“TOSHKENT IRRIGATSIYA VA QISHLOQ XO‘JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI” MILLIY TADQIQOT UNIVERSITETI

# Fan: Materiallar qarshiligi

## Mavzu 06: Oddiy shakllarning inersiya momentlari

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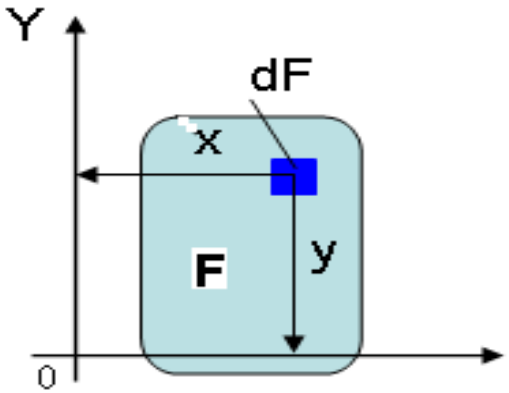
Mexanika va kompyuterli modellashtirish kafedrasi dotsenti

## Reja:

1. Tekis shakllarning geometrik xarakteristika-larini qanday qilib o'zgartirish yoki oshirish mumkin?
2. Parallel o'qlarga nisbatan tekis shakllarning geometrik xarakteristikalari qanday aniqlanadi?
3. Bosh inersiya o‘qlari va bosh inersiya momentlari?
4. Tekis shaklning o‘qlarga nisbatan inersiya va qarshilik momentlari.

# Tekis shakllarning statik va inertsiya momentlari




1-rasm

$$S_x = \int_F y dF \qquad S_y = \int_F x dF \qquad (1)$$

$$S_x = F \cdot Y_c, \quad S_y = F \cdot X_c \qquad (2)$$

$I_x = \int_F y^2 dF$ - tekis shaklning x o'qiga nisbatan inertsiya momenti; (3)

$I_y = \int_F x^2 dF$ - tekis shaklning u o'qiga nisbatan inertsiya momenti; (4)

$I_{xy} = \int_F xy dF$ - tekis shaklnig markazdan kochirma inertsiya momenti; (5)

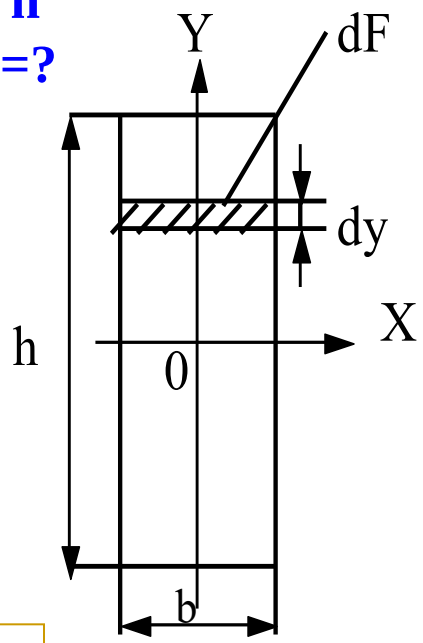
$I_\rho = \int_F \rho^2 dF$ - tekis shaklning qutb inertsiya momenti. (6)

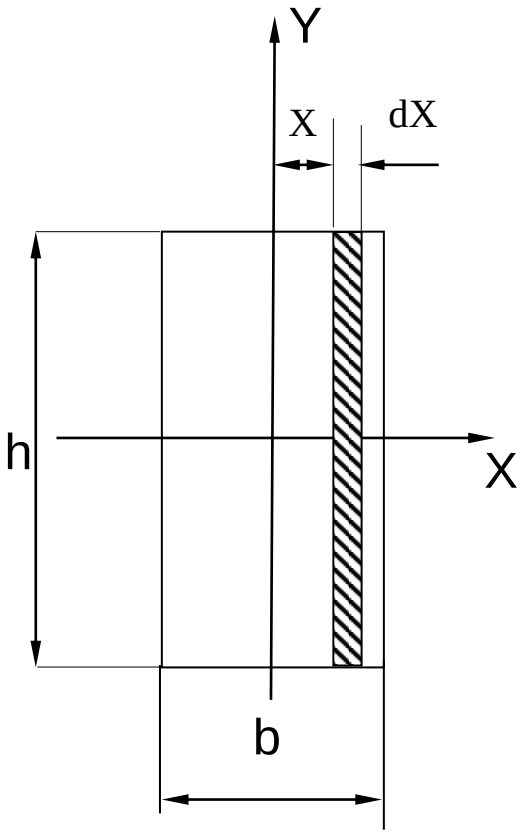
# Oddiy shakllarning inersiya momentlari
## (markazdan o'tgan o‘qlarga nisbatan)

**a) *To'gri to'rtburchak***

**berilgan: b, h**

**t.k.: $I_x$=? $I_y$=?**




$$dF = hdX$$

$$dF = bdY$$

$$I_Y = \int_F X^2 dF$$

$$I_X = \int_F Y^2 dF$$

$$I_X = \int_{-h/2}^{h/2} bY^2 dY = b\int_{-h/2}^{h/2} Y^2 dY = \frac{bh^3}{12}$$

$$I_Y = \int_{-b/2}^{b/2} hX^2 dX = h\int_{-b/2}^{b/2} X^2 dX = \frac{hb^3}{12}$$

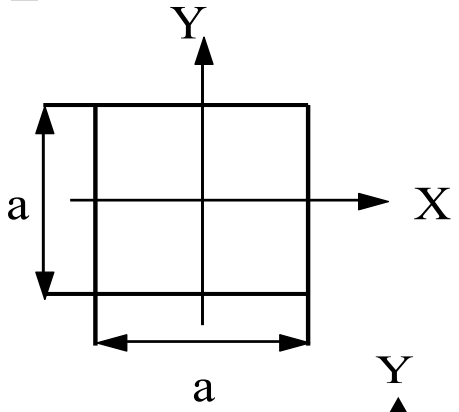
# Oddiy shakllarning inersiya momentlari

***Isboti:***

$$I_x = \int_F y^2 \cdot dF = \int_{-h/2}^{h/2} y^2 \cdot b \cdot dy = b \int_{-h/2}^{h/2} y^2 \cdot dy = \frac{by^3}{3}\Bigg|_{-h/2}^{h/2} =$$

$$= \frac{b}{3}\left[\left(\frac{h}{2}\right)^3 - \left(-\frac{h}{2}\right)^3\right] = \frac{b}{3}\left[\frac{h^3}{8} + \frac{h^3}{8}\right] = \frac{2bh^3}{24} = \frac{bh^3}{12}$$

***b)Kvadrat:***

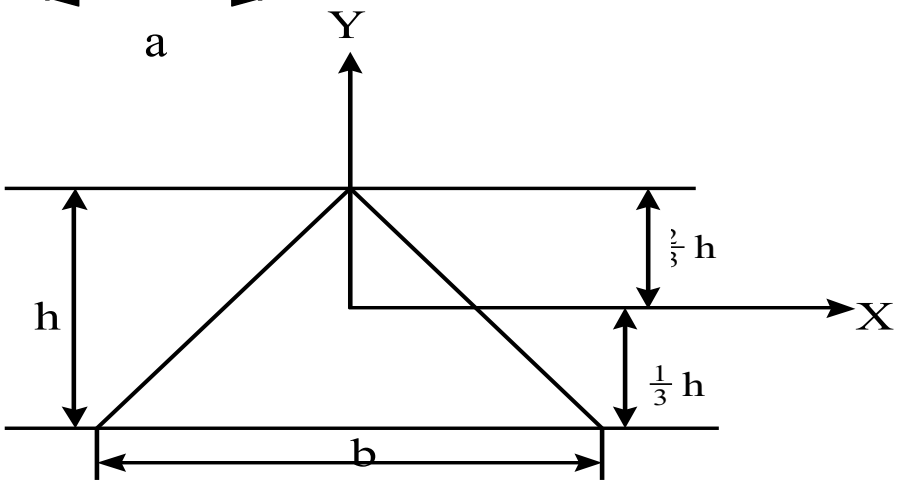



$$I_x = I_y = \frac{a^4}{12}$$

***v) Uchburchak***

berilgan: b, h

t.k.: $I_x$=? $I_y$=?

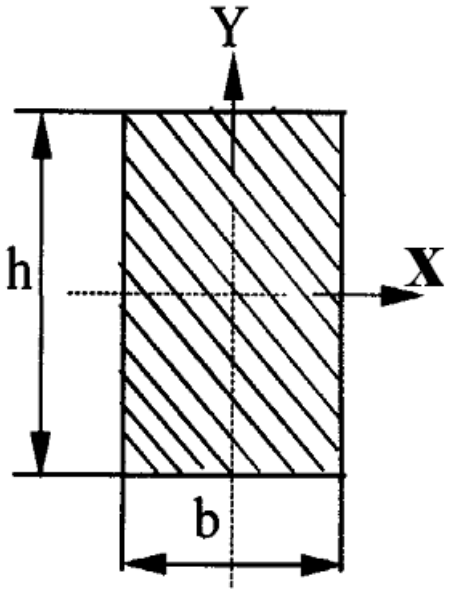



$$I_x = \frac{b\,h^3}{36}$$

$$I_y = \frac{b\,h^3}{48}$$

# Oddiy shakllarning inersiya momentlari

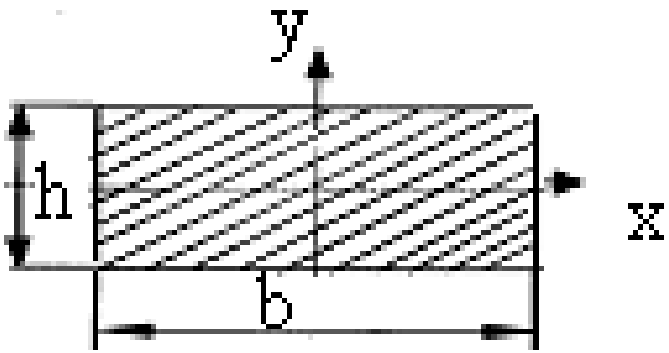
**Misol**:




Berilgan: h=8 sm,

b=4 sm, F=32 $sm^2$

$$I_x = \frac{bh^3}{12} = \frac{4 \cdot 8^3}{12} = \frac{2048}{12} = 170{,}667\ (sm^4)$$



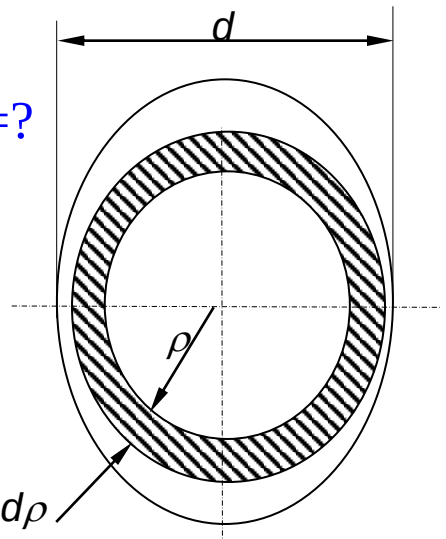

$$I_x = \frac{bh^3}{12} = \frac{8 \cdot 4^3}{12} = \frac{512}{12} = 42{,}667\ (sm^4)$$

# Oddiy shakllarning inersiya momentlari

## *g) Doira*

berilgan: d

t.k.: Ir=? Ix=? Iy=?




$$dF = 2\pi\rho d\rho$$

$$I_\rho = \int_F \rho^2 dF$$

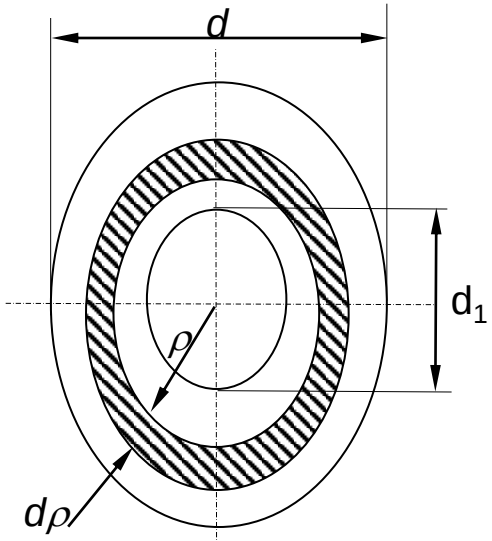
$$I_P = \int_F \rho^2 dF = 2\pi \int_0^{d/2} \rho^3 d\rho = \frac{2\pi d^4}{64} = \frac{\pi d^4}{32}$$

$$I_X = I_Y = \frac{I_P}{2} = \frac{\pi d^4}{64}$$

## *d) Xalqa*

berilgan: d, $d_1$

t.k.: Ix=? Iy=?




$$dF = 2\pi\rho d\rho$$

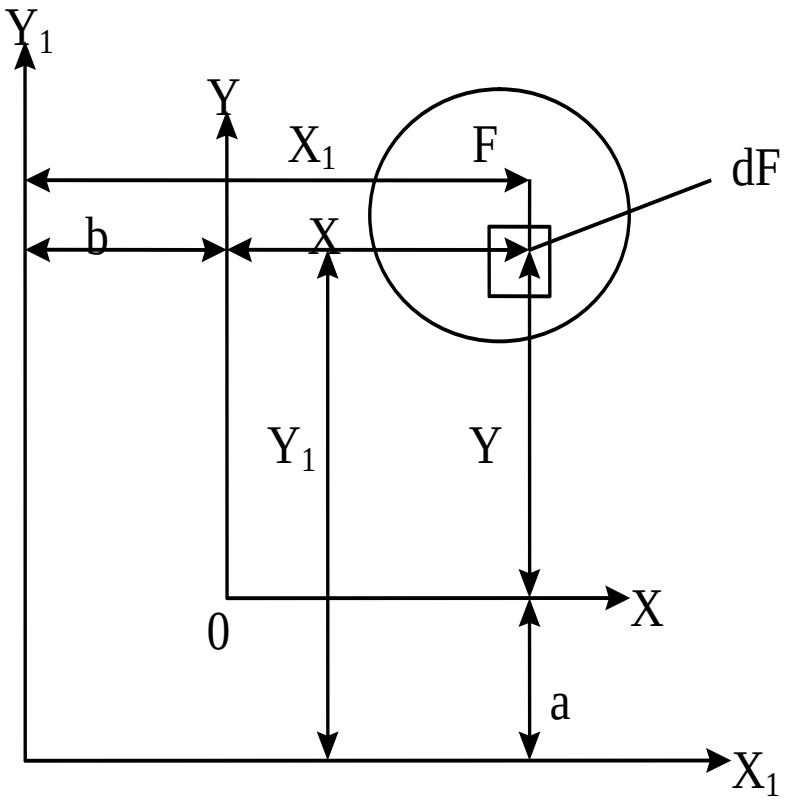
$$I_\rho = \int_A \rho^2 dF$$

$$I_P = \int_A \rho^2 dF = 2\pi \int_{d_1/2}^{d/2} \rho^3 d\rho = \frac{2\pi d^4}{64} - \frac{2\pi d_1^4}{62} = \frac{\pi d^4}{64}\left(1 - \frac{d_1^4}{d^4}\right) = \frac{\pi d^4}{32}\left(1 - \alpha^4\right)$$

$$I_X = I_Y = \frac{I_P}{2} = \frac{\pi d^4}{64}\left(1 - \alpha^4\right)$$

# Parallel o‘qlarga nisbatan inersiya momentlari o’rtasidagi bog’lanish

Bizga ***F*** yuzaga ega bo'lgan tekis shaklning ***x*** va ***y*** o‘qlariga nisbatan inersiya momentlari $I_x$, $I_y$, $I_{xy}$, $I_r$ berilgan bulsin. Bu o‘qlariga parallel bulgan va ulardan ***a*** va ***b*** masofadan o'tgan $x_1$ , $y_1$ o‘qlariga nisbatan ***F*** yuzaga ega bulgan tekis shaklni inersiya momentlarini topish talab qilinsin.




$$x_1 = x + b, y_1 = y + a$$

$$I_X = \int_F Y^2 dF; \quad S_X = \int_F Y dF = 0; \quad F = \int_F dF$$

$$I_{x_1} = \int_F y_1^2 \cdot dF, \quad I_{y_1} = \int_F x_1^2 \cdot dF, \quad I_{x_1 y_1} = \int_F x_1 \cdot y_1 \cdot dF$$

$$I_{y_1} = \int_F Y_1 dF = \int_F (y + a)^2 dF =$$

$$= \int_F y^2 dF + 2a \int_F y dF + a^2 \int_F dF =$$

$$= I_x + 2aS_x + a^2 \cdot F$$

bu erda

$$\int_F ydF = S_x$$

$$\int_F xdF = S_y$$

$$\int_F dF = F$$

bulganligi uchun

$$I_{x_1} = I_x + 2a\,S_x + a^2 F$$

$$I_{y_1} = I_y + 2b\,S_y + b^2 F$$

$$I_{x_1y_1} = I_{xy} + a\,S_y + b\,S_x + abF$$

agarda $Ox$ va $Oy$ o‘qlari shaklni ogirlik markazidan o'tgan bo'lsa bu holda formulalar ancha soddalashib, quyidagi ko'rinishga keladi ***(chunki $S_x=0$; $S_y=0$ bo'lgani uchun)***

$$I_{x_1} = I_x + a^2 \cdot F, \quad I_{y_1} = I_y + b^2 \cdot F,$$

$$I_{x_1y_1} = I_{xy} + a \cdot b \cdot F, \quad I_{\rho_1} = I_\rho + \left(a^2 + b^2\right) \cdot F$$

## *Xulosa*

1.Tekis kesim yuzining ixtiyoriy o‘qqa nisbatan inersiya momenti, shu o‘qqa parallel ravishda kesim og‘irlik markazidan o‘tuvchi markaziy o‘qqa nisbatan olingan inersiya momenti bilan kesim yuzaning o‘qlar orasidagi masofa kvadratiga ko‘paytmasining yig‘indisiga teng.

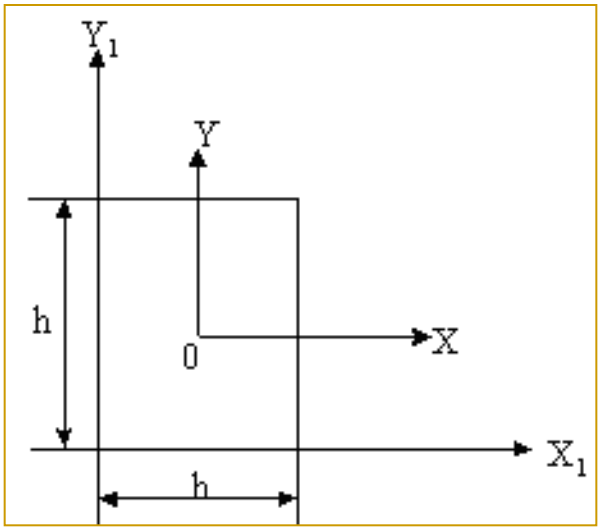
2.Tekis kesim yuzining ixtiyoriy o‘qqa nisbatan markazdan qochirma inersiya momenti, shu o‘qqa parallel ravishda kesim og‘irlik markazidan o‘tuvchi markaziy o‘qlarga nisbatan olingan markazdan qochirma inersiya momenti bilan kesim yuzining o‘qlarga nisbatan olingan koordinatalariga ko‘paytmasining yig‘indisiga teng.

**Misol:**

Berilgan: h=4 sm

b=2 sm, F=bh=8 $sm^2$

$$I_x = \frac{bh^3}{12} = \frac{2 \cdot 4^3}{12} = \frac{96}{12} = 8\ (sm^4)$$




$$I_y = \frac{b^3 h}{12} = \frac{2^3 \cdot 4}{12} = \frac{32}{12} = 2{,}667\ (sm^4)$$

$$I_{x_1} = I_x + \left(\frac{h}{2}\right)^2 \cdot F = \frac{bh^3}{12} + \left(\frac{h}{2}\right)^2 \cdot b \cdot h = \frac{2 \cdot 4^3}{12} + \left(\frac{4}{2}\right)^2 \cdot 2 \cdot 4 =$$

$$= 8 + 32 = 40\ (sm^4)$$

$$I_{y_1} = I_y + \left(\frac{b}{2}\right)^2 \cdot F = \frac{hb^3}{12} + \left(\frac{b}{2}\right)^2 b \cdot h = \frac{4 \cdot 2^3}{12} + \left(\frac{2}{2}\right)^2 \cdot 2 \cdot 4 = 2{,}667 + 8 = 10{,}667\ (sm^4)$$

# Bosh inersiya o‘qlari va bosh inersiya momentlari

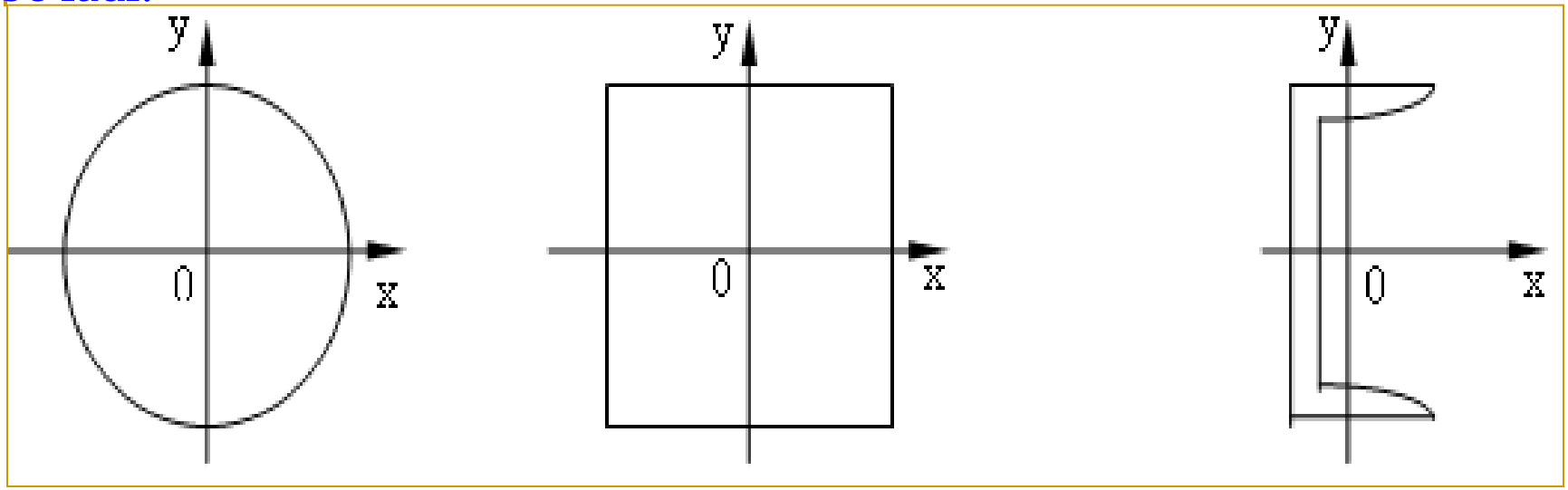
Bosh inersiya momentlari o’qlardan biriga nisbatan *max* bo’lsa, ikkinchisiga nisbatan *min* bo’ladi va qo’yidagicha topiladi:

$$I_{\substack{\max\\ \min}} = \frac{I_x + I_y}{2} \pm \frac{1}{2}\sqrt{\left(I_x - I_y\right)^2 + 4I_{xy}^2}$$

Bosh o’qlarning vaziyati quyidagi formula orqali topiladi:

$$tg2\alpha = \frac{2I_{xy}}{I_y - I_x}$$

Tekis shaklning har qanday simmetrik o‘qi uning bosh o‘qlaridan biri bo'ladi.




Agarda simmetrik o‘q bitta bo'lsa, unga perpendkulyar turgan ikkinchi o‘q ham bosh o‘q hisoblanadi.

Quyidagi kattaliklar tekis shaklning o‘qlarga nisbatan inersiya radiuslari deyiladi.

$$i_x = \sqrt{\frac{I_x}{F}}, \quad i_y = \sqrt{\frac{I_y}{F}}, \quad (sm)$$

***Misol.*** To‘g‘ri to‘rtburchak va teng yonli burchakdan tashkil topgan tekis kesim yuzasi ***chizmada*** berilgan. Tekis kesim yuzasi bosh inersiya momentlari, bosh inersiya o‘qlar holati va inersiya radiuslari aniqlansin?.

***Yechish.*** **1.** Murakkab kesim yuzasini oddiy yuzalarga ajratiladi (prokat profillari va og‘irlik markazi ma’lum bo‘lgan oddiy kesimlarga) va tartib bilan raqamlanadi, qaralayotgan murakkab tekis kesim yuzi ikki qismdan iborat bo‘lib ular quyidagilardir:

***Berilgan:***

a). To‘g‘ri to‘rtburchak $h = 14sm,\ b = 4sm,\ F_1 = 56sm^2$

b). Teng yonli burchak 140x140x10 uchun GOST 8509-72 dan

$$b = 14\,sm;\quad d = 1{,}0\,sm;\quad R = 1{,}4\,sm;\quad r = 0{,}46sm;\quad z_0 = 3{,}82\,sm.$$

$$F_2 = 27{,}9\,sm^2;\ I_x = I_y = 512\,sm^4;\ I_u = I_{0\max} = 814\,sm^4;\ I_v = I_{0\min} = 211\,sm^4.$$

**2.** Murakkab kesim yuzani masshtabda chizilib barcha kerakli o‘lchamlar ko‘rsatiladi. Kesimning $C_1; C_2$ og‘irlik markazlaridan bir biriga parallel bo‘lgan $x_1C_1y_1$ va $x_2C_2y_2$ koordinatalar tizimi o‘tkaziladi.

3. Murakkab kesim yuzasini $x0y$ koordinatalar tizimiga joylashtiriladi va to‘g‘ri to‘rtburchak og‘irlik markazi $C_1$ koordinatalari $x_1=2\ sm;\ y_1=7\ sm$ teng ekanligi, teng yonli burchak $C_2$ og‘irlik markazi koordinatalari $x_2=4+z_0=4+3,82=7,82\ sm,\ \ y_2=z_0=3,82\ sm$ teng ekanligi aniqlanadi.

4. Murakkab kesimning $C$ og‘irlik markazi koordinatalari quyidagi formulalardan aniqlanadi:

$$x_c=\frac{F_1x_1+F_2x_2}{F_1+F_2}=\frac{56\cdot 2+27,9\cdot 7,82}{56+27,9}=3,935\,sm.$$

$$y_c=\frac{F_1y_1+F_2y_2}{F_1+F_2}=\frac{56\cdot 7+27,9\cdot 3,82}{56+27,9}=5,943\,sm.$$

5.Murakkab kesim $C$ og‘irlik markazlaridan $x_1C_1y_1$, $x_2C_2y_2$ koordinatalar sistemasiga parallel bo‘lgan $x_cCy_c$ koordinatalar tizimi o‘tkaziladi.

6. Oddiy kesimlarning inersiya momentlarini aniqlab olinadi:

a) To‘g‘ri to‘rtburchakli kesim uchun $C_1x_1$ , $C_1y_1$ bosh o‘qlar bo‘lgani uchun bosh markaziy inersiya momentlari qiymatlari quyidagilarga teng bo‘ladi:

$$I_{x1}^{I}=\frac{4\cdot 14^3}{12}=1365,33333sm^4;\quad I_{y1}^{I}=\frac{4^3\cdot 14}{12}=165,33333sm^4;\quad I_{x1y1}^{I}=0.$$

b) Teng yonli burchak uchun $C_2x_c$ , $C_2y_c$ o‘qlariga nisbatan inersiya momentlari qiymati quyidagiga teng:

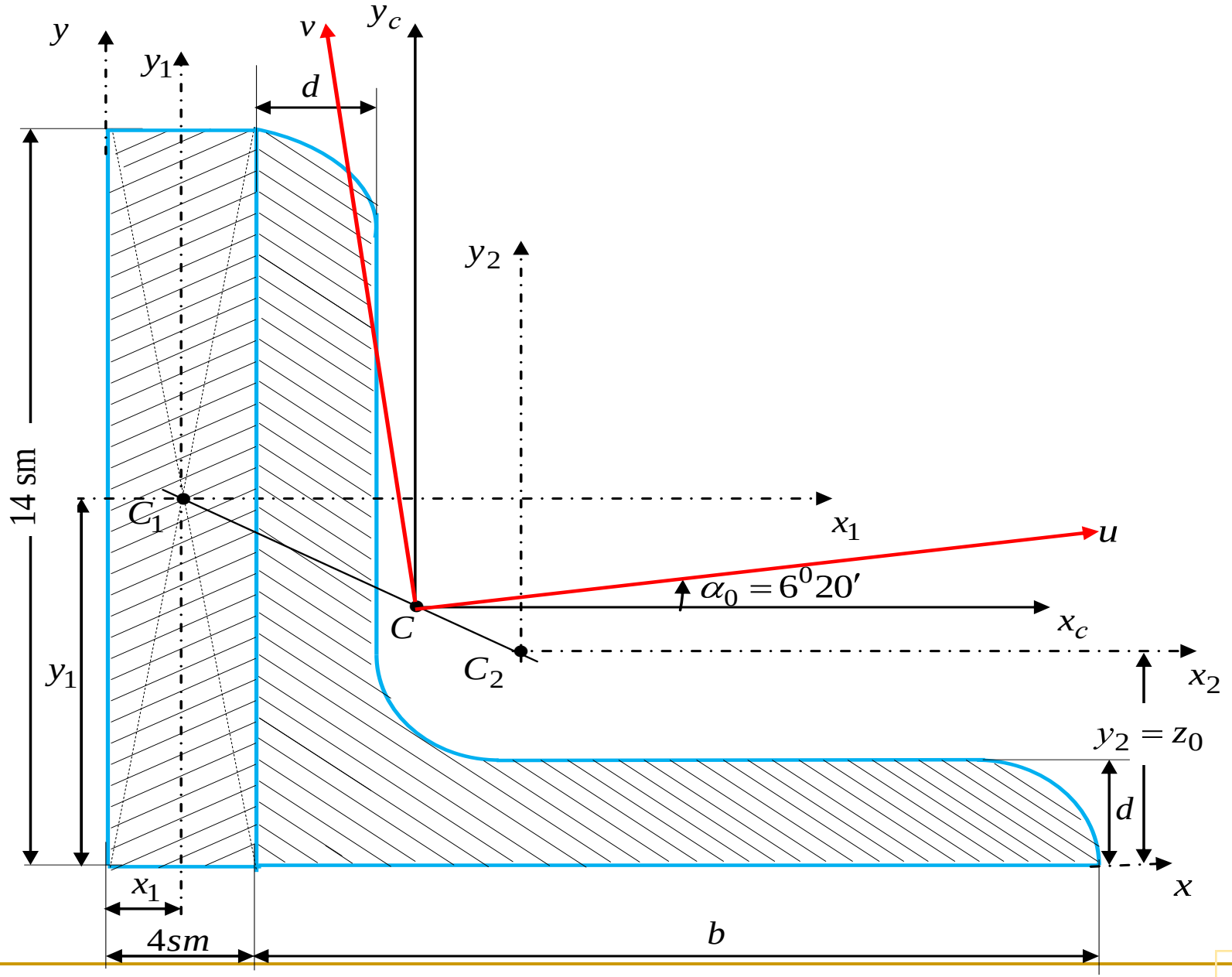
$$I_{x2}^{II} = I_{y2}^{II} = 301{,}5\,sm^4$$

Markazdan qochirma inersiya momentini quyidagi formuladan aniqlaymiz:

$$I_{x2y2}^{II} = \frac{I_{u2} - I_{v2}}{2}\sin 2\alpha = \frac{814-211}{2}\sin 2\left(-45^0\right) = \frac{603}{2}(-1) = -301{,}5\,sm^4.$$

**7.** $x_cCy_c$ koordinata tizimida $C_1$ , $C_2$ nuqtalarning koordinatalari aniqlanadi:

$$a_1 = (y_1 - y_c) = 7 - 5{,}943 = 1{,}057\,sm;$$
$$a_2 = -(y_c - y_2) = -(5{,}943 - 3{,}82) = -2{,}123\,sm;$$
$$b_1 = -(x_c - x_1) = -(3{,}935 - 2) = -1{,}935\,sm;$$
$$b_2 = (x_2 - x_c) = (7{,}82 - 3{,}935) = 3{,}885\,sm.$$




*Chizma. To‘g‘ri to‘rtburchak va burchakdan tashkil topgan murakkab kesim bosh o‘qlar holati.*

8. Yuqorida keltirilgan formulalardan foydalanib markaziy o‘qlariga nisbatan inersiya momentlari hisoblanadi:

$$I_{xc} = I_{xc}^{I} + I_{xc}^{II} = \left[I_{x1}^{I} + a_1^2 A_1\right] + \left[I_{x2}^{II} + a_2^2 A_2\right] =$$

$$= \left[1365{,}33333 + (1{,}057)^2 \cdot 56\right] + \left[512 + (-2{,}123)^2 \cdot 27{,}9\right] = 2065{,}648 sm^4.$$

$$I_{yc} = I_{yc}^{I} + I_{yc}^{II} = \left[I_{y1}^{I} + b_1^2 A_1\right] + \left[I_{y2}^{II} + b_2^2 A_2\right] =$$

$$= \left[165{,}33333 + (-1{,}935)^2 \cdot 56\right] + \left[512 + (3{,}885)^2 \cdot 29{,}7\right] = 1328{,}111 sm^4.$$

$$I_{xcyc} = I_{xcyc}^{I} + I_{xcyc}^{II} = \left[I_{x1y1}^{I} + a_1 b_1 A_1\right] + \left[I_{x2y2}^{II} + a_2 b_2 A_2\right] =$$

$$= \left[0 + (1{,}057)(-1{,}935) \cdot 56\right] + \left[-301{,}5 + (-2{,}123)(3{,}885) \cdot 27{,}9\right] = -43{,}152\, sm^4.$$

9. $x_cCy_c$ koordinata o‘qlariga nisbatan markaziy bosh o‘qlarning burilish burchagi topiladi:

$$tg2\alpha_0 = -\frac{2I_{xcyc}}{I_{xc} - I_{yc}} = -\frac{-86{,}2041}{2065{,}648 - 1328{,}111} = 0{,}11702.$$

$$2\alpha_0 = 6^0 40', \quad \alpha_0 = 3^0 20'$$

10. Bosh markaziy $uCv$ tizimi $x_cCy_c$ koordinata tizimiga nisbatan soat millari yo‘nalishiga teskari yo‘nalish bo‘yicha $\alpha_0 = 3^0 20'$ burchakka buriladi.

11. Bosh inersiya momentlarini aniqlanadi:

$$I_{\max} = \frac{I_{xc} + I_{yc}}{2} + \frac{1}{2}\sqrt{\left(I_{xc} - I_{yc}\right)^2 + 4I_{xcyc}^2} = \frac{2065{,}648 + 1328{,}111}{2} +$$

$$+ \frac{1}{2}\sqrt{\left(2065{,}64 - 1328{,}111\right)^2 + 4\cdot\left(-43{,}152\right)^2} = 2068{,}164 sm^4.$$

$$I_{\min} = \frac{I_{xc} + I_{yc}}{2} - \frac{1}{2}\sqrt{\left(I_{xc} - I_{yc}\right)^2 + 4I_{xcyc}^2} = \frac{2065{,}648 + 467{,}7}{2} -$$

$$- \frac{1}{2}\sqrt{\left(2065{,}648 - 1328{,}111\right)^2 + 4\cdot\left(-43{,}152\right)^2} = 1325{,}595 sm^4.$$

12. Hisob natijalari ikki usulda tekshiriladi:

1. O‘zaro perpendikulyar bo‘lgan o‘qlar koordinata bosh atrofida ixtiyoriy burchakka burilganda bu o‘qlarga nisbatan olingan inersiya momentlarining yig‘indisi teng va o‘zgarmas miqdor ekanligi tekshiriladi, yani

$$I_{\max} + I_{\min} = I_{xc} + I_{yc} = I_{\rho};$$

$$2068{,}164 + 1325{,}59 = 2065{,}648 + 1328{,}111; \qquad 3393{,}754 = 3393{,}759.$$

2. Bosh o‘qlarning to‘g‘ri topilganligiga ishonch hosil qilish uchun markazdan qochirma inersiya momentining nolga tengligi tekshiriladi, ya’ni

$$I_{uv} = \frac{I_{xc} - I_{yc}}{2}\sin 2\alpha + I_{xcyc}\cos 2\alpha =$$

$$= \frac{2065{,}648 - 1328{,}111}{2} \cdot 0{,}1164 + (-43{,}152) \cdot 0{,}9931 = 0.$$

13. Murakkab kesimning bosh inersiya radiuslari hisoblanadi:

$$i_u = \sqrt{\frac{I_u}{A}} = \sqrt{\frac{2068{,}164}{83{,}9}} = 4{,}965 sm. \quad i_v = \sqrt{\frac{I_v}{A}} = \sqrt{\frac{1325{,}595}{83{,}9}} = 3{,}975 sm.$$

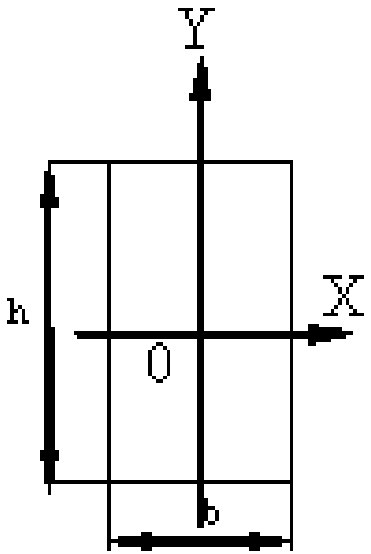
# Tekis shakllarning qarshilik momentlari

Konstruksiya elementlarida xosil buladigan kuchlanishni topish qarshilik momenti degan geometrik xarakteristika ishlatiladi.

u quyidagicha topiladi:

$$W_x = \frac{I_x}{Y_{max}}; \qquad W_y = \frac{I_y}{X_{max}}$$

a) To'gri to'rtburchak:

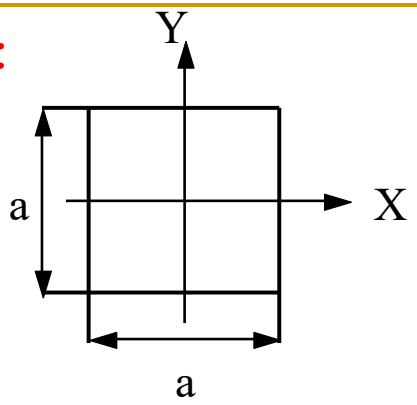



o‘qlarga nisbatan qarshilik momentlari deyiladi. Bu erda $Y_{max}$, $X_{max}$ tekis shaklning markazidan eng uzoqda joylashgan nuqta koordinatasi.

$$W_x = \frac{I_x}{Y_{max}} = \frac{\frac{bh^3}{12}}{\frac{h}{2}} = \frac{bh^2}{6}; \quad W_x = \frac{bh^2}{6}(sm^3)$$

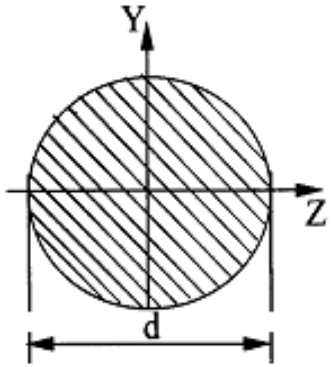
$$W_y = \frac{hb^2}{6}(sm^3)$$

b) Kvadratda:




$$W_x = W_y = \frac{a^3}{6}\left(sm^3\right)$$

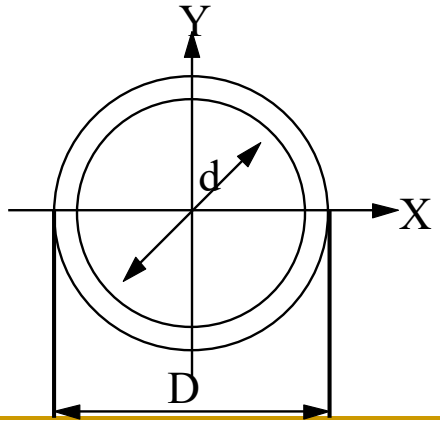
v) Doira:




$$W_x = W_y = \frac{\pi d^3}{32}\left(sm^3\right)$$

$$W_p = \frac{\pi d^3}{16}\left(sm^3\right)$$

g) Xalqa:




$$W_x = W_y = \frac{\pi D^3}{32}(1-\alpha)\left(sm^3\right)$$

$$W_p = \frac{\pi D^3}{16}\left(1-\alpha^4\right)\left(sm^3\right)$$

# TAKRORLASH UCHUN SAVOLLAR

1. O’qlar parallel kuchirilganda inertsiya momentlari qanday o’zgaradi?
2. Parallel o‘qlarga nisbatan inersiya momentlar nimaga teng?
3. Bosh inersiya o‘qlari va bosh inersiya momentlari deb nimaga aytiladi?
4. Tekis shaklning o‘qlarga nisbatan inersiya radiuslari qanday topiladi?
5. Tekis shakllarning qarshilik momentlari qanday topiladi?

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