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Longitudinal vibrations of underground pipelines of finite length in medium surrounded by soil with different properties along pipeline length

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Abstract. An analysis of the dynamic response of an underground main pipeline under a longitudinal wave propagating in soil along the pipe is given in the article. The problem of the longitudinal wave impact on a pipeline of finite length, interacting with soil according to the elastic-viscous law, is considered. The ends of the pipeline are fixed to massive nodes that interact with the medium according to linear laws. Along the length of the pipeline, the coefficients of the elastic and viscous pipeline-soil interaction change depending on the coordinate. In this article, the influence of the coefficients of elastic and viscous interaction of the "pipe-soil" system is studied when these coefficients are coordinate functions. The variability of the values of the coefficients along the length of the pipeline leads to a change in displacements from 0 to 15% and strain from 0 to 18%, compared with the case when these coefficients are constant. Depending on the length of the pipeline, the response of the pipeline to seismic action is different. This is especially evident at the boundary points. Considering the weight of nodes leads largely to a decrease in the strain of the pipeline relative to the soil strain at the boundary points.

1 Introduction

Underground pipelines are a key component of critical life support systems such as water supply, gas and liquid fuels, sewerage, electricity, and telecommunications. Their interaction with the soil structure caused by seismic waves has a crucial impact on the behavior of the pipeline and, when integrated throughout the pipeline network, on the entire system's performance [1, 2].

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In [2], the pipe slippage relative to the surrounding soil during the seismic wave propagation was taken into account for the first time; the differential equation of longitudinal oscillations of the pipeline was derived, and its solution was obtained for a finite and semi-infinite pipeline under harmonic and impulse loads.

It was experimentally established that the law of interaction of underground structures with various soils in the general case is non-linear. The parameters characterizing the non-linear, elastic, plastic, and viscous properties of the underground pipeline interaction with soil were determined [3].

In [4], studies of the influence of elastic-plastic interaction properties on seismic vibrations of an underground pipeline system were performed, for which a system with a finite number of degrees of freedom was chosen as the design scheme.

In [5], based on wave theory, a one-dimensional coupled problem of the seismic resistance of underground pipelines under seismic impacts was formulated. In [6], to describe the dynamic strain of soil, the elastoviscoplastic G.M. Lyakhov model was used. The system of partial differential equations of hyperbolic type, which describes the wave process, was solved by the method of characteristics and the method of finite differences using an implicit scheme. The change in wave parameters over time for different sections of the soil layer was obtained by a numerical solution.

In [7-16], various mechanical and mathematical models were analyzed, and several topical problems of underground and surface structures were solved.

The influence of the coefficients of elasticity and viscosity of the pipeline interaction at the contact with soil on the stress-strain state of the underground pipeline was studied in detail in [17-18]. In [19], the effect of inertia forces on the deformed state of an underground pipeline was studied. The effect of a seismic wave on an underground pipeline was considered in [20]. Accumulated experience demonstrates that direct simulation of real objects in the general case is not ensuring the required quality of the relevant analytical models [21].

Consider the problem of the impact of a longitudinal wave on a pipeline of finite length, interacting with soil according to the elastic-viscous law. The ends of the pipeline are fixed to massive nodes that interact with the medium according to linear laws (Fig. 1). Along the length of the pipeline, the coefficients of elastic and viscous interaction between the pipeline and the soil change depending on the coordinate.

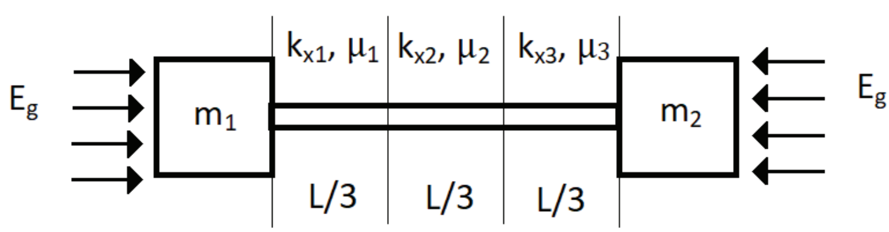


Fig. 1. Scheme of longitudinal wave flow past pipeline of finite length coupled with massive nodes

To simplify the statement of the problem, we accept the following aspect - the presence of a pipeline does not affect the wave field near it.

Since the wave field behind the surface wavefront depends on the depth of the soil medium, let us consider the average displacements of the particles of the soil medium along the axis of the pipeline.

The outer surface of the pipeline is in contact with soil along the pipeline axis according to the elastic-viscous law, and the ends of the pipeline are coupled with massive nodes through elastic elements. With these assumptions, the equation of longitudinal oscillations of the pipeline and the boundary conditions are written in the following form:

$$\rho F\frac{\partial^2 u}{\partial t^2}-EF\frac{\partial^2 u}{\partial x^2}+\pi Dk_x(x)\left(u-u_g\right)+\frac{\pi D\mu(x)}{H}\left(\frac{\partial u}{\partial t}-\frac{\partial u_g}{\partial t}\right)=0,\quad 0<x<L, \tag{1}$$

$$m_1\frac{\partial^2 u}{\partial t^2}=EF\frac{\partial u}{\partial x}-E_gF_1\frac{\partial u_g}{\partial x}\qquad \text{at}\quad x=0, \tag{2}$$

$$m_2\frac{\partial^2 u}{\partial t^2}=-EF\frac{\partial u}{\partial x}+E_gF_1\frac{\partial u_g}{\partial x}\qquad \text{at}\quad x=L. \tag{3}$$

The initial conditions are zero, i.e.,

$$u=0,\ \ \frac{\partial u}{\partial t}=0\qquad \text{at}\quad t=0, \tag{4}$$

where $u(x,t)$ is the longitudinal displacement of an arbitrary section of the pipeline; $u_g(x,t)=A\cdot\sin\omega(t-x/C_p)H(t-x/C_p)$ is the movement of soil particles behind the front of a wave propagating at a velocity C_p; A is the maximum ground displacement; $\omega-$ is the angular velocity of seismic wave vibration determined by the formula $\omega=2\pi/T$; C_p is the velocity of wave propagation in soil; $H(z)-$is the Heaviside function; E and $\rho-$are Young's modulus and the density of pipeline material; E_g is Young's modulus of soil; F_1 is the cross-sectional area of massive nodes in the form of a parallelepiped; F and $L-$ are the cross-sectional area and length of the pipeline; $k_x-$is the coefficient of elastic interaction of the "pipe-soil" system; m_1 and $m_2-$are the masses of nodes rigidly fixed to the pipeline in sections $x=0,\ x=L$.
We pass to dimensionless variables using the following formulas:

$$\bar{\bar{u}}=u/A,\ \bar{u}_g=u_g/A,\ \bar{x}=x/a,\ \ \bar{t}=t/b, \tag{5}$$

where $a=\sqrt{EF/K},\quad b=\sqrt{m/K}$.
Taking into account (5), we rewrite equation (1) in the following form:

$$\frac{\partial^2\bar{u}}{\partial\bar{t}^2}-\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\alpha\eta\cdot\frac{\partial\bar{u}}{\partial\bar{t}}+\xi\bar{u}=\alpha\cdot\eta\frac{\partial\bar{u}_g}{\partial\bar{t}}+\xi\bar{u}_g, \tag{6}$$

Here $\alpha = M / (b \cdot K)$.
We write boundary conditions (2) and (3) in dimensionless variables

$$\frac{\partial^2 \overline{u}}{\partial \overline{t}^2} = \frac{EFb^2}{m_1 a} \cdot \frac{\partial \overline{u}}{\partial \overline{x}} - \frac{E_g F_1 b^2}{m_1 a} \cdot \frac{\partial \overline{u}_g}{\partial \overline{x}} \quad \text{at} \quad \overline{x} = 0, \tag{7}$$

$$\frac{\partial^2 \overline{u}}{\partial \overline{t}^2} = -\frac{EFb^2}{m_2 a} \cdot \frac{\partial \overline{u}}{\partial \overline{x}} + \frac{E_g F_1 b^2}{m_2 a} \cdot \frac{\partial \overline{u}_g}{\partial \overline{x}} \quad \text{at} \quad \overline{x} = L / a, \tag{8}$$

2 Methods

To solve equation (6), we use the following implicit scheme of the finite difference method of the second-order accuracy [22]:

$$\frac{\partial^2 \overline{\overline{u}}}{\partial \overline{t}^2} \approx \frac{\overline{\overline{u}}_i^{j+1} - 2\overline{\overline{u}}_i^j + \overline{\overline{u}}_i^{j-1}}{\tau^2}, \quad \frac{\partial^2 \overline{\overline{u}}}{\partial \overline{x}^2} \approx \frac{\overline{\overline{u}}_{i+1}^{j+1} - 2\overline{\overline{u}}_i^{j+1} + \overline{\overline{u}}_{i-1}^{j+1}}{h^2}, \tag{9}$$

where τ and h are the time step and coordinate step.

The partial derivative concerning time, in coordinate and displacement, is approximated as follows:

$$\frac{\partial \overline{\overline{u}}}{\partial \overline{t}} \approx \frac{\overline{\overline{u}}_i^{j+1} - \overline{\overline{u}}_i^{j-1}}{2 \cdot \tau}, \quad \frac{\partial \overline{u}}{\partial \overline{x}} \approx \frac{\overline{u}_{i+1}^j - \overline{u}_{i-1}^j}{2 \cdot h} \quad \overline{\overline{u}} \approx \frac{\overline{\overline{u}}_i^{j+1} + \overline{\overline{u}}_i^{j-1}}{2}. \tag{10}$$

Approximations of the differentials of the function concerning time and coordinate (9) and (10) of differential equations (6–8) take the following form:

$$\begin{aligned} &b_2 \cdot \overline{\overline{u}}_{i-1}^{j+1} + b_1 \cdot \overline{\overline{u}}_i^{j+1} + b_2 \cdot \overline{\overline{u}}_{i+1}^{j+1} = \\ &= 2 \cdot \overline{\overline{u}}_i^j + b_3 \cdot \overline{\overline{u}}_i^{j-1} + b_4 \cdot (\overline{u}g_i^{j+1} + \overline{u}g_i^{j-1}) + b_5 \cdot (\overline{u}g_i^{j+1} - \overline{u}g_i^{j-1}). \end{aligned} \tag{11}$$

Here $b_1 = 1 + 2\tau^2 / h^2 + \alpha\tau / 2 + \tau^2 / 2$; $b_2 = -\tau^2 / h^2$; $b_3 = -1 - \tau^2 / 2 + \alpha\tau / 2$; $b_4 = \tau^2 / 2$; $b_5 = \alpha\tau / 2$.

We approximate the boundary conditions (7) and (8) in the following form:

$$\begin{aligned} &\text{a)} \quad \overline{u}_0^{j+1} - 2\overline{u}_0^j + \overline{u}_0^{j-1} = f_1(-3\overline{u}_0^j + 4\overline{u}_1^j - \overline{u}_2^j) - f_2(-3\overline{u}_{g0}^j + 4\overline{u}_{g1}^j - \overline{u}_{g2}^j), \\ &\text{б)} \quad \overline{u}_N^{j+1} - 2\overline{u}_N^j + \overline{u}_N^{j-1} = -f_3(\overline{u}_{N-2}^j - 4\overline{u}_{N-1}^j + 3\overline{u}_N^j) + f_4(\overline{u}_{gN-2}^j - 4\overline{u}_{gN-1}^j + 3\overline{u}_{gN}^j), \end{aligned} \tag{12}$$

Here $f_1 = \dfrac{E \cdot F \cdot b^2 \cdot (\Delta \overline{t})^2}{2 \cdot m_1 \cdot a \cdot \Delta \overline{x}}$; $f_2 = \dfrac{E_g \cdot F_1 \cdot b^2 \cdot (\Delta \overline{t})^2}{2 \cdot m_1 \cdot a \cdot \Delta \overline{x}}$; $f_3 = \dfrac{E \cdot F \cdot b^2 \cdot (\Delta \overline{t})^2}{2 \cdot m_2 \cdot a \cdot \Delta \overline{x}}$; $f_4 = \dfrac{E_g \cdot F_1 \cdot b^2 \cdot (\Delta \overline{t})^2}{2 \cdot m_2 \cdot a \cdot \Delta \overline{x}}$.

So, we determine the following displacement ratios for the end points of the underground pipeline $\overline{u}_0^{j+1}$ and $\overline{u}_N^{j+1}$

$$\text{a)} \quad \overline{u}_0^{j+1} = 2\overline{u}_0^j - \overline{u}_0^{j-1} + f_1(-3\overline{u}_0^j + 4\overline{u}_1^j - \overline{u}_2^j) - f_2(-3\overline{u}_{g0}^j + 4\overline{u}_{g1}^j - \overline{u}_{g2}^j),$$
$$\text{b)} \quad \overline{u}_N^{j+1} = 2\overline{u}_N^j - \overline{u}_N^{j-1} - f_3(\overline{u}_{N-2}^j - 4\overline{u}_{N-1}^j + 3\overline{u}_N^j) + f_4(\overline{u}_{gN-2}^j - 4\overline{u}_{gN-1}^j + 3\overline{u}_{gN}^j).$$

The implicit scheme is absolutely stable. It can be reduced to a system of linear algebraic equations (SLAE) with a tridiagonal matrix solved by the sweep method. To solve the SLAE, we need to know the values of $\overline{\overline{u}}_i^{j-1}$, $\overline{\overline{u}}_i^j$, $i = \overline{1, N-1}$, $j = 1, 2, \ldots$ on the lower time layers. Consider a pipeline fixed at both ends into the ground.
For $j = 0$, taking into account zero initial conditions, we have

$$\overline{u}\big|_{t=0} = 0, \ \Rightarrow \overline{\overline{u}}_i^0 = 0, \ \left.\frac{\partial \overline{u}}{\partial t}\right|_{t=0} = 0 \ \Rightarrow \left.\frac{\partial \overline{\overline{u}}}{\partial \overline{t}}\right|_i^0 = \frac{1}{\tau}\left(\overline{\overline{u}}_i^1 - \overline{\overline{u}}_i^0\right) \Rightarrow \overline{\overline{u}}_i^1 = 0,$$
$$i = 0, \ldots, N.$$

For $i = 1$, taking into account (12), we have:

$$b_1 \cdot \overline{\overline{u}}_1^{j+1} + b_2 \cdot \overline{\overline{u}}_2^{j+1} = = -b_2 \cdot \overline{\overline{u}}_0^{j+1} + 2 \cdot \overline{\overline{u}}_1^j + b_3 \cdot \overline{\overline{u}}_1^{j-1} + b_4 \cdot (\overline{u}g_1^{j+1} + \overline{u}g_1^{j-1}) + b_5 \cdot (\overline{u}g_1^{j+1} - \overline{u}g_1^{j-1}). \tag{13}$$

For $i = N-1$, taking into account (12), we have:

$$b_2 \cdot \overline{\overline{u}}_{N-2}^{j+1} + b_1 \cdot \overline{\overline{u}}_{N-1}^{j+1} = = -b_2 \cdot \overline{\overline{u}}_N^{j+1} + 2 \cdot \overline{\overline{u}}_{N-1}^j + b_3 \cdot \overline{\overline{u}}_{N-1}^{j-1} + b_4 \cdot (\overline{u}g_{N-1}^{j+1} + \overline{u}g_{N-1}^{j-1}) + b_5 \cdot (\overline{u}g_{N-1}^{j+1} - \overline{u}g_{N-1}^{j-1}). \tag{14}$$

Consider the sweep method of a simple and efficient algorithm for solving systems of linear algebraic equations with tridiagonal matrices [22]:

$$\begin{aligned}
&B_1 \cdot \bar{\bar{u}}_1^{j+1} + C_1 \cdot \bar{\bar{u}}_2^{j+1} = D_1,\\
&A_2 \cdot \bar{\bar{u}}_1^{j+1} + B_2 \cdot \bar{\bar{u}}_2^{j+1} + C_2 \cdot \bar{\bar{u}}_3^{j+1} = D_2,\\
&\quad\cdots\quad\cdots\quad\cdots\quad\cdots\quad\cdots\quad\cdots\quad\cdots\quad\cdots\\
&A_k \cdot \bar{\bar{u}}_{k-1}^{j+1} + B_k \cdot \bar{\bar{u}}_k^{j+1} + C_k \cdot \bar{\bar{u}}_{k+1}^{j+1} = D_k,\\
&\quad\cdots\quad\cdots\quad\cdots\quad\cdots\quad\cdots\quad\cdots\quad\cdots\quad\cdots\\
&A_{N-2} \cdot \bar{\bar{u}}_{N-3}^{j+1} + B_{N-2} \cdot \bar{\bar{u}}_{N-2}^{j+1} + C_{N-2} \cdot \bar{\bar{u}}_{N-1}^{j+1} = D_{N-2},\\
&A_{N-1} \cdot \bar{\bar{u}}_{N-2}^{j+1} + B_{N-1} \cdot \bar{\bar{u}}_{N-1}^{j+1} = D_{N-1}.
\end{aligned} \tag{15}$$

We transform the first equation of system (15) into the following form:

$$\bar{\bar{u}}_1^{j+1} = \alpha_1 \cdot \bar{\bar{u}}_2^{j+1} + \beta_1 \tag{16}$$

where $\alpha_1 = -C_1 / B_1,\ \beta_1 = D_1 / B_1$.

We substitute the expression obtained for $\bar{\bar{u}}_1^{j+1}$ into the second equation of the system:

$$A_2 \cdot (\alpha_1 \cdot \bar{\bar{u}}_2^{j+1} + \beta_1) + B_2 \cdot \bar{\bar{u}}_2^{j+1} + C_2 \cdot \bar{\bar{u}}_3^{j+1} = D_1 .$$

Let us transform this equation into the form

$$\bar{\bar{u}}_2^{j+1} = \alpha_2 \cdot \bar{\bar{u}}_3^{j+1} + \beta_2 , \tag{17}$$

where $\alpha_2 = -C_2 / (B_2 + A_2 \cdot \alpha_1),\ \beta_2 = (D_2 - A_2\beta_1) / (B_2 + A_2 \cdot \alpha_1)$.

Expression (17) is substituted into the third equation of the system, and so on.

At the $k-$th step of this process, $(1 < k < N)$ $k-$th equation of the system is transformed to the form:

$$\bar{\bar{u}}_k^{j+1} = \alpha_k \cdot \bar{\bar{u}}_{k+1}^{j+1} + \beta_k \tag{18}$$

where $\alpha_k = -C_k / (B_k + A_k \cdot \alpha_{k-1}),\ \beta_k = (D_k - A_k\beta_{k-1}) / (B_k + A_k \cdot \alpha_{k-1})$.

At the $(N-1)$-th step, the substitution of expression $\bar{\bar{u}}_{N-2}^{j+1} = \alpha_{N-2} \cdot \bar{\bar{u}}_{N-1}^{j+1} + \beta_{N-2}$ into the last equation gives

$$A_{N-1} \cdot (\alpha_{N-2} \cdot \bar{\bar{u}}_{N-1}^{j+1} + \beta_{N-2}) + B_{N-1} \cdot \bar{\bar{u}}_{N-1}^{j+1} = D_{N-1} .$$

From here, it is possible to determine the value of $\bar{\bar{u}}_{N-1}^{j+1}$:

$$\bar{\bar{u}}_{N-1}^{j+1} = \beta_{N-1} = \frac{D_{N-1} - A_{N-1}\beta_{N-2}}{B_{N-1} + A_{N-1} \cdot \alpha_{N-2}}$$

The values of the remaining unknowns $\bar{\bar{u}}_k^{j+1}$ for $k = N-2, N-3, ..., 1$ are now easily calculated using formula (18).

The transformations performed to make it possible to organize the calculations of the sweep method in two stages [22].

The forward course of the sweep method (a forward sweep) consists in calculating the sweep coefficients α_k $(1 < k < N-1)$ and β_k $(1 < k < N-1)$. For $k = 1$, the coefficients are calculated by the following formulas:

$$\alpha_1 = -C_1/\gamma_1 , \ \beta_1 = D_1/\gamma_1 , \ \gamma_1 = B_1 , \tag{19}$$

and for $k = 2, 3, ..., N-2-$ the coefficients are calculated by recursive formulas:

$$\alpha_k = -C_k/\gamma_k , \ \beta_k = D_k - A_k\beta_{k-1}/\gamma_k , \ \gamma_k = B_k + A_k \cdot \alpha_{k-1} . \tag{20}$$

For $k = N-1$, the forward sweep ends with the calculation of

$$\beta_{N-1} = D_{N-1} - A_{N-1}\beta_{N-2}/\gamma_{N-1} , \ \gamma_{N-1} = B_{N-1} + A_{N-1} \cdot \alpha_{N-2} . \tag{21}$$

The backward course of the sweep method (a backward sweep) gives the values of the unknowns. First, it is assumed that $\bar{\bar{u}}_{N-1}^{j+1} = \beta_{N-1}$. Then the values of the remaining unknowns are calculated by the following formula:

$$\bar{\bar{u}}_k^{j+1} = \alpha_k \cdot \bar{\bar{u}}_{k+1}^{j+1} + \beta_k , \ k = N-2, N-3, ..., 1 \tag{22}$$

Calculations are conducted in descending order of values k from $N-2, N-3, ..., 1$ to 1.

The method can be implemented when $B_1 \neq 0, \ B_k + A_k \cdot \alpha_{k-1} \neq 0,$ $k = 2, \ldots, N-1$.

Considering the connection between the sweep method and the Gauss method, we can say that this condition is satisfied, for example, when the matrix of system (15) is a matrix with diagonal dominance, i.e., $|C_1| < |B_1|$, $|A_N| < |B_N|$, $|A_i| + |C_i| < |B_i|$, $i = 2, \ldots, n-1$ [22].

3 Results and discussion

To analyze the dynamic response of an underground main pipeline under the action of a wave in soil, it is assumed that the elastic pipe has a finite length. Consider a solid steel pipe. Pipe characteristics are given in Table 1. Soil characteristics are given in Table 2, and the characteristics of the well are given in Table. 3.

Table 1. Characteristics of a steel pipe

Diameter, m	Thickness, m	Young modulus, 10^8 kN/m^2	Length, m	Density, kg/m^3
0.2	0.01	2.1	200.0	7800.0

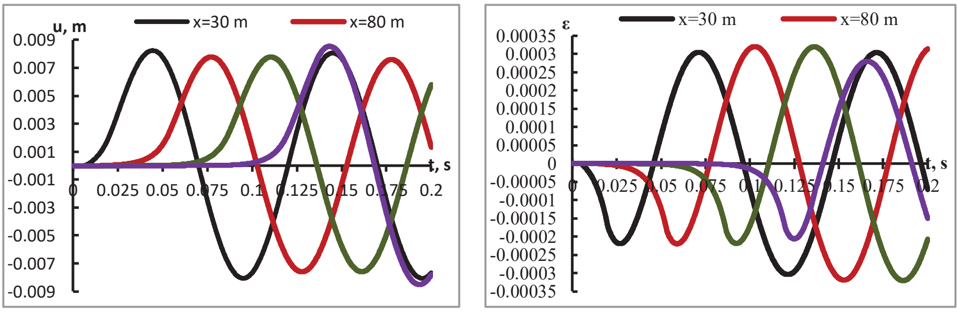
Table 2. Soil characteristics

Elastic resistance of soil k_x,10^7 N/m^3	Viscous resistance of soil μ, kN·f/m^2	Apparent wave propagation velocity,C_p, m/s	Period of fundamental vibrations *T*, s	Vibration amplitude *A*, m	Young modulus, 10^4 kN/m^2
0.5–4	0–100	1500.0	0.1	0.01	4.0

Table 3. Characteristics of the wells

Mass of the 1st well m_1, kg	Mass of the 2nd well m_2, kg	Cross section of the well F_1, m^2	Depth of laying H, m
100.0	100.0	1.0	1.0

Fig. 2 shows the change in displacement (a) and strain (b) of a 200-meter underground pipeline over time. Here the results for pipeline sections $x = 30$ m, $x = 80$ m, $x = 130$ m and $x = 180$ m are compared.



a *b*

Fig. 2. Change in displacement (a) and strain (b) over time ($k_x = 10^4 kN/m^3$, $\mu =$ 20 kN·s/m^2)

Fig. 3 shows the change in displacement and strain along the coordinate. In this case, a pipeline 200 m long is considered.

The values of the coefficient of elastic interaction along the length of the pipeline are considered constant and equal to $k_x = 10^4 kN/m^3$, and the values of the coefficient of viscous interaction are considered a function of the coordinate, determined by the formula (23).

$$\mu(x) = \begin{cases} 20 \quad \text{at} \quad x \in (0,100] \\ 100 \quad \text{at} \quad x \in (100,150] \quad kN{\cdot}s/m^2 \\ 100 \quad \text{at} \quad x \in (150,200] \end{cases} \tag{23}$$

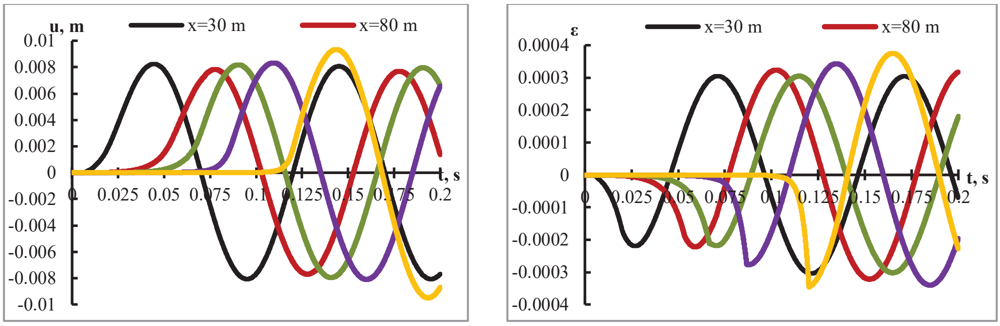
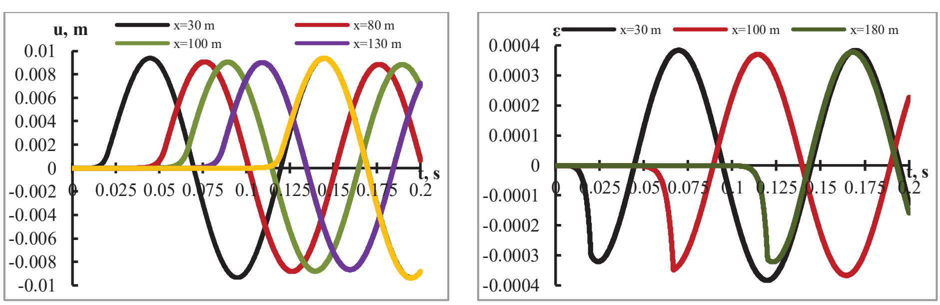


Fig. 3. Change in displacements (a) and strains (b) over time

Now we consider the case in which the values of the coefficient of elastic interaction in different sections along the length of the pipeline change according to formula (24). A pipeline 200 m long is located between massive wells. The coefficient of viscous interaction along the length of the pipeline is considered constant and equal to $\mu = 200\ \text{kN}\cdot s/m^2$.

$$k_x(x) = \begin{cases} 4\cdot 10^4 & \text{at } x \in [0,50] \\ 10^4 & \text{at } x \in (50,150] \quad kN/m^3 \\ 3\cdot 10^4 & \text{at } x \in (150,200] \end{cases} \tag{24}$$

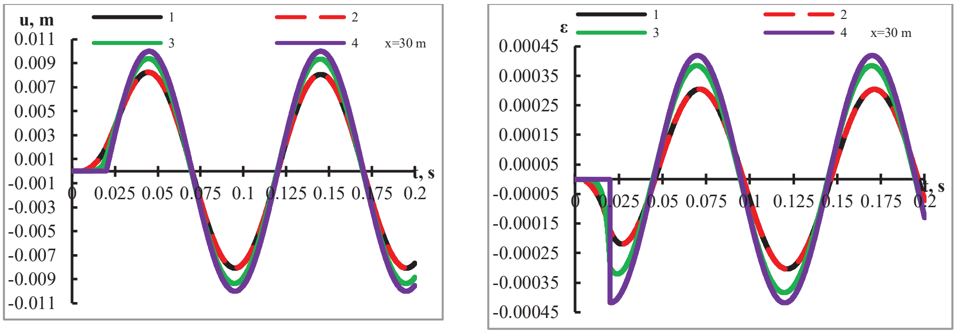
Fig. 4 shows the change in displacements and strains over time. The displacements and strains are greater in the first section than in the other sections.



a *b*

Fig. 4. Change in displacements (a) and strains (b) over time

Fig. 5 shows the case in which the coefficients of elastic and viscous interaction between the pipeline and soil are functions of the coordinate. Figure 5 shows the change in displacements and strains in an underground pipeline over time. Figures 5 (a) and (b) show a comparison of the results for different combinations of functions $k_x(x)$ and $\mu(x)$.



a *b*

Fig. 5. Changes in displacements (a) and strains (b) over time: 1 is $k_x = 10^4 kN/m^3$ and $\mu =$ 20 kN$\cdot s/m^2$; 2 is $k_x = 10^4 kN/m^3$ and $\mu(x)$ is determined by formula (23); 3 is $k_x(x)$ determined by formula (24) and $\mu =$ 20 kN$\cdot s/m^2$; 4 is wave in soil.

Let us assume that the coefficient $\mu =$ 200 kN$\cdot s/m^2$ is constant along the length of the pipeline. To analyze the influence of a change in the elastic interaction coefficient k_x, we consider two cases shown in Fig. 6,7, the first case is when $k_x(x)$ is determined by the formula (10), and the second case is when the value of the coefficient $k_x = 10^4 kN/m^3$ is constant.

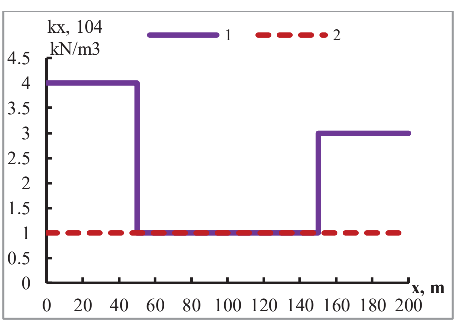


Fig. 6. Change in function $k_x(x)$:
1 is $k_x(x)$ determined by formula (24);
2 is $k_x = 10^4 kN/m^3$

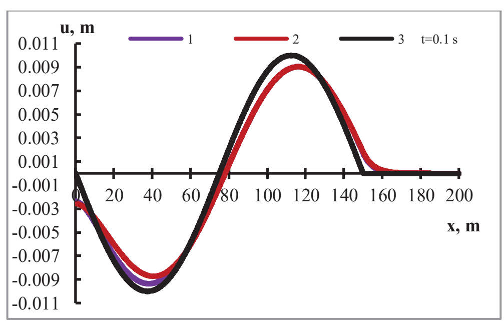
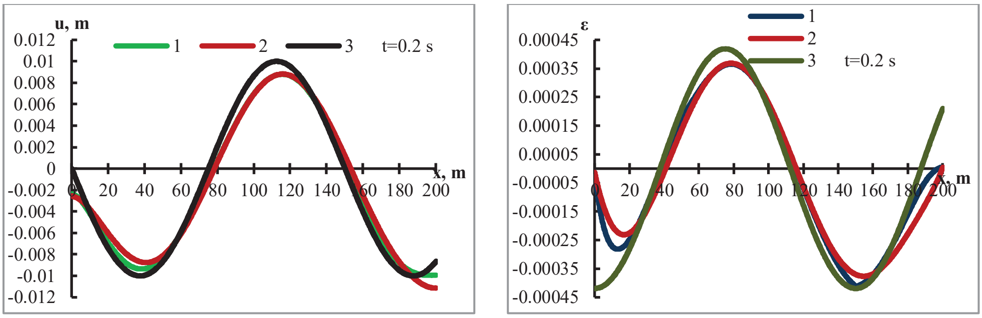


Fig. 7. Change in displacements over time: 1 is $k_x(x)$ determined by formula (24); 2 is $k_x = 10^4 kN/m^3$

In the area from 0 to 50 meters, due to the greater value of the coefficient of elasticity, a difference in the results of the above cases is seen; the pipeline displacement is approaching the soil displacement value. This is confirmed by the results shown in Fig. 8a.

 https://doi.org/10.1051/e3sconf/202340104060



a *b*

Fig. 8. Change in displacements (a) and strains (b) along the coordinate: 1 is $k_x(x)$ determined by formula (10); 2 is $k_x = 10^4 kN/m^3$; 3 is wave in soil

At the boundaries of the underground pipeline, the results of displacements (Fig. 8, a) and strains (Fig. 10, b) differ depending on the value of the elastic interaction coefficient $k_x(x)$.

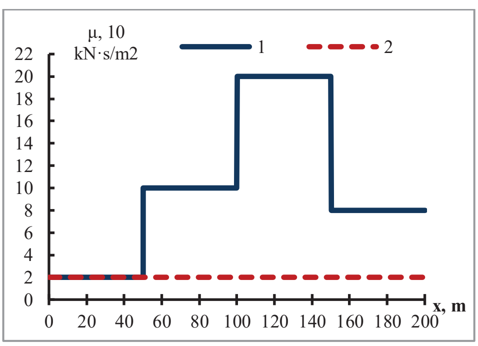


Fig. 9. Change in viscosity coefficient of interaction along coordinate.

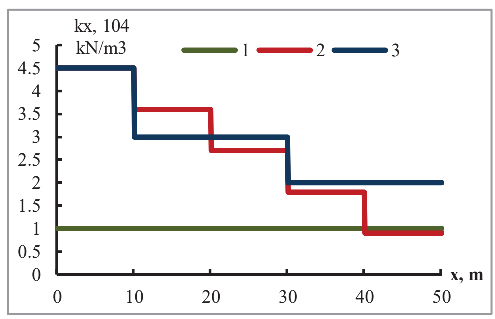


Fig. 10. Change in elasticity coefficient of interaction along coordinate

Fig 9 shows functions $\mu_1(x)$ and $\mu_2(x)$. Function $\mu_1(x)$ is determined by formula (25), and function $\mu_2(x) =$ 20 kN$\cdot s/m^2 = const$.

$$\mu_1(x) = \begin{cases} 20 \text{ at } x \in [0,50] \\ 100 \text{ at } x \in (50,100] \\ 200 \text{ at } x \in (100,150] \\ 80 \text{ at } x \in (100,150] \end{cases} \text{kN}\cdot s/m^2 \qquad (25)$$

Fig. 11 shows that in the interval with large values of the viscous interaction coefficient at $\mu_1(x)$, the displacement is 13% greater than $\mu_2(x)$. At the boundary points $x = 0$ and $x = L$, the displacements and strains of the pipeline and soil differ greatly (see Figs. 11, a and b).

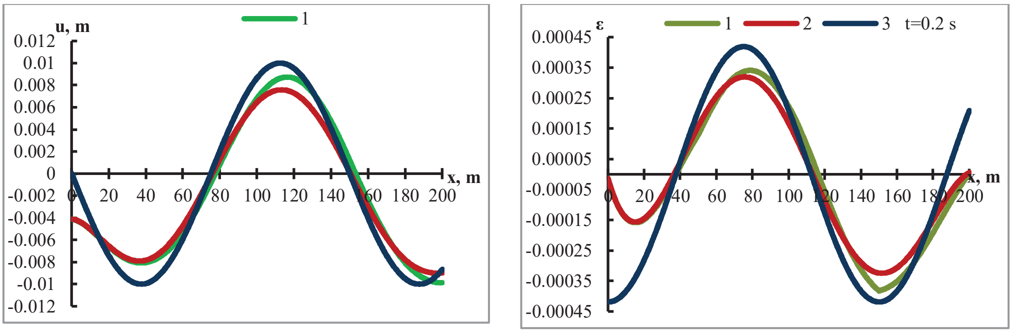


Fig. 11. Change in displacements (a) and strains (b) along the coordinate: 1 is for $\mu_1(x)$; 2 is for $\mu_2(x)$; 3 is wave in soil

Consider a pipeline 50 m long (see Fig. 12). We assume that the viscosity coefficient is $\mu = 50$ kN·s/m^2, and remains constant along the length of the pipeline. Still, the coefficient of elastic interaction varies depending on the coordinate. Figure 12 compares results for three cases: the first for $k_{x1} = 10^4 kN/m^3$, the second for $k_{x2}(x)$, and the third for $k_{x3}(x)$. Functions $k_{x2}(x)$ and $k_{x3}(x)$ are determined from formulas (26) and (27), respectively. The graph of functions $k_{x1}(x)$, $k_{x2}(x)$ and $k_{x3}(x)$ is shown in Fig.9.

$$k_{x2}(x) = \begin{cases} 4.5\cdot 10^4 \text{ at } x\in[0,10] \\ 3.6\cdot 10^4 \text{ at } x\in(10,20] \\ 2.7\cdot 10^4 \text{ at } x\in(20,30] \\ 1.8\cdot 10^4 \text{ at } x\in(30,40] \\ 0.9\cdot 10^4 \text{ at } x\in(40,50] \end{cases} kN/m^3 \tag{26}$$

$$k_{x3}(x) = \begin{cases} 4.5\cdot 10^4 \text{ at } x\in[0,10] \\ 3\cdot 10^4 \text{ at } x\in(10,30] \\ 2\cdot 10^4 \text{ at } x\in(30,50] \end{cases} kN/m^3 \tag{27}$$

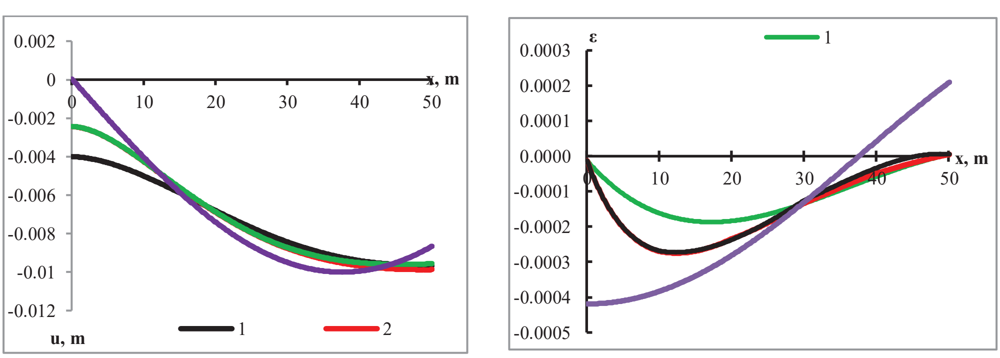


Fig. 12. Change in displacements (a) and strains (b) along the coordinate: 1 is for k_{x1}; 2 is for $k_{x2}(x)$; 3 is for $k_{x3}(x)$; 4 is wave in soil

If the length of the pipeline L is less than the wavelength in soil λ, $L/\lambda<1$, then at the boundary sections $x=0$ and $x=L$ the movement of the pipeline differs significantly from the movement of soil at $L/\lambda>1$.

An underground pipeline, when the soil moves in the form of a traveling sine wave, experiences strains close to the strains in the soil. An increase in the elastic interaction coefficient leads to an increase in strain in the underground pipeline.

An increase in the viscosity coefficient of interaction causes a reduction in the relative movement between the pipeline and the soil.

4 Conclusions

The influence of the coefficients of elastic and viscous interaction of the "pipe-soil" system was studied when these coefficients are coordinate functions. The variability of the values of the coefficients along the length of the pipeline leads to a change in displacement from 0 to 15% and strain from 0 to 18%, compared with the case when these coefficients are constant.

The response of the pipeline to seismic impact differs depending on the length of the pipeline. This is especially evident at the boundary points. At $L/\lambda<1$, the relative displacement is greater than at $L/\lambda>1$.

Taking into account the mass of nodes leads to a large extent, to a decrease in the strain of the pipeline relative to the strain of soil at the boundary points.

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