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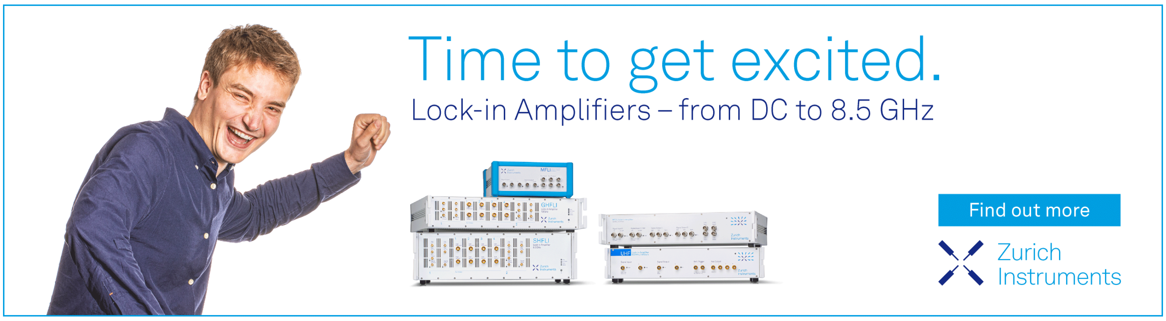

Impact of a plane compression wave on a cylindrical shell filled with fluid

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Abstract. A nonstationary strain of elastic cylindrical shell filled with an ideal compressible fluid under the action of a plane stepwise compression wave propagating in an external elastic medium is investigated. It is assumed that the shell is infinite, and the front of the incident wave is parallel to its axis. An expansion in a Fourier series in terms of natural vibration modes and finite differences in the radial coordinate in time is used to solve the problem. The asymptotic behavior of excitations as $t \to \infty$ (t – is time) is investigated similarly to. The approach makes it possible to obtain reliable and complete information about the basic physical patterns of the nonstationary strain of a cylindrical shell filled with fluid over the entire impact range of an elastic wave on the system.

INTRODUCTION

Theoretical estimates of the dynamics of underground structures are based on solutions to the problems of elastic wave diffraction. Among these classes of problems, the most difficult is a nonstationary formulation. The study of Baron and Parnes [1] initiated the analysis of nonstationary problems of elastic wave diffraction by cavities (supported and unsupported ones). To solve the problems, the required functions were expanded in a Fourier series in the circumferential coordinate; the Laplace transform in time was used. The inverse transform was conducted for zero and second forms at $p \to 0$ (p is the transform parameter). Thus, a solution was obtained that describes the stress state as $t \to \infty$. In [2], to solve this problem, a numerical inversion of the Laplace transform was used by the expansion of the original in series in Jacobi polynomials. Diffraction by rigid inclusions was considered in [3, 4], where the problems were reduced to the numerical solution of the Volterra integral equation of the second kind concerning the displacement potentials. To solve nonstationary problems of the medium, V.D. Kubenko has developed a method based on using the integral Laplace transform in time and its inversion using the Volterra equations. Some aspects of nonstationary diffraction of plane expansion and shear waves were considered in[6-8]. Along with this, in [9-12], the dynamic behavior and wave phenomena in various systems were investigated by the finite element method (FEM), taking into account the features of the structures.

Thus, most of the results of nonstationary problems were obtained for rigid inclusions, only at initial or large values of time, which does not allow obtaining complete information over the entire time interval. It should be noted that L.I. Slepyan presented various formulations of nonstationary problems and original approaches for their solution in his monograph [13].

The following approach to solving nonstationary wave diffraction problems by deformable inclusions was proposed in [14]. The motion of the shell and the elastic medium is expanded in a Fourier series in the angular coordinate. The resulting system of one-dimensional equations is solved numerically using an explicit finite-difference scheme. Zero derivatives in these equations are replaced by a three-point approximation, and the time step is chosen equal to the steps along the radial coordinate, which minimizes the numerical variance. In parallel with this, asymptotic solutions were constructed for each harmonic as $t \to \infty$. The proposed comprehensive

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approach allows one to obtain reliable and complete information about the basic physical patterns of the nonstationary strain of a cylindrical shell.

Based on this approach, an impact of a plane acoustic pressure wave on a cylindrical shell filled with fluid was considered in [15]. Similar nonstationary problems of diffraction of plane elastic compression and shear waves were studied in [16-18].

The proposed study investigates the nonstationary strain of a cylindrical shell filled with fluid under the impact of a plane stepwise compression wave propagating in an external elastic medium.

Problem statement

Consider the impact of a stepwise plane compression wave on an infinitely long elastic shell filled with an ideal compressible fluid and surrounded by an elastic medium. The wave front is considered parallel to the shell axis, thereby reducing the problem to a flat formulation.

In the polar coordinate system (r, θ) related to the cylinder, the stresses and displacements in the incident wave touching the frontal point with coordinates $r = R$, $\theta = 0$ at time point $t = 0$ are given in the following form

$$\begin{aligned}
&\sigma_r^o = \sigma[\varepsilon - 1 - (\varepsilon + 1)Cos2\theta]H_o(z)/2 \\
&\sigma_{r\theta}^o = \sigma(\varepsilon + 1)Sin2\theta H_o(z)/2 \\
&u_r^o = -\frac{\sigma}{\rho_1 c_1^2} zCos\theta\, H_o(z) \\
&u_\theta^o = \frac{\sigma}{\rho_1 c_1^2} zSin\theta\, H_o(z) \\
&z = c_1 t - R + rCos\theta, \quad \varepsilon = -\nu_1/(1-\nu_1)
\end{aligned}$$

where H_o is the Heaviside function, σ is the stress at the front of the wave propagating in the direction z, R is the shell radius, $\rho_{1,}\ \nu_1$ are the density and Poisson's ratio of the medium, respectively, c_1 is the velocity of the expansion wave.

The motion of an elastic medium is described by wave equations for scalar (φ) and vector (ψ) potentials, the flow of a fluid is described by a wave equation for the velocity potential (φ_0) (the dots on top denote time derivatives).

$$\ddot{\varphi} = c_1^2 \Delta\varphi, \quad \ddot{\psi} = c_2^2 \Delta\psi, \quad \ddot{\varphi}_0 = c_0^2 \Delta\varphi_0 \tag{1}$$

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2},$$

where c_z is the shear wave velocity, c_0 is the speed of sound in fluid.

The shell motion is described by linear equations of the classical Kirchhoff - Love theory

$$\begin{aligned}
&c^{-2}\ddot{\upsilon} = R^{-2}\left[\frac{\partial^2\upsilon}{\partial^2\theta} + \frac{\partial w}{\partial\theta} + \delta\left(\frac{\partial^2\upsilon}{\partial\theta^2} + \frac{\partial^3 w}{\partial\theta^3}\right)\right] + \beta(\sigma_{r\theta}^o + \sigma_{r\theta})/_{r=R} \\
&c^{-2}\ddot{w} = -R^{-2}\left[\frac{\partial\upsilon}{\partial\theta} + w + \delta\left(\frac{\partial^4 w}{\partial\theta^4} - \frac{\partial^3\upsilon}{\partial\theta^3}\right)\right] + \beta(\sigma_r^o + \sigma_r - P)/_{r=R}
\end{aligned} \tag{2}$$

$$\delta = h^2/12R^2,\ \beta = \left(\rho c^2 h\right)^{-1},\ P = -\rho_o \frac{\partial \varphi_o}{\partial t}$$

where υ, w are the displacements of the shell in tangential (θ) and radial (r) directions, h, ρ are the thickness and density of the shell material, respectively, c is the speed of sound in a thin plate, P is the pressure in the internal medium caused by the shell motion, ρ_o is the density of a fluid.

It is required to determine the stresses and displacements in the shells under zero initial conditions and the following boundary conditions on $r = R$:

$$R^{-1}\frac{\partial \varphi}{\partial \theta} - \frac{\partial \psi}{\partial r} + u_\theta^o = \upsilon, \quad \frac{\partial \varphi}{\partial r} + R^{-1}\frac{\partial \psi}{\partial \theta} + u_r^o = w \tag{3}$$

Potentials ϕ and ψ should, in addition, be zero outside the expanding region bounded by the front of excitations. On the surface of the shell $(r = R)$, the condition of equality of the radial velocities of the shell and fluid is satisfied

$$\frac{\partial \varphi_o}{\partial r} = \frac{\partial w}{\partial t} \tag{4}$$

SOLUTION TO THE PROBLEM

To solve the problem, the Fourier series expansion by angle θ is used. The equations of motion (1), (2) for the m-th mode of vibration (m = 0, 1, 2,….) take the following form

$$\begin{aligned} c^{-2}\ddot{\upsilon}_m &= -\alpha^2(1+\delta)\upsilon_m - \alpha R^{-1}\left(1+m^2\delta\right)w_m + \beta\left(\sigma^0_{r,\theta,m} + \sigma_{r,\theta,m}\right)/_{r=R} \\ c^{-2}\ddot{w}_m &= -\alpha R^{-1}\left(1+m^2\delta\right)\upsilon_m - R^{-2}\left(1+m^4\delta\right)w_m + \beta\left(\sigma^0_{r,m} + \sigma_{r,m} - P_m\right)/_{r=R} \end{aligned} \tag{5}$$

$$\begin{aligned} c_1^{-2}\ddot{\varphi}_m &= \frac{\partial^2 \varphi_m}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi_m}{\partial r} - \frac{m^2}{r^2}\varphi_m \\ c_2^{-2}\ddot{\psi}_m &= \frac{\partial^2 \psi_m}{\partial r^2} + \frac{1}{r}\frac{\partial \psi_m}{\partial r} - \frac{m^2}{r^2}\psi_m \end{aligned} \tag{6}$$

$$\begin{aligned} c_0^{-2}\ddot{\varphi}_{0,m} &= \frac{\partial^2 \varphi_{0,m}}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi_{0,m}}{\partial r} - \frac{m^2}{r^2}\varphi_{0,m} \\ \alpha &= m/R \end{aligned} \tag{7}$$

The Laplace transform in time with parameter p is applied to the systems of equations (the transformed values are indicated by the upper subscript L). A solution in images is obtained in the following sequence [16]: the solution of equations (6), (7) is found taking into account the radiation conditions and boundary conditions (3), (4); the total stresses in the external medium and the pressure in fluid acting on the shell are calculated. The solution to the problem and the analysis of the results obtained in the absence of fluid are presented in [16,17].

To calculate the pressure in the fluid, equation (7) is solved taking into account (4) and the boundedness of the solution on $r = 0$; then, we obtain

$$\varphi^L_{0,m} = C_m I_m\left(pc_0^{-1}r\right), \quad C_m = \frac{pw_m^L}{I_m^{'}(pc_0^{-1}r)}, \quad I' = \left(pc_0^{-1}r\right) = \frac{\partial I_m\left(pc_0^{-1}r\right)}{\partial r}$$

where I_m are the modified Bessel functions of the first kind.

Then, the expression for the pressure in the fluid acting on the shell takes the following form

$$P^L = \rho_0 p^2 \frac{I_m(pc_0^{-1}R)}{I_m^{'}(pc_0^{-1}R^{-1})} w_m^L \tag{8}$$

Thus, we obtain the exact solution of the problem in images by adding the pressure of fluid (8) to the total radial stress in the medium acting on the shell and solving the system of equations (5) with respect to w_m^L, υ_m^L.

We search the asymptotics of the solution for large values of time from the beginning of the process. As $p \to 0$ in the expressions obtained, we determine the asymptotics $(t \to \infty)$ of the Fourier coefficients for each harmonic. For $m > 2$, the asymptotics gives zero values of these coefficients: in the course of time, the first three forms $(m = 0, 1, 2)$ turn out to be the determinant forms.

The asymptotics of the solution is given as:

$$w = -\frac{\sigma r^2}{\rho c^2 h(1+\gamma+\gamma_0)} - \frac{\sigma t}{\rho_1 c_1} Cos\theta - \frac{2R(2+\gamma)\sigma}{\rho_1 c_1^2 B} Cos2\theta$$

$$\upsilon = \frac{\sigma t}{\rho_1 c_1} Sin\theta + \frac{2R(2+\gamma)\sigma}{\rho_1 c_1^2 B} Sin2\theta \tag{9}$$

$$\sigma_\theta^u = -\frac{\sigma R}{(1+\gamma+\gamma_0)h} + \frac{2R(1+\varepsilon)\sigma}{hB} Cos2\theta$$

$$\gamma = 2\rho_1 c_2^2 R / (\rho c^2 h), \; \gamma_0 = 2\rho_0 c_0^2 R / (\rho c^2 h)$$

$$B = \gamma(1-\varepsilon) + 3 - \varepsilon + 4\delta(3 - \varepsilon + 9\gamma^{-1} + 3\varepsilon\gamma^{-1})$$

Here σ_θ^u is the chain stress in the shell, calculated by the following formula

$$\sigma_\theta^u = \rho c^2 R^{-1}\left(w + \frac{\partial \upsilon}{\partial \theta}\right)$$

From (9), it follows that the first form describes the motion of the shell as a solid unit. Zero and second forms determine the strains and stresses in the shell. The fluid contributes to zero forms only, increasing the radial fluid of the shell over γ_0.

NUMERICAL SOLUTION

To reveal the error of asymptotic estimates over a finite time interval and to determine the coefficient of dynamics and the limits of applicability of these estimates in specific cases with real parameters of the shell and media, a numerical solution was conducted.

The motion of an elastic medium is written by the dynamic equations of the theory of elasticity in displacements. After expansion in a Fourier series in the angular coordinate, the equations are rewritten in finite-difference form. An explicit "cross" scheme is used. The numerical variance appearing in the area of frontal discontinuities due to space and time discretization is minimized by the appropriate approximation of equations and boundary conditions [14] and by the optimal choice of the parameters of the difference grid. A detailed description of this algorithm for an elastic medium is given in [17].

In calculations in the domain $(r < 1)$ in the equation for φ_0, a three-point approximation of the term with zero derivative is applied $(m^2/r^2)\varphi_0(t = n\tau, r = jh_0) \approx (m^2/4r^2)(\varphi_{0,j+1}^n + 2\varphi_{0,j}^n + \varphi_{0,j-1}^n)$, where $\varphi_{0,j}^n$ is the value of the function φ_0 at grid nodes, τ, h_0 are the grid steps in time and space. The boundary condition on the surface and the fluid pressure is approximated by one-sided differences. At the point $r=0$ where the wave equation has a singularity, the potential φ_0 must satisfy the following conditions [15].

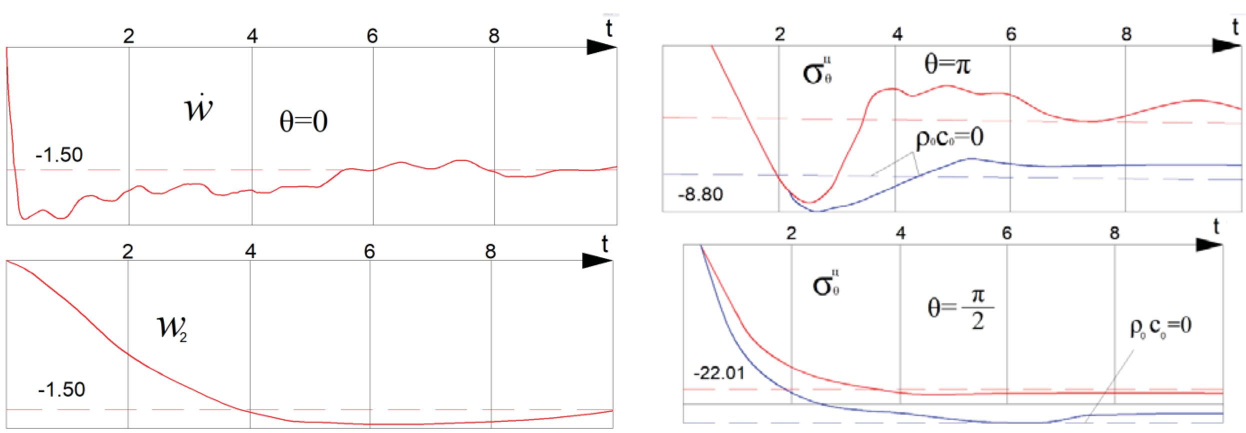
$$m \neq 0: \quad \varphi_{0,m} = 0$$
$$m = 0: \quad \frac{\partial^2 \varphi_{0,0}}{\partial t^2} = 2c_0^2 \frac{\partial^2 \varphi_{0,0}}{\partial r^2}, \quad \frac{\partial \varphi_{0,0}}{\partial r} = 0$$

Calculations have shown that at $h_1 = 0.05R$ (h_1 is the step of the difference grid along the radial coordinate in an elastic medium), a completely satisfactory accuracy is achieved; with a further decrease in h_1, the change in the results is observed in the third significant digit. When calculating the sums of the Fourier series, 11 terms were retained $(m = 0,.....10)$; an increase in the number of forms did not lead to a change in the results by more than 3%. In calculations ρ_1, c_1, R were taken as the unit of measurement.

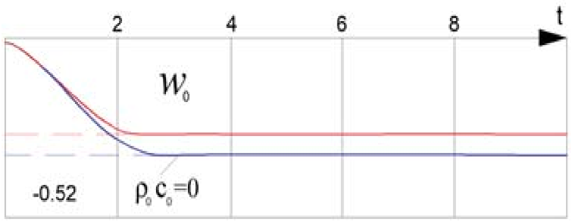
The figure shows the results for $\tau = h_1 = 0.05R$, $h_0 = 0.1$, and the following parameters of the shell and medium $E = 23$, $\rho = 2.9$, $h = 0.04$, $\nu = \nu_1 = 0.25$, $\rho_0 = 0.4$, $c_0 = 0.7$. The numbers in the figures correspond to the maximum values taken by the sought for values for the considered time. Dashed lines correspond to asymptotics (9).

The presence of fluid reduces zero form of displacement, and its effect on other forms is negligible. The internal medium increases the inertia of the system, as a result of which the level of $\dot{w}$ in the shell with fluid is lower than without fluid. As in the case $\rho_0 c_0 = 0$, the shell velocity becomes equal with time to the particle velocity in the incident wave (Fig.1a).

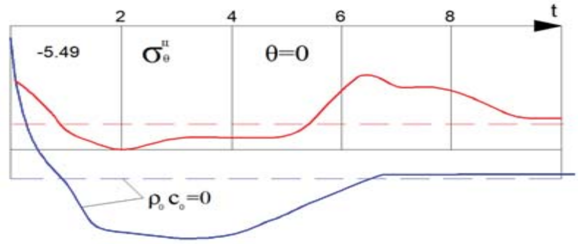
Analysis of oscillograms of the chain stresses shows that the maximum compressive stress occurs at the side point $(\theta = \pi/2)$. The difference between the numerical solution and the asymptotics for $t \geq 4$ is insignificant. For comparison, the results for the case $\rho_0 c_0 = 0$ are shown. The internal medium significantly reduces the maximum value of σ_θ^u at the frontal point $\theta = 0$. Its decrease depends on $\rho_0 c_0^2$ and increases with an increase in the modulus of volumetric compression of the fluid. At the shaded point $(\theta = \pi)$, the first excitations are caused by elastic waves $(t_1 = \pi R / c)$; the basic excitations appear after the acoustic wave $(t_2 = 2R / c_0)$ arrival. At $t > t_2$, the amplitudes of the chain stresses sharply decrease and tend to the asymptotic value (9) (Fig.1b).



(a) (b)



(a)



(b)

FIGURE 1. a) oscillograms of $w_0, w_2, \dot{w}$, b) oscillograms of chain stresses.
__________ numerical solution, --------------- asymptotic solution (9), __________ $\rho_0 c_0 = 0$

CONCLUSION

The results obtained allow us to draw the following conclusions.

1. For sufficiently large time values, the strains and chain stresses in the shell are determined by zero and second forms. The fluid contributes to zero modes of vibration only, increasing the radial fluid of the shell by γ_0.

2. The developed difference scheme that minimises the numerical variance allows us to accurately describe frontal discontinuities. Comparison of two solutions shows that at $(t \geq 6R/c_1)$, the asymptotic solution completely describes the stress-strain state of the shell.

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