TOSHKENT IRRIGATSIYA VA QISHLOQ XO'JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI

FAN: NAZARIY MEXANIKA

MAVZU 04

Statikaning asosiy teoremasi va fazoviy kuchlar sistemasining muvozanat sharti.

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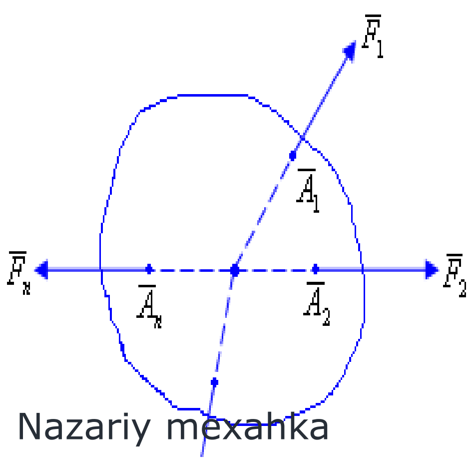
Nazariy va qurilish mexanikasi kafedrasi dotsenti

Reja:

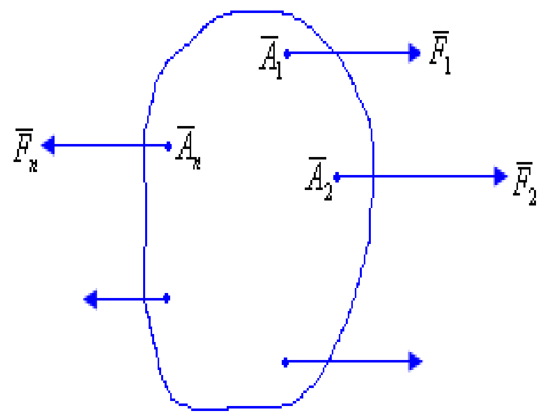
1. **Kuchni o'ziga nisbatan parallel ko'chirishga oid lemma.**
2. **Fazodagi ixtiyoriy joylashgan kuchlar sistemasini bir nuqtaga keltirish.**
3. **Fazodagi kuchlar sistemasi bosh vektori va bosh momentining analitik ifodalari.**

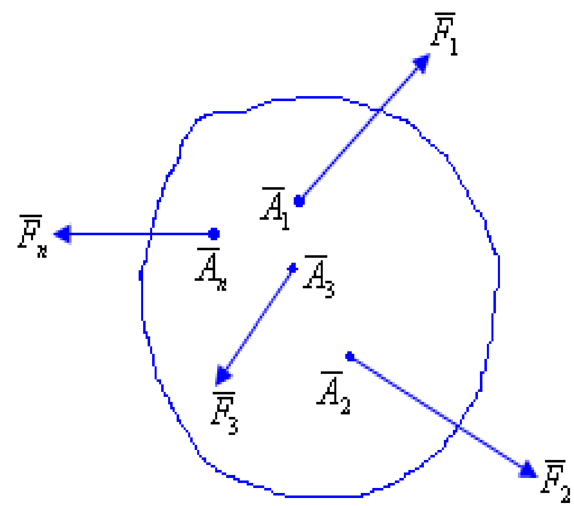
Statika qismida asosan ikkita asosiy masala yechiladi:

1. Jismga ta'sir etuvchi kuchlarni sodda holga keltirish.
2. Hosil qilingan kuchlar sistemasi uchun muvozanat shartlarini aniqlash.



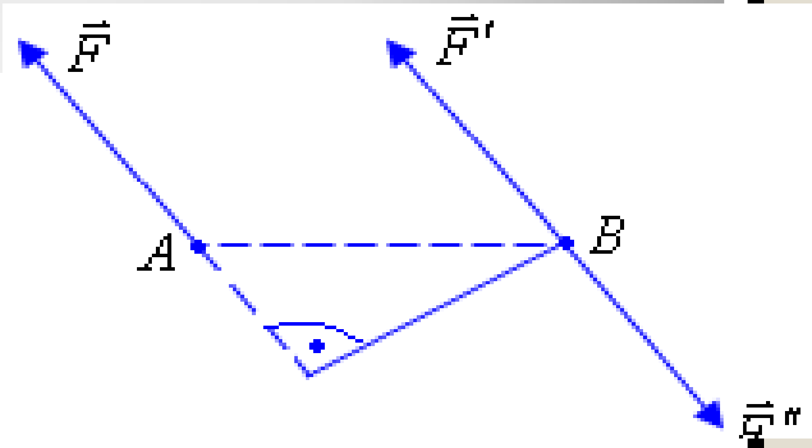
Nazariy mexahka





Kuchni parallel ko'chirish haqidagi lemma

***Puanso lemmasi*. Jismning biror nuqtasiga qo'yilgan kuch, jismda olingan ixtiyoriy keltirish markaziga qo'yilgan xuddi shunday kuchga va momenti berilgan kuchning keltirish markaziga nisbatan momentiga teng juftga ekvivalent bo'ladi.**



Statikaning asosiy teoremasi. (Puanso teoremasi)

1. Jismga qo'yilgan kuchlar sistemasining geometrik yig'indisiga, berilgan kuchlarning *bosh vektori* deyiladi, ya'ni

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n = \sum_{k=1}^{n} \vec{F}_k \ .$$

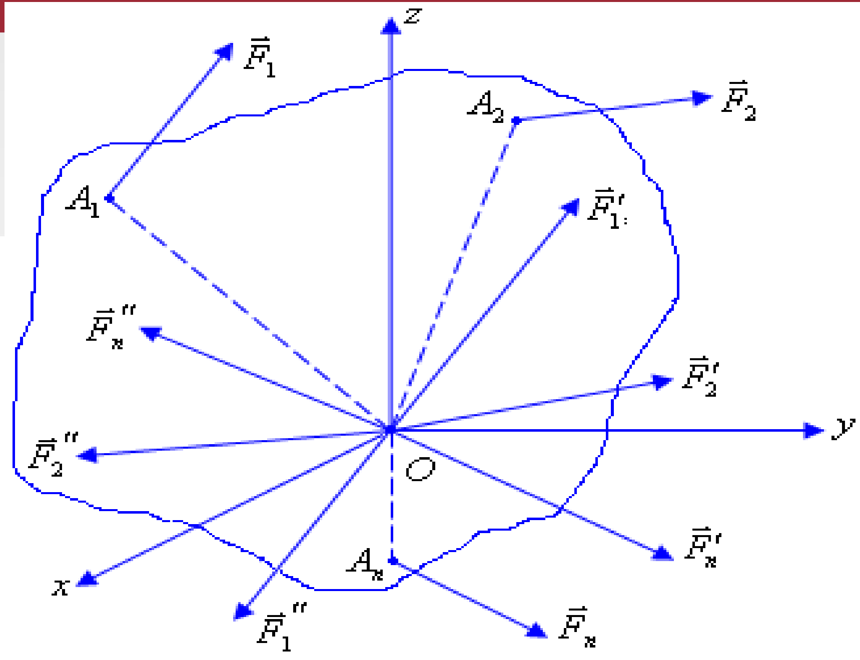
2. Jismga qo'yilgan kuchlardan biror markazga nisbatan olingan momentlarning geometrik yig'indisiga, berilgan kuchlarning ***bosh momenti*** deyiladi, ya'ni:

$$\vec{M}_0 = \vec{m}_0(\vec{F}_1) + \vec{m}_0(\vec{F}_2) + \ldots + \vec{m}_0(\vec{F}_n) = \sum_{k=1}^{n} \vec{m}_0(\vec{F}_k) \, .$$

.

Puanso teoremasi. Fazoda ixtiyoriy joylashgan kuchlar sistemasini biror markazga keltirish natijasida bu kuchlar sistemasi keltirish markaziga qo'yilgan bosh vektorga teng bitta kuchga va momenti ga teng bo'lgan bitta juft kuch momentiga ekvivalent bo'ladi.

Shunday qilib, qattiq gismning nuqtalariga qo'yilgan fazoviy kuchlarni ixtieriy bitta markazga keltirish natigasida, bu kuchlar bitta bosh bektorga va bitta bosh momentga ekvivalnt boladi, y'ni



$$\vec{R} = \vec{F}_1' + \vec{F}_2' + ... + \vec{F}_n'$$

$$(\vec{F}_1, \vec{F}_2, ..., \vec{F}_n) \sim (\vec{F}, \vec{M}_0),$$

Kuchlarning bosh vektori va bosh momentini analitik aniqlash

$$F_x = F_{1x} + F_{2x} + \ldots + F_{nx} = \sum_{k=1}^{n} F_{kx},$$

$$F_y = F_{1y} + F_{2y} + \ldots + F_{ny} = \sum_{k=1}^{n} F_{ky},$$

$$F_z = F_{1z} + F_{2z} + \ldots + F_{nz} = \sum_{k=1}^{n} F_{kz}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{\left(\sum_{k=1}^{n} F_{kx}\right)^2 + \left(\sum_{k=1}^{n} F_{ky}\right)^2 + \left(\sum_{k=1}^{n} F_{kz}\right)^2}$$

$$\cos(\vec{F}, \hat{\vec{i}}) = \frac{F_x}{F}, \quad \cos(\vec{F}, \hat{\vec{j}}) = \frac{F_y}{F}, \quad \cos(\vec{F}, \hat{\vec{k}}) = \frac{F_z}{F}.$$

Bosh momentning moduli va yo'nalishini aniqlash.

$$M_{0x} = m_x(\vec{F}_1) + m_x(\vec{F}_2) + \ldots + m_x(\vec{F}_n) = \sum_{k=1}^{n} m_x(\vec{F}_k),$$

$$M_{0y} = m_y(\vec{F}_1) + m_y(\vec{F}_2) + \ldots + m_y(\vec{F}_n) = \sum_{k=1}^{n} m_y(\vec{F}_k)$$

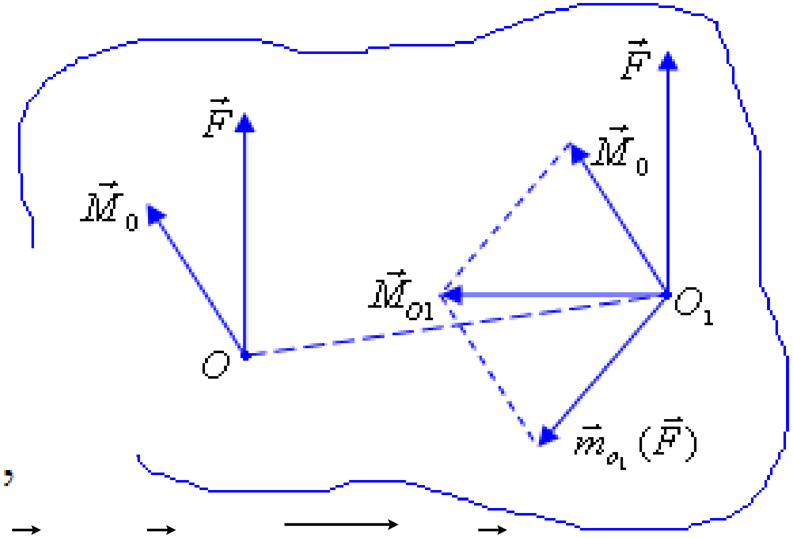
$$M_{0z} = m_z(\vec{F}_1) + m_z(\vec{F}_2) + \ldots + m_z(\vec{F}_n) = \sum_{k=1}^{n} m_z(\vec{F}_k)$$

$$M_0 = \sqrt{M_{0x}^2 + M_{0y}^2 + M_{0z}^2}$$

$$\cos(\vec{M}_0, \hat{\ } \vec{i}) = \frac{M_{0x}}{M}, \quad \cos(\vec{M}_0, \hat{\ } \vec{j}) = \frac{M_{0y}}{M}, \quad \cos(\vec{M}_0, \hat{\ } \vec{k}) = \frac{M_{0z}}{M}.$$

Keltirish markaziga qo'yilgan bosh vektor bilan bosh moment orasidagi burchakni aniqlash.

$$\vec{F}\cdot\vec{M}_0 = F\cdot M_0\cdot\cos(\vec{F},\overset{\wedge}{}\vec{M}_0)$$



$$\cos(\vec{F},\overset{\wedge}{}\vec{M}_0) = \frac{\vec{F}\cdot\vec{M}_0}{F\cdot M_0} = \frac{F_x\cdot M_{0x} + F_y\cdot M_{0y} + F_z\cdot M_{0z}}{F\cdot M_0},$$

$$\vec{M}_{O1} = \vec{M}_0 + \vec{m}_{O1}(\vec{F}) = \vec{M}_0 + \overrightarrow{O_1O}\times\vec{F} = \vec{M}_0 - \overrightarrow{OO_1}\times\vec{F}$$

Demak, *bosh vektor va bosh momentni biror yangi keltirish markaziga ko'chirish natijasida, bosh vektor o'zgarmay qoladi va bosh moment eski keltirish markaziga qo'yilgan bosh vektordan yangi keltirish markaziga nisbatan olingan momentga o'zgaradi.*

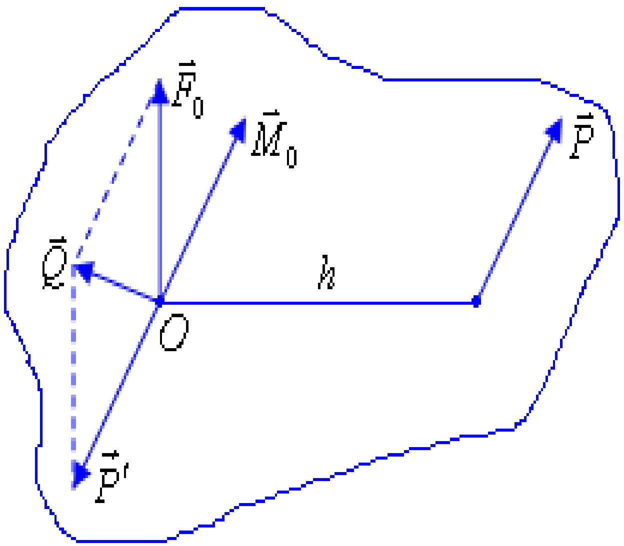
Fazoviy kuchlar sistemasining muvozanat shartlari

Teorema. Fazoviy kuchlar sistemasi muvozanatda bo'lishi uchun bu kuchlarning bosh vektori va bosh momenti nolga teng bo'lishi zarur hamda yetarlidir.

Avvalo *yetarli* shartni isbotlaymiz, ya'ni berilgan kuchlar sistemasini nolga ekvivalent ekanligini ko'rsatamiz

$$(\vec{F}_1, \vec{F}_2, ..., \vec{F}_n) \sim (\vec{F}_0, \vec{M}_0) \sim 0.$$

Endi teoremani *zaruriy* shartini isbotlaymiz, ya'ni berilgan kuchlarning nolga ekvivalentlik shartidan, ularning muvozanatda bo'lishini ko'rsatamiz.



$$\vec{F}_0 = 0,\ \vec{M}_0 = 0$$

№	Kuchlarning turlari	Tekislikdagi kuchlar sistemasi uchun muvozanat tenglamalari	Fazoviy kuchlar sistemasi uchun muvozanat tenglamalari
1.	Ta'sir chiziqlari bir nuqtada kesishuvchi kuchlar sistemasi.	$\sum_{k=1}^{n} F_{kx} = 0;\ \sum_{k=1}^{n} F_{ky} = 0$	$\sum_{k=1}^{n} F_{kx} = 0;\ \sum_{k=1}^{n} F_{ky} = 0;\ \sum_{k=1}^{n} F_{kz} = 0$
2.	Parallel kuchlar sistemasi $(\vec{F}_k // O_x)$	$\sum_{k=1}^{n} F_{kx} = 0;$ $\sum_{k=1}^{n} m_o(\vec{F}_k) = 0$	$\sum_{k=1}^{n} F_{kx} = 0;\ \sum_{k=1}^{n} m_y(\vec{F}_k) = 0;$ $\sum_{k=1}^{n} m_z(\vec{F}_k) = 0$
3.	Ta'sir chiziqlari ixtiyoriy yo'nalgan kuchlar sistemasi	$\sum_{k=1}^{n} F_{kx} = 0;\ \sum_{k=1}^{n} F_{ky} = 0$ $\sum_{k=1}^{n} m_o(\vec{F}_k) = 0$	$\sum_{k=1}^{n} F_{kx} = 0;\ \sum_{k=1}^{n} m_x(\vec{F}_k) = 0$ $\sum_{k=1}^{n} F_{ky} = 0;\ \sum_{k=1}^{n} m_y(\vec{F}_k) = 0$ $\sum_{k=1}^{n} F_{kz} = 0;\ \sum_{k=1}^{n} m_z(\vec{F}_k) = 0$

Statikaning invariantlari va markaziy vint o'qi tenglamasi

Berilgan kuchlar sistemasining invarianti deb, keltirish markazi o'zgarganda o'zgarmay qoluvchi kattaliklarga aytiladi.

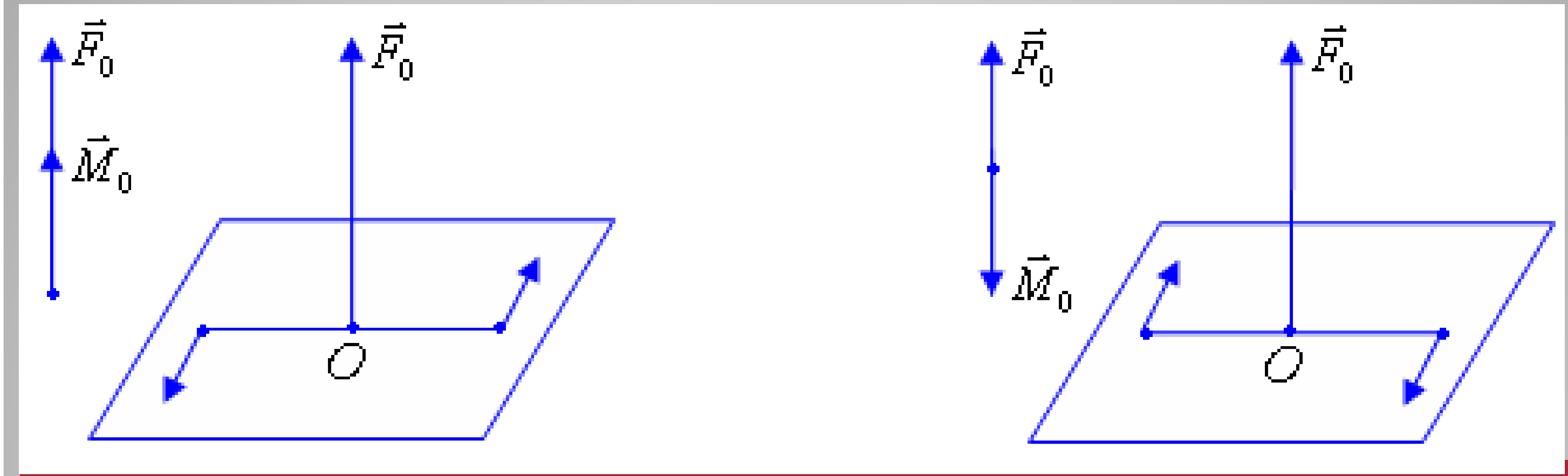
$$J_1 = F_0^2 = F_x^2 + F_y^2 + F_z^2$$

$$J_2 = \vec{M}_0 \cdot \vec{F} = M_{0x} \cdot F_x + M_{0y} \cdot F_y + M_{0z} \cdot F_z$$

$$M_{01} \cdot \cos(\widehat{\vec{M}_{01}, \vec{F}_0}) = M_0 \cdot \cos(\widehat{\vec{M}_0, \vec{F}_0}).$$

$$\frac{M_{0x} - (y \cdot F_z - z \cdot F_y)}{F_x} = \frac{M_{0y} - (z \cdot F_x - x \cdot F_z)}{F_y} = \frac{M_{Oz} - (x \cdot F_y - y \cdot F_x)}{F_z}$$

- Agar bosh vektor va bosh moment o'zaro kollinear bo'lsa, u holda ularni *dinamik vint* yoki *dinama* deb ataladi.



Teorema. Agar berilgan kuchlar sistemasi uchun statikaning ikkinchi invarianti noldan farqli bo'lsa, u holda kuchlar sistemasini dinamik vintga keltirish mumkin.

1. Berilgan kuchni parallel ko'chirish haqidagi lemmani qanday tushunasiz?
2. Statikaning asosiy teoremasi qanday hollarda qo'llaniladi?
3. Kuchlarning bosh vektorining moduli va yo'nalishi qanday aniqlanadi?
4. Kuchlarning bosh momentining moduli va yo'nalishi qanday aniqlanadi?
5. Parallel kuchlar sistemasining muvozanat tenglamalari bilan kesishuvchi kuchlar sistemasining muvozanat tenglamalari orasida qanday o'xshashlik va farqlar bor?
6. Ixtiyoriy yo'nalgan kuchlar sistemasining muvozanat tenglamalari ko'rinishini o'zgartirish mumkinmi?
7. Statikaning invariantlarini qanday tushunasiz?

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E'TIBORINGIZ UCHUN RAHMAT!

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