

Investigation of hydraulic resistance of pulsating flows of viscous fluid in elastic pipe

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Abstract. In recent years, flexible pipes made of synthetic polymer materials are being rapidly introduced into practice. Pulsating currents are important in driving water and other liquids in main pipes. This paper examines the pulsating currents of a viscous incompressible fluid in an elastic pipe. The necessary hydrodynamic parameters will be determined by solving the problem, such as distributions of pressure, velocity, flow rate, propagation velocity of the pressure pulse wave, and their attenuation. For the first time in this article, the decrease in hydraulic resistance in a pulsating flow through pipes due to the elasticity of the wall will be determined. The dependence of the dimensionless value of the pressure pulse wave on the vibrational number α is studied. The pulse wave speed is compared with the speed of Moens-Korteweg, c_∞ and significant differences between them are found. The dependence of the reciprocal value of attenuation, related to the wavelength, on the vibrational number α is also studied; it is shown that the attenuation is free at lower values of the Womersley vibrational parameter, practically equals zero, and at large values of which it asymptotically approaches unity [1-9].

1 Introduction

Recently, the intensive introduction into practice of flexible pipelines made of polymeric synthetic materials is of great importance [1-3], pulsating fluid flows in elastic pipes. In this regard, studies of pulsating flows of a viscous fluid in pipelines, taking into account the elastic properties of the wall, have become relevant. Also in this area, pulsating fluid flows in pipes are of no small importance, taking into account the various mechanical properties of the wall [12–20]. Proceeding from this consideration, in this article, pulsating flows of a viscous fluid in an elastic pipe will be investigated. By solving the problem, the necessary hydrodynamic parameters will be determined, such as distributions of pressure, velocity, flow rate, propagation velocity of the pressure pulse wave and their attenuation. For the first time in this article, the decrease in hydraulic resistance in a pulsating flow through pipes due to the elasticity of the wall will be determined.

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2 Methods

Let us formulate a simplified problem, which is of no small importance in studying the pulsating flow of a viscous fluid in pipes with elastic walls [2–17]. To do this, consider that the relative amplitude of the wall deformation to the radius is too small compared to unity, and so on $\frac{\Delta R}{R} \ll 1$. And also, the flow of liquid occurs in a long pipeline, so that

$\varepsilon = \frac{R}{L} \ll 1$. Then, neglecting small quantities from the system of equations for the flow of a viscous fluid, we have

$$\begin{cases} \frac{\partial v_x}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 v_x}{\partial r^2} + \frac{1}{r} \frac{\partial v_x}{\partial r} \right), \\ \frac{\partial p}{\partial r} = 0, \quad \frac{\partial v_x}{\partial x} + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} = 0. \end{cases} \quad (1)$$

To deform the pipeline wall based on the accepted assumption of small wall deformations, it is sufficient to use the Lightfoot equation [1-3].

$$\rho_\omega h \frac{\partial^2 u_r}{\partial t^2} = (p - p_c) - \frac{E h u_r}{R^2 (1 - \nu_1^2)}, \quad (2)$$

where u_r is the ratio of radial deformation ΔR to the radius of the pipe to rest; p_c is ambient pressure; ρ_ω is pipe wall density; h is wall thickness; E is modulus of elasticity; R is radius of the middle surface of the pipe wall; ν_1 is Poisson's ratio.

The left side of the equation expresses the inertia of the pipe wall; however, they are negligible, so we neglect them. Then (2) has the form

$$p - p_c = \frac{E h u_r}{R^2 (1 - \nu_1^2)}. \quad (3)$$

Note that fluid adhesion and pipe wall permeability are determined by the boundary conditions for the velocity components:

$$v_x = 0, \quad v_r = \frac{\partial u_r}{\partial t} \text{ at } r = R. \quad (4)$$

If the wall deformation is small, then we can assume that

$$v_r \Big|_{r=R+u} = v_r \Big|_{r=R} \quad (5)$$

Differentiating equations (3) concerning the variable t , taking into account (5), we write

$$\frac{\partial \bar{p}}{\partial t} = \frac{E h \mathcal{G}_{r=R}}{R^2 (1 - \nu_1^2)} \quad (6)$$

where $\bar{p} = p - p_c$.

By integrating the continuity equation from 0 to R we find

$$\frac{\partial \bar{V}_x}{\partial x} = -\frac{2}{R} g_{r=R} \quad (7)$$

where \bar{V}_x is the average flow velocity.

Then the relationship between pressure and average velocity is described by the equation

$$\frac{\partial \bar{p}}{\partial t} = -\frac{\bar{E}h}{2R} \frac{\partial \bar{V}_x}{\partial x}$$

or

$$\frac{\partial \bar{P}}{\partial t} = -\frac{\bar{E}h}{2R} \frac{\partial \bar{V}_x}{\partial x}, \quad (8)$$

where $\bar{p} = p - p_c$, $\bar{E} = \frac{E}{1 - \nu_1^2}$.

Thus, the simplified system of equations for the motion of a viscous fluid in pipes with elastic walls will take the final form:

$$\begin{cases} \frac{\partial V_x}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 V_x}{\partial r^2} + \frac{1}{r} \frac{\partial V_x}{\partial r} \right), \\ \frac{\partial p}{\partial r} = 0, \quad \frac{\partial V_x}{\partial x} + \frac{\partial V_r}{\partial r} + \frac{V_r}{r} = 0, \\ \frac{\partial \bar{p}}{\partial t} = -\frac{\bar{E}h}{2R} \frac{\partial \bar{V}_x}{\partial x}. \end{cases} \quad (9)$$

To solve a simplified problem, under the conditions that in the initial and final sections of the pipe, the fluid pressure is given in a complex form, as is done in the previous paragraph, which corresponds to the case under consideration, and so on

$$\begin{aligned} p &= \sum_{n=1}^N p_{n0} \exp(in\omega t) \quad \text{at } x=0, \\ p &= \sum_{n=1}^N p_{nL} \exp(in\omega t) \quad \text{at } x=L. \end{aligned} \quad (10)$$

Here p_{n0} and p_{nL} are oscillation amplitudes; ω is circular frequency of oscillations; n is harmonic number.

We seek the solution to the system of equations (10) in the form

$$V_x(x, r, t) = \bar{V}_x(r) e^{in\omega t}, \quad p_x(x, t) = \bar{p}(x) e^{in\omega t}.$$

Then the system of equations takes the following form:

$$\frac{\partial^2 \tilde{V}_x}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{V}_x}{\partial r} - \frac{i n \omega}{\nu} \tilde{V}_x = \frac{1}{\rho \nu} \frac{\partial \tilde{p}}{\partial x}, \quad (11)$$

$$\frac{\partial \tilde{p}}{\partial r} = 0, \quad \frac{\partial \tilde{V}_x}{\partial x} + \frac{\partial \tilde{V}_r}{\partial r} + \frac{\tilde{V}_r}{r} = 0, \quad (12)$$

$$i n \omega \frac{1}{a} \bar{p} = -\frac{\partial \bar{V}_x}{\partial x}, \quad (13)$$

where $a = \frac{\bar{E}h}{2R}$,

Solution of the system of equations (11) taking into account the boundary conditions (10) written in the form

$$\bar{V}(x, r) = \frac{1}{\rho(i n \omega)} \left(-\frac{\partial \bar{p}}{\partial x} \right) \left(1 - \frac{I_0 \left(\sqrt{\frac{i n \omega}{\nu}} r \right)}{I_0 \left(\sqrt{\frac{i n \omega}{\nu}} R \right)} \right) \quad (14)$$

Multiplying both parts of formula (14) by $2r/R^2$ and integrating from 0 to R we obtain

$$\bar{V}(x) = \frac{1}{i n \omega \rho} \left(-\frac{\partial \bar{p}}{\partial x} \right) \left(1 - \frac{2J_1 \left(i^{3/2} \alpha_n \right)}{i^{3/2} \alpha_n J_0 \left(i^{3/2} \alpha_n \right)} \right) \quad (15)$$

Where $\alpha_n^2 = \frac{n \omega}{\nu} R^2$

Denote

$$z = \left[\frac{1}{i n \omega \rho} \left(1 - \frac{2J_1 \left(i^{3/2} \alpha_n \right)}{i^{3/2} \alpha_n J_0 \left(i^{3/2} \alpha_n \right)} \right) \right]^{-1} \quad (16)$$

Then

$$\frac{\partial \bar{p}}{\partial x} = -z \bar{V}_x(x) \quad (17)$$

Differentiating (17) concerning x and substituting its value from equation (13) into place $\frac{\partial \bar{V}_x(x)}{\partial x}$, we obtain equations for determining the pressure

$$\frac{\partial^2 \bar{p}}{\partial x^2} - \frac{i n \omega}{a} z \bar{p} = 0 \quad (18)$$

For this equation, the boundary conditions are

$$\begin{aligned} \bar{p} &= \sum_{n=1}^N \bar{p}_{n0} \quad \text{at } x=0, \\ \bar{p} &= \sum_{n=1}^N \bar{p}_{nL} \quad \text{at } x=L. \end{aligned} \quad (19)$$

The solution of equation (18), taking into account the boundary conditions (19), has the following form:

$$\bar{p}(x) = \sum_{n=1}^N \left[\bar{p}_{n0} \frac{\text{sh} \sqrt{\frac{i n \omega}{a}} z L \left(1 - \frac{x}{L}\right)}{\text{sh} \sqrt{\frac{i n \omega}{a}} z L} + \bar{p}_{nL} \frac{\text{sh} \sqrt{\frac{i n \omega}{a}} z L \frac{x}{L}}{\text{sh} \sqrt{\frac{i n \omega}{a}} L} \right], \quad (20)$$

$$\bar{V}_x(x) = \sum_{n=1}^N \left[p_{n0} \frac{\text{ch} \sqrt{\frac{i n \omega}{a}} z L \left(1 - \frac{x}{L}\right)}{\text{sh} \sqrt{\frac{i n \omega}{a}} z L} - \bar{p}_{nL} \frac{\text{sh} \sqrt{\frac{i n \omega}{a}} z L \frac{x}{L}}{\text{sh} \sqrt{\frac{i n \omega}{a}} z L} \right] \sqrt{\frac{i n \omega}{a z}}. \quad (21)$$

3 Results and Discussion

From the obtained formulas (20) and (21), it can be seen that the velocity and pressure essentially depend on the complex parameter $\sqrt{\frac{i n \omega}{a}} z L$. Therefore, the complex parameter will be denoted by $\bar{\chi} + \bar{\beta} i$, so on.

$$\sqrt{\frac{i n \omega}{a}} z L = \bar{\chi} + \bar{\beta} i \quad (22)$$

For simplicity, let's take $n = 1$. Then

$$\sqrt{\frac{i\omega}{a}} zL = \bar{\chi} + \bar{\beta}i \quad (23)$$

Here

$$a = \frac{\bar{E}h}{2R}, \quad \bar{E} = \frac{E}{(1+\nu_1^2)}, \quad \alpha = \sqrt{\frac{\omega}{\nu}} R, \quad z = \left[\frac{1}{i\omega\rho} \left(1 - \frac{2J_1(i^{3/2}\alpha)}{i^{3/2}\alpha J_0(i^{3/2}\alpha)} \right) \right]^{-1}.$$

Separating the real and imaginary parts of expression (23), we obtain

$$\bar{\chi} = \pm\omega\sqrt{\frac{\rho}{a}}L^4\sqrt{M_2^2 + N_2^2} \sin\frac{\varphi}{2}, \quad \bar{\beta} = \pm\omega\sqrt{\frac{\rho}{a}}L^4\sqrt{M_2^2 + N_2^2} \cos\frac{\varphi}{2},$$

where $\varphi = \text{arctg} \frac{N_2}{M_2}$.

Here $J_0(i^{3/2}\alpha) = \text{ber}_0\alpha + i\text{bei}_0\alpha$, $J_1(i^{3/2}\alpha) = \text{ber}_1\alpha + i\text{bei}_1\alpha$,

$$\frac{J_1(i^{3/2}\alpha)}{J_0(i^{3/2}\alpha)} = M_1 + N_1i$$

$$M_1 = \frac{\text{ber}_1\alpha\text{ber}_0\alpha + \text{bei}_1\alpha\text{bei}_0\alpha}{\text{ber}_0^2\alpha + \text{bei}_0^2\alpha}, \quad N_1 = \frac{-\text{ber}_1\alpha\text{bei}_0\alpha + \text{ber}_0\alpha\text{bei}_1\alpha}{\text{ber}_0^2\alpha + \text{bei}_0^2\alpha},$$

$$\frac{2}{i^{3/2}\alpha} = \frac{2}{i\sqrt{i}\alpha} = -\frac{\sqrt{2}}{\alpha}(1+i)$$

$$\begin{aligned}
& -\frac{\sqrt{2}}{\alpha}(1+i)(M_1+N_1i) = -\frac{\sqrt{2}}{\alpha}(M_1-N_1) - \frac{\sqrt{2}}{\alpha}(M_1+N_1)i \\
& \quad 1 + \frac{\sqrt{2}}{\alpha}(M_1-N_1) + \frac{\sqrt{2}}{\alpha}(M_1+N_1)i, \\
z = i\omega\rho & \left[1 - \frac{2J_1(i^{3/2}\alpha)}{i^{3/2}\alpha J_0(i^{3/2}\alpha)} \right]^{-1} = i\omega\rho \left[\frac{1}{\left(1 + \frac{\sqrt{2}}{\alpha}(M_1-N_1)\right) + \left(\frac{\sqrt{2}}{\alpha}(M_1+N_1)i\right)} \right] = \\
& = i\omega\rho \left[\frac{1 + \frac{\sqrt{2}}{\alpha}(M_1-N_1) - \frac{\sqrt{2}}{\alpha}(M_1+N_1)i}{\left(1 + \frac{\sqrt{2}}{\alpha}(M_1-N_1)\right)^2 + \left(\frac{\sqrt{2}}{\alpha}(M_1+N_1)\right)^2} \right], \\
M_2 & = \frac{1 + \frac{\sqrt{2}}{\alpha}(M_1-N_1)}{\left(1 + \frac{\sqrt{2}}{\alpha}(M_1-N_1)\right)^2 + \left(\frac{\sqrt{2}}{\alpha}(M_1+N_1)\right)^2}, \\
N_2 & = \frac{-\frac{\sqrt{2}}{\alpha}(M_1+N_1)}{\left(1 + \frac{\sqrt{2}}{\alpha}(M_1-N_1)\right)^2 + \left(\frac{\sqrt{2}}{\alpha}(M_1+N_1)\right)^2}.
\end{aligned}$$

Then

$$z = i\omega\rho(M_2 + N_2i),$$

$$\sqrt{\frac{i\omega z}{a}}L = i\omega\sqrt{\frac{\rho}{a}}L(M_2 + N_2i)^{\frac{1}{2}} = \pm i\omega\sqrt{\frac{\rho}{a}}L^{\frac{1}{2}}\sqrt{M_2^2 + N_2^2} \left(\cos\frac{\varphi}{2} + i\sin\frac{\varphi}{2} \right).$$

Here χ , as previously mentioned, is the coefficient that characterizes the attenuation of oscillations and $\frac{1}{\beta}$ is characterizes the dimensionless velocity of the pulse wave. If we

denote the speed of propagation of a pulse wave through C , then $\frac{c}{c_\infty} = \frac{\omega L}{c_\infty \beta}$, where

$c_\infty = \sqrt{\frac{Eh}{2\rho R}}$ is the Moens-Korteweg formula; E is modulus of elasticity; h is wall thickness; ρ is liquid density; R is pipe radius, L is pipe length.

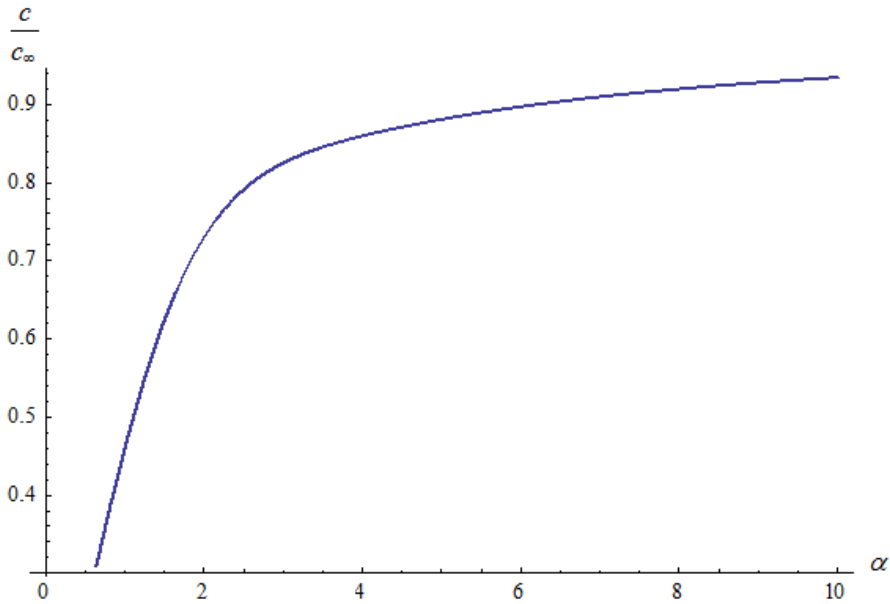


Fig.1. Dependence of the dimensionless value of the phase velocity on the vibrational number α

Based on the obtained formulas, we will analyze the pulse wave propagation velocity and wave attenuation depending on the oscillatory Reynolds number.

Based on the obtained formulas, we will analyze the pulse wave propagation velocity and wave attenuation depending on the oscillatory Reynolds number. On fig. 1. shows the dependence of the dimensionless value of the pressure pulse wave on the vibrational number α . It was revealed that the propagation velocity of a pressure pulse wave increases with an increase in the elasticity modulus of the surrounding tissue and an increase in the wavelength.

Here, too, the speed of the pulse wave is compared with the Moens-Korteweg speed, c_∞ and significant differences between them are revealed at lower values of the Womersley vibrational parameter, at large values of which no significant differences are observed.

On fig. 2. shows the dependence of the reciprocal of the attenuation, related to the wavelength, on the vibrational number α .

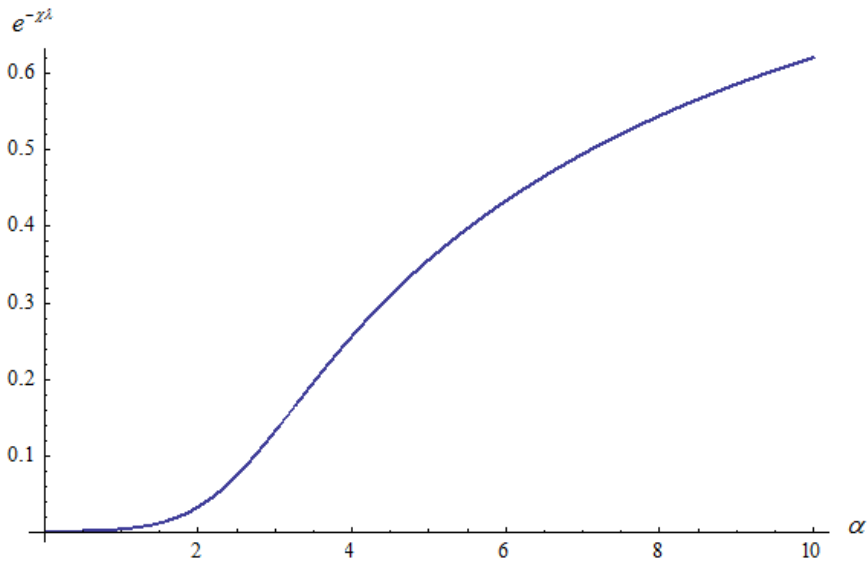


Fig. 2. Dependence of the reciprocal value of attenuation referred to wavelength from the vibrational number α .

The results show that the attenuation waves at lower values of the Womersley vibrational parameter is practically equal to zero, and at large values, it asymptotically approaches unity.

4 Conclusion

The dependence of the dimensionless value of the pressure pulse wave on the vibrational number is studied α . It was revealed that the propagation velocity of a pressure pulse wave increases with an increase in the elasticity modulus of the surrounding tissue and an increase in the wavelength. The speed of the pulse wave is compared with the speed of

Moens-Korteweg C_∞ , and revealed significant differences between them occur at lower values of the Womersley vibrational parameter, at higher values of which no significant differences are observed. The dependence of the reciprocal value of attenuation, related to the wavelength, on the vibrational number α is also investigated; it is shown that the attenuation of the wave at lower values of the Womersley vibrational parameter is practically equal to zero, and at its larger values, asymptotically approaches unity [3-12].

The presented simplified model is suitable for determining the propagation velocity of a pulse wave and pulse damping. However, it is unacceptable to determine the hydraulic resistance in an elastic pipe since, in this case, the impedance $\left(-\frac{\partial p}{\partial x}\right)/Q$ does not depend on the coefficient of elasticity of the wall.

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