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Nonstationary Deformation of Cylindrical shells Under a Plane Pressure Wave

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Abstract. Nonstationary deformation of a cylindrical shell located in an infinite elastic medium is studied under plane pressure wave with a front parallel to the shell axis. A combined solution method is used in the paper. Separating the angular coordinate by the Fourier method, the plane problem is reduced to a series of one-dimensional problems for each harmonic. Applying the Laplace transform over time to these systems of equations, the exact solution in the images is obtained. Asymptotic solutions of the stress-strain state at large values of time $t \to \infty$ are obtained. In parallel with this, a truncated system is numerically solved using an explicit finite-difference scheme and a method for minimizing numerical dispersion, which gives an accurate description of front discontinuities. A comparison of both solutions made it possible to determine the applicability limits of the asymptotic forms and to obtain an assessment of dynamic state of shell and medium during the entire time of interaction.

1. Introduction

Elastic wave diffraction is a classical problem of the dynamics of deformable bodies, and its solution requires the involvement of a complex mathematical apparatus.

The possibilities for analyzing the dynamic behavior of massive bodies and structures subject to seismic, explosive and shock loads, and the stress-strain state of the surrounding medium based on full-scale experiments are significantly limited. In this situation, the calculation methods of the wave mechanics of a deformable rigid body are becoming important. Theoretical assessment of the dynamics of structures should be based on the solutions of non-stationary problems of wave diffraction on bodies of different geometry of the contour.

There are two approaches to solving these problems: stationary and non-stationary ones. The widest range of diffraction problems in an elastic medium is considered for steady waves. By restricting ourselves to steady wave motions, we get the opportunity to separate one of the independent variables t - time, by extracting the exponential factor $e^{-i\omega t}$ (ω is the circular frequency); this simplifies the solution of the problem. At the same time, the study of steady waves is important for many applied problems. Moreover, if it is possible to calculate the response of an elastic medium to steady excitations in a wide frequency range, this makes it possible to study transient processes using the apparatus of the Fourier integral transform. Without going into details, we will point out that a review of these papers is given in [1].

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The principles of unsteady problems analysis of elastic waves diffraction on cavities (supported and unsupported ones) were laid by Baron and Parnes [2]. To solve the problem, the functions were expanded in a Fourier series in the circumferential coordinate; the Laplace transform in time was used. The inverse transform was carried out for the zero and second modes at $p \to 0$ (p is the transform parameter). Thus, a solution that describes the stress state at $t \to \infty$ was obtained. In [3], to solve this problem, the numerical inversion of the Laplace transform was used by expanding the original in series in Jacobi polynomials. In [4-6], a diffraction on rigid inclusions was considered. In [4-5] the problems are reduced to a numerical solution of the integrals of Volterra equations of the second kind with respect to displacement potentials, while in [6] an approximate solution for calculating the total force acting on a rigid cylinder is obtained for the initial points of time. To solve non-stationary problems in the medium, V.D. Kubenko developed a method based on the use of the Laplace integral transform in time and its inversion using Volterra equations [7]. Some aspects of the unsteady diffraction of plane waves (expansion and shear) are considered in [8–11, 14].

Dynamic behavior and wave phenomena in various systems are studied in [16–21] taking into account design features of structures.

Most of the results are obtained for rigid inclusions, and only for initial or long time intervals, which does not allow obtaining complete information over the entire time interval. It should be noted that various statements of non-stationary problems and original methods to solve them are described by Slepyan L.I. in his monograph [12].

With results obtained in [13], it became possible to apply the following approach to solving non-stationary problems of wave diffraction on rigid and deformable inclusions. In this paper, the equations of motion of the shell and the elastic medium in displacements differ in the Fourier series in angular coordinate. The resulting system of one-dimensional equations is solved numerically using an explicit finite-difference scheme.

Zero derivatives in the equations are replaced by a three-point approximation and the time step is chosen equal to the step on radial coordinate; this minimizes numerical dispersion [13]. These solutions determine the stresses in the shell and medium over the entire interaction interval. In addition, the same problem is solved by analytical methods to obtain asymptotic solutions. Numerical and asymptotic solutions supplement each other. A comparison of the two solutions allows us to determine the applicability limits of asymptotic solutions.

2. Maths statement of the problem

Let us consider the action of a stepped plane pressure wave on an infinitely long elastic cylindrical shell surrounded by an elastic medium. The wave front is considered to be parallel to the shell axis, so, the problem is reduced to a plane statement.

In the polar coordinate system (r , θ) related to the cylinder, the stresses and displacements in the incident wave, tangent to the frontal point with coordinates $r = R$, $\theta = 0$, time $t = 0$, are set in the form:

$$\sigma_\theta^0 = \sigma\left[\varepsilon - 1 + (\varepsilon + 1)Cos20\right]H_0(z)/2 \qquad \sigma_r^0 = \sigma\left[\varepsilon - 1 - (\varepsilon + 1)Cos20\right]H_0(z)/2$$

$$\sigma_{r\theta}^0 = \sigma\left[(\varepsilon + 1)Sin20\right]H_0(z)/2 \qquad u_r^0 = -\frac{\sigma}{\rho_1 c_1^2} zCos\theta H_0(z)$$

$$u_\theta^0 = -\frac{\sigma}{\rho_1 c_1^2} zSin\theta H_0(z)$$

$$z = c_1 t - R + rCos\theta, \qquad \varepsilon = -\nu_0/(1 - \nu_0)$$

where H_0 is the Heaviside function, σ - the stress at the front of the wave propagating in direction z, R - the shell radius, c_1 - the velocity of the expansion waves, ν_0 - the Poisson's ratio of the medium.

The motion of an elastic medium is described by wave equations for the scalar (φ) and vector (ψ) displacements potentials (the dot over the symbol denotes the time derivatives)

$$\ddot{\varphi}^{\Sigma} = c_1^2 \Delta \varphi^{\Sigma}, \qquad \ddot{\psi}^{\Sigma} = c_2^2 \Delta \psi^{\Sigma} \tag{1}$$

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial \theta^2}$$

where: c_2 is the shear wave velocity, Σ — here and hereinafter denotes the values corresponding to the total field which consists of excitations in the direct wave (φ^0) and the wave reflected and radiated from the shell surface $(\varphi^{\Sigma} = \varphi^0 + \varphi, \quad \psi^{\Sigma} = \psi^0 + \psi)$.

Shell motion is described by linear equations of the classical Kirchhoff – Love theory.

$$\begin{gathered} c^{-2}\ddot{\upsilon} = R^{-2}\left[\frac{\partial^2 \upsilon}{\partial \theta^2} + \frac{\partial w}{\partial \theta} + \delta\left(\frac{\partial^2 \upsilon}{\partial \theta^2} - \frac{\partial^3 w}{\partial \theta^2}\right)\right] + \beta \sigma_{r\theta}^{\Sigma} /_{r=R} \\ c^{-2} w = -R^{-2}\left[\frac{\partial \upsilon}{\partial \theta} + w + \delta\left(\frac{\partial^4 \upsilon}{\partial \theta^4} - \frac{\partial^2 w}{\partial \theta^2}\right)\right] + \beta \sigma_r^{\Sigma} /_{r=R} \\ \delta = h^2/(12R^2), \ \beta = \rho c^2 h \end{gathered} \tag{2}$$

where υ, w are the tangential and normal shell displacements,

h and ρ – its thickness and density, c - the velocity of sound in a thin plate.

The issue of interaction conditions of structural elements with the medium is a separate topic of investigation [22, 23] and here we restrict ourselves to one of the limiting cases of elastic interaction, i.e. to a sliding contact, when the boundary condition is written in the following form.

$$r = R: \qquad w = \frac{\partial \varphi}{\partial r} + \frac{1}{r}\frac{\partial \psi}{\partial \theta} + u_r^0, \sigma_r^{\Sigma} \quad \sigma_{r\theta}^{\Sigma} = \sigma_{r\theta} + \sigma_{r\theta}^0 = 0 \tag{3}$$

The problem is to determine the stresses and displacements in the shell and medium under zero initial conditions. The potentials φ and ψ must be equal to zero outside the expanding region restricted by the excitation front.

To solve the problem, expansion in the Fourier series in angle is used. The equations of motion (1), (2) for the m-th mode of vibrations $(m = 0, 1, 2, ...)$ take the following form:

$$\begin{gathered} c^{-2}\ddot{\upsilon}_m = -\alpha^2(1+\delta)\upsilon_m - \alpha R^{-1}(1+m^2\delta)w_m = 0 \\ c^{-2} w_m = -\alpha R^{-1}(1+m^2\delta)\upsilon_m - R^{-2}(1+m^4\delta)w_m + \beta \sigma_{r,m}^{\Sigma} /_{r=R} = 0 \end{gathered} \tag{4}$$

$$c_1^{-2}\ddot{\varphi}_m = \frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi_m}{\partial r} - \frac{m^2}{r^2}\varphi_m$$
$$c_2^{-2}\ddot{\psi}_m = \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi_m}{\partial r} - \frac{m^2}{r^2}\psi_m \qquad (5)$$

The boundary conditions (3) are transformed to:

$$r = R: \quad w_m = \frac{\partial\varphi_m}{\partial r} + mR^{-1}\psi_m + u^0_{r_1 m}, \quad \sigma^{\Sigma}_{r\theta,m} = \sigma_{r\theta,m} + \sigma^0_{r\theta,m} = 0 \qquad (6)$$

3. Solution of the problem

We apply to the system (4) - (6) the Laplace transform in time with the parameter p (the transformed values are denoted by superscript L). The solution of equation (5) in the images, taking into account the radiation and boundary conditions (6), will take the form:

$$\varphi = AK_m(pr/c_1), \quad \psi^L_m = BK_m(pr/c_2),$$
$$A_m = \left\{\left(u^{0,L}_{r,m} - w_m\right)\left[\left(\rho_1\rho^2 + m^2R^{-2}\right)K^{(0)}_{m,2} - 2\rho_1 c_2^2 R^{-1}K^{(1)}_{m,2}\right] + \sigma^{o,L}_{r\theta,m}\cdot mR^{-1}K^{(0)}_{m,2}\right\}\Delta^{-1}_m$$
$$B_m = \left\{\left(u^{0,L}_{r,m} - w_m\right)\rho_1 c_2^2 mR^{-1}\left(K^{(0)}_{m,1}R^{-1} - K^{(1)}_{m,1}\right) - \sigma^{o,L}_{r\theta,m}K^{(1)}_{m,1}\right\}\Delta^{-1}_m$$
$$\Delta_m = 2\rho_1 c_2^2 R^{-1}K^{(1)}_{m,1}K^{(1)}_{m,2} - 2\rho_1 c_2^2 m^2 R^{-2}K^{(1)}_{m,1}K^{(1)}_{m,2} - \rho_1\rho^2 K^{(1)}_{m,1}K^{(0)}_{m,2}$$

$$K^{(n)}_{m,q} = \frac{d^n}{dr^n}K_{m,q}\left(\frac{r\rho}{c_q}\right)/_{r=R} \quad \left(q = 1, 2;\ n = 0,1\right) \qquad (7)$$

where K_m is the Macdonald function of the m-th order.

To obtain the images of harmonics of the Fourier series of displacements and stresses in a direct wave, we first apply the Laplace transform, then expand into a Fourier series using the relation

$$H^L_o(p) = p^{-1}\exp\left(Rp\left(1 - rR^{-1}\cos\theta\right)/c_1\right)$$
$$I_m\left(R_p/c_1\right) = \frac{1}{2\pi}\int_0^{2\pi}\exp\left(Rp\cos\theta/c_1\right)\cos(m\theta)d\theta$$

where I_m are the modified Bessel functions of the first kind of the m-th order; the sought for values are obtained at $r = R$ (ρ_1 - is the density of the medium).

$$
\begin{aligned}
&\left[\sigma^{o}_{\theta,m}\right]^{L}=\sigma e_m e^{-\gamma}p^{-1}\left[(\varepsilon+1)I^{(2)}_{m,1}-I^{(0)}_{m,1}\right]\\
&\left[\sigma^{o}_{2,m}\right]^{L}=\sigma e_m e^{-\gamma}p^{-1}\left[\varepsilon I^{(0)}_{m,1}-(\varepsilon+1)I^{(2)}_{m,1}\right]\\
&\left[\sigma^{o}_{r\theta,m}\right]^{L}=\sigma e_m e^{-\gamma}p^{-1}\gamma^{-1}m(\varepsilon+1)\left[I^{(1)}_{m,1}-\gamma^{-1}I^{(0)}_{m,1}\right]\\
&\left[u^{o}_{r,m}\right]^{L}=\sigma e_m e^{-\gamma}\left(\rho_1 c_1 p^2\right)^{-1}I^{(1)}_{m,1}\\
&\left[u^{o}_{\theta,m}\right]^{L}=\sigma e_m e^{-\gamma}\left(\rho_1 c_1 \gamma p^2\right)^{-1}mI^{(0)}_{m,1}\\
&I^{(n)}_{m,q}=\frac{d^n}{dr^n}I_m\left(\frac{rp}{c_q}\right)/_{r=R}\quad (q=1,2),(n=0,1,2)\\
&e_0=1,\ e_m=2\quad (m\neq 0),\ \gamma=Rpc_1^{-1}
\end{aligned}
\tag{8}
$$

Entering (7) into the stress expression through potentials and taking into account relations (8), we determine the values of the total stresses acting on the shell in the medium. To obtain this solution, we use the expression for Vronskian cylinder functions, and the recurrence relations for I_m, K_m [24].

$$
\begin{aligned}
&\sigma^{\Sigma,L}_{r,m}=\sigma^{0,L}_{r,m}+\sigma^{L}_{r,m}=\bar{\sigma}^{L}_{r,m}+2\rho_1 c_2^2 R^{-1}\left(m^2-1-F_m\right)w^L_m\\
&F_m=\left[\left(\rho_1 p^4 R/2c_2^2+2\rho_1 p^2 m^2 R^{-1}\right)K^{(0)}_{m,1}K^{(0)}_{m,2}-\rho_1 p^2(K^{(0)}_{m,1}K^{(1)}_{m,2}+m^2K^{(1)}_{m,1}K^{(0)}_{m,2})\right]\Delta_m^{-1}
\end{aligned}
\tag{9}
$$

$$\sigma^{\Sigma,L}_{r\theta,m}=\sigma^{0,L}_{r,m}+\sigma^{L}_{r\theta,m}=0$$

where $\bar{\sigma}^{L}_{r,m}$ are the stresses on the surface of a rigid stationary cylinder, which take the following form:

$$\bar{\sigma}^{L}_{r,m}=\frac{\sigma e_m e^{-\gamma}}{pR\Delta_m}\left[2\rho_1 c_2^2 R^{-1}K^{(1)}_{m,2}-\left(\rho_1 p^2+2\rho_1 c_2^2 m^2 R^{-2}\right)K^{(0)}_{m,2}\right]\tag{10}$$

3.1. Fixed cylinder. Assuming that $w=0$ (9) we obtain the exact solution in the images of the diffraction problem of a stepped pressure wave on a fixed cylinder under sliding contact, which has the form (10). To reverse the resulting expression in explicit form is not possible. We will seek the asymptotic form of the stress field at large values of time from the beginning of the process. Assuming p in formula (10) small, we hold the first terms of the series in the Bessel functions expansions. As is shown in [12], if the asymptotic form of images $(p\to 0)$ has a singular point of algebraic - logarithmic type

$$f^L \sim p^k/\ln p\quad (k\neq 0,1,2)$$

then the asymptotic form of the original $(t\to\infty)$ is

$$f\ \sim t^{-k-1}/\ln t\tag{11}$$

With vanishing $p\to 0$ and $m=1$ using for formula (11), we obtain the following asymptotic representations at $t\to\infty$

$$\bar{\sigma}_{r,0}=-\sigma,\ \bar{\sigma}_{r,0}=0,\ \bar{\sigma}_{r,1}=\frac{2(\varepsilon+1)}{(\varepsilon+3)}\ \frac{2c_1 t\sigma}{R\ln(t\alpha)}\tag{12}$$

$$\bar{\sigma}_{r\theta,1} = 0,\ \bar{\sigma}_{r,2} = -\frac{\sigma(\varepsilon+1)}{\varepsilon+5},\ \bar{\sigma}_{r\theta,1} = 0,\ \bar{\sigma}_{\theta,m} = -\varepsilon\bar{\sigma}_{r,m}\ \ (m = 0,1)$$

$$\bar{\sigma}_{r,m} = \bar{\sigma}_{\theta,m} = 0(m > 3)\quad \alpha = \frac{2c_1}{RC}\quad \frac{(1+\varepsilon)^{\frac{1}{\varepsilon+3}}}{2}$$

where $C = 1{,}781072418$ is the Euler constant.

At $m \geq 3$ all parameters of excitations asymptotically tend to zero. According to the formulas

$$F = \int_0^{2\pi} (\bar{\sigma}_{r\theta} Sin\theta - \bar{\sigma}_r Cos\theta)\, Rd\theta\,, \quad I = \int_0^t Fdt$$

we calculate the total force acting on the fixed cylinder in the direction of wave motion, and its momentum,

$$F = \frac{4\pi c_1 t\sigma}{\ln(t\alpha)} \cdot \frac{\varepsilon+1}{\varepsilon+3}\,, \qquad I = \frac{2\pi c_1 t^2\sigma}{\ln(t\alpha)} \cdot \frac{\varepsilon+1}{\varepsilon+3} \tag{13}$$

It can be seen from (12) and (13) that, for sufficiently large values of time, the main contribution to stresses is made by the first mode of stress and the total force grows with time $t/\ln t$. The expression for the total force and momentum (13) coincides with the previously obtained result for the rigid contact [15].

3.2 Thin-walled shell. Substituting (9) in (4), we solve the system of algebraic equations for the sought for functions

$$w_m^L = \frac{N_4\bar{\sigma}_{rm}}{\beta(N_1N_4 - N_2N_3)}\,, \qquad \upsilon_m = -\frac{N_3\bar{\sigma}_{r_1m}}{\beta(N_1N_4 - N_2N_3)} \tag{14}$$

$$N_1 = p^2c^{-2} + (1+m^4\delta)R^{-2} - \gamma_0(m^2 - 1 - F_m)R^{-2}$$

$$N_2 = N_3 = mR^{-2}(1+m^2\delta) \qquad N_4 = p^2c^{-2} + m^2R^{-2}(1+\delta)$$

$$F_m = p^2c^{-2} + R^{-2}(1+m^2\delta) - \gamma_0(m^2-1)R^{-2} - F_m\beta^{-1}, \qquad \gamma_0 = \frac{2\rho_1 c_2^2 R}{\rho c^2 h}$$

Formulas (9), (14) completely determine the solution to the problem in the images. As before, we determine the asymptotic form ($t \to \infty$) of the Fourier coefficients for each harmonics.

At $m > 2$ asymptotic form gives zero values of these coefficients: over time, the first three modes ($m = 0, 1, 2$) are determinant. The movement of the shell as a solid whole corresponds to the first mode

$$w_1 = -\upsilon_1 = -t\sigma / \rho_1 c_1 \tag{15}$$

The same mode determines the total force F and its momentum I their asymptotic forms are

$$F = 0\,, \qquad I = M\rho / \rho_1 c_1 \tag{16}$$

$M = 2\pi R\rho h$ is the shell mass.

It follows from (15) and (16) that, over time, the shell velocity becomes equal to the particles velocity in the incident wave. The total stresses in the medium at the boundary with the shell (as well as the stresses in the shell) moving together with the surrounding medium ($m = 1$), and therefore the amplitudes of the total force F tend to zero. The momentum remains a finite value and is equal to the product of the shell mass and the velocity of the medium particles.

The asymptotic form of zero and second harmonics completely determines the stresses in the medium and shell at a relatively large interval of time from the beginning of the interaction:

$$m = 0: \qquad w_0 / R = -(R/h)(1+\gamma_0)^{-1}\sigma / \rho c^2; \qquad \sigma^{ц}_{\theta,0} = -(R/h)(1+\gamma_0)^{-1}\sigma$$

$$\sigma^{\Sigma}_{r,0} = -\sigma(1+\gamma_0)^{-1}, \qquad \sigma^{\Sigma}_{\theta,0} = \sigma(\varepsilon - \gamma_0(1+\gamma_0)^{-1}),$$

$$\sigma^{\Sigma}_{r\theta,0} = v_0 = 0$$

$$m = 2: \qquad w_2 / R = -(8 - h^2R^{-2}/6)B^{-1}\sigma / \rho_1 c_1^2 \qquad v_2 / R = (4 + h^2R^{-2}/3)B^{-1}\sigma / \rho_1 c_1^2$$

$$\sigma^{ц}_{\theta,2} = (R/h)(1+\varepsilon)(h^2R^{-2}/2\gamma_0)B^{-1}\sigma \tag{17}$$

$$\sigma^{изг}_{\theta,2} = 3(1+\varepsilon)B^{-1}\sigma / 2\gamma_0$$

$$\sigma^{\Sigma}_{r\theta,2} = \sigma^{\Sigma}_{r,2} = 0, \qquad \sigma^{\Sigma}_{\theta,2} = 2(1+\varepsilon)\sigma$$

$$m > 2 \qquad w_m = \upsilon_m = 0$$

Here $\sigma^{ц}_{\theta}, \sigma^{изг}_{\theta}$ are the chain and bending stresses in the shell, calculated by formulas

$$\sigma^{ц}_{\theta} = \frac{ER^{-1}}{1-v^2}\left(w + \frac{\partial v}{\partial \theta}\right); \quad \sigma^{изг}_{\theta} = \frac{Eh}{2(1-v^2)R^2}\left(\frac{\partial v}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2}\right)$$

It follows from the asymptotic form (17) that the interaction with surrounding elastic medium reduces the amplitudes of axisymmetric displacements and stresses. With a decrease in the main dimensionless parameter of the problem γ_0 , the influence on the medium weakens and at $\gamma_0 = 0$ disappears: a medium with zero rigidity (ideal compressible fluid) does not change the amplitudes and stresses in the shell, compressed by all-round pressure.

A qualitative difference between the influence of an elastic medium compared with an acoustic one is that in the latter case, the displacements and stresses corresponding to the second mode are asymptotically equal to zero, while in the case of an elastic medium ($\gamma_0 \neq 0$), their contribution to the total amplitudes increases with γ_0 and can become the determinant one.

We briefly analyze the asymptotic forms for the limit cases of shell interaction with an elastic medium, i.e. rigid and sliding contacts. The main difference is observed in the second mode($m = 2$). In the case of a sliding contact, the chain stresses in the second mode are small ($\sigma^{ц}_{\theta,2} \sim 0(h)$) and the main contribution is made by the zero form. Therefore, at $t \to \infty$ the nature of $\sigma^{ц}_{\theta}$ distribution tends to axisymmetric one.

4. Algorithm of numerical solution and analysis of results

At a rigid contact, the chain stress is determined by the sum of the zero and second forms [15]. This leads to an inhomogeneous distribution of stresses along the circumference. At the lateral point ($\theta = \pi / 2$), the compressive stresses add up in two forms and determine the maximum value of $\sigma^{ц}_{\theta}$, and at $\gamma_0 >> 1$ in the vicinity of these points the zones of tensile stresses appear. Bending stresses are determined by the second form, and their amplitude is by an order of magnitude lower than that of chain ones. The value $\sigma_o^{изг}$ in the case of sliding contact is slightly greater than at the rigid contact.

Next, it is necessary to find out the error of the asymptotic estimates over a finite time interval, determine the dynamics coefficients, and establish the limits of applicability of these estimates in specific cases with real parameters of the shell and medium. Numerical solution serves to this goal.

The numerical solution is carried out as follows. Medium motion is written down by the dynamic equations of the theory of elasticity in displacements, which after expansion into a Fourier series in angular coordinate takes the following form.

$$\frac{\partial^2 u_{r,m}}{\partial t^2} = c_1^2\left[\frac{\partial^2 u_{r,m}}{\partial r^2} + \frac{1}{r}\frac{\partial u_{r,m}}{\partial r}\right] - \frac{c_1^2 + c_2^2 m^2}{r^2}u_{r,m} + \frac{\left(c_1^2 - c_2^2\right)m}{r}\frac{\partial u_{\theta,m}}{\partial r} - \frac{\left(c_1^2 + c_2^2\right)m}{r^2}u_{\theta,m}$$

$$\frac{\partial u_{\theta,m}}{\partial t^2} = c_2^2\left[\frac{\partial^2 u_{\theta,m}}{\partial r^2} + \frac{1}{r}\frac{\partial u_{\theta,m}}{\partial r}\right] - \frac{c_2^2 + c_1^2 m^2}{r^2}u_{\theta,m} - \frac{\left(c_1^2 - c_2^2\right)m}{r}\frac{\partial u_{r,m}}{\partial r} - \frac{\left(c_1^2 + c_2^2\right)m}{r^2}u_{r,m}$$

An explicit finite difference scheme is applied to the solution of these equations, and a method of minimizing numerical dispersion (MND) is used [13]. The essence of the method is as follows: in these equations, the first and second derivatives are replaced by central differences, the zero derivatives are approximated by three points.

$$w_i = \frac{w_{i+1} + 2w_i + w_{i-1}}{4}, \quad w(u_r, u_\theta).$$

Such an approximation increases the stability limit of the scheme and allows one to choose a time step equal to a step along the radial coordinate. The MND method makes it possible to exactly describe the frontal discontinuities in solving numerous non-stationary problems [25].

Calculations showed that at $h_1 = 0{,}05R$ (h_1 is the step of the difference grid) quite satisfactory accuracy is achieved; at a further decrease in h_1, the change in results is observed in the third significant figure. When calculating the sums of the Fourier series, 11 terms were held ($m = 0, ..., 10$), an increase in the number of forms did not lead to a change in results by more than 3%. In calculations, ρ_1, c_1 R were taken as the units of measurement.

Figures. 1 - 4 present the results of calculations for the following parameters of the shell and medium $E = 23, \rho = 2.9, \quad h = 0.04, \quad \nu = \nu_1 = 0{,}25$.

Oscillograms of individual forms of chain stresses show that the zero form at $t \approx 3$ reaches the statics (Fig. 1). This corresponds to the time when the incident wave completely flows over the shell. Chain stresses of the second form are oscillatory in nature and at $t \to \infty$ tend to zero (Fig. 2). As a result, for sufficiently large values of time, the main contribution is made by the zero form and the stress state becomes axisymmetric (Fig. 3). A change in the oscillogram curvature corresponds to the arrival of waves propagating along the shell in the circumferential direction. With these parameters, the numerical solution is 10% higher than the asymptotic solution. With a decrease in shear rigidity of the medium, the chain stresses increase. The dynamic coefficient depends on γ and increases with its. The amplitude of the bending stresses is by an order of magnitude less than the chain ones, and their maximum value does not significantly exceed the asymptotic form (Fig. 4).

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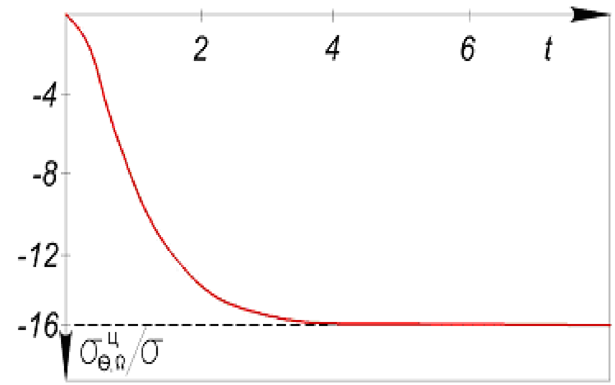

Figure 1. Oscillogram of chain stresses in zero mode
_____ numerical solution
- - - - asymptatic solution (17)

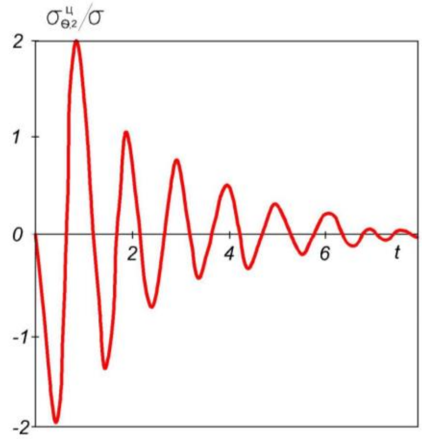

Figure 2. Oscillogram of chain stresses in the second mode

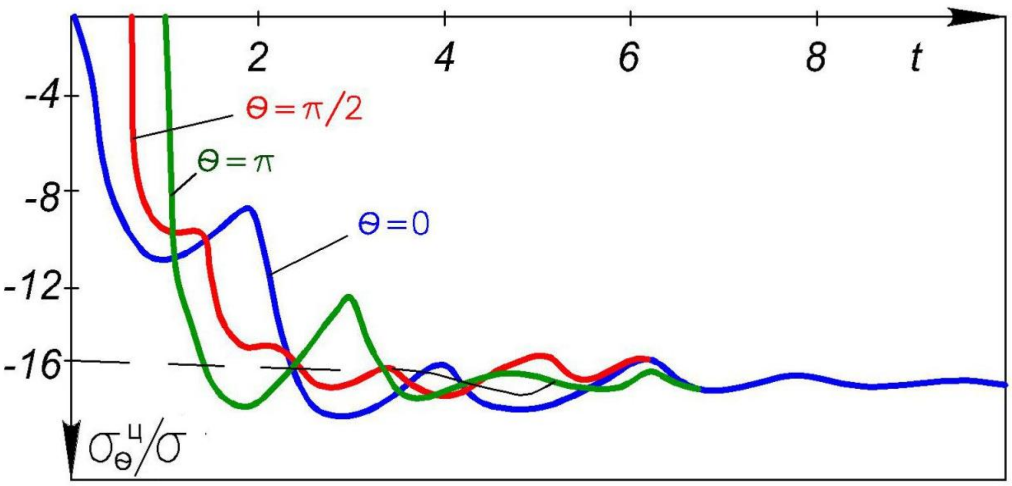

Figure 3. Oscillogram of chain stress

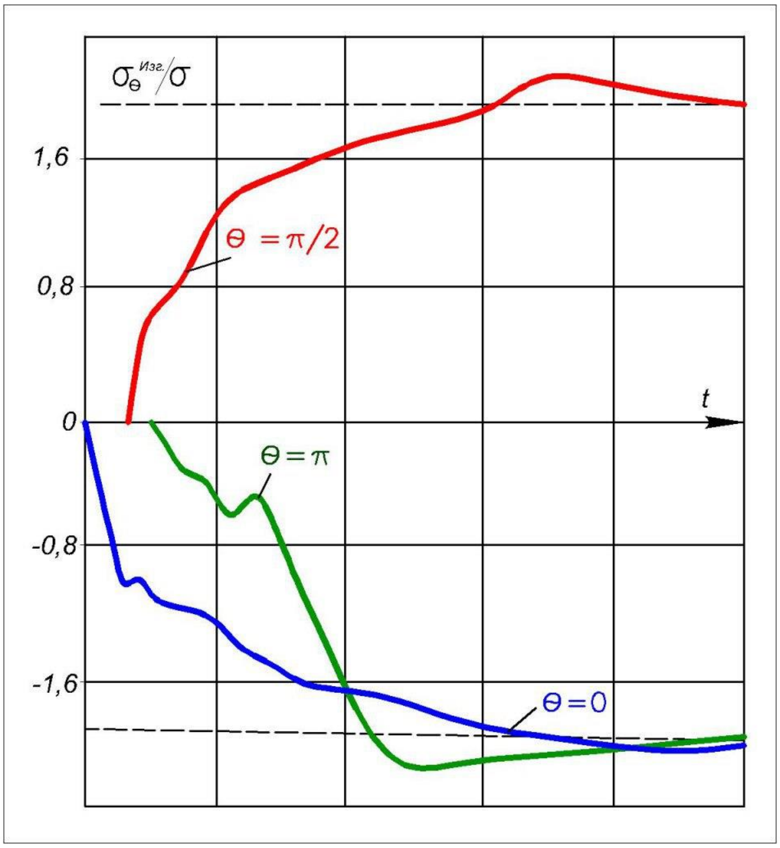

Figure 4. Oscillogram of bending stress

5. Conclusion

A numerical-analytical study of plane wave unsteady interaction with cylindrical shell allows us to draw the following conclusions:

1. An asymptotic solution to the problem is obtained. The main contribution is made by zero mode. The chain stresses are by an order of magnitude greater than the bending stresses.
2. The proposed difference scheme, built on the basis of minimizing numerical dispersion, allows us to accurately describe the frontal discontinuities. A comparison of the two solutions shows that, at $t \geq 6R/c_1$ the asymptotic solution completely describes the stress-strain state of the shell.
3. The maximum chain stresses are 10% greater than the asymptotic solution. The dynamic coefficient depends on γ and increases with it.

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