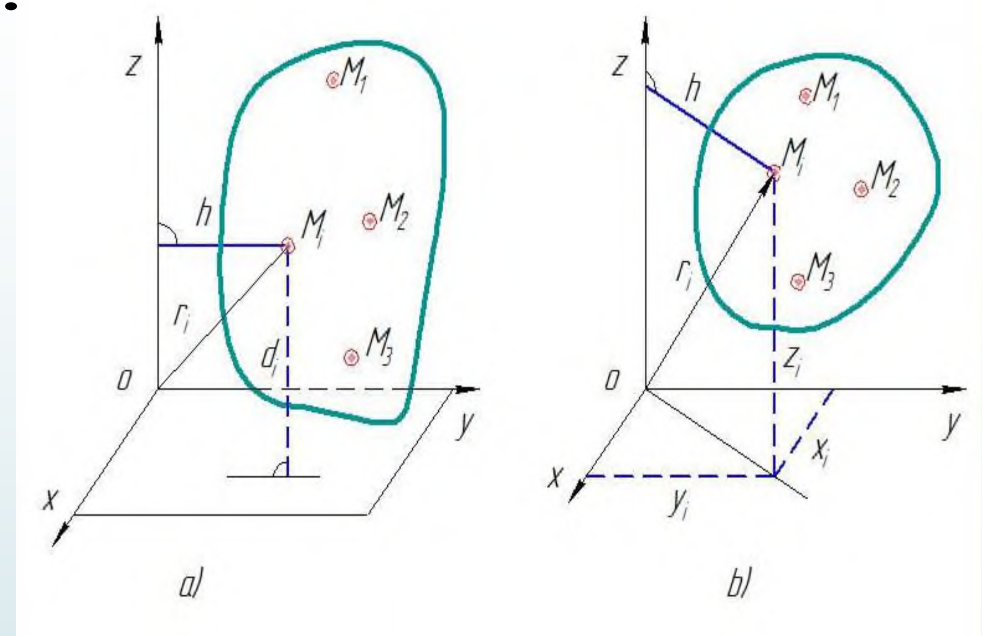
$$S_{Oxy} = \sum_{i=1}^{n} m_i z_i, \quad S_{Oyz} = \sum_{i=1}^{n} m_i x_i, \quad S_{Oxz} = \sum_{i=1}^{n} m_i y_i.$$

Jismning inersiya momenti

$$I_O=\sum_{i=1}^{n} m_i r_i^2,\quad I_z=\sum_{i=1}^{n} m_i h_i^2,\quad I_{Oxy}=\sum_{i=1}^{n} m_i d_i^2.$$

$$I_{xy}=\sum_{i=1}^{n} m_i x_i y_i,\quad I_{xz}=\sum_{i=1}^{n} m_i x_i z_i,\quad I_{yz}=\sum_{i=1}^{n} m_i y_i z_i$$

$$I_O=\sum_{i=1}^{n}\Delta m_i r_i^2=\sum_{i=1}^{n}\Delta m_i\left(x_i^2+y_i^2+z_i^2\right)$$



$$I_{Oyz}=\sum_{i=1}^{n}\Delta m_i x_i^2$$

$$I_{Oxy}=\sum_{i=1}^{n}\Delta m_i z_i^2$$

$$I_{Oxz}=\sum_{i=1}^{n}\Delta m_i y_i^2$$

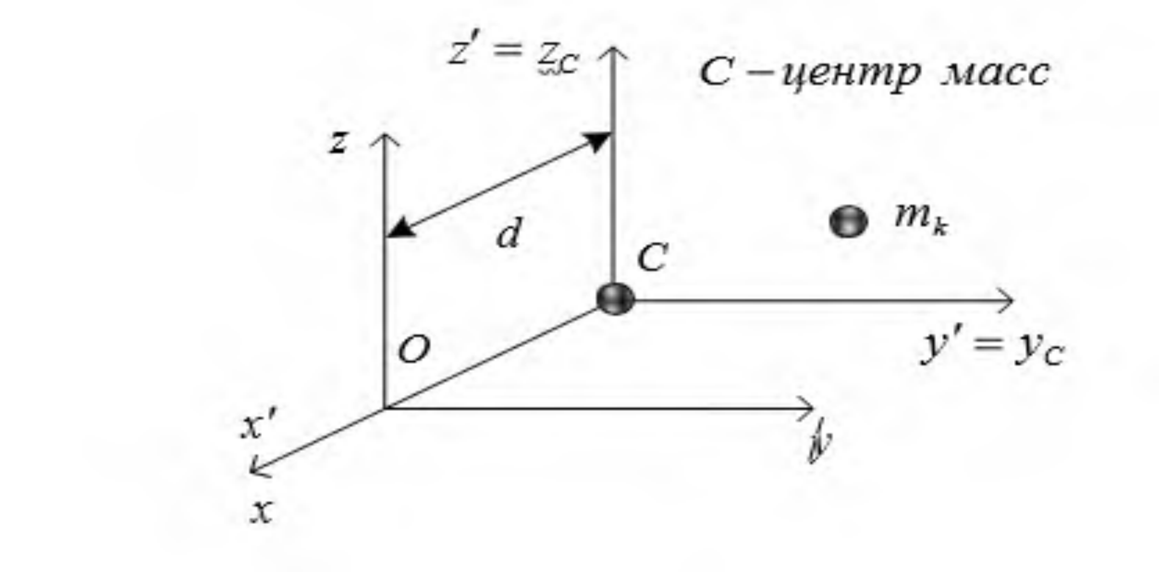
$$\begin{cases} I_{Ox}=\int\limits_{(M)}(y^2+z^2)dm=\int\limits_{(\vartheta)}\rho\cdot(y^2+z^2)d\vartheta, \\ I_{Oy}=\int\limits_{(M)}(x^2+y^2)dm=\int\limits_{(\vartheta)}\rho\cdot(x^2+z^2)d\vartheta; \\ I_{Oz}=\int\limits_{(M)}(x^2+y^2)dm=\int\limits_{(\vartheta)}\rho\cdot(x^2+y^2)d\vartheta. \end{cases}$$

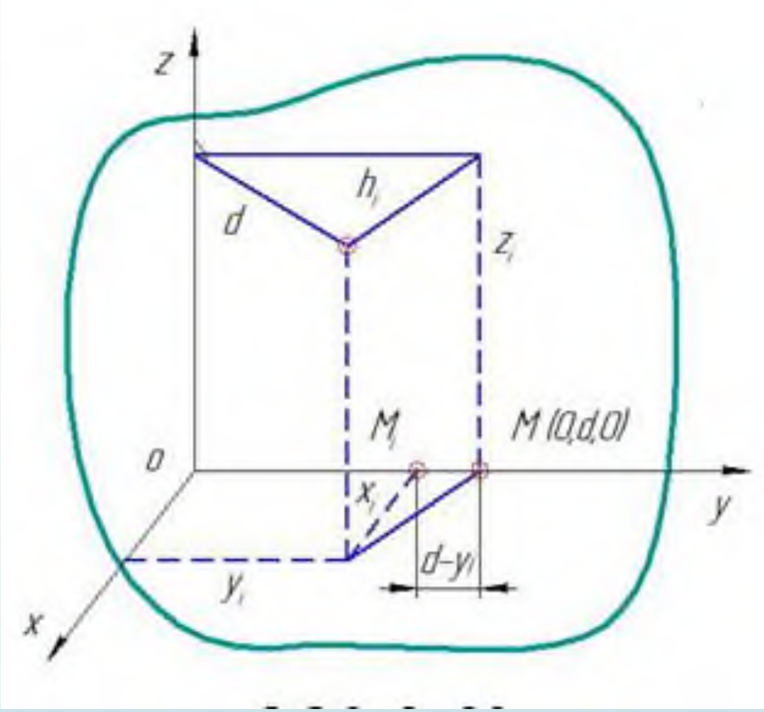
$$\Delta m=\rho\Delta\vartheta_i$$

$$I_0=\int\limits_{(M)} r^2dm=\int\limits_{(\vartheta)}\rho\cdot r^2 d\vartheta$$

<u>Shteyner teoremasi.</u> ***Qattiq jismning biror o'qqa nisbatan inersiya momenti deb jismning massalar markazidan berilgan o'qqa parallel ravishda o'tuvchi o'qqa nisbatan inersiya momentiga va jism massasining ushbu o'qlar orasidagi masofa kvadrati ko'paytmasining yig'indisiga teng.***

$$I_{z_1} = I_{Cz} + M \cdot d^2$$

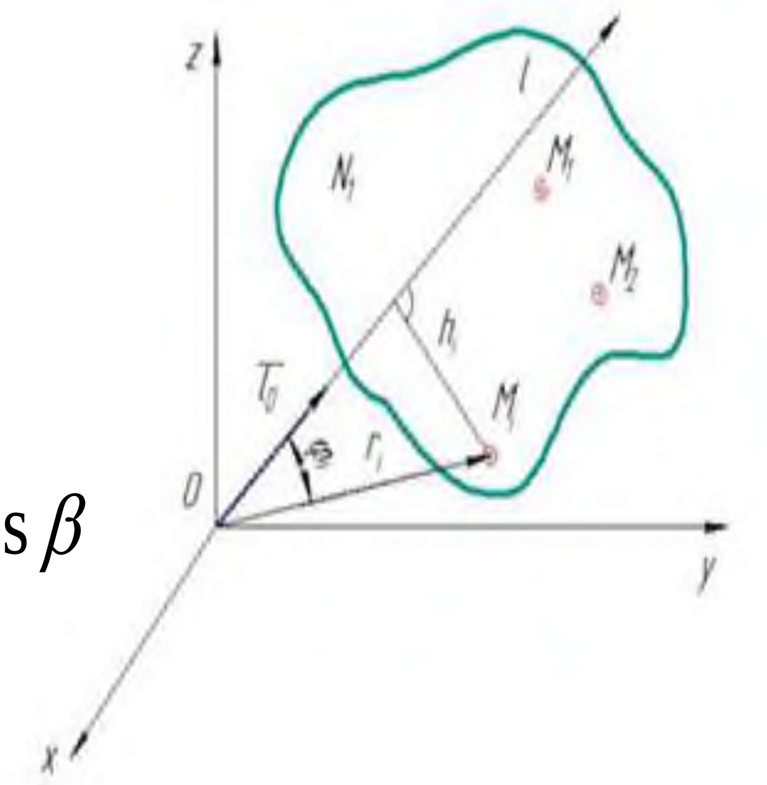




Jismning berilgan nuqtadan o'tuvchi ixtiyoriy o'qqa nisbatan inersiya momenti

$$I_\ell = \sum_{i=1}^{n} \Delta m_i \cdot h_i^2$$

$$J_\ell = J_x \cos^2\alpha + J_y \cos^2\beta + J_z \cos^2\gamma - 2J_{xy}\cos\alpha\cos\beta - 2J_{yz}\cos\beta\cos\gamma - 2J_{zx}\cos\gamma\cos\alpha$$

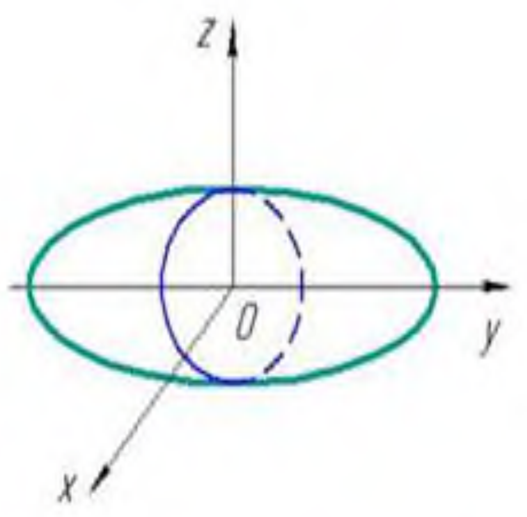


$$J_\ell = J_x \cos^2\alpha + J_y \cos^2\beta + J_z \cos^2\gamma$$

Inersiya ellipsoidi

Biror qattiq jism va markazi ixtiyoriy O nuqtada bo'lgan $\ell_1, \ell_2, \ldots, \ell_n$ o'qlar dastasi berilgan. Jismning dasta o'qlariga nisbatan inersiya momentlari mos ravishda $I_{\ell_1}, I_{\ell_2}, \ldots, I_{\ell_n}$ bo'lsin. Inersiya momentlaridan iborat ushbu sonlar to'plamining geometrik talqinini izlaymiz. Dasta markazini koordinatalar sistemasining boshi deb qabul qilamiz. Dastaning ixtiyoriy o'qini olib, uni ℓ orqali belgilaymiz. Jismning ushbu o'qqa nisbatan inersiya momenti I_ℓ bo'lsin. ℓ o'qda koordinatalar boshidan boshlab $OM = \dfrac{1}{\sqrt{I_\ell}}$ kesma ajratamiz Agar dastaning barcha o'qlari ustida ham mos inersiya momentlaridan tuzilgan shunday kesmalar ajratilsa, kesmalarning uchlari qandaydir sirtni tashkil qiladi.

Ushbu sirtni aniqlaylik. M nuqtaning koordinatalarini x, y, z orqali belgilaymiz.



ℓ o'qning yo'naltiruvchi kosinuslari α, β, γ bo'lsin. U holda

$$\alpha = \frac{x}{OM} = x\sqrt{I_\ell}, \beta = \frac{y}{OM} = y\sqrt{I_\ell}, \gamma = \frac{z}{OM} = z\sqrt{I_\ell}$$

ℓ o'qning yo'naltiruvchi kosinuslari α, β, γ bo'lsin. U holda

$$\alpha = \frac{x}{OM} = x\sqrt{I_\ell}, \beta = \frac{y}{OM} = y\sqrt{I_\ell}, \gamma = \frac{z}{OM} = z\sqrt{I_\ell}$$

Bu kattaliklarni quyidagi formulaga qo'yamiz,

$$J_\ell = J_x \cos^2\alpha + J_y \cos^2\beta + J_z \cos^2\gamma - 2J_{xy}\cos\alpha\cos\beta - 2J_{yz}\cos\beta\cos\gamma - 2J_{zx}\cos\gamma\cos\alpha$$

va I_l ga qisqartiramiz

$$I_x x^2 + I_y y^2 + I_z z^2 - 2I_{xy}xy - 2I_{xz}xz - 2I_{yz}yz = 1$$

bu tenglama ikkinchi tartibli sirtning, ya'ni *ellipsoidning* tenglamasidir. Bu ellipsoidga *inersiya ellipsoidi* deyiladi. Koordinatalar boshidan o'tgan o'qlar ellipsoidaning *bosh o'qlari* deyiladi. Agar koordinata o'qlari ellipsoidning bosh o'qlaridan iborat bo'lsa, inersiya ellipsoidning tenglamasi

$$I_x x^2 + I_y y^2 + I_z z^2 = 1$$

Bu holda inersiya ellipsoidining bosh o'qlari *jismning inersiya bosh o'qlari* deyiladi. Jismning inersiya bosh o'qlariga nisbatan I_x, I_y, I_z inersiya momentlari *inersiya bosh momentlari* deyiladi. Ko'ramizki, jismning inersiya bosh o'qlariga nisbatan markazdan qochuvchi inersiya momentlari nolga teng ekan. Agar jismning inersiya bosh o'qlari jismning massalar markazidan o'tsa, u o'qqa *inersiya markaziy bosh o'qi* deyiladi.

Inersiya bosh o'qlarining quyidagi hossalari mavjud:

1. Inersiya markaziy bosh o'qi ushbu o'qning ixtiyoriy nuqtasiga nisbatan bosh inersiya o'qi bo'ladi.

2, Jismning simmetriya o'qi uning inersiya markaziy bosh o'qi bo'ladi.

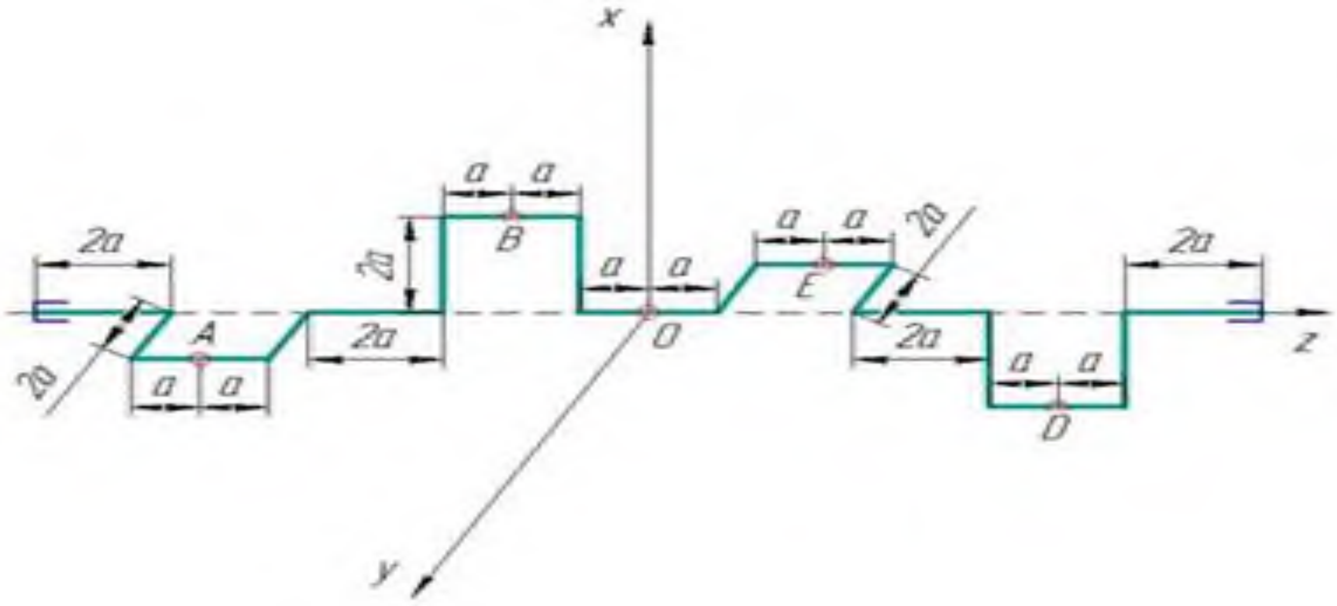
3, Jismning simmetriya tekisligiga tik bo'lgan har qanday o'q ushbu tekislik bilan kesishish nuqtasiga nisbatan inersiya bosh o'qidan iborat bo'ladi.

Statika qismida sistemaga ta'sir etuvchi kuchlarni tashqi va ichki kuchlarga ajratish to'g'risidagi qisqacha to'xtalgan edik. Ma'lumki, mexanik sistemaga ta'sir etuvchi kuchlar shu sistema tarkibiga kirmaydigan jismlar orqali qo'yilgan bo'lsa, unday kuchlar tashqi kuchlar, sistema nuqtalarining o'zaro ta'sir kuchlari esa ichki kuchlar deyiladi. Tashqi kuchlarni yuqori indeksda «e» harfni, ichki kuchlarni esa yuqori indeksda «*i*» harfni (frantsuzcha exterieur – tashqi va interieur – ichki so'zlarning boshlang'ich harflari) qo'yish bilan belgilaymiz; F^e - tashqi kuch, F^i - ichki kuch.

Масала.

$$m_A = m_B = m_E = m_D = m$$

J_{xy}, J_{xz}, J_{yz}



$$J_{yz} = \sum_{k=1}^{n} m_k y_k z_k = m_A y_A z_A + m_B y_B z_B + m_E y_E z_E + m_D y_D z_D =$$

$$= m_A \cdot 2a(-6a) + m_E(-2a) \cdot 2a = 12 m_A \cdot a^2 - 4 m_E \cdot a^2 = -16 m \cdot a^2$$

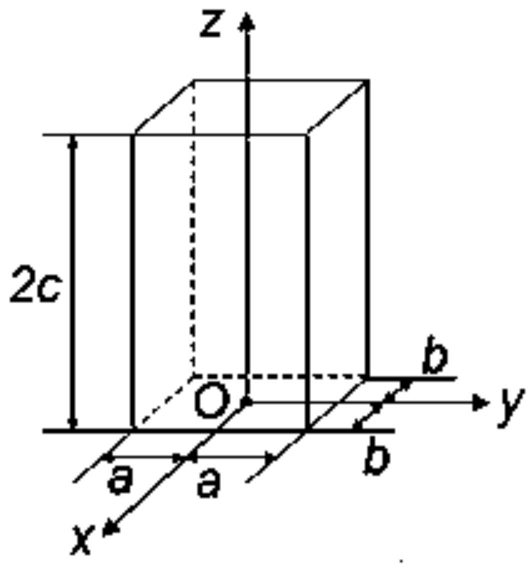
$$J_{xy} = \sum_{k=1}^{n} m_k x_k y_k = m_A x_A y_A + m_B x_B y_B + m_E x_E y_E + m_D x_D y_D = 0$$

$$J_{xz} = \sum_{k=1}^{n} m_k x_k z_k = m_A x_A z_A + m_B x_B z_B + m_E x_E z_E + m_D x_D z_D =$$

$$= m_B(-2a) \cdot 2a + m_D \cdot 6a(-2a) = -4 m_B \cdot a^2 - 12 m_D \cdot a^2 = -16 m \cdot a^2$$

Масала.

$$J_x = \int (y^2 + z^2) dm$$



$$dm = \frac{M}{V} dV = \frac{M}{2a \cdot 2b \cdot 2c} dxdydz = \frac{M}{8a \cdot b \cdot c} dxdydz$$

$$J_x = \frac{M}{8a \cdot b \cdot c} \int_0^{2c} \int_{-a}^{a} \int_{-b}^{b} (y^2 + z^2) dxdydz = \frac{M}{8a \cdot b \cdot c} \left(\int_0^{2c} \int_{-a}^{a} (y^2 + z^2) \cdot x \Big|_{-b}^{b} \right) dydz =$$

$$= \frac{M}{8a \cdot b \cdot c} \int_0^{2c} \left(\frac{1}{3} 2b \cdot y^3 \Big|_{-a}^{a} + 2b \cdot z^2 \cdot y \Big|_{-a}^{a} \right) dz = \frac{M}{8a \cdot b \cdot c} \int_0^{2c} \left(\frac{2b \cdot 2a^3}{3} + 2b \cdot 2a \cdot z^2 \right) dz =$$

$$= \frac{M}{8a \cdot b \cdot c} \left(\frac{4b \cdot a^3 \cdot z}{3} \Big|_0^{2c} + 4b \cdot a \cdot \frac{z^3}{3} \Big|_0^{2c} \right) = \frac{M}{8a \cdot b \cdot c} \left(\frac{8b \cdot a^3 \cdot c}{3} + 4b \cdot a \cdot \frac{(2c)^3}{3} \right) = \frac{M}{3} (a^2 + 4c^2)$$

$$J_y = \frac{M}{3} (b^2 + 4c^2)$$

$$J_z = \int_M (x^2 + y^2) dm = \frac{M}{8a \cdot b \cdot c} \int_0^{2c} \int_{-a}^{a} \int_{-b}^{b} (x^2 + y^2) dxdydz = \frac{M}{8a \cdot b \cdot c} \int_0^{2c} \int_{-a}^{a} \left(\frac{1}{3} 2b^3 + y^2 \cdot 2b \right) dydz =$$

$$= \frac{M}{8a \cdot b \cdot c} \int_0^{2c} \left(\frac{2b^3}{3} \cdot 2a + \frac{2b}{3} \cdot 2a^3 \right) dz = \frac{M}{8a \cdot b \cdot c} \left(\frac{2b^3 \cdot 2a \cdot 2c}{3} + \frac{2b \cdot 2a^3 \cdot 2c}{3} \right) = \frac{M}{3} (a^2 + b^2)$$

Masala. $m_1, m_2 . m_3$, d

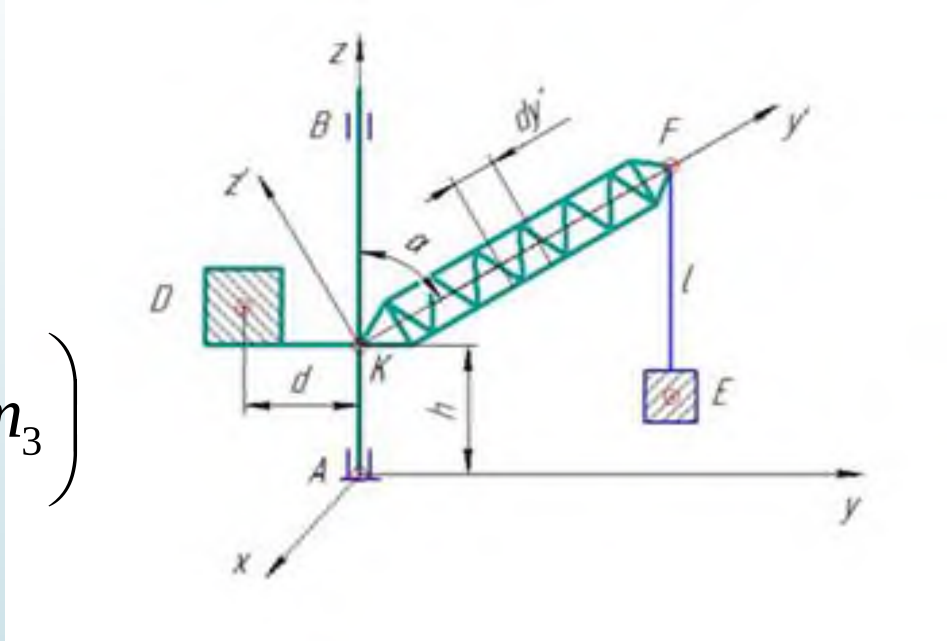
$$J_z = J_z^{KF} + J_z^{D} + J_z^{E} \qquad J_z^{D} = m_2 d^2 \qquad J_z^{E} = m_3 L^2 \sin^2\alpha$$

$$J_z^{KF} = J_{x'}^{KF} \cos\alpha_1 + J_{y'}^{KF} \cos\alpha_2 + J_{z'}^{KF} \cos\alpha_3$$

$$J_{x'y'} = J_{x'z'} = J_{y'z'} = 0 \qquad \alpha_1 = 90^\circ$$

$$\alpha_3 = 90^\circ - \alpha \qquad \cos\alpha_3 = \sin\alpha \qquad J_{y'}^{KF} = 0$$

$$J_{z'}^{KF} = \frac{1}{3} m_2 L^2 \qquad J_z = m_2 d^2 + L^2 \sin^2\alpha \cdot \left(\frac{m_1}{3} + m_3\right)$$



$$J_{yz} = J_{yz}^{KF} + J_{yz}^{D} + J_{yz}^{E} \qquad J_{yz}^{D} = -m_2 dh$$

$$J_{yz}^{E} = m_3 L \cdot \sin\alpha \cdot (h - \ell + L\cos\alpha)$$

$$J_{yz}^{KF} \Rightarrow dy' \qquad dm = \left(\frac{m_1}{L}\right) dy' \qquad y = y' \cdot \sin\alpha \qquad z = y'\cos\alpha + h$$

$$J_{yz}^{KF} = \int yzdm = \frac{m_1}{L}\int_0^l (h + y'\cos\alpha)\cdot y'\cdot\sin\alpha\cdot dy' = m_1 L\cdot\sin\alpha\cdot\left(\frac{h}{2} + \frac{L}{3}\cos\alpha\right)$$

$$J_{yz} = L\cdot\sin\alpha\cdot\left[\frac{m_1 L}{3}\cos\alpha + m_3(L\cdot\cos\alpha - \ell)\right] + h\cdot\left(m_3 L\cdot\sin\alpha + \frac{m_1 L}{2}\cdot\sin\alpha - m_2 d\right)$$

$$m_3 L\cdot\sin\alpha + \frac{m_1 L}{2}\cdot\sin\alpha - m_2 d = 0$$

$$J_{yz} = L\cdot\sin\alpha\cdot\left[\frac{m_1 L}{3}\cos\alpha + m_3(L\cdot\cos\alpha - \ell)\right] = \frac{3M_3 + M_1}{6}\cdot L^2\cdot\sin^2\alpha - M_3 L\cdot\ell\cdot\sin\alpha$$

1. **Moddiy nuqtalar sistemasi deb nimaga aytiladi? (Misollar bilan tushuntiring).**
2. **Sistema massasi va sistema massalar markazi qanday aniqlanadi?**
3. **Jismning inersiya momenti tushunchasi nima uchun kiritiladi?**
4. **Jismning inersiya momenti qanday turlarga bo'linadi?**
5. **Jismning parallel o'qlariga nisbatan inersiya momentini tushuntiring.**
6. **Bir jinsli oddiy jismlarning inersiya momentini hisoblashni tushuntiring.**
7. **Jismning berilgan nuqtadan o'tuvchi o'qqa nisbatan inersiya momenti qanday aniqlanadi?**
8. **Bosh inersiya o'qlari va markaziy bosh inersiya o'qlari tushunchalari nima uchun kiritiladi?**

Inersiya ellipsoidasini tushuntiring.

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E'TIBORINGIZ UCHUN RAHMAT!

HUSANOV Q.

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