

# Pulsating flow of stationary elastic-viscous fluids in flat-wall channel

*A. Sh. Begjanov*<sup>1</sup>, *Z. K. Shukurov*<sup>2</sup>, and *B. Sh. Yuldoshev*<sup>3\*</sup>

<sup>1</sup>Urgench State University, Urgench, Uzbekistan

<sup>2</sup>Kattakurgan branch of Samarkand State University, Samarkand, Uzbekistan

<sup>3</sup>“Tashkent Institute of Irrigation and Agricultural Mechanization Engineers” National Research University, Tashkent, Uzbekistan

**Abstract.** In this article, a specific problem of pulsating flow of visco-elastic fluids in a flat channel in a stationary state was solved. The main goal is to study the movement of visco-elastic fluids based on simplified mathematical models and to determine the existing hydrodynamic laws and hydrodynamic effects in the pulsating flow of a Newtonian fluid based on the obtained results.

## 1 Introduction

Stationary oscillating (pulsating) flows, in which transient processes occur in the flow of liquids, are particularly interesting in science, technical and technological processes. In such processes, even if the fluid movement occurs in a stationary mode due to the presence of oscillatory motion, the considered process consists of a periodic function of time. In this case, it is considered that the fluid fluctuations occur in the same state in each period. Therefore, when solving fluid flow problems, periodic functions of time can be used, which makes it much easier to solve a system of differential equations. Many scientific and practical studies have been devoted to the pulsating flows of Newtonian fluids in flat channels and cylindrical pipes by domestic and foreign scientists. In particular, in research studies [1-7], the stationary oscillation flows of pulsating viscous fluids in channels and pipes have been sufficiently studied. Scientific research on the pulsating flow of viscous fluids was applied to the circulatory system of biomechanics [8-12]. In this area, the research works conducted by applying the generalized topological models of Shulman-Khusid [13-20] to the pulsating flow of visco-elastic fluids are insufficient. It is known that the generalized topological models of Shulman-Husid play an important role in characterizing the pulsating behavior of polymer fluids, turbid water mixtures, and other similar fluids [21-24].

## 2 Methods

In formulating the problem, the distance between the walls of a flat channel is defined as  $2h$ , and the length of the channel is defined as  $L$ . Here  $L$  is large enough that  $h/L=1$ , the

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\*Corresponding author: [baxtiyor\\_yuldashev68@mail.ru](mailto:baxtiyor_yuldashev68@mail.ru)

condition is satisfied. The flow is stabilized in such cases, and the transverse velocity value is found using the continuity equation. And channel axes are defined as follows: Axis  $\mathcal{X}$  is directed along the middle of the channel in a horizontal direction and is called the longitudinal axis, and the axis  $\mathcal{Y}$  is taken in a vertical direction perpendicular to the axis  $\mathcal{X}$  and is referred to as the vertical axis. The mathematical model of the problem is as follows:

$$\left\{ \begin{array}{l} \frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial \tau}{\partial y}, \quad \frac{\partial u}{\partial x} + \frac{\partial g}{\partial y} = 0, \\ \tau = \sum_{k=1}^{\infty} \tau_k, \quad \frac{\partial \tau_k}{\partial t} + \frac{g_k}{\lambda_k} \tau_k = 2p_k D_{21}, \\ \frac{\partial p_k}{\partial t} + \frac{g_k}{\lambda_k} p_k = \frac{\eta_k}{\lambda_k^2} f_k, \quad D_{21} = \frac{1}{2} \frac{\partial u}{\partial y}. \end{array} \right. \quad (1)$$

Since the pulsating flows of visco-elastic liquids are considered at small deformations of liquid particles, functions  $f_k$  and  $g_k$  can be taken as invariant  $f_k = 1$ ,  $g_k = 1$  based on the work [10-12]. Then equations (1) can be expressed in form (2) by performing mathematical operations:

$$\left\{ \begin{array}{l} i\omega u_1 = -\frac{1}{\rho} \frac{\partial p_1}{\partial x} + \frac{1}{\rho} \frac{\partial \tau_1}{\partial y}, \\ \frac{\partial u_1}{\partial x} + \frac{\partial g_1}{\partial y} = 0, \\ \tau_1 = \sum_{k=1}^{\infty} \tau_{1k}, \quad i\omega \tau_{1k} + \frac{1}{\lambda_k} \tau_{1k} = p_k \frac{\partial u_1}{\partial y}, \\ i\omega p_k + \frac{1}{\lambda_k} p_k = \frac{\eta_k}{\lambda_k^2}. \end{array} \right. \quad (2)$$

Since the flow is symmetrical about the longitudinal axis of the flat channel, the boundary conditions are formulated as follows:

$$\begin{aligned} y = h \text{ да } u_1 = 0, v_1 &= \frac{h\gamma^*}{\eta} (p_1 - p_c), \\ y = 0 \text{ да } \frac{\partial u_1}{\partial y} &= 0, v_1 = 0, \\ x = 0 \text{ да } p_1 &= p_1^0, \\ x = L \text{ да } p_1 &= p_1^L. \end{aligned} \quad (3)$$

Solving the resulting system of equations (2) using boundary conditions (3), we express the solution by changing the form as follows:

$$p_1(x) = p_1^0 \frac{sh\sqrt{k} z_0 L \left(1 - \frac{x}{L}\right)}{sh\sqrt{k} z_0 L} + p_1^L \frac{sh\sqrt{k} z_0 L \frac{x}{L}}{sh\sqrt{k} z_0 L}, \tag{4}$$

$$\langle u_1(x) \rangle = p_1^0 \frac{ch\sqrt{k} z_0 L \left(1 - \frac{x}{L}\right)}{sh\sqrt{k} z_0 L} - p_1^L \frac{sh\sqrt{k} z_0 L \frac{x}{L}}{sh\sqrt{k} z_0 L} \sqrt{\bar{z}}. \tag{5}$$

The found formulas (4) and (5) represent the formulas for finding the pressure and averaged velocities of the fluid along the longitudinal axis in a flat channel with a permeable wall, respectively. By analyzing the characteristics of the magnitude in the argument of the hyperbolic sine and hyperbolic cosine functions in these formulas, it is possible to determine the propagation speed of pulse waves and their attenuation along the longitudinal axis. From the found solutions (4) and (5), the formula defining the distribution law of the longitudinal velocity profile in the pulsating flow of a Newtonian fluid is derived. For this, it is sufficient to set the relaxation coefficient in a visco-elastic fluid equal to zero.

To obtain calculation results using the found formulas (4) and (5), one begins by analyzing the characteristics of the quantities in the argument of the hyperbolic sine and hyperbolic cosine functions. It is known that these quantities are one of the main factors of wave propagation. With the help of these arguments, it was noted above that it is possible to determine the propagation speed of pulse waves and its attenuation along the longitudinal axis. Taking this into account, below, we present the analysis results of these quantities for a Newtonian fluid. The found formulas (4) and (5) express the change of pressure and velocity along the longitudinal axis, and since these formulas mainly depend on the complex parameter  $\sqrt{k} z_0 L$ , expressing it in this form:

$$\sqrt{k} z_0 L = \bar{\chi} + \bar{\beta}i. \tag{6}$$

We distinguish the real and abstract parts of  $Z_0$  as follows:

$$z_0 = \left[ \frac{3}{i\alpha_0^2} \left( 1 - \frac{\sin\left(i^{\frac{3}{2}}\alpha_0\right)}{\left(i^{\frac{3}{2}}\alpha_0\right)\cos\left(i^{\frac{3}{2}}\alpha_0\right)} \right) \right]^{-1} = \frac{\bar{R}}{3} + \frac{\bar{L}}{3}i,$$

$$\bar{R} = \frac{\alpha_0^2 (A_1^2 + B_1^2)}{(A_2^2 + B_2^2)} B_2,$$

$$\bar{L} = \frac{(A_1^2 + B_1^2)\alpha_0^2}{A_2^2 + B_2^2} A_2.$$

Here

$$A_1 = \bar{A}\bar{M}_1 + \bar{B}M_1, \quad B_1 = \bar{A}M_1 - \bar{B}\bar{M}_1,$$

$$A_2 = (A_1^2 + B_1^2) - A_1C - B_1D, \quad B_2 = (B_1C - A_1D),$$

$$C = \sin M_1 ch \bar{M}_1, \quad D = -\cos M_1 sh \bar{M}_1,$$

$$\bar{A} = \sin M_1 sh \bar{M}_1, \quad \bar{B} = \cos M_1 ch \bar{M}_1, \quad M_1 = \frac{\alpha_0}{\sqrt{2}}, \quad \bar{M}_1 = \frac{\alpha_0}{\sqrt{2}},$$

$$i^{\frac{3}{2}} \alpha_0 = \frac{\alpha_0}{\sqrt{2}}(1-i) = M_1 - \bar{M}_1 i.$$

Now, putting the value of  $z_0$  and  $\bar{k}$  into formula  $\sqrt{\bar{k}} z_0 L = \bar{\chi} + \bar{\beta} i$ ,  $\bar{\chi}$ ,  $\bar{\beta}$  is found:

$$\sqrt{\bar{k}} z_0 L = L \sqrt{\frac{3\gamma^*}{h^2}} \sqrt{\frac{1}{3}} \sqrt[4]{\bar{R}^2 + \bar{L}^2} \left( \cos \frac{\varphi}{2} + i \sin \frac{\varphi}{2} \right) = \sqrt{\frac{\gamma^*}{h^2}} L \sqrt[4]{\bar{R}^2 + \bar{L}^2} \left( \cos \frac{\varphi}{2} + i \sin \frac{\varphi}{2} \right).$$

Here  $\alpha_0 = \sqrt{\frac{\omega}{v}} h$ ,  $v = \frac{\eta}{\rho}$ ,  $\varphi = \text{arctg} \frac{\bar{L}}{\bar{R}}$ .

From this formula,  $\bar{\chi}, \bar{\beta}$  is found accordingly. That is

$$\bar{\chi} = \sqrt{\frac{\gamma^*}{h^2}} L (\sqrt[4]{\bar{R}^2 + \bar{L}^2} \cos \frac{\varphi}{2}),$$

$$\bar{\beta} = \sqrt{\frac{\gamma^*}{h^2}} L (\sqrt[4]{\bar{R}^2 + \bar{L}^2} \sin \frac{\varphi}{2}).$$

Here,  $\bar{\chi}$  is a dimensionless coefficient that determines the attenuation of the wave along the length;  $1/\bar{\beta} - OX$  is a dimensionless quantity characterizing the propagation speed of the pulse wave along the axis.

### 3 Results and Discussion

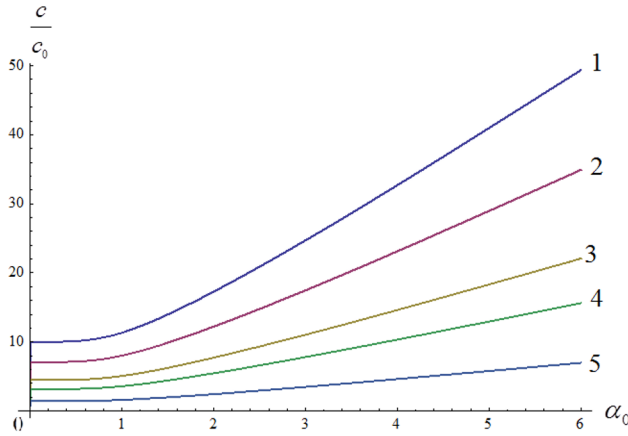
The propagation speed of the pulse wave is determined by the following formula  $c = \omega L / \bar{\beta}$ . From this formula, the dimensionless form of the propagation speed of the pulse wave is found as follows:

$$c = \frac{\omega L}{\bar{\beta}} = \frac{\omega L}{\sqrt{\frac{\gamma^*}{h^2}} L (\sqrt[4]{\bar{R}^2 + \bar{L}^2} \sin \frac{\varphi}{2})} = \frac{v}{h} \sqrt{\frac{1}{\gamma^*}} \alpha_0^2 \left( \sqrt[4]{\bar{R}^2 + \bar{L}^2} \sin \frac{\varphi}{2} \right)^{-1},$$

$$\frac{c}{c_0} = \alpha_0^2 \sqrt{\frac{1}{\gamma^*}} \left( \sqrt[4]{\bar{R}^2 + \bar{L}^2} \sin \frac{\varphi}{2} \right)^{-1}. \tag{7}$$

Here  $c_0 = 5 \cdot v / h$  is the base pulse wave propagation speed.

Based on the formula (7) determined as a result of solving the problem, the propagation speed of the pulse wave depending on the vibration frequency parameter, is depicted in Figure 1.



**Fig. 1.** Variation of pulse wave propagation speed at different values of wall permeability coefficient  $\gamma^*$  depending on vibration frequency parameter 1-0.01; 2-0.02; 3; 0.05; 4-0.1;5-0.5

The variation of the magnitude, which is the inverse of the magnitude of the wave attenuation obtained concerning the wavelength, depending on the parameter of the oscillation frequency, is determined by the following formula:  $\frac{\partial \ln p}{\partial x} = -\frac{\bar{\chi}}{L}$ .

Here  $\bar{\chi}$  characterizes the decay of the wave.

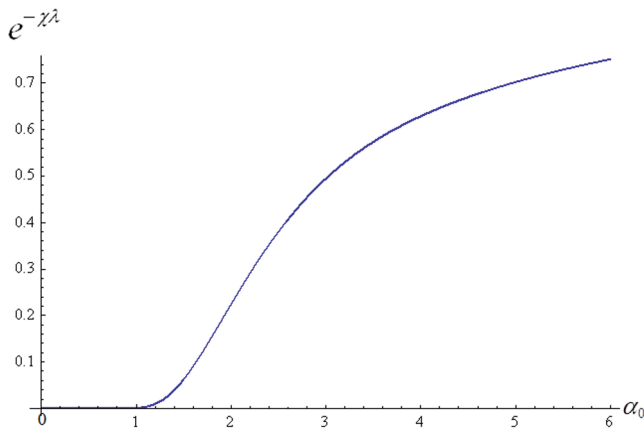
Integrating  $\frac{\partial \ln p}{\partial x} = -\frac{\bar{\chi}}{L}$ , we get:

$$\ln p = -\frac{\bar{\chi}}{L}x + \ln c \quad \text{and} \quad x = 0 \text{ et } p = p_0, \quad \text{for being } \ln p_0 = \ln c, \quad \text{and the result is as follows:}$$

$$\frac{p}{p_0} = e^{-\bar{\chi}\lambda}$$

Here  $\lambda = x/L$ .

This formula above represents the inverse of the magnitude of the wave attenuation taken concerning the wavelength.



**Fig. 2.** Variation of magnitude, which is inverse of magnitude of wave attenuation obtained concerning wavelength, as function of oscillation frequency parameter.

## 4 Conclusions

Figure 1 shows the pulse wave propagation speed variation depending on the vibration frequency parameter. At sufficiently small values of the vibration frequency parameter, it was found that the propagation velocity of the pulse wave can be found by formula  $c_0 = 5\nu/h$ , and this formula was adopted as the formula representing the propagation velocity of the base pulse wave. It is shown in the figure that the propagation speed of the pulse wave does not significantly differ from the propagation speed of the base pulse wave at small values of the vibration frequency parameter. At large values of the vibration frequency parameter, the propagation speed of the pulse wave is significantly different from the base speed, and its increase ensures an increase in the propagation speed of the pulse wave.

It can be seen from Fig. 2. that at small values of the vibration frequency parameter, wave extinction almost does not occur, while at its large values, the wave extinction index increases significantly. Although the variation of the magnitude, which is inverse to the magnitude of the wave attenuation obtained concerning the wavelength, depending on the oscillation frequency parameter, is caused by the wall permeability coefficient, it is almost independent of its variation.

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