

# Investigation of nonlinear oscillations of high-rise flue gas stacks under various kinematic effects

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**Abstract.** A detailed analysis of the current state of the problem is presented in the article. A mathematical model is given to assess the dynamic behavior of high-rise structures, considering geometric nonlinearity (nonlinear dependence of strains on displacements) under periodic kinematic effects and the own weight of the structure. The unsteady forced oscillations of the high-rise flue gas stack of the Novo-Angrenskaya HPP were studied considering the geometric dimensions of the stack and the physical and mechanical characteristics of the material of the structure. The influence of dissipation in the material, leading to rapid or gradual damping of vibrations depending on the frequency spectrum of natural vibrations in one direction or another, as well as geometric nonlinearity, leading to an increase in the amplitude of vibrations at a low energy dissipation in the structure, and resonant impact, causing significant displacements of the structure. The predominant effect of energy dissipation in the material on the pattern of the dynamic behavior of a high-rise structure is observed compared to geometric nonlinearity.

## 1 Introduction

High-rise flue gas stacks and cooling towers used for environmental protection are unique and important structures of industrial and energy complexes. Such structures are mainly located in seismically active regions of our country.

Lately, insufficient attention has been paid to studying their dynamic behavior and strength. Therefore, it is required to develop an effective calculation method and evaluate these structures' dynamic behavior and strength under various impacts, considering design features and nonlinear strain.

In the existing building codes of some countries, an elastic conical cantilever with a constant inclination is used as a calculation model for such structures. This model does not consider such features of structures as real geometry, design features, large strains, and nonlinear deformation of the material, which directly impact the value of dynamic behavior and strength of structures.

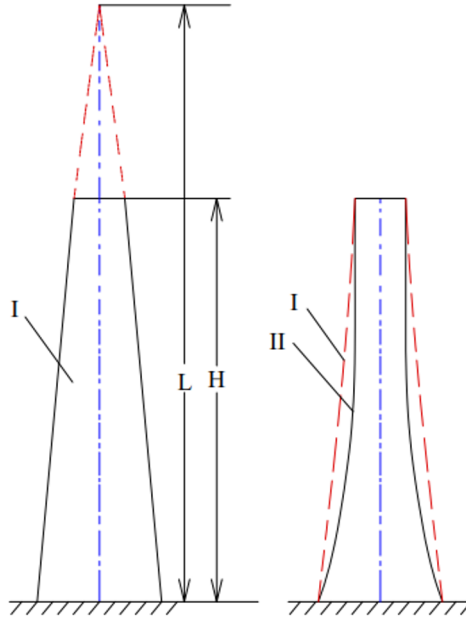
In designing and constructing high-rise structures such as reinforced concrete flue gas

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stacks, constructive solutions for structures with a variable inclination of the generatrix and a variable wall thickness are used.

Generally, dynamic calculations of high-rise flue gas stacks are conducted in accordance with building codes and regulations, where a conical cantilever beam with a constant generatrix inclination is used as a design scheme. If we analyze the actual dimensions of the structure and the dimensions of the accepted design scheme (Fig. 1), then we can see a large deviation of these dimensions from each other (Table 1). This leads to large errors in specific calculations.



**Fig.1.** Design scheme of high-rise flue gas stack: I is design scheme used; II is real geometric dimensions

**Table 1.** Difference between dimensions of real structures and design schemes

Stack elevation of Novo-Angrenskaya HPP	Outer diameter according to design scheme - I	Real outer diameter according to scheme - II	Difference in diameters
325	16.5	16.5	0
275	19.5	16.5	3
235	21.9	16.5	5.4
190	24.6	19.2	5.4
115	29.7	23.8	5.9
65	32.1	28.8	3.3
35	33.9	32.4	1.5
0	38	38	0

The question arises about the correct choice of the structure model, which allows for adequately reflecting the real geometry of the structure, considering the variability of the thickness of the annular cross-section and the inclination of the generatrix.

Along with this, materials and structures exhibit the properties of geometric and physical nonlinearity even under low impacts. Moreover, if the physically nonlinear

deformation of the material leads to a decrease in the amplitude of forced vibrations, then in the case of geometrically nonlinear deformation of the structure, its increase is possible, especially in the resonant mode of vibrations of the structure with low dissipation.

The nonlinear dynamics of an axisymmetric body subjected to impulsive loading were considered earlier in [1]. The structure's dynamic behavior was studied, taking into account the material's physical nonlinearity and dissipative properties. The article presents numerical results that show the nonlinear behavior of a body under the influence of an impulsive load. Mathematical models and calculation methods are given, based on the equations of motion of a rigid body, considering the material's physical nonlinearity. Using numerical methods, the oscillation characteristics and dynamic parameters of the body are obtained under various impulse loads.

Recently, several publications have been published where the stress-strain state and dynamic behavior of high-rise structures are considered.

These scientific articles are devoted to the following issues:

- modeling of spatial natural vibrations of axisymmetric systems, such as cylinders, cones, and spheres, is considered in [2]. A new modeling method is based on the theory of Legendre functions and spherical harmonics, which allows calculating the spectra of natural frequencies and vibration modes for axisymmetric systems with arbitrary boundary conditions.

- in [3], the use of dynamic dampers to reduce the vibrations of high-rise buildings was studied. The use of viscoelastic materials for the production of damping elements and the mathematical models used for their calculation are described. The simulation results showed that using dynamic dampers could significantly reduce the vibration amplitude of the building and increase its stability under conditions of strong external impacts, such as earthquakes [25].

- in [4], forced vibrations of axisymmetric bodies of a non-homogeneous structure are considered. A technique for modeling such bodies and analyzing their dynamic behavior in the presence of external influences are given. A mathematical model of an axisymmetric body is proposed, presented as a finite element model, considering the non-homogeneous properties of materials. Several numerical experiments demonstrated the proposed method's effectiveness for calculating forced vibrations of axisymmetric non-homogeneous systems.

- modeling of spatial natural oscillations of viscoelastic acyclic systems is considered in [5]. A mathematical model of the motion of such systems is proposed, and the results of the numerical simulation of oscillations using the finite element method are presented.

- in [6], the mathematical modeling of a rod protected by a system of dynamic shock absorbers under the action of kinematic excitations is considered. Magnetorheological shock absorbers are used as shock absorbers; they have variable stiffness and damping properties depending on the external magnetic field. A mathematical model of the rod and shock absorber system is presented, and its dynamic characteristics are studied under various kinematic excitations.

- in [7], a study of a nonlinear dynamic analytical model of high-rise buildings subject to seismic excitations is presented, considering geometric nonlinearity. The article describes a model that considers the deviation of the structure in a plane and the discrepancy between displacements and rotation angles between floors. A numerical study of the model is conducted on the example of a 20-story building; the acceleration, and deformation of the building are calculated at various levels of seismic excitations, and it is noted that an account for geometric nonlinearity significantly affects the dynamic characteristics of high-rise buildings.

- the study in [8] considers geometric nonlinearity in the dynamic analysis of tall buildings under the influence of wind loads. A nonlinear model is applied to analyze the dynamic characteristics of a 15-story building and reveal the significant influence of

geometric nonlinearity on the dynamic response of the building.

- in [9], a force analogy method is presented for nonlinear dynamic analysis of tall buildings under the influence of seismic excitations. The method is applied to analyzing a 76-story building, and its results are compared with those obtained using other methods. The study shows that the force analogy method allows for obtaining more accurate results in the nonlinear dynamic analysis of tall buildings.

- in [10], a method of nonlinear seismic analysis of high-rise buildings is presented, considering the soil-structures interaction and geometric nonlinearity. The behavior of a 30-story building under various seismic loads was studied, and the importance of considering soil conditions in the design of high-rise buildings in areas with high seismic activity was revealed.

- in [11], nonlinear seismic analysis of high-rise buildings was studied, considering both geometric and physical nonlinearities. A numerical model was developed to study the behavior of high-rise buildings under seismic loads, and the influence of various parameters such as height-to-width ratio, structural rigidity, and earthquake intensity was investigated.

- in [12], the influence of geometrical nonlinearity on the dynamic response of high-rise buildings during earthquakes is considered. A model of a building is used, considering nonlinear deformations, and a new method for calculating the dynamic characteristics of a building during an earthquake is proposed. A numerical study was carried out based on a building with 30 floors, analytical solutions were obtained, and a comparison of the results with other calculation methods was given. It is shown that geometric nonlinearity has an important influence on the dynamic response of a building during earthquakes, and the proposed method is effective in modeling such effects.

- the study in [13] investigates the effect of geometric nonlinearity on the seismic response of high-rise buildings subject to random ground vibrations. Numerical modeling was used, and dynamic analysis of buildings was conducted, considering geometric nonlinearity and random seismic excitations. The results show that geometric nonlinearity leads to higher dynamic compliance of buildings and increased vibration amplitude.

- in [14], applying a nonlinear analysis method for assessing the seismic behavior of tall buildings is considered, considering geometric and physical nonlinearity. The authors describe a model that shows the effects of nonlinearity in calculating the response of a building to seismic impact, such as the nonlinearity of the material and changes in the geometry of the structure under deformation. The results show that accounting for the nonlinearity gives more exact estimates of structure behavior under seismic impact.

- in [15], the nonlinear dynamic behavior of tall buildings under the action of seismic loads was investigated, considering the geometric and physical nonlinearities. Numerical modeling was conducted based on the finite element approach using the OpenSees software. As a result of the study, data were obtained on strains, stresses, accelerations, and force parameters of tall buildings under various impacts. The analysis showed a significant influence of geometric and physical nonlinearity on the dynamic response of high-rise buildings, which is important in the design and construction of such objects [24].

- in [16], methods of nonlinear analysis of high-rise buildings were considered, considering the geometric and physical nonlinearity under the action of seismic loads. The study applied two methods of nonlinear analysis: the finite element method and the pseudo-dynamic testing method. The study showed that accounting for nonlinearity allows for obtaining more accurate results than using linear analysis methods.

- in [17], the influence of axial deformations on the dynamic response of high-rise buildings is studied, considering geometric nonlinearity. The authors used the finite element method and conducted a nonlinear analysis of several models of buildings of various configurations and heights. The study results showed that axial deformations affect the dynamic response of buildings, and an account for geometric nonlinearity is important in

assessing these effects.

- the authors of [18] investigate the nonlinear dynamic response of various configurations of high-rise buildings to seismic loads. Two types of buildings are considered: rectangular and L-shaped ones. Using the finite element method and ANSYS software, analysis was conducted for various parameters, including the weight of the building, its rigidity, height, and the distance between floors. The results show that strains and stresses in a building increase with an increase in the building's weight, rigidity, and height and a decrease in the distance between floors.

- in [21], [23], the influence of vertical loads on the nonlinear dynamic response of high-rise buildings is considered, considering the geometric nonlinearity. The study uses the ANSYS software based on the finite element method. The article presents the results of numerical modeling of various types of buildings, showing that vertical loads significantly impact the dynamic response of buildings, and an account for geometric nonlinearity is important for the reliable prediction of the behavior of buildings under seismic loads.

- in [20], the nonlinear seismic behavior of steel moment frames with beam-to-column connections is considered. Numerical modeling of various types of beam and column connections is conducted, and their seismic stability is evaluated. The study results show that the behavior of frames depends on the type of connection and that the correct choice of connection can significantly increase their seismic resistance. Particular attention is paid to the effect of plastic deformations, which can occur during strong earthquakes and lead to structure failure.

- in [21], the nonlinear seismic behavior of tall buildings is studied, considering the geometric nonlinearity. Numerical simulations were performed to evaluate the influence of various parameters, such as column rigidity, on the dynamic response of buildings. The results showed that geometric nonlinearity could lead to a significant change in the dynamic response of the building, especially in buildings with high column rigidity.

The above is a review of several scientific articles published over the past 15-20 years, which shows that each approach, when solving specific problems, has its advantages and disadvantages and is used in solving specific practical tasks.

To meet the increasing requirements for the operating conditions of high-rise structures (stacks, cooling towers, etc.) located in areas of high seismic activity, it is necessary to consider the different natures of the nonlinearity and energy dissipation in the material and structure.

Based on this analysis, it can be noted that today the development of mathematical models, solution methods, and the study of the dynamics of high-rise structures is a relevant and necessary problem worldwide that needs to be solved.

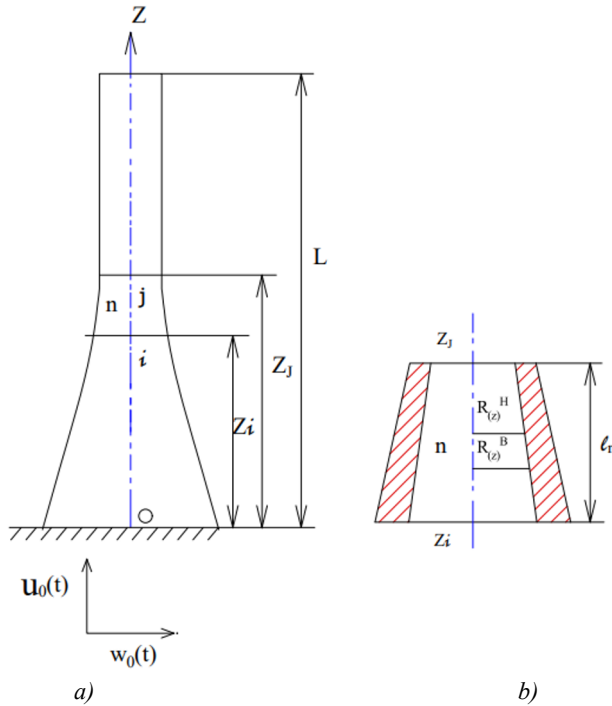
## **2 Methods**

### **2.1 Mathematical model**

A high-rise structure under consideration is modeled by a rod with a straight axis, variable inclination, and wall thickness (Fig. 2), which oscillates due to the dynamic impact of different frequencies. It is assumed that the cross-sections under vibrations remain plane and perpendicular to the axis of the rod (structure). It is also assumed that a longitudinal force acts in the sections of the rod, which is the weight of the part of the structure located above.

Considering the problem of vibrations of a high-rise structure, it is necessary to consider the nonlinear properties of the structure's material, its design features (variable cross-section and inclination), non-uniform axial load, dynamic impact of the different frequency

spectrums, etc. Here we consider the nonlinearity related to large displacements of the structure and a significant change in its geometry, i.e., geometric nonlinearity. In this case, the deformations include the first derivative of the displacement and the second derivative  $\varepsilon = \varepsilon(u', u'^2)$ .



**Fig.2.** Calculation model of high-rise structure (a) with variable inclination and wall thickness (b)

Accounting for any type of nonlinearity complicates the exact solution to the problem for a rod structure, especially with a variable cross-section. Known analytical solutions concern mainly a rod of constant cross-section. Given the complexity of the problem posed, a numerical solution method is used - the finite element method (FEM), based on the variational minimum principle of the total energy.

To simulate the straining process in high-rise structures under various dynamic influences, taking into account their real geometry and geometric nonlinearity, we take the calculation scheme shown in Fig.2. The lower part of the structure ( $z = 0$ ) rests on a rigid foundation and oscillates under the action of a kinematic impact  $u(0, t) = u_0(t)$ ;  $w(0, t) = w_0(t)$ . The task is to determine the fields of displacements and internal force factors that arise at various structure points under kinematic effects.

To describe the dynamic process occurring in the structure, the following functional is used:

$$L = \int_0^t (T - U - \Pi) dt \tag{1}$$

and corresponding kinematic conditions

$$z = 0 : u(0, t) = u_0(t); w(0, t) = w_0(t) \quad (2)$$

Here:  $T$  represents the kinetic energy of the rod model of the structure,  $U$  is the potential energy;  $\Pi$  is the potential of external forces: of axial weight load -  $Q(z)$  on the longitudinal displacement of section "z" and of horizontal inertial load resulting from kinematic effect with acceleration  $\ddot{w}_0(t)$  at the base of the stack.

Let the cantilever rod under consideration, fixed in the base, perform bending vibrations in the direction of the least rigidity, determined by the horizontal deflection -  $w$  and longitudinal displacements -  $u$ . Then the potential energy of deformation under bending of the rod  $U$ , the kinetic energy  $T$ , the potential of gravity forces (axial compressive force  $Q(z)$  - the weight of the part of the structure located above), and inertial forces from the kinematic effect at the base  $\Pi$ , related to the deflection  $w$  and longitudinal displacements  $u$  of the rod are defined in the following way:

$$U = \frac{1}{2} \int_0^L EF(z) \left( \frac{\partial u}{\partial z} \right)^2 dz + \frac{1}{2} \int_0^L EJ(z) \left( \frac{\partial^2 w}{\partial z^2} \right)^2 dz$$

$$T = \frac{1}{2} \int_0^L \rho F(z) \left[ \left( \frac{\partial u}{\partial t} \right)^2 + \left( \frac{\partial w}{\partial t} \right)^2 \right] dz \quad (3)$$

$$\Pi = - \int_0^L \rho g(L-z)u dz + \int_0^L \rho \ddot{w}_0(t)w dz$$

Here  $u$ ,  $w$  are longitudinal and transverse displacements of the rod section;  $z$  is the coordinate of the section,  $J(z)$  is the moment of inertia of the annular section;  $F(z)$  is the cross-sectional area,  $L$  is the structure's height.

The terms in the last expression represent the potential of gravity and inertial forces from the kinematic effect, respectively.

To obtain the equations for the rod oscillations, we use the Lagrange equations with generalized coordinates  $q_i$  in the longitudinal and transverse directions:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial}{\partial q_i} (T - U) = Q_i, \quad (i = 1, 2) \quad (4)$$

Substituting expressions for potential and kinetic energies (3) and generalized external axial and transverse inertial forces  $Q_i$  into (4) leads to two independent equations representing the equations of forced longitudinal and transverse vibrations of the rod under axial and horizontal loading

$$- \frac{\partial}{\partial z} \left( EF(z) \frac{\partial u}{\partial z} \right) + \rho F(z) \frac{\partial^2 u}{\partial t^2} = Q_1(z, t)$$

$$\frac{\partial^2}{\partial z^2} \left( EJ(z) \frac{\partial^2 w}{\partial z^2} \right) + \rho F(z) \frac{\partial^2 w}{\partial t^2} = Q_2(z, t) \quad (5)$$

When solving the coupled system (5), it is necessary to set three boundary conditions at each end of the rod: one for the first equation (longitudinal vibrations) and two for the second (bending vibrations):

Rigid fixing at the base -

$$z=0: u=0; w = 0; \frac{\partial w}{\partial s} = 0;$$

free upper end -  $z=L$ :

$$EF\left(\frac{\partial u}{\partial s}\right) = 0; \frac{\partial}{\partial z} EJ\left(\frac{\partial^2 w}{\partial z^2}\right) = 0; EJ\left(\frac{\partial^2 w}{\partial z^2}\right) = 0 \tag{6}$$

Thus, the problem of forced unsteady longitudinal-transverse oscillations of a rod (structure) is reduced to solving a system of equations (5) with boundary conditions at the ends of the rod (6) and initial conditions

$$t=0: u=0, w=w_0, \dot{w} = \dot{w}_0 \tag{7}$$

The solution to equations (5) under boundary and initial conditions (6) and (7) can be simple when a homogeneous beam with a constant inclination and thickness is considered within the framework of a linear formulation.

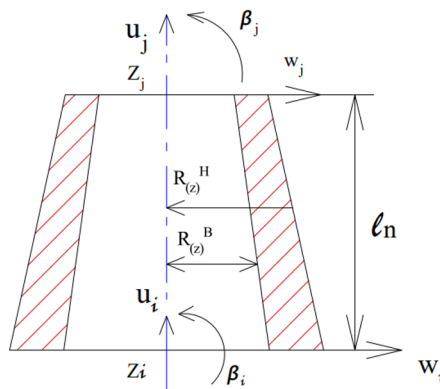
When considering a beam of variable thickness, considering various kinds of nonlinearity, a numerical method should be used.

Recently, the FEM has been widely used to solve variational problems in the mechanics of a deformable rigid body. It is characterized by a wide range of applicability, invariance with respect to the structure's geometry and materials' mechanical characteristics.

The general scheme of the FEM consists in discretizing the model (a rod) under study by finite elements connected at a finite number of nodal points, the displacements of which satisfy the condition of the minimum of the total energy functional.

### 2.2 Solution method

To solve the variational problem (1) under kinematic conditions (2), we use the FEM with a discretization of the computational domain (Fig. 2) by finite elements in the form of a truncated cone of variable thickness (Fig. 3).



**Fig. 3.** Rod element with positive direction of longitudinal ( $u_i, u_j$ ), bending ( $w_i, w_j$ ), and angular ( $\beta_i, \beta_j$ ) displacements.



The rod is approximated by a set of  $N$  rod elements of finite dimensions (Fig. 3), each of which works in compression and bending. At the junctions of the elements, generalized displacements are introduced, determined as a result of minimizing the functional (1) according to the debugged FEM algorithm. Based on the obtained displacements of the nodal points, the displacements of any point inside the element, its strain, and stresses are approximated. In this case, for each  $n$ -th element, changes in the cross-sectional area and moment of inertia are determined by the following formulas:

$$F^n(z) = \pi \left[ (R^H(z))^2 - (R^B(z))^2 \right], \quad J^n(z) = \frac{\pi}{4} \left[ (R^H(z))^4 - (R^B(z))^4 \right] \quad (8)$$

where  $R^H(z) = R_i^H - i^H z$ ,  $R^B(z) = R_i^B - i^B z$  are the outer and inner radii of the annular section, respectively;  $i^H$ ,  $i^B$  are the outer and inner inclinations of the element wall, respectively.

Finite-element discretization of the considered problem of joint longitudinal-transverse oscillations of an elastic rod is performed by rod elements (Fig. 4) with linear  $u_i$ ,  $u_j$ ,

$w_i$ ,  $w_j$  and angular  $\beta_i = \frac{\partial w_i}{\partial z}$ ,  $\beta_j = \frac{\partial w_j}{\partial z}$  displacements.

Further use of the FEM procedure reduces the considered variational problem (1) and (2) to a system of resolving differential equations

$$[M] \{\ddot{q}\} + [K] \{q\} = \{Q(t)\} \quad (9)$$

where  $[M]$  and  $[K]$  are the general matrices of mass and stiffness of the structure, formed from the matrices of mass and stiffness of individual elements (Fig. 3),  $\{q\}$  is the total vector of nodal displacements of structures, and  $\{Q(t)\}$  is the total nodal load acting on the structure, formed by the loads acting on individual elements.

The stiffness matrix of an individual element  $[K^e]$  that characterizes the relationship between internal nodal forces and nodal displacements is determined by the general formula

$$[K^n] = \int_{z_i, R^H(z)}^{z_j, R^B(z)} [D]^T [E] dz \quad (10)$$

Here, rectangular matrices  $[D]$  and  $[E]$  connect the displacements of the element nodes with strain

$$\{\varepsilon\} = [D] \{q\} \quad (11)$$

and stresses (internal forces) with strain, according to Hooke's law

$$\{\sigma\} = [E] \{\varepsilon\} \quad (12)$$

Below, depending on the consideration of one or another type of nonlinearity, the matrices  $[D]$  and  $[E]$  are supplemented with nonlinear components.

Taking into account the disconnection of the longitudinal and transverse strains of the

rod element, the general matrices of the stiffness and masses of the element can be represented as

$$[K^n] = \begin{bmatrix} K_{lon} & 0 \\ 0 & K_{tr} \end{bmatrix}, [M^n] = \begin{bmatrix} M_{lon} & 0 \\ 0 & M_{tr} \end{bmatrix} \quad (13)$$

Here, the submatrices  $[M_{lon}]$ ,  $[K_{lon}]$  are the matrices of the masses and stiffness of the rod element under conditions of longitudinal strain, and  $[M_{tr}]$ ,  $[K_{tr}]$  - under bending.

The stiffness and mass matrices of the rod under tension-compression conditions with longitudinal nodal displacements at the ends  $u_i$  and  $u_j$  have the order (2x2) and are obtained as a result of a linear approximation of displacements in the element

$$u = a_1 + a_2 z \quad (14)$$

The law of change in the deflection of the rod element is taken as a cubic approximation representing the integral of the differential equation for the bending of a beam loaded in nodal sections ( $EIw''(z)=0$ ):

$$w = a_3 + a_4 z + a_5 z^2 + a_6 z^3 \quad (15)$$

Matrices of bending stiffness and masses of a rod with nodal displacements at the ends  $w_i, w_j$  and rotations at the ends  $\beta_i, \beta_j$  are of the fourth order.

The nodal load in section  $z$  under the own weight of the part of the stack located above the section, entering the right side of the first equation (4), increases linearly toward the base of the stack and is determined by the following formula

$$Q_l = \rho g(L-z) \quad (16)$$

The nodal inertial load over the entire height of the structure, entering the right side of the second equation (5), is determined by the base's acceleration and the structure's mass attributable to this node.

When solving the problem of unsteady forced vibrations of a rod (a structure), in addition to the boundary conditions, the initial conditions are also considered. The presented matrix system of differential equations (9), where the unknowns are the displacements of the connecting nodes of the rod elements, is solved by the Newmark step-by-step method [24]. This method is based on expansions of unknown node displacements  $q(t_i+\tau)$  and their derivatives  $\dot{q}(t_i+\tau)$  into  $\tau$  power series (an integration step):

$$q(t_i+\tau) = q_i + \tau \dot{q}_i + \frac{\tau^2}{2} \ddot{q}_i + \alpha \tau^3 \ddot{\ddot{q}}_i \quad (17)$$

$$\dot{q}(t_i+\tau) = \dot{q}_i + \tau \ddot{q}_i + \beta \tau^2 \ddot{\ddot{q}}_i$$

where  $\alpha$  and  $\beta$  are chosen under condition  $\beta \geq 0,5$ ;  $\alpha \geq 0,25(\beta+0,5)^2$ , which ensures the unconditional convergence of the integration process.

Application of this method allows at each  $(i+1)$ -th time step to obtain the following algebraic system of equations for nodal displacements  $q_{i+1}$

$$[A]\{q_{i+1}\}=\{P_{i+1}\} \quad (18)$$

where

$$[A]=[K]+[C]\beta/(\alpha\tau)+[M]/(\alpha\tau^2) \quad (19)$$

$$\{P_{i+1}\}=\{R_{i+1}\}+[M]\left[\frac{\{q_i\}}{\alpha\tau^2}+\frac{\{\dot{q}_i\}}{\alpha\tau}+\left(\frac{1}{2\alpha}-1\right)\{\ddot{q}_i\}\right]+[C]\left[\frac{\beta\{q_i\}}{\alpha\tau}+\left(\frac{\beta}{\alpha}-1\right)\{\dot{q}_i\}+\frac{\tau}{2}\left(\frac{\beta}{\alpha}-2\right)\{\ddot{q}_i\}\right]\{q_i\} \quad (20)$$

where  $\{\dot{q}_i\}$ ,  $\{\ddot{q}_i\}$  are the displacements, velocities, and accelerations of the nodal points obtained at the previous  $i$ -th step.

System (18) with subsequent formulas (19) - (20) is given in the general form and contains the dissipation matrix  $[C]$ . In this paper, where the dynamics of an elastic model of a structure without dissipation is considered,  $[C]=0$  is assumed.

The procedure for generating global stiffness and mass matrices for the entire rod, and nodal inertial and weight load vectors, is performed automatically in the developed program. When forming the stiffness matrix, boundary conditions (6) at the ends of the column are considered.

Using the described methodology and algorithm, a computer program was compiled to evaluate the dynamic behavior of real high-rise structures under various dynamic effects, considering their real geometry and nonlinear strain.

In a geometrically nonlinear formulation (nonlinear dependence of strains on displacements), unsteady forced oscillations of the Novo-Angrenskaya HPP high-rise flue gas stack were studied under horizontal harmonic acceleration of the base during 8 s, changing according to the following law

$$\ddot{w}_0 = \begin{cases} \sin(2\pi\alpha t), & \text{at } t \leq 8\text{sec} \\ 0, & \text{at } t > 8\text{sec} \end{cases} \quad (21)$$

where  $\omega$  is the frequency of external influences applied to the structure's base. When solving this problem, the own weight of the structure is also taken into account.

To obtain the resolving equations of motion, we use the dependences of strains on displacements and maintain nonlinear terms.

Then, in the case of longitudinal deformation of the rod, taking into account the geometric nonlinearity, we have

$$\sigma = EF(z)\left(1 + \frac{1}{2} \frac{\partial u}{\partial z}\right) \frac{\partial u}{\partial z} \quad (22)$$

The dependence of the bending moment ( $M$ ) on the curvature  $\left(\frac{1}{\rho} = \frac{\partial^2 w}{\partial z^2}\right)$ , taking into account the geometric nonlinearity under bending of the rod, can be represented by the differential expression

$$M_{cur} = EJ(z)\left(1 + \frac{1}{2} \frac{\partial^2 w}{\partial z^2}\right) \frac{\partial^2 w}{\partial z^2} \quad (23)$$

Then dependence (11) ceases to be linear

$$\{\varepsilon\} = [D_q(q)]\{q\} \quad (24)$$

where the matrix components  $[D_q(q)]$  are power functions of the nodal displacement vector components, and its elements are determined by the following formula

$$d_{i,j}(q) = d_{ij} + \frac{\partial d_{ij}}{\partial q_j} q_j, \quad i, j = 1, \dots, 6 \quad (25)$$

Here  $d_{ij}$  is an element of the matrix  $[D]$  in formula (11).

By substituting (25) into (24) and, then, into (10), we obtain the stiffness matrix of the element, taking into account the geometric nonlinearity, which is the sum of the linear part (presented above) and the nonlinear part, depending on the longitudinal deformation acquired at each step and the change in curvature radius of the axis of the rod

$$[K] = \begin{bmatrix} K_{lon} & 0 \\ 0 & K_{tr} \end{bmatrix} + \begin{bmatrix} K_{lon} \left( \frac{\partial u}{\partial z} \right) & 0 \\ 0 & K_{tr} \left( \frac{\partial^2 w}{\partial z^2} \right) \end{bmatrix} \quad (26)$$

The resolving system of nonlinear differential equations obtained in the course of finite element discretization has the following form

$$[M]\{\ddot{q}\} + [K]\{q\} = \{P(t)\} - [K_{non}(q)]\{q\} \quad (27)$$

with an additional allowance for viscosity, the system of differential equations (27) takes the following form

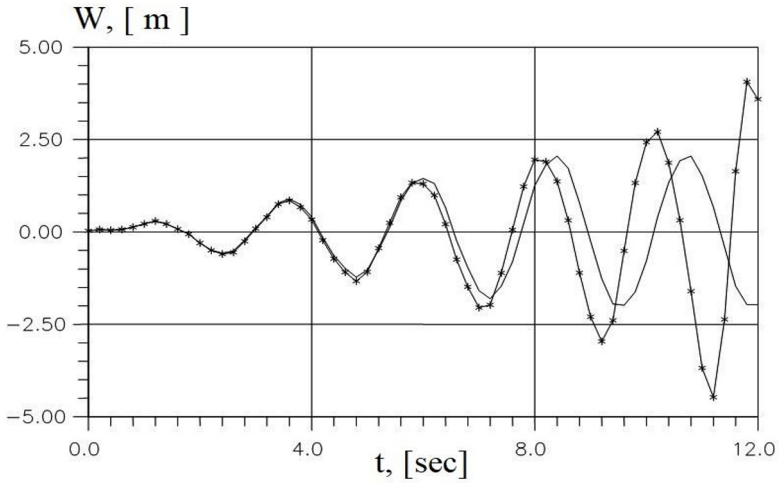
$$[M]\{\ddot{q}\} + \eta[K]\{\dot{q}\} + [K]\{q\} = \{P(t)\} - [K_{non}(q)]\{q\} \quad (28)$$

Both systems (27) and (28) are solved by an iterative method based on the Newmark method for solving systems of differential equations. In this case, the value of the right parts of systems (27) and (28) depends on the stress-strain state, expressed through the nodal displacements determined in the previous step.

### 3 Results and discussion

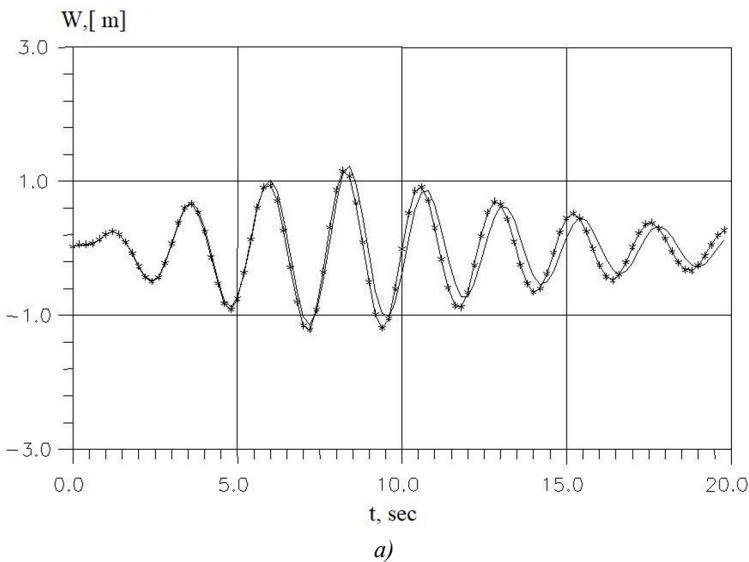
When solving the problem, zero initial conditions  $t = 0: \{q_0\} = 0; \{\dot{q}_0\} = 0$  and kinematic effect (21) with a duration of 8 seconds and  $\omega = 0.4$  Hz are used. The estimated time for the whole process is 20 sec. The system of nonlinear differential equations (27) and (28) is solved by the indicated iterative method. The calculations were made for the high-rise flue gas stack of the Novo-Angrenskaya HPP.

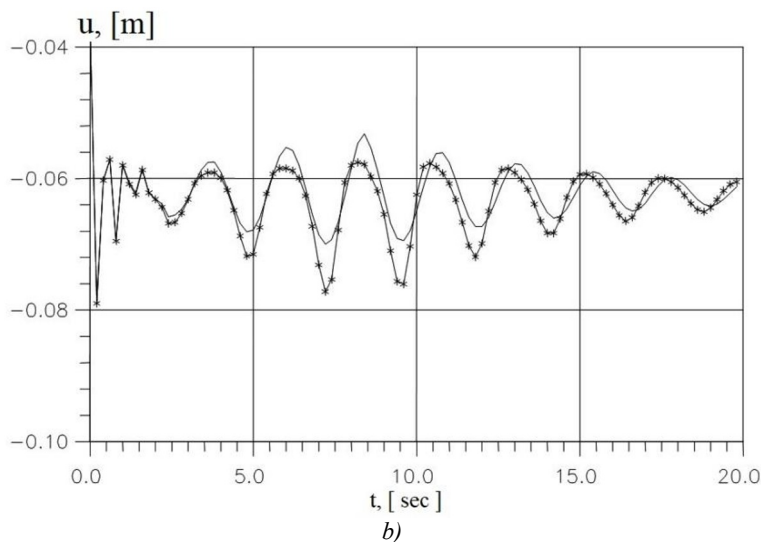
Figure 4 shows the transverse displacements of the top point of the stack under the harmonic impact specified above, obtained considering the geometric nonlinearity (line —\*—\*—). Here, for comparison, the displacements of the same point of a linear-elastic stack are given without considering the dissipation (———).



**Fig. 4.** Transverse displacements of top point of chimney obtained in geometrically nonlinear (—\*—) and linear elastic (—) formulations under horizontal kinematic effect (21)

A comparative analysis of the results obtained shows that an account for the geometric nonlinearity leads to a gradual increase in the amplitude of oscillations even after the cessation of the impact (from 8 to 12 sec in Fig. 4). The additional allowance for viscosity under the same dynamic action completely compensates for the unlimited increase in the amplitudes of a stack under geometrically nonlinear strain, bringing its behavior closer to that of an elastic stack with dissipative properties. This can be seen from the results presented in Fig. 5, where the thin line (—) shows the transverse (a) and longitudinal (b) displacements of the elastic stack, and the line with asterisks (—\*—) shows the corresponding displacements of the stack, taking into account geometrically nonlinear strain.





**Fig. 5.** Transverse (a) and longitudinal (b) displacements of top point of stack in linear-elastic (—) and geometrically nonlinear (—\*—\*) formulations, taking into account dissipative properties of material under kinematic effect (21)

The results presented in Fig. 4 are explained by a significant increase in the amplitude of stack displacements without dissipation under resonant action with the frequency of natural oscillations. The values of the amplitude and deformation of the stack at the time of termination of the kinematic load, in this case, large, therefore the right side of systems (27) or (28), containing the reactive loads from the nodal displacements obtained in the previous step ( $[K_{\text{non}}(q)]\{q\}$ ) when growing, increases the displacement at the next step of the iterative process even after the termination of the kinematic effect  $P(t)$ . The presence of dissipation (Fig. 5) reduces the absolute level of oscillations, and the nonlinear term has practically no effect on the magnitude of the displacements.

Thus, the presence of geometric nonlinearity in some frequency ranges (resonant ones) for a structure with low dissipative properties leads to a significant increase in the oscillation amplitudes. The presence of dissipation in the structure compensates for the increase in the amplitude of oscillations after the termination of the impact. It leads to the disappearance of high-frequency harmonics.

## 4 Conclusions

1. A detailed review of known publications related to the assessment of the dynamic behavior of various high-rise structures is given in the article.
2. A mathematical model of a high-rise structure was developed considering its design features, non-uniform axial load, dynamic impact of different frequency spectra, and geometric nonlinearity associated with large displacements of the structure and a significant change in its geometry.
3. A technique was developed for calculating a high-rise structure, considering dissipation and geometric nonlinearity.
4. The dynamic behavior of a high-rise structure (the flue gas stack of the Novo-Angrenskaya HPP) under kinematic impact at the base was studied.
5. The influence of dissipation in the material, which leads to rapid or gradual damping

of vibrations depending on the frequency spectrum of natural vibrations in one direction or another, was revealed;

6. Geometric nonlinearity, leading to an increase in the amplitude of oscillations with low energy dissipation in the structure and resonant action, causes significant displacements of the structure;

7. Non-stationary kinematic effects of different periods could cause resonant oscillations or a superposition of oscillations with different harmonics.

8. The predominant effect of energy dissipation in the material on the pattern of the dynamic behavior of a high-rise structure was determined in comparison with geometric nonlinearity.

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