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Nonlinear integro-differential equations of bending of physically nonlinear viscoelastic plates

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Abstract. An important feature of real materials is the nonlinear nature of the relationship between stress and strain. Taking into account such features of deformation of structural materials in the calculation allows us to bring theoretical forecasts closer to their real behavior. Physical nonlinearity is due to the calculation in the calculation of the nonlinear dependence between the components of the generalized stresses and strains characterizes the work of the structural material in the elastoplastic region. Many structural elements of modern technology are made in the form of plates of various shapes. The widespread use of plates is explained by their high strength, considerable rigidity and relatively low weight, which, along with the manufacturability of manufacturing, distinguishes them among other structural elements. A constant trend in the development of technology is an increase in the intensity of dynamic effects, accompanied by an increase in the level of vibration.

1. Introduction

Reduction in material consumption of structures while maintaining their strength and reliability is an important task. This problem can be solved by a more complete consideration of factors affecting the actual picture of strain: physical nonlinearity of the material, viscoelastic properties of the material, aging factors.

Physical nonlinearity is associated with the calculation of real properties of the materials. Recently, numerous studies have been published devoted to the stress-strain state of viscoelastic plates and shells.

However, the issues of considering the structural orthotropy of plates while taking into account physical nonlinearity of the material remain relevant; these issues are of particular interest in the problems of viscoelastic properties of the material [3, 4-9, 10, 11-20].

2. Statement of the problem. The basic equations.

Consider a flat plate of constant thickness under the action of a continuous transverse load *q (x, y, t).* Let *w (x, y, t)* be the deflection of the plate. The task is to derive equations to determine deflection *w (x, y, t)* assuming that the plate material follows the law of physically nonlinear viscoelasticity.

To derive the nonlinear integro-differential equation of bending of the physically nonlinear viscoelastic plate, we will proceed from the expressions for extension and shear, which gives the geometry of small strains of the plate elements. These expressions can be used in linear theory as well.

According to Kirchhoff's hypotheses, we accept

$$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2};\quad \varepsilon_y = -z\frac{\partial^2 w}{\partial y^2};\quad y_{xy} = -2z\frac{\partial^2 w}{\partial x\partial y};\quad \gamma_{xz} = \gamma_{yz} = 0. \tag{1}$$

According to [1], the relationships between stresses and strains can be represented as

$$\left.\begin{aligned}
&\sigma_x - \sigma_0\delta_y = 2G_0(1-\lambda_1 R^*)\left[1+\sum_{n=1}^{\infty}\gamma_{2n}\,(1-\lambda_1 R^*)^{2n}\psi_0^{2n}(x,y,t)\right](\varepsilon_x - \varepsilon_0);\\
&\sigma_x - \sigma_y = 2G_0(1-\lambda_1 R^*)\left[1+\sum_{n=1}^{\infty}\gamma_{2n}\,(1-\lambda_1 R^*)^{2n}\psi_0^{2n}(x,y,t)\right](\varepsilon_x - \varepsilon_y);\\
&\tau_{xy} = 2G_0(1-\lambda_1 R^*)\left[1+\sum_{n=1}^{\infty}\gamma_{2n}\,(1-\lambda_1 R^*)^{2n}\psi_0^{2n}(x,y,t)\right]\gamma_{xy}.
\end{aligned}\right\} \tag{2}$$

Following G. Cauderer [2], we accept

$$\varepsilon_z = -\frac{v_0}{1-v_0}z\nabla^2 w; \tag{3}$$

$$\varepsilon_0 = -\frac{1}{3}\left(\frac{1-2v_0}{1-v_0}\right)z\nabla^2 w; \tag{4}$$

$$\psi_0^2 = \frac{8}{9}\left[v_1\left(w_{xx}^2 + w_{yy}^2\right) + v_2 w_{xx}w_{yy} + 3w_{xy}^2\right]z^2 = \lambda(x,y,t)z^2, \tag{5}$$

where $v_0 -$ is the Poisson ratio,

$$v_1 = \frac{v_0}{(1-v_0)^2} + 1;\quad v_2 = \frac{2v_0}{(1-v_0)^2} - 1.$$

Note that dependences (3) and (4) take place in the plane-stressed state of the plate. Substituting these expressions into relations (2), for the stresses σ_x, σ_y and τ_{xy} , we obtain

$$\left.\begin{aligned}
&\sigma_x = -\frac{2G_0}{1-v_0}(1-\lambda_1 R^*)\left[1+\sum_{n=1}^{\infty}\gamma_{2n}\,(1-\lambda_1 R^*)^{2n}\lambda^{2n}z^{2n}\right]\left(\frac{\partial^2 w}{\partial x^2} + v_0\frac{\partial^2 w}{\partial y^2}\right)z;\\
&\sigma_y = -\frac{2G_0}{1-v_0}(1-\lambda_1 R^*)\left[1+\sum_{n=1}^{\infty}\gamma_{2n}\,(1-\lambda_1 R^*)^{2n}\lambda^{2n}z^{2n}\right]\left(\frac{\partial^2 w}{\partial y^2} + v_0\frac{\partial^2 w}{\partial x^2}\right)z;\\
&\tau_{xy} = 2G_0(1-\lambda_1 R^*)\left[1+\sum_{n=1}^{\infty}\gamma_{2n}\,(1-\lambda_1 R^*)^{2n}\lambda^{2n}z^{2n}\right]\frac{\partial^2 w}{\partial x\partial y}z.
\end{aligned}\right\} \tag{7}$$

3. Solution methods. Obtaining integro-differential equations of bending of a physically nonlinear viscoelastic plate.

In order to make equilibrium conditions for any element of the plate, introduce the moments reduced to a unit length

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_x z dz;\quad M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_y z dz;\quad M_{xy} = -M_{yx} = \int_{-\frac{h}{2}}^{\frac{h}{2}}\tau_{yx} z dz \tag{8}$$

and transverse forces

$$Q_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xz} dz; \qquad Q_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{yz} dz. \tag{9}$$

Then the equilibrium equations have the form

$$\left.\begin{aligned} &\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} = Q_x; \\ &\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} = Q_x; \\ &\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = -q(x,y,t). \end{aligned}\right\} \tag{10}$$

Excluding Q_x and Q_y from these equations, we obtain

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -q(x,y,t). \tag{11}$$

From equations (7) for bending and torsional moments we have

$$\begin{aligned} M_x &= -D_0(1-\lambda_1 R^*)\left(\frac{\partial^2 w}{\partial x^2} + v_0 \frac{\partial^2 w}{\partial y^2}\right) - 3D_0(1-\lambda_1 R^*) \times \\ &\times \left[\sum_{n=1}^{\infty} \frac{\gamma_{2n} h^{2n}}{2^{2n}(2n+3)} (1-\lambda_1 R^*)^{2n} \lambda^{2n}\right]\left(\frac{\partial^2 w}{\partial x^2} + v_0 \frac{\partial^2 w}{\partial y^2}\right); \\ M_y &= -D_0(1-\lambda_1 R^*)\left(\frac{\partial^2 w}{\partial y^2} + v_0 \frac{\partial^2 w}{\partial x^2}\right) - 3D_0(1-\lambda_1 R^*) \times \\ &\times \left[\sum_{n=1}^{\infty} \frac{\gamma_{2n} h^{2n}}{2^{2n}(2n+3)} (1-\lambda_1 R^*)^{2n} \lambda^{2n}\right]\left(\frac{\partial^2 w}{\partial y^2} + v_0 \frac{\partial^2 w}{\partial x^2}\right); \\ M_{xy} &= -D_0(1-v_0)(1-\lambda_1 R^*)\frac{\partial^2 w}{\partial x \partial y} - 3D_0(1-v_0)(1-\lambda_1 R^*) \times \\ &\times \left[\sum_{n=1}^{\infty} \frac{\gamma_{2n} h^{2n}}{2^{2n}(2n+3)} (1-\lambda_1 R^*)^{2n} \lambda^{2n}\right]\frac{\partial^2 w}{\partial x \partial y}, \end{aligned} \tag{12}$$

where $D_0 = \dfrac{G_0 h^3}{6(1-\nu_0)}$ – is the bending stiffness of plate,

$$\lambda^2(x,y,t) = \frac{8}{9}\left[v_1\left(w_{xx}^2 + w_{yy}^2\right) + v_2 w_{xx} w_{yy} + 3w_{xy}^2\right]. \tag{13}$$

Substituting (12) into equilibrium equation (11), we obtain the following integro-differential equation for the bending of a physically nonlinear viscoelastic plate

$$\begin{aligned} &D_0(1-\lambda_1 R^*)\nabla^4 w + 3D_0(1-\lambda_1 R^*)F(x.y.t)\nabla^4 w + 6D_0(1-\lambda_1 R^*) \times \\ &\times \left[F_x \frac{\partial}{\partial x}\nabla^2 w + +F_y \frac{\partial}{\partial y}\nabla^2 w + 0.5\left(F_{xx} + v_0 F_{yy}\right) w_{xx} + \right. \\ &\left. +0.5\left(F_{yy} + v_0 F_{xx}\right) w_{yy} + (1-v_0) F_{xy} w_{xy}\right] = q(x,y,t), \end{aligned} \tag{14}$$

$$F(x,y,t) = \sum_{n=1}^{\infty} \frac{\gamma_{2n} h^{2n}}{2^{2n}(2n+3)} (1-\lambda_1 R^*)^{2n} \lambda^{2n}(x,y,t). \tag{15}$$

At $F(x,y)$ =0, only the first term remains on the left-hand side of equation (14), and equation (14) coincides with the equation of bending of an elastic-viscous plate (Hooke's law).

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If instead of rectangular coordinates *x* and *y,* use the polar coordinates $x = r\cos\varphi\,, y = r\sin\varphi$ after converting the derivatives of *w* along the *x* and y coordinates into derivatives of *w* along the *r* and *φ* coordinates, for the strains $\varepsilon_r, \varepsilon_\varphi$ and $\gamma_{r\varphi}$ instead of formulas (1), we obtain the following

$$\begin{gathered}\varepsilon_r = -zw_{rr};\\ \varepsilon_\varphi = z\left(\frac{1}{r}w_r + \frac{1}{r^2}w_{\varphi\varphi}\right);\\ \gamma_{r\varphi} = -2z\left(\frac{1}{r}w_{r\varphi} + \frac{1}{r^2}w_\varphi\right),\end{gathered} \tag{16}$$

where the subscripts in the deflection *w* mean the corresponding derivatives along the coordinates *r* and *φ*. Introducing (16) into (7), for stresses in polar coordinates we have

$$\left.\begin{gathered}\sigma_r = -\frac{2G_0}{1-v_0}(1-\lambda_1 R^*)\left[1+\sum_{n=1}^{\infty}\gamma_{2n}(1-\lambda_1 R^*)^{2n}\lambda^{2n}z^{2n+1}\right]\times\\ \times\left(w_{rr} + v_0\left(\frac{1}{r}w_r + \frac{1}{r^2}w_{\varphi\varphi}\right)\right);\\ \sigma_\varphi = -\frac{2G_0}{1-v_0}(1-\lambda_1 R^*)\left[1+\sum_{n=1}^{\infty}\gamma_{2n}(1-\lambda_1 R^*)^{2n}\lambda^{2n}z^{2n+1}\right]\times\\ \times\left(\frac{1}{r}w_r + \frac{1}{r^2}w_{\varphi\varphi} + v_0 w_{rr}\right);\\ \tau_{r\varphi} = -2G_0(1-\lambda_1 R^*)\left[1+\sum_{n=1}^{\infty}\gamma_{2n}(1-\lambda_1 R^*)^{2n}\lambda^{2n}z^{2n+1}\right]\times\\ \times\left(\frac{1}{r}w_{r\varphi} - \frac{1}{r^2}w_\varphi\right).\end{gathered}\right\} \tag{17}$$

By entering the notation

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2},$$

The following is obtained for the average extension and the square of the shear strain intensity instead of (4) and (5):

$$\begin{gathered}\varepsilon_0 = -\frac{1}{3}\left(\frac{1-2v_0}{1-v_0}\right)z\nabla^2 w;\\ \psi_0^2 = \frac{8}{9}\left\{v_1\left[w_{rr}^2 + \frac{1}{r^2}\left(w_r + \frac{1}{r}w_{\varphi\varphi}\right)^2\right]+\right.\\ \left.+\frac{v_2}{r}w_{rr}\left(w_r + \frac{1}{r}w_{\varphi\varphi}\right) + \frac{3}{r^2}\left(w_{rr} + \frac{1}{r}w_\varphi\right)^2\right\}z^2.\end{gathered} \tag{18}$$

For bending and torsional moments we get

$$\left.\begin{aligned}
M_r &= -D_0(1-\lambda_1 R^*)\left[1+\sum_{n=1}^{\infty}\frac{\gamma_{2n}h^{2n}}{2^{2n}(2n+3)}(1-\lambda_1 R^*)^{2n}\lambda^{2n}(r,\varphi,t)\right]\times \\
&\times\left(w_{rr}+v_0\left(\frac{1}{r}w_r+\frac{1}{r^2}w_{\varphi\varphi}\right)\right. \\
M_\varphi &= -D_0(1-\lambda_1 R^*)\left[1+\sum_{n=1}^{\infty}\frac{\gamma_{2n}h^{2n}}{2^{2n}(2n+3)}(1-\lambda_1 R^*)^{2n}\lambda^{2n}(r,\varphi,t)\right]\times \\
&\times\left(\frac{1}{r}w_r+\frac{1}{r^2}w_{\varphi\varphi}+v_0 w_{rr}\right); \\
M_{r\varphi} &= -D_0(1-v_0)\left[1+\sum_{n=1}^{\infty}\frac{\gamma_{2n}h^{2n}}{2^{2n}(2n+3)}(1-\lambda_1 R^*)^{2n}\lambda^{2n}(r,\varphi,t)\right]\times \\
&\times\left(\frac{1}{r}w_{r\varphi}-\frac{1}{r^2}w_\varphi\right); \\
\lambda(r,\varphi,t) &= \frac{8}{9}\left\{v_1\left[w_{rr}^2+\frac{1}{r^2}\left(w_r+\frac{1}{r}w_{\varphi\varphi}\right)^2\right]+\right. \\
&\left.+\frac{v_2}{r}w_{rr}\left(w_r+\frac{1}{r}w_{\varphi\varphi}\right)+\frac{3}{r^2}\left(w_{r\varphi}+\frac{1}{r}w_\varphi\right)^2\right\}.
\end{aligned}\right\} \quad (19)$$

Then the equation of bending of a physically nonlinear viscoelastic plate takes the form

$$\begin{gathered}
D_0\nabla^4 w+3D_0\left[\sum_{n=1}^{\infty}\frac{\gamma_{2n}h^{2n}}{2^{2n}(2n+3)}(1-\lambda_1 R^*)^{2n}\lambda^{2n}(r,\varphi,t)\nabla^4 w\right]+ \\
+6D_0\left[F_r\frac{\partial}{\partial r}\nabla^2 w+\frac{2}{r^2}F_\varphi\frac{\partial}{\partial\varphi}\nabla^2 w+0.5\left(F_{rr}+\frac{v_0}{r}F_r+\frac{v_0}{r^2}F_{\varphi\varphi}\right)w_{rr}+\right. \\
+\left(\frac{1}{r}F_r+\frac{1}{r^2}F_{\varphi\varphi}+v_0F_{rr}\right)\left(\frac{1}{r}w_r+\frac{1}{r^2}w_{\varphi\varphi}\right)+ \\
\left.+\frac{2(1-v_0)}{r^2}\left(F_{r\varphi}-\frac{1}{r}F_\varphi\right)\left(w_{r\varphi}-\frac{1}{r}w_\varphi\right)\right]=(1+\lambda_1\Gamma^*)q(r,\varphi,t),
\end{gathered} \quad (20)$$

where

$$F(r,\varphi,t)=\sum_{n=1}^{\infty}\frac{\gamma_{2n}h^{2n}}{2^{2n}(2n+3)}(1-\lambda_1 R^*)^{2n}\lambda^{2n}(r,\varphi,t). \quad (21)$$

For an axisymmetrically loaded round plate, the bending equation (20) takes the form

$$\begin{gathered}
D_0\nabla^4 w+3D_0\left[\sum_{n=1}^{\infty}\frac{\gamma_{2n}h^{2n}}{2^{2n}(2n+3)}(1-\lambda_1 R^*)^{2n}\lambda^{2n}(r,t)\right]\nabla^4 w+ \\
+6D_0\left[F_r\frac{\partial}{\partial r}\nabla^2 w+0.5\left(F_{rr}+\frac{v_0}{r}F_r\right)w_{rr}+\left(\frac{1}{r}F_r+v_0F_{rr}\right)\frac{1}{r}w_r\right]= \\
=(1+\lambda_1\Gamma^*)q(r,t),
\end{gathered} \quad (22)$$

where

$$\psi_0^2=\frac{8}{9}\left[v_1\left(r^2w''^2+w'^2+v_2rw'w''\right)\right]\frac{z^2}{r^2}; \quad (23)$$

$$F(r,t)=\sum_{n=1}^{\infty}\frac{\gamma_{2n}h^{2n}}{2^{2n}(2n+3)}(1-\lambda_1 R^*)^{2n}\lambda^{2n}(r,t); \quad (24)$$

$$\lambda^2(r,t)=\frac{8}{9r^2}\left[v_1\left(r^2w''^2+w'^2+v_2rw'w''\right)\right]. \quad (25)$$

Practically useful formulas are derived immediately for the particular case when the extension function $\chi(\varepsilon_0)$ and the shear function $\gamma(\psi_0^2)$ can be represented with sufficient accuracy in the form

$$\chi(\varepsilon_0) \equiv 1; \quad \gamma(\psi_0^2) = 1 + \gamma_2\, \psi_0^2. \tag{26}$$

For this case, in series (15), (21) and (24), it is necessary to be limited to one term, i.e. for the function F we have

$$F = \frac{\gamma h^2}{20}(1 - \lambda_1 R^*)^2 \lambda^2(x, y, t); \tag{27}$$

$$\lambda^2(x, y, t) = \frac{8}{9}\left[v_1\left(w_{xx}^2 + w_{yy}^2\right) + v_2 w_{xy} w_{yy} + 3w_{xy}^2\right]; \tag{28}$$

and in the axisymmetric case, λ is determined by formula (25)

$$\left.\begin{aligned} F(r, \varphi, t) &= \frac{\gamma_2 h^2}{20}(1 - \lambda_1 R^*)^2 \psi_0^2(x, y, t); \\ \psi_0^2 &= \frac{8}{9}\left\{v_1\left[\left(w_{rr}^2 + \frac{1}{r^2}\left(w_r + \frac{1}{r} w_{\varphi\varphi}\right)\right)\right] + \right. \\ &\left. + \frac{v_2}{r} w_{rr}\left(w_r + \frac{1}{r} w_{\varphi\varphi}\right) + \frac{3}{r^2}\left(w_{r\varphi} - \frac{1}{r} w_\varphi\right)^2\right\} z^2. \end{aligned}\right\} \tag{29}$$

From equations (14), (20) and (22), as a particular case, the equations of bending of thin plates of linear theory are obtained.

In equations (14), (20) and (22), assuming that

$$\left.\begin{aligned} q(x, y, t) &= -N_x \frac{\partial^2 w}{\partial x^2} - 2N_x \frac{\partial^2 w}{\partial x \partial y} - N_y \frac{\partial^2 w}{\partial x^2}; \\ q(x, y, t) &= q_0(x, y, t) - p\frac{\partial^2 w}{\partial t^2}, \end{aligned}\right\} \tag{30}$$

we obtain the equations of stability and vibrations of thin physically nonlinear viscoelastic plates.

4. Discussion of results

As a specific example, consider the bending of physically nonlinear viscoelastic freely supported thin plates. In this case, the boundary conditions are:

$$\begin{aligned} \frac{\partial^2 w}{\partial x^2} &= 0 \quad at \quad x = \pm a; \\ \frac{\partial^2 w}{\partial y^2} &= 0 \quad at \quad y = \pm b. \end{aligned} \tag{31}$$

The sought for solution satisfying the boundary conditions (31) is taken in the form

$$w(x, y, t) = A(t) \sin\frac{n\pi\, x}{a} \sin\frac{m\pi\, y}{b}. \tag{32}$$

Substituting function (32) into equation (15) and applying the Bubnov-Galerkin method to determine coefficient $A(t)$, we obtain

$$\begin{aligned} &(\alpha_n^2 + \beta_m^2)^2 \frac{ab}{4} A(t) + 3A(t)(1 - \lambda_1 R^*)^2 A^2(t)\{(\alpha_n^2 + \beta_m^2)^2 a_{nm} + \\ &+6\left[a_{nm}^x + a_{nm}^y + 0.5\left(a_{nm}^{xx} + v_0 a_{nm}^{yy}\right) + \right. \\ &\left. +0.5\left(a_{nm}^{yy} + v_0 a_{nm}^{xx}\right) + (1 - v_0) a_{nm}^{xy}\right]\} = \frac{1}{D_0}(1 - \lambda_1 \Gamma^*)_{q_{nm}}(t). \end{aligned} \tag{33}$$

The following notation is introduced in formula (33):

$$a_{nm} = \int_0^a \int_0^b F(x, y) sin^2 \alpha_n x\, sin^2 \beta_m y dx dy, \qquad \alpha_n = \frac{n\pi}{a}, \qquad \beta_m = \frac{m\pi}{b};$$

$$q_n(t) = \int_0^a \int_0^b q(x,y,t) \sin\alpha_n x \sin\beta_m y dxdy;$$
$$a_{nm}^x = (\alpha_n^2 + \beta_m^2)\alpha_n \int_0^a \int_0^b F_x(x,y) \sin 2\alpha_n x\, sin^2\beta_m y dxdy;$$
$$a_{nm}^y = (\alpha_n^2 + \beta_m^2)\beta_m \int_0^a \int_0^b F_x(x,y) sin^2\alpha_n x \sin 2\beta_m y dxdy;$$
$$a_{nm}^{xx} = \alpha_n^2 \int_0^a \int_0^b \left(F_{xx} + v_0 F_{yy}\right) sin^2\alpha_n x\, sin^2\beta_m y dxdy; \quad (34)$$
$$a_{nm}^{yy} = \beta_m^2 \int_0^a \int_0^b \left(F_{yy} + v_0 F_{xx}\right) sin^2\alpha_n x\, sin^2\beta_m y dxdy;$$
$$a_{nm}^{xy} = \frac{\alpha_n\beta_m}{4} \int_0^a \int_0^b F_{xy} \sin 2\alpha_n x \sin 2\beta_m y dxdy,$$

where

$$F(x,y) = \frac{8}{9}\{[v_1(\alpha_n^4 + \beta_m^4) + v_2\alpha_n^2\beta_m^2] sin^2\alpha_n x sin^2\beta_m y + $$
$$+3\alpha_n^2\beta_m^2 cos^2\alpha_n x cos^2\beta_m y\};$$
$$F_x(x,y) = \frac{8}{9}\{[v_1(\alpha_n^4 + \beta_m^4) + v_2\alpha_n^2\beta_m^2] sin^2\alpha_n x cos^2\beta_m y -$$
$$-3\alpha_n^2\beta_m^2 sin^2\alpha_n x cos^2\beta_m y\};$$
$$F_y(x,y) = \frac{8}{9}\{[v_1(\alpha_n^4 + \beta_m^4) + v_2\alpha_n^2\beta_m^2] sin^2\alpha_n x cos^2\beta_m y -$$
$$-3\alpha_n^2\beta_m^2 sin^2\beta_m y cos^2\alpha_n x\};$$
$$F_{xy}(x,y) = -\frac{8}{9}\{[v_1(\alpha_n^4 + \beta_m^4) + v_2\alpha_n^2\beta_m^2 + 3\alpha_n^2\beta_m^2] \sin 2\alpha_n x sin\, 2\beta_m y\}$$

The last equation can be represented as

$$A(t)[1 + 3(1 - \lambda_1 R^*)^2 A^2(t)] \frac{1}{(\alpha_n^2 + \beta_m^2)^2}\{(\alpha_n^2 + \beta_m^2)^2 a_{nm} +$$
$$+6\left[a_{nm}^x + a_{nm}^y + 0.5\left(a_{nm}^{xx} + v_0 a_{nm}^{yy}\right) + 0.5\left(a_{nm}^{yy} + v_0 a_{nm}^{xx}\right) + \right. \quad (35)$$
$$\left. +(1 - v_0)a_{nm}^{xy}\right]\} = \frac{1}{D_0(\alpha_n^2 + \beta_m^2)^2}(1 + \lambda_1\Gamma^*)q_n(t).$$

Equation (35) can be solved using the method of successive approximations [2], of power series. Using the method of successive approximations, we obtain

$$A^{(0)}(t) = \frac{(1 + \lambda_1\Gamma^*)q_n(t)}{D_0(\alpha_n^2 + \beta_m^2)^2};$$
$$A^{(n)}(t) = \frac{A^{(0)}}{1 + 3(1 - \lambda_1 R^*)^2 A^{2(n-1)}(t) \cdot \frac{1}{(\alpha_n^2 + \beta_m^2)^2}\{(\alpha_n^2 + \beta_m^2)^2 a_{nm} + } \quad (36)$$
$$\overline{+6\left[a_{nm}^x + a_{nm}^y + 0.5\left(a_{nm}^{xx} + v_0 a_{nm}^{yy}\right) + 0.5\left(a_{nm}^{yy} + v_0 a_{nm}^{xx}\right) + (1 - v_0)a_{nm}^{xy}\right.};$$

$$q_n(t) = \int_0^a \int_0^b q(x, y, t) \sin \alpha_n x \sin 2 \beta_m ydxdy.$$

Substituting $A(t)$ according to (36) into (32), we obtain an approximate solution to the problem of bending of freely supported thin rectangular plates.

5. Conclusions

The integro-differential equations of bending of a physically nonlinear viscoelastic plate are derived. The obtained equations are the generalized equations of the nonlinear theory of plates and shells. As an example, to illustrate the theory of plates and shells, the solutions for the bending of physically nonlinear viscoelastic freely supported thin plates are given.

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